A Multi-Server Queuing-Inventory System with Attraction-Retention Mechanisms for Impatient Customers and Catastrophes in Warehouse

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Abstract

This paper presents a multi-server Markovian queuing-inventory system (MQIS) that incorporates attraction-retention (AR) mechanisms for impatient customers and models catastrophic inventory losses within a warehouse setting. The system consists of C identical servers, a limited waiting area, and a storage capacity of Q items. Periodic disruptions may destroy all inventory in the system, compelling waiting customers either to remain until stock is replenished or to exit the system. A subset of servers may take joint vacations when no customers are waiting. To analyze this queuing-inventory system (QIS), we derive balance equations using a three-dimensional continuous-time Markov chain framework, solving for steady-state solutions through a recursive method. We then derive performance metrics and identify special-case queuing-inventory models within the broader system. A cost-loss model is formulated to optimize the service rate and server vacation strategies, minimizing overall costs. A genetic algorithm is employed to conduct a cost analysis. We collected primary data from the Ethio Telecom district head office in Arba Minch, Ethiopia to validate our theoretical findings. The empirical analysis serves a dual purpose: to investigate performance measure sensitivity to parameter variations and to discuss an optimization problem aimed at minimizing expected total cost (ETC) while assessing the impacts of AR mechanisms and catastrophic events on ETC.

Keywords: Multi-Server System; Customer Attraction and Retention; Warehouse Disruptions; Impatient Customer Dynamics; Inventory Management Strategies.

1 Introduction

Queuing-inventory systems (QIS) combine queuing theory and inventory control to improve service quality in production, healthcare, and hospitality. In QIS, each customer served departs the system, reducing inventory by one unit at the service completion point. For example, Ethio Telecom service centers in Ethiopia operate with waiting lines, where customers are served on a first-come, first-served ($FCFS$) basis, consuming on-hand inventory items. Such integrated queuing systems have applications in supply chain management,[1] vehicle maintenance,[2] and medical services.[3] Markovian QIS models are widely used for designing telecommunication and service systems, often incorporating advanced features like server vacations, customer retrials, impatience, and catastrophic events impacting inventory.

Sigman and Simchi-Levi[4] and Melikov and Molchanov[5] independently introduced the QIS concept, prompting extensive research. In traditional queuing theory, servers are typically assumed always to be available;

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however, in practical settings, servers may become unavailable for periods due to maintenance, repairs, supplementary tasks, or breaks. Such unavailable periods are termed "vacations." Researchers have studied QIS models with server vacations to optimize server idle time. In multi-server QIS, vacation models often assume all servers take a synchronous vacation, meaning they leave and return to the system together. In real-world scenarios, however, organizations generally prefer to keep some servers active (either idle or busy) while others are on vacation.

Daniel and Ramanarayanan[6] first introduced the vacation concept within a QIS. Since then, QIS vacation policies have been widely studied. For example, Divya et al.[7] analyzed a single-server QIS with impatient customers and a server that takes vacations to restock inventory. They derived expressions for performance measures using probability-generating functions. Nithya et al.[8] observed that breaks improve employee performance, analyzing a four-dimensional stochastic QIS with multiple server vacations and a state-dependent arrival process. Recently, Yue et al.[9] studied synchronous server vacations in a multi-server QIS, where all servers simultaneously vacation when inventory is depleted. However, this synchronous policy may not be suitable in some cases.

In contrast, Jegannathan et al.[10] investigated a multi-server QIS with asynchronous vacations and customer retrials, where each of the C servers independently starts or ends vacation based on system conditions. Service times are exponentially distributed, and if servers encounter insufficient customers or items, they may initiate another vacation. The system manages inventory with an (s, Q) replenishment policy, and stationary probabilities are derived using matrix geometric approximation.

Modern QIS models also consider customer attraction-retention (AR) strategies, essential in competitive business environments. Offering promotions or improved service quality can attract new customers, but increased arrival rates can lead to service delays. Customers who become impatient may abandon the queue without service, impacting total costs. Balancing attraction and retention is crucial for QIS models aiming to minimize total costs.

