H-Max Single-Valued Neutrosophic Distance Measure in Medical Diagnosis

Ngan Thi Roan [1,*] and Florentin Smarandache [2]

[1] Hanoi University of Natural Resources and Environment, Hanoi, VIETNAM; roanngan@gmail.com

[2] Dept. Math and Sciences, University of New Mexico, Gallup, NM, USA; smarand@unm.edu

* Correspondence: roanngan@gmail.com

Abstract

As an instance of a neutrosophic set, a single-valued neutrosophic set (SVNS) can be used to handle uncertainty, inaccuracy, indeterminacy, and inconsistency. In this paper, a new not-so-sophisticated distance measure between two SVNSs is defined by considering the cross-assessment between characteristic functions for the forward and backward differences. Furthermore, a single-valued neutrosophic similarity measure, a single-valued neutrosophic entropy measure, and their basic properties are presented and proven. In addition, an application to medical diagnosis is shown to illustrate the effective applicability of the proposals.

Keywords: Single-Valued Neutrosophic Distance, similarity measure, entropy measure, medical diagnosis

1.Introduction

In order to approach uncertainty, in 1965, the fuzzy set (FS) theory was introduced by Zadeh [1] and has achieved a great success in many real applications. Zadeh defined the degree of membership of a FS A defined on a universe X by one real value $\mu_A(x) \in [0,1]$. However, in many real-life systems, it may be very difficult to assign the membership value for a FS to handle imprecise, incomplete, and inconsistent information. In 1983, Atanassov [2] proposed a generalization of FS, named the intuitionistic fuzzy set (IFS) on a universe X, where the truth membership, $\mu_A(x) \in [0,1]$, is considered as well as the falsity membership, $\nu_A(x) \in [0,1]$, for

DOI: 10.5281/zenodo.5164640

Received:February 02, 2021, Accepted: June 04, 2021

description of an object. They are tied together by condition $\mu_A(x)+\nu_A(x)\in[0,1]$ for each $x\in X$. In a belief system, the inconsistent and indeterminate information is not handled satisfactorily by IFS. In 1999, the concept of neutrosophic set (NS) was originally introduced by Smarandache [3]. This is a branch of philosophy and is a mathematical tool for studying the neutralities. A NS is represented independently by a truth membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$, where $T_A(x)$, $I_A(x)$, and $F_A(x)$ are real standard or nonstandard subsets of $]^{-}0,1^{+}[$. Some of the recent developments on how to use neutrosophic data can be mentioned as: in 2018 a mathematical model for clustering the neutrosophic big data sets was determined [4]; in 2019 a Python tool for bipolar neutrosophic matrices is presented in [5]; in 2020 a novel social choice theory based multi-criteria decision making method under neutrosophic environment was introduced [6]; and a smart agriculture mechanism model equipped with neutrosophy theory was studied [7]. In order to apply NS in real scientific and engineering areas, Wang et al [8] introduced a single-valued neutrosophic set (SVNS) and it has received considerable attention for various researchers. Some applications of SVNSs in decision-making can be found in [9-16].

A distance measure is an effective tool in decision-making problems, such as, intuitionistic fuzzy and picture fuzzy distance measures have been studied for use in medical diagnosis and Controlling Network Power Consumption [17-21]. From 2013 to present, several distance measures of SVNSs were introduced such as Hausdorff distance [22], Cosine similarity [23] measures, and the distance measures of Ye [24], Aydoğdu [25], and Huang [9]. In 2016, the measure of Huang [9] has solved the limitation of Ye's measure [24] in some cases where it is necessary to distinguish the differences between several objects, but it is quite complicated in calculation. Regarding the need to distinguish the differences, in an intuitionistic fuzzy environment, the H-max distance measure of Ngan et al. [18] has solved well by considering the cross-assessment between characteristic functions. From a comprehensive consideration of the problem, this paper focuses on proposing a new distance measure of SVNSs and its application in medical diagnosis for the purpose of increasing accuracy and reducing computation time. This research based on some of the following main motivations.

- H-max distance measure of intuitionistic fuzzy sets in decision making [18] and H-max distance measure of the picture fuzzy sets [21].
- The single-valued neutrosophic weighted distance measure, the single-valued neutrosophic weighted similarity measure, and the single-valued neutrosophic entropy measure of Huang [9].
- The entropy measure is used to calculate the weights of attributes in decision-making problem [26].

The main contribution of the proposal is to extend the H-max distance measure to SVNSs, thereby building the formula of entropy measure of SVNSs and they are included in the new diagnostic method.

DOI: 10.5281/zenodo.5164640

In Section 2, some basic concepts are reviewed. In Section 3, the notions of the distance measure, the similarity measure, and the entropy measure are introduced. In Section 4, an application to medical diagnosis given to show the effectiveness of the new distance measure applied in medical diagnosis. A conclusion is set in section 5.

2. Preliminaries

Definition 1. [7] Let X be a universe set and $x \in X$. A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$, where $T_A(x)$, $I_A(x)$, and $F_A(x)$ are real standard of nonstandard subsets of $]^-0,1^+[$, and there is no restriction on the sum of them. Hence

$$T_A(x):\ x \to\ ]^-0,1^+[\,,\ \ I_A(x):\ x \to\ ]^-0,1^+[\,,\ \ F_A(x): \mathrm{x} \to\ ]^-0,1^+[\,, \text{ and}$$

$$^-0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+.$$

To make it easier to access science and engineering problems, a subclass of a NS which is the concepts of SVNS was proposed by Wang et al. [8].