The vacation policy in our study differs from existing models. Specifically, D servers (where $0 < D < C$) take a vacation together when no customers are waiting at the service completion point, while the remaining $C - D$ servers are always available, either serving customers (when inventory is available) or idling (when inventory is zero). If, upon returning, fewer than $C - D$ customers are in line, the D servers take another vacation. Since only a subset of servers can vacation simultaneously, our model uses an asynchronous vacation policy. Inventory management follows a $(0, q, Q)$ policy: when inventory drops to q, an order is placed to restock up to Q units $(q < Q)$.

Catastrophic events such as fires, floods, and human errors can destroy inventory entirely. Following a catastrophe, all inventory in storage becomes unavailable, necessitating immediate restocking. This scenario is termed a QIS with catastrophes. Melikov et al.[12] studied single-server QIS with catastrophes, using matrix-analytic methods to determine steady-state distributions. However, few studies consider total inventory destruction. We extend this work to a multi-server MQIS with catastrophes, AR mechanisms for impatient customers, and asynchronous server vacations. Here, C removable servers operate under an asynchronous vacation policy, and catastrophic events occur in the warehouse at a rate γ.

The rest of this paper is organized as follows. Section 2 presents model assumptions and a detailed description. Sections 3 and 4 outline the research methodology and analytical framework, respectively. Section 5 discusses data collection, while Section 6 covers numerical results and analysis. Conclusions are provided in Section 7.

2 Description of the Model

We consider a finite-capacity multi-server MQIS incorporating attraction-retention (AR) mechanisms for impatient customers and accounting for catastrophic events in the warehouse. The model assumptions and notations used throughout this paper are as follows:

1. Customers arrive at the service system, which is attached to an inventory, according to a Poisson process with rate λ ($\lambda \geq 0$). Upon arrival, each customer joins the queue with probability b_n or balk with

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probability $(1 - b_n)$. For $n \leq C - D$, $b_n = 1 - \gamma$. Additionally, customer attraction mechanisms (e.g., rewards, coupons) increase the arrival rate by β, representing the percentage increase due to these incentives. While this increase can lead to service delays, potentially causing customer impatience, retention mechanisms (e.g., high-quality service, customer relationship efforts) help retain customers at a rate of r.

2. The system consists of C removable servers, a limited capacity waiting room for up to N customers, and a maximum inventory size of Q. Each customer requires one inventory item for service, reducing the on-hand inventory by one unit upon service completion.

3. Customers form a single queue, served on a first-come, first-served (FCFS) basis. Once service begins, it continues without interruption unless a catastrophe occurs. Service times are independent and identically distributed random variables following an exponential distribution with density function $s(t) = \mu e^{-\mu t}$, $t \geq 0$, where μ is the service rate.

4. After joining the queue, each customer waits for a random amount of time T before service begins. If service has not started by then, the customer may become impatient and leave the queue without service. This waiting time T is a random variable with density function $f(t) = \alpha e^{-\alpha t}$, $t \geq 0$, where α is the reneging rate. Let n denote the number of customers in the system. If $n \leq C - D$ and inventory is available, customers receive immediate service, and reneging does not occur. For $n > C - D$, $(n - C + D)$ customers wait in the queue. Due to attraction mechanisms, the arrival rate may increase, potentially inducing higher customer reneging. Retention mechanisms mitigate this, producing customer retention at r. Thus, the average reneging rate $R(n)$ for n customers in the system is:

$$R(n) = \begin{cases} (n - C + D)(1 - r)\alpha, & C - D < n < N \\ 0, & 0 \leq n \leq C - D \end{cases}$$

5. When the system becomes empty, D servers ($D < C$) initiate a vacation for a random period V. If they return to find no waiting customers, the D servers begin another vacation. The vacation time V follows an exponential distribution with density function $v(t) = \xi e^{-\xi t}$, $t \geq 0$, where ξ is the vacation rate.

6. An $(0, q, Q)$ ordering policy manages inventory. When the inventory level falls to q, an order is placed to replenish inventory up to Q ($q < Q$), where q is the reorder level. The order delivery time is a random variable with an exponential distribution parameter η ($\eta > 0$). The relationship between the number of servers on duty and the reorder level is $C - D < q$. Upon each service completion, one inventory item is used.

7. Catastrophic events occur in the warehouse following a Poisson process with rate γ. Each catastrophe instantly empties the warehouse, setting on-hand inventory to zero. Customers whose service is interrupted by a disaster may either rejoin the queue or leave the system. During this period, servers become inoperative, and a restoration process begins.

8. During the restoration period, new customers continue to arrive. Restoration times are independent and identically distributed, following an exponential distribution with parameter κ.

3 Research Methodology

3.1 Research Design

This study aims to reduce cost losses associated with warehouse catastrophes and impatient customer behaviors in QIS models. To achieve an optimal total cost, it is essential to both attract new arrivals and retain potential reneging customers by improving service quality. This motivation led us to analyze a queuing system with attached inventory and to incorporate attraction-retention mechanisms aimed at minimizing cost loss. In addition, an asynchronous vacation policy for dedicated C-removable servers is adopted to improve service quality. The analysis is structured on building a mathematical model of the QIS, assuming a Markov process. The impact of key parameters on various performance measures is discussed using empirical data from Ethio Telecom's service center in Arba Minch, Ethiopia.

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3.2 Data Collection Method

The study uses primary data collected through direct observation at the Ethio Telecom Arba Minch district head office. Data, including customer arrival times and service completion times, were recorded daily on a data sheet over two weeks, excluding Sundays. Collection took place within Ethiopia's standard workweek for 48 hours, specifically from 8:30 am to 12:30 pm and 1:30 pm to 5:30 pm, for a total of 12 days. Arrival and service times for each customer were recorded using a stopwatch function on a mobile phone.

3.3 Time Measurements and Data Extraction

From the recorded time measurements, essential data were extracted according to the following process. Each customer entering the Ethio Telecom service center was assigned an index, $i = 1, 2, 3, \ldots, n$, corresponding to the following time measures:

- (i) Customer arrival time
- (ii) Service completion time
- (iii) Inter-arrival time between consecutive arrivals
- (iv) Number of customers in the queue
- (v) Number of servers on vacation
- (vi) Number of reneged customers
- (vii) Waiting time in the queue
- (viii) Service time
- (ix) Waiting time in the system

Each of these parameters was averaged over the eight hours of data collection per day to obtain daily mean values. Using MATLAB, the average operating characteristics of the QIS were estimated by applying equations derived from the $M/M/C/N$ MQIS model.

3.4 Data Analysis Method

Both analytical and numerical analyses were conducted. The balance equations governing the MQIS in terms of steady-state probabilities and study parameters were formulated. A recursive method was employed to derive closed-form expressions for the steady-state probabilities associated with system size. Upon completing data collection, the data were checked for completeness and exported to SPSS version 20 for further analysis to obtain expected values. Based on the recorded data, numerical analysis was conducted to examine the effects of various parameters on the MQIS performance.

4 Analysis of the Model

In this section, we carry out an analytical analysis of a finite-capacity multi-server $MQIS$ model that incorporates attraction-retention mechanisms for impatient customers, alongside the impact of catastrophes in the warehouse and random lead times.

Let the state of the system at time t be described by the following random variables:

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- $X(t)$: denotes the system size (number of customers),
- $Y(t)$: describes the inventory level,
- $Z(t)$: indicates the status of the server, defined as

$$Z(t) = \begin{cases} 0, & \text{if } D \text{ servers are on vacation at time } t \\ 1, & \text{if all } C \text{ servers are available for service at time } t \end{cases}$$

The stochastic process

$$\Phi(t) = \{X(t), Y(t), Z(t);\ t \geq 0\}$$

is modeled as a three-dimensional Markov process with a state space given by:

$$\Omega = \{(n, s, 0) : 0 \leq n \leq C - D,\ 0 \leq s \leq Q\} \cup \{(n, s, 1) : C - D + 1 \leq n \leq N,\ 0 \leq s \leq Q\}$$

For the process $\Phi(t)$, the steady-state probability distribution is defined as:

$$p_{i,n,s} = \lim_{t \to \infty} \Pr\{X(t) = n, Y(t) = s, Z(t) = i\}, \quad i = 0, 1$$