Definition 2. [8] Let X be a universe set and $x \in X$. A SVNS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$, and can be denoted by $A = \{< x, T_A(x), I_A(x), F_A(x) >| x \in X\}$. For each point x in X, $T_A(x), I_A(x), F_A(x) \in [0,1]$.

The inclusion, equality, and complement relations are defined by Wang et al. [8] for SVNSs A, B as follows:

$A \subseteq B$ if and only if $T_A(x) \le T_B(x)$, $I_A(x) \ge I_B(x)$ and $F_A(x) \ge F_B(x)$ for any x in X.

$A = B$ if and only if $T_A(x) = T_B(x)$, $I_A(x) = I_B(x)$ and $F_A(x) = F_B(x)$ for any x in X.

$A^c = \{< x, F_A(x), 1 - I_A(x), T_A(x) \mid x \in X\}$ where A^c denotes the complement of A.

Definition 3. [9] A mapping, E : SVNSs → [0, 1], is an entropy measure if the conditions (e1)-(e4) are satisfied.

(e1) $E(A) = 0$ if and only if A or A^c is a crisp set;

(e2) $E(A) = 1$ if and only if $A = A^c$, i.e., $T_A(x_i) = F_A(x_i)$ and $I_A(x_i) = 0.5$ for all $x_i \in X$.

(e3) $E(A) \le E(B)$ if A is less fuzzy than B, i.e., if

DOI: 10.5281/zenodo.5164640

$T_A(x_i) \le T_B(x_i)$, $F_A(x_i) \ge F_B(x_i)$ for $T_B(x_i) \le F_B(x_i)$ and $I_A(x_i) = I_B(x_i) = 0.5$, or

$T_A(x_i) \ge T_B(x_i)$, $F_A(x_i) \le F_B(x_i)$ for $T_B(x_i) \ge F_B(x_i)$ and $I_A(x_i) = I_B(x_i) = 0.5$;

(e4) $E(A) = E(A^c)$.

In 2016, Huang [9] introduced the weighted distance measure on SVNSs as follows.

$$d_\lambda(A,B) = \left[\sum_{j=1}^{n} \omega_j \left(\sum_{i=1}^{4} \beta_i \varphi_i(x_j) \right)^{\lambda} \right]^{1/\lambda}$$

where $\lambda > 0$, $\beta_i \in [0,1]$ and $\sum_{i=1}^{4} \beta_i = 1$, $\omega_j \in [0,1]$ and $\sum_{j=1}^{n} \omega_j = 1$,

$$\varphi_1(x_j) = \frac{|T_A(x_j) - T_B(x_j)|}{3} + \frac{|I_A(x_j) - I_B(x_j)|}{3} + \frac{|F_A(x_j) - F_B(x_j)|}{3},$$

$$\varphi_2(x_j) = \max\left\{ \frac{2 + T_A(x_j) - I_A(x_j) - F_A(x_j)}{3}, \frac{2 + T_B(x_j) - I_B(x_j) - F_B(x_j)}{3} \right\}$$
$$- \min\left\{ \frac{2 + T_A(x_j) - I_A(x_j) - F_A(x_j)}{3}, \frac{2 + T_B(x_j) - I_B(x_j) - F_B(x_j)}{3} \right\},$$

$$\varphi_3(x_j) = \frac{|T_A(x_j) - T_B(x_j) + I_B(x_j) - I_A(x_j)|}{2}, \varphi_4(x_j) = \frac{|T_A(x_j) - T_B(x_j) + F_B(x_j) - F_A(x_j)|}{2}.$$

In 2018, by reasoning about the need for the cross-evaluation, Ngan et al. [18] defined the H-max distance measure on IFSs by

$$d_{Hm}(A,B) = \frac{1}{3m} \sum_{i=1}^{m} \left(|\mu_1(x_i) - \mu_2(x_i)| + |\nu_1(x_i) - \nu_2(x_i)| + \max\{\mu_1(x_i), \nu_2(x_i)\} - \max\{\mu_2(x_i), \nu_1(x_i)\} \right).$$

In 2020, Smarandache and Ngan et al. [21] introduced the picture fuzzy distance measure based on $d_{Hm}(A,B)$ as

$$D_0(x,y) = \frac{1}{3}\left(|x_1 - y_1| + |x_2 - y_2| + |x_3 - y_3| + \left|\max\{x_1, y_3\} - \max\{x_3, y_1\}\right| \right), \forall x, y \in P^*,$$

where $P^* = \{x = (x_1, x_2, x_3) \mid x_1, x_2, x_3, \sum_{i=1}^{3} x_i \in [0,1]\}$.

DOI: 10.5281/zenodo.5164640

Note that a picture fuzzy set, proposed by Cuong [27], is considered as a special case of neutrosophic sets when when all three components, $T_A(x)$, $I_A(x)$, and $F_A(x)$, are dependent, i.e., $T_A(x)+I_A(x)+F_A(x)\in[0,1]$.

Remark 1. We agree with the argument of Huang [9], that is, some existing measures give an equal rating that is not tight. Suppose in a match, there are three players whose performances are represented by three SVNSs $A=<0.8,0,0.8>$, $B=<0.7,0,0.7>$, and $C=<0.7,0,0.9>$ on $X=\{x\}$. Obviously, the distance between A and B is more closer than the distance between A and C, such as, by the distance of Huang [9], we obtain that $d(A,B)=0.06$ < $d(A,C)=0.14$. However, this distance measure is quite cumbersome. Furthermore, by the distance and similarity measures introduced in [22-25], we have $d(A,B)=d(A,C)$, $s(A,B)=s(A,C)$, and $s(A,B)<s(A,C)$, which are not consistent with our intuition (see Table 1).

Table 1. The values of the distance and similarity measures between $A,B,$ and C.

[25]	[23]	[24]	[22]	[9]
$s(A,B)$=0.93	$s(A,B)$=0.53	$d(A,B)=0.08$	$d(A,B)=0.1$	$d(A,B)=0.06$
=	<	=	=	<
$s(A,C)$=0.93	$s(A,C)$=0.56	$d(A,C)=0.08$	$d(A,C)=0.1$	$d(A,C)=0.14$

3. H-max single-valued neutrosophic distance measure

Definition 4. Let A and B be two SVNSs in a universe of discourse $X=\{x_1,x_2,...,x_n\}$, $A=\{<x_i,T_A(x_i),I_A(x_i),F_A(x_i)>|\, x_i\in X\}$ and $B=\{<x_i,T_B(x_i),I_B(x_i),F_B(x_i)>|\, x_i\in X\}$. The H-max single-valued neutrosophic weighted distance measure is defined by

$$\begin{aligned} d_{H-N}(A,B)=\sum_{i=1}^{n}\chi_i\Big(&\omega_1\left|T_A(x_i)-T_B(x_i)\right|+\omega_2\left|I_A(x_i)-I_B(x_i)\right|+\omega_3\left|F_A(x_i)-F_B(x_i)\right| \\ &+\omega_4\left|\max\{T_A(x_i),I_B(x_i)\}-\max\{I_A(x_i),T_B(x_i)\}\right| \\ &+\omega_5\left|\max\{T_A(x_i),F_B(x_i)\}-\max\{F_A(x_i),T_B(x_i)\}\right|\Big) \end{aligned}$$

where $\omega_j\in(0,1)$ for $j=1,2,3,4,5,$ and $\sum_{j=1}^{5}\omega_j=1,$ $\chi_i\in[0,1]$ for $i=1,2,...,n$ and $\sum_{i=1}^{n}\chi_i=1.$

Remark 2. Revisiting Remark 1, we have $d_{H-N}(A,B)<d_{H-N}(A,C)$, where $A=<0.8,0,0.8>$, $B=<0.7,0,0.7>$, and $C=<0.7,0,0.9>$. Specifically, when $\omega_j=\frac{1}{5}, j=1,2,3,4,5$, we have $d_{H-N}(A,B)=0.06$ and $d_{H-N}(A,C)=0.08$. Furthermore, the proposed distance measure is less cumbersome than that of Huang [9], However, its complexity is intentional. Considering the case

DOI: 10.5281/zenodo.5164640

Huang [9] mentioned, we have $d_{H-N}(A,B)=d_{H-N}(A,C)$, where $A=<0.9,0,0.3>$, $B=<0.7,0,0.1>$, and $C=<0.7,0,0.5>$. This means, in this case, our proposed distance measure is not suitable to distinguish the differences between three players.

Proposition 1. The distance measure $d_{H-N}(A,B)$ satisfies the following properties:

(d1) $0\le d_{H-N}(A,B)\le 1$;

(d2) $d_{H-N}(A,B)=0$ if and only if $A=B$;

(d3) $d_{H-N}(A,B)=d_{H-N}(B,A)$;

(d4) If $A\subseteq B\subseteq C$, C is an SVNS in X, then $d_{H-N}(A,B)\le d_{H-N}(A,C)$ and $d_{H-N}(B,C)\le d_{H-N}(A,C)$.

Proof.

(d1) For $i=1,2,...,n$, since $|T_A(x_i)-T_B(x_i)|$, $|I_A(x_i)-I_B(x_i)|$, $|F_A(x_i)-F_B(x_i)|$ $\in[0,1]$, and $|\max\{T_A(x_i),I_B(x_i)\}-\max\{I_A(x_i),T_B(x_i)\}|$, $|\max\{T_A(x_i),F_B(x_i)\}-\max\{F_A(x_i),T_B(x_i)\}|$ $\in[0,1]$, hence $0\le d_{H-N}(A,B)\le 1$.

(d2) $d_{H-N}(A,B)=0\Leftrightarrow\begin{cases}T_A(x_i)=T_B(x_i)\\ I_A(x_i)=I_B(x_i)\\ F_A(x_i)=F_B(x_i)\end{cases},\forall i=1,2,...,n\Leftrightarrow A=B.$

(d3) The symmetry of measure $d_{H-N}(A,B)$ with respect to their argument is obvious.

(d4) Let $A\subseteq B\subseteq C$, we have $T_A(x_i)\le T_B(x_i)\le T_C(x_i)$, $I_A(x_i)\ge I_B(x_i)\ge I_C(x_i)$ and $F_A(x_i)\ge F_B(x_i)\ge F_C(x_i)$ for $i=1,2,...,n$. Then,

$$\begin{aligned}&|T_A(x_i)-T_B(x_i)|\le|T_A(x_i)-T_C(x_i)|,\\&|I_A(x_i)-I_B(x_i)|\le|I_A(x_i)-I_C(x_i)|,\\&|F_A(x_i)-F_B(x_i)|\le|F_A(x_i)-F_C(x_i)|.\end{aligned}$$

Moreover, we have

$$\max\{T_C(x_i),I_A(x_i)\}\ge\max\{T_B(x_i),I_A(x_i)\}\ge\max\{T_A(x_i),I_B(x_i)\}\ge\max\{T_A(x_i),I_C(x_i)\},$$

$$\max\{T_C(x_i),F_A(x_i)\}\ge\max\{T_B(x_i),F_A(x_i)\}\ge\max\{T_A(x_i),F_B(x_i)\}\ge\max\{T_A(x_i),F_C(x_i)\},$$

hence

DOI: 10.5281/zenodo.5164640

$$\left|\max\{T_B(x_i), I_A(x_i)\} - \max\{T_A(x_i), I_B(x_i)\}\right| \le \left|\max\{T_C(x_i), I_A(x_i)\} - \max\{T_A(x_i), I_C(x_i)\}\right|,$$

$$\left|\max\{T_B(x_i), F_A(x_i)\} - \max\{T_A(x_i), F_B(x_i)\}\right| \le \left|\max\{T_C(x_i), F_A(x_i)\} - \max\{T_A(x_i), F_C(x_i)\}\right|.$$

Thus, $d_{H-N}(A,B) \le d_{H-N}(A,C)$. Similarly, we also have $d_{H-N}(B,C) \le d_{H-N}(A,C)$.

Definition 5. Let A and B be two SVNSs in a universe of discourse $X = \{x_1, x_2, ..., x_n\}$, $A = \{< x_i, T_A(x_i), I_A(x_i), F_A(x_i) >| x_i \in X\}$ and $B = \{< x_i, T_B(x_i), I_B(x_i), F_B(x_i) >| x_i \in X\}$. The H-max single-valued neutrosophic weighted similarity measure is defined by

$$s_{H-N}(A,B) = 1 - d_{H-N}(A,B).$$

Proposition 2. The similarity measure $s_{H-N}(A,B)$ satisfies the following properties:

(s1) $0 \le s_{H-N}(A,B) \le 1;$

(s2) $s_{H-N}(A,B) = 1$ if and only if $A = B;$

(s3) $s_{H-N}(A,B) = s_{H-N}(B,A);$

(s4) If $A \subseteq B \subseteq C$, C is an SVNS in X, then $s_{H-N}(A,B) \ge s_{H-N}(A,C)$ and $s_{H-N}(B,C) \ge s_{H-N}(A,C)$.

Proposition 3. $s_{H-N}(A, A^c)$ is an entropy measure for a SVNS A.

Proof.

(e1) Let A be a crisp set, i.e., $T_A(x_i) = 1, I_A(x_i) = F_A(x_i) = 0$ or $F_A(x_i) = 1, T_A(x_i) = I_A(x_i) = 0$ for all $x_i \in X$. No matter in which cases (A or A^c is a crisp set), then $s_{H-N}(A, A^c) = 0$. On the other hand, let $s_{H-N}(A, A^c) = 0$, we can easily get A or A^c is a crisp set.

(e2) From (s2) in Proposition 2, we obtain that $s_{H-N}(A, A^c) = 1$ if and only if $A = A^c$.

(e3) Let A is less fuzzy than B, assume that $T_A(x_i) \le T_B(x_i)$, $F_A(x_i) \ge F_B(x_i)$ for $T_B(x_i) \le F_B(x_i)$ and $I_A(x_i) = I_B(x_i) = 0.5$, then

$$T_A(x_i) \le T_B(x_i) \le F_B(x_i) \le F_A(x_i),$$

DOI: 10.5281/zenodo.5164640

$$\max\{T_A(x_i),0.5\} \le \max\{T_B(x_i),0.5\} \le \max\{F_B(x_i),0.5\} \le \max\{F_A(x_i),0.5\},$$ and

$$d_{H-N}\left(A,A^c\right)=\sum_{i=1}^{n}\chi_i\left((\omega_1+\omega_3+\omega_5)\left|T_A(x_i)-F_A(x_i)\right|+\omega_4\left|\max\{T_A(x_i),0.5\}-\max\{0.5,F_A(x_i)\}\right|\right)$$

$$d_{H-N}\left(B,B^c\right)=\sum_{i=1}^{n}\chi_i\left((\omega_1+\omega_3+\omega_5)\left|T_B(x_i)-F_B(x_i)\right|+\omega_4\left|\max\{T_B(x_i),0.5\}-\max\{0.5,F_B(x_i)\}\right|\right)$$

We obtain that $d_{H-N}\left(A,A^c\right)\ge d_{H-N}\left(B,B^c\right)$ and then $s_{H-N}\left(A,A^c\right)\le s_{H-N}\left(B,B^c\right)$. The other case can be also proved by the same way.

(e4) Since (s3) in Proposition 2, we have $E(A)=E(A^c)$.

4. An application of the H-Max Single-Valued Neutrosophic Distance Measure to medical diagnosis

4.1. The proposed method named Two-d_{H-N}.

The considered problem: Consider a medical dataset with m records of m corresponding patients P_i, $i=1,2,...,m$, and with n attributes S_j, $j=1,2...,n$, of a disease D with k disease classes labeled C_t, $t=1,2,...,k$, of D. Find the disease label (diagnosis) for each patient P_i.

The method Two-d_{H-N}:

- **Step 1.** For each attribute, choose three suitable fuzzìication functions (based on experts) representing the truth membership function T, the indeterminacy-membership function I, and the falsity-membership function F.
  From the original data and the fuzzification functions, built two single-valued neutrosophic matrices (the elements of the matrices are the single-valued neutrosophic values):

  (M1): Matrix of the attributes on patients (P_i and S_j are the i^th row and the j^th column of (M1), respectively, $i=1,2,...,m; j=1,2...,n$).

  (M2): Matrix of the classification results. (M2) is a $k\times n$ matrix (C_t is the t^th row of (M2), $t=1,2,...,k$).

- **Step 2.** Calculate the entropy of the attributes $E\left(S_j\right)$.

DOI: 10.5281/zenodo.5164640

- **Step 3.** Calculate the similarity $s_{H-N}\left(P_i, C_t\right)$ between the attributes of each patient (P_i) with all of the disease classes C_t, $t=1,2,...,k$, where the weight of each attribute is corresponding to its entropy.
- **Step 4.** For the i^th^ patient, find the highest similarity value $\hat{s}_{H-N}\left(P_i, C_t\right)=s_{H-N}\left(P_i, C_{t_0}\right)$, $t_0 \in [1,k]$.Conclude that the classification label of the i^th^ patient is t_0.

The time complexity:

- The time complexity of Step 1: $O(m.n)+O(k.n)$
- The time complexity of Step 2: $O(n)$
- The time complexity of Step 3: $O(m.k)$
- The time complexity of Step 4: $O(m)$

Thus, the time complexity of the method Two-$d_{H\text{-}N}$ is $O(m.n+k.n+n+m.k+m)$.

4.2. Numeric example

In this section, a numeric example for the method Two-$d_{H\text{-}N}$ made on 5 patients, which are the 36^th^ patient, the 57^th^ patient, the 77^th^ patient, the 97^th^ patient, and the 525^th^ patient, randomly selected of the ILPD (Indian Liver Patient Dataset) Data Set taken from UCI.

Table 2 shows information of the dataset, where there are 10 considerd attributes: Age, Gender, Total Bilirubin (TB), Direct Bilirubin (DB), Alkaline Phosphotase (ALP), Alamine Aminotransferase (ALT), Aspartate Aminotransferase (AST), Total proteins (TP), Albumin (ALB), and Albumin and Globulin Ratio (A/G). There are 2 classification labels: 1 (liver patient) and 2 (non-liver patient).

Table 2. Records of the 36^th^ patient, the 57^th^ patient, the 77^th^ patient, the 97^th^ patient, and the 525^th^ patient of the ILPD dataset

No	Age	Gender	TB	DB	ALP	ALT	AST	TP	ALB	A/G	Label
36	30	Male	1.3	0.4	482	102	80	6.9	3.3	0.9	1
57	33	Male	0.8	0.2	198	26	23	8	4	1	2
77	31	Male	0.9	0.2	518	189	17	5.3	2.3	0.7	1
97	39	Male	3.8	1.5	298	102	630	7.1	3.3	0.8	1
525	29	Male	0.8	0.2	156	12	15	6.8	3.7	1.1	2

DOI: 10.5281/zenodo.5164640

The method Two-$d_{H\text{-}N}$ includes the following steps:

- **Step 1:** In this step, the data is reviewed and evaluated before the fuzzification process.

Table 3. Data preprocessing

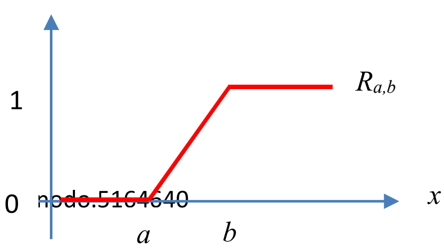
	S_1 (TB) (normal: 0.8-1.2)	S_2 (DB) (normal: 0.2-0.4)	S_3 (ALP) (normal: 20-140)	S_4 (ALT) (normal: 29-33)	S_5 (AST) (normal: 23-33)	S_6 (ALB) (normal: 3.5-4.8)	S_7 [A/G= ALB/(TP-ALB)] (normal: 1-1.5)
P_1	1.3	0.4	482	102	80	3.3	0.9
P_2	0.8	0.2	198	26	23	4	1
P_3	0.9	0.2	518	189	17	2.3	0.7
P_4	3.8	1.5	298	102	630	3.3	0.8
P_5	0.8	0.2	156	12	15	3.7	1.1
	Max=32.6 Max(2)=5.3	Max=17.1 Max(2)=2.3	Max=2110 Max(2)=486	Max=2000 Max(2)=119	Max=2946 Max(2)=178	Min=1.7 Min(2)=2.2	Min=0.3 Min(2)=0.5

In Table 3:

- Select 7 required attributes out of the original 10 attributes.
- Given the normal threshold of each attribute (obtained from expert knowledge).
- Differentiate the correlation between attributes with disease classes. Specifically, the orange background indicates that if a patient has liver disease, this attribute will increase; the green background indicates that if a patient has liver disease, this attribute is reduced.
- The last row: Specify the maximum values (Max) and the minimum values (Min) of attributes; and the maximum value (Max(2)) and the minimum values (Min(2)) corresponding to label classification 2 (non-liver patientfrom the ILPD dataset.

Fuzzification process: Two fuzzification functions are used, which are Right-function and Left-function.

$$R_{a,b}(x)=\begin{cases}0 & : x\le a\\ \dfrac{x-a}{b-a} & : a<x\le b,\\ 1 & : b<x\end{cases}\qquad L_{a',b'}(x)=\begin{cases}1 & : x\le a'\\ \dfrac{x-b'}{a'-b'} & : a'<x\le b'\\ 0 & : b'<x\end{cases}$$



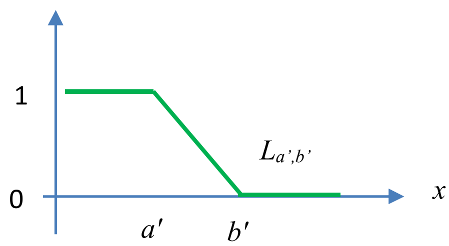


Figure 1. Graphs of the fuzzification functions

The attributes on patients:

$$S_1 =< T_1(x), I_1(x), F_1(x) >=< R_{1.2,5.3}(x), L_{0.2,3}(x), L_{0.6,4}(x) >$$

$$S_2 =< T_2(x), I_2(x), F_2(x) >=< R_{0.4,2.3}(x), L_{0.1,1}(x), L_{0.15,1.5}(x) >$$

$$S_3 =< T_3(x), I_3(x), F_3(x) >=< R_{140,486}(x), L_{80,250}(x), L_{100,400}(x) >$$

$$S_4 =< T_4(x), I_4(x), F_4(x) >=< R_{33,119}(x), L_{5,60}(x), L_{30,100}(x) >$$

$$S_5 =< T_5(x), I_5(x), F_5(x) >=< R_{33,178}(x), L_{10,100}(x), L_{23,150}(x) >$$

$$S_6 =< T_6(x), I_6(x), F_6(x) >=< L_{2.2,3.5}(x), R_{2,4}(x), R_{3,5}(x) >$$

$$S_7 =< T_7(x), I_7(x), F_7(x) >=< L_{0.5,1}(x), R_{0.3,1.5}(x), R_{0.8,2.5}(x) >$$

Two single-valued neutrosophic matrices (M1) and (M2): (see Tables 4 and 5)

Table 4. Matrix of the attributes on patients

(M1)	S_1	S_2	S_3	S_4	S_5	S_6	S_7
P_1	<0.02,0.6,0.7>	<0,0.6,0.8>	<0.9,0,0>	<0.8,0,0>	<0.3,0.2,0.5>	<0.1,0.6,0.1>	<0.2,0.5,0.08>
P_2	<0,0.7,0.9>	<0,0.8,0.9>	<0.1,0.3,0.6>	<0,0.6,1>	<0,0.8,1>	<0,1,0.5>	<0,0.5,0.1>
P_3	<0,0.7,0.9>	<0,0.8,0.9>	<1,0,0>	<1,0,0>	<0,0.9,1>	<0.9,0.1,0>	<0.6,0.3,0>
P_4	<0.6,0,0.05>	<0.5,0,0>	<0.4,0,0.3>	<0.8,0,0>	<1,0,0>	<0.1,0.6,0.1>	<0.4,0.4,0>
P_5	<0,0.7,0.9>	<0,0.8,0.9>	<0.04,0.5,0.8>	<0,0.8,1>	<0,0.9,1>	<0,0.8,0.3>	<0,0.6,0.2>

Table 5. Matrix of the classification results

(M2)	$\hat{S}_1$	$\hat{S}_2$	$\hat{S}_3$	$\hat{S}_4$	$\hat{S}_5$	$\hat{S}_6$	$\hat{S}_7$
La := 1	<1,0,0>	<1,0,0>	<1,0,0>	<1,0,0>	<1,0,0>	<1,0,0>	<1,0,0>
La := 2	<0,1,1>	<0,1,1>	<0,1,1>	<0,1,1>	<0,1,1>	<0,1,1>	<0,1,1>

where $\hat{S}_j$, $j = 1, 2, 3, 4, 5$, are the attribute characteristics of the disease classes.

DOI: 10.5281/zenodo.5164640

- **Step 2:** Calculating the entropy of the attributes:

$$E\left(S_1\right) = s_{H-N}(S_1, S_1^{\,c}) = 0.31; \quad E\left(S_2\right) = s_{H-N}(S_2, S_2^{\,c}) = 0.26;$$

$$E\left(S_3\right) = s_{H-N}(S_3, S_3^{\,c}) = 0.35; \quad E\left(S_4\right) = s_{H-N}(S_4, S_4^{\,c}) = 0.12;$$

$$E\left(S_5\right) = s_{H-N}(S_5, S_5^{\,c}) = 0.18; \quad E\left(S_6\right) = s_{H-N}(S_6, S_6^{\,c}) = 0.57$$

$$E\left(S_7\right) = s_{H-N}(S_7, S_7^{\,c}) = 0.76$$

where $\chi_i = \frac{1}{5}, i = 1,2,3,4,5$ and $\omega_j = \frac{1}{5}, j = 1,2,3,4,5.$

- **Step 3:** Calculating the similarity $s_{H-N}\left(P_i,(\mathrm{La}:=1)\right)$ and $s_{H-N}\left(P_i,(\mathrm{La}:=2)\right)$, $i = 1,2,3,4,5,$ where $\omega_j = \frac{1}{5}, j = 1,2,3,4,5$ and $\chi_i = \dfrac{E\left(S_i\right)}{\sum_{i=1}^{5} E\left(S_i\right)}$, i.e.,

$$\chi_1 = 0.12,\ \chi_2 = 0.1, \chi_3 = 0.14, \chi_4 = 0.05, \chi_5 = 0.07, \chi_6 = 0.22, \chi_7 = 0.3.$$

$$s_{H-N}\left(P_1,(\mathrm{La}:=1)\right) = 0.43,\ s_{H-N}\left(P_1,(\mathrm{La}:=2)\right) = 0.415,$$

$$s_{H-N}\left(P_2,(\mathrm{La}:=1)\right) = 0{,}17,\ s_{H-N}\left(P_2,(\mathrm{La}:=2)\right) = 0.59,$$

$$s_{H-N}\left(P_3,(\mathrm{La}:=1)\right) = 0.62,\ s_{H-N}\left(P_3,(\mathrm{La}:=2)\right) = 0.33,$$

$$s_{H-N}\left(P_4,(\mathrm{La}:=1)\right) = 0.59,\ s_{H-N}\left(P_4,(\mathrm{La}:=2)\right) = 0.266,$$

$$s_{H-N}\left(P_5,(\mathrm{La}:=1)\right) = 0.15,\ s_{H-N}\left(P_5,(\mathrm{La}:=2)\right) = 0.7,$$

- **Step 4.** Making decisions:

- Since $s_{H-N}\left(P_1,(\mathrm{La}:=1)\right) > s_{H-N}\left(P_1,(\mathrm{La}:=2)\right)$, we obtain that the classification label of 1st patient is 1. That means P_1 is a liver patient.
- Since $s_{H-N}\left(P_2,(\mathrm{La}:=1)\right) < s_{H-N}\left(P_2,(\mathrm{La}:=2)\right)$, we obtain that the classification label of 2nd patient is 2. That means P_2 is a non-liver patient.
- Since $s_{H-N}\left(P_3,(\mathrm{La}:=1)\right) > s_{H-N}\left(P_3,(\mathrm{La}:=2)\right)$, we obtain that the classification label of 3th patient is 1. That means P_3 is a liver patient.

DOI: 10.5281/zenodo.5164640

- Since $s_{H-N}\left(P_4,(\mathrm{La}:=1)\right) > s_{H-N}\left(P_4,(\mathrm{La}:=2)\right)$, we obtain that the classification label of 4th patient is 1. That means P_4 is a liver patient.
- Since $s_{H-N}\left(P_5,(\mathrm{La}:=1)\right) < s_{H-N}\left(P_5,(\mathrm{La}:=2)\right)$, we obtain that the classification label of 5th patient is 2. That means P_2 is a non-liver patient.

These conclusions and the classification results of Table 2 are the same. Also on the dataset ILPD, Ngan et al. [19] tested 14 diagnostic methods based on the considered intuitionistic fuzzy distance measures. The best mean absolute error obtained from their experiment is 0.28.

5. Conclusions

In this paper, based on the H-max distance measure on IFSs , a new distance mearsure on SVNSs is proposed. Further, a single-valued neutrosophic similarity measure, a single-valued neutrosophic entropy measure, and their basic properties are presented and proven. In addition, an application to medical diagnosis is shown to illustrate the effective applicability of the proposals. There, the proposed diagnostic method named Two-$d_{H\text{-}N}$ and a numerical example are clarified in detail. The diagnostic results according to the proposed method and the actual results are the same. Furthermore, the diagnostic method is relatively simple to perform. In the future, we will test the proposed diagnostic method on real datasets and compare the performance with related methods. Furthermore, we will develop the distance measure for neutrosophic sets.

Funding: "This research received no external funding"

Conflicts of Interest: "The authors declare no conflict of interest."

References

[1] L. A. Zadeh, Fuzzy Sets, *Informtion and Control* , vol 8, pp. 338 – 353, 1965.
[2] K. Atanassov, Intuitionistic fuzzy sets*, Fuzzy Sets and Systems,* vol 20, pp. 87–96, 1986.
[3] F. Smarandache, A unifying field in logics. neutrosophy: neutrosophic probability, set and logic, *Rehoboth: American Research Press*, 1999.
[4] F. Taş, S. Topal, F. Smarandache, Clustering neutrosophic data sets and neutrosophic valued metric spaces, *Symmetry*, vol 10(10), 430, 2018.
[5] S. Topal, S. Broumi, A. Bakali, M. Talea, F. Smarandache, A python tool for implementations on bipolar neutrosophic matrices, *Infinite Study*, 2019.
[6] S. Topal, A. Çevik, F. Smarandache, A New Group Decision Making Method With Distributed Indeterminacy Form Under Neutrosophic Environment: An Introduction to Neutrosophic Social Choice Theory, *IEEE Access*, vol 8, pp. 42000-42009, 2020.

[7] S. Topal, F. Tas, S. Broumi, O. A. Kirecci, Applications of Neutrosophic Logic of Smart Agriculture via Internet of Things. International Journal of Neutrosophic Science, vol 12(2), pp. 105-115, 2020

[8] H. Wang, F. Smarandache, Y. Q. Zhang, R. Sunderraman, Single valued neutrosophic sets, *Multispace Multistruct*, vol 4, pp. 410–413, 2010.

[9] H. L. Huang, New distance measure of single□valued neutrosophic sets and its application, *International Journal of Intelligent Systems*, vol 31(10), pp. 1021-1032, 2016.

[10] S. Ridvan, Multi-criteria neutrosophic decision making method based on score and accuracy funtions under neutrosophic environment, *Eprint Arxiv*, 2014.

[11] J. Ye, A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets, *Journal of Intelligent & Fuzzy Systems,* vol 26, pp. 2459–2466, 2014.

[12] J. Ye, Single valued neutrosophic cross-entropy for multicriteria decision making problems, *Applied Mathematical Modelling,* vol 38, pp. 1170–1175, 2014.

[13] H. Zhang, J. Wang, X. Chen, Interval neutrosophic sets and their application in multicriteriadecision making problems, *International Journal of Intelligent Systems*, vol 31(5), pp. 1–15, 2014.

[14] A. A. Salama, M. Fazaa, M. Yahya, M. Kazim, A suggested diagnostic system of corona virus based on the neutrosophic systems and deep learning, *International Journal of Neutrosophic Science*, vol 9, 54, 2020.

[15] F. Smarandache, D. Stanujkic, D. Karabasevic, G. Popovic, A novel approach for assessing the reliability of data contained in a single valued neutrosophic number and its application in multiple criteria decision making, *International Journal of Neutrosophic Science,* vol 11(1), 22, 2020.

[16] J. Ding, W. Bai, C. Zhang, A New Multi-Attribute Decision Making Method with Single-Valued Neutrosophic Graphs, *International Journal of Neutrosophic Science*, vol 11(2) , pp. 76-86, 2020.

[17] R. T. Ngan, L. H. Son, M. Ali, δ-equality of intuitionistic fuzzy sets: a new proximity measure and applications in medical diagnosis, *Applied Intelligence*, vol 48(2), pp. 499-525, 2018.

[18] R. T. Ngan, L. H. Son, B. C. Cuong, M. Ali, H-max distance measure of intuitionistic fuzzy sets in decision making, *Applied Soft Computing*, vol 69, pp. 393-425, 2018.

[19] R. T. Ngan, L. H. Son, M. Ali, D. E. Tamir, N. D. Rishe, A. Kandel, Representing complex intuitionistic fuzzy set by quaternion numbers and applications to decision making, *Applied Soft Computing*, vol 87, 105961, 2020.

[20] L. H. Son, R. T. Ngan, M. Ali, H. Fujita, M. Abdel-Basset, N. L. Giang, G. Manogaran, M. K. Priyan, A new representation of intuitionistic fuzzy systems and their applications in critical decision making, *IEEE Intelligent Systems*, vol 35(1), pp. 6-17, 2019.

[21] F. Smarandache, N. T. Roan, S. C. Arnau, M. A. Díaz, J. M. M. Rubio, P. Lopez, F. Andujar, L. H. Son, V. V. Manh, The picture fuzzy distance measure in controlling network power consumption, *Journal of Fuzzy Extension & Applications*, vol 1(3), 148, 2020.

[22] S. Broumi, F. Smarandache, Several similarity measures of neutrosophic sets, *Infinite Study*, 2013.

[23] J. Ye, Vector similarity measures of simplified neutrosophic sets and their application in multicriteria decision making, *Infinite Study*, 2014.

DOI: 10.5281/zenodo.5164640

[24] J. Ye, Clustering methods using distance-based similarity measures of single-valued neutrosophic sets, *Journal of Intelligent Systems*, vol 23(4), pp. 379-389, 2014.
[25] A. Aydoğdu, On similarity and entropy of single valued neutrosophic sets, *General mathematics notes*, vol 29(1), pp. 67-74, 2015.
[26] R. T. Ngan, L. H. Son, B. C. Cuong, A New Entropy of Dynamical Systems in Complex Intuitionistic Fuzzy Environment, *Proceeding of the 11th International Conference on Internet (ICONI 2019), Hanoi, Vietnam*, pp. 61-65, 2019.
[27] B. C. Cuong, V. Kreinovich, Picture Fuzzy Sets-a new concept for computational intelligence problems, *In 2013 third world congress on information and communication technologies (WICT 2013), IEEE*, pp. 1-6, 2013.

DOI: 10.5281/zenodo.5164640