We assume that there are no customers in the system and that the level of inventory is Q at time $t = 0$. Using the Markov process and the state-transition diagram in Figure 1, the $MQIS$ with catastrophes in the warehouse is governed by the set of balance equations (1 - 11).

$$-\lambda(1+\beta)p_{0,0,0} + (\mu + \eta + \kappa)p_{0,1,s} = 0;\ s = 0, 1, 2, \cdots, q, q+1, \cdots, Q,\ n = 0 \tag{1}$$

$$\begin{aligned} &\lambda(1+\beta)p_{0,n-1,s} - [\lambda(1+\beta) + n\mu + \eta + \kappa]\, p_{0,n,s} + [(n+1)\mu + \eta + \kappa]\, p_{0,n+1,s} \\ &= 0;\ s = 0, 1, 2, \cdots, q, q+1, \cdots, Q,\ 1 \leq n < C - D \end{aligned} \tag{2}$$

$$\begin{aligned} &\lambda(1+\beta)p_{0,C-D-1,s} - [\lambda(1+\beta)b_{C-D,s} + (C-D)\mu + \eta + \kappa)]p_{0,C-D,s} + \\ &[(C-D)\mu + \eta + \kappa) + \alpha]p_{0,C-D+1,s} + [(C-D+1)\mu + \eta + \kappa]p_{1,C-D+1,s} = 0; \\ &s = 1, 2, \cdots, q, q+1, \cdots, Q,\ n = C - D \end{aligned} \tag{3}$$

$$\begin{aligned} &\lambda(1+\beta)b_{n-1,s}p_{0,n-1,s} - [\xi + \lambda(1+\beta)b_{n,s} + (C-D)\mu + \eta + \kappa) + (n+ \\ &D - C)\alpha(1-r)]p_{0,n,s} + [(C-D)\mu + \eta + \kappa) + (n+1+D-C)\alpha(1-r)] \\ &p_{0,n+1,s} = 0;\ s = 1, 2, \cdots, q, q, \cdots, Q, C - D < n < N \end{aligned} \tag{4}$$

$$\begin{aligned} &\lambda(1+\beta)b_{N-1,s}p_{0,N-1,s} - [\xi + (C-D)\mu + \eta + \kappa + (N+D-C)\alpha(1-r)] \\ &p_{0,N,s} = 0;\ s = 1, 2, \cdots, q, q+1, \cdots, Q,\ n = N \end{aligned} \tag{5}$$

$$\begin{aligned} &\xi p_{0,C-D+1,s} - [\lambda(1+\beta) + (C-D+1)\mu + \eta + \kappa]p_{1,C-D+1,s} + [(C-D+2)\mu + \\ &\eta]p_{1,C-D+2,s} = 0;\ s = 1, 2, \cdots, q, q+1, \cdots, Q,\ n = C - D + 1 \end{aligned} \tag{6}$$

$$\begin{aligned} &\xi p_{0,n,s} + \lambda(1+\beta)p_{1,n-1,s} - [\lambda(1+\beta) + n\mu + \eta + \kappa]p_{1,n,s} + [(n-1)\mu + \eta + \kappa] \\ &p_{1,n+1,s} = 0; s = 1, 2, \cdots, q, q+1, \cdots, Q,\ C - D + 2 \leq n \leq C - 1 \end{aligned} \tag{7}$$

$$\begin{aligned} &\xi p_{0,C,s} + \lambda(1+\beta)p_{1,C-1,s} - [\lambda(1+\beta)b_{C,s} + (C\mu + \eta + \kappa)]p_{1,C,s} + [C\mu + \eta + \\ &\kappa + \alpha(1-r)]p_{1,C+1,s} = 0;\ s = 1, 2, \cdots, q, q+1, \cdots, Q,\ n = C \end{aligned} \tag{8}$$

$$\begin{aligned} &\xi p_{0,n,s} + \lambda(1+\beta)b_{n-1,s}p_{1,n-1,s} - [\lambda(1+\beta)b_{n,s} + (C\mu + \eta + \kappa) + (n-C)\alpha \\ &(1-r)]p_{1,n,s} + [(C\mu + \eta + \kappa) + (n+1-C)\alpha(1-r)]p_{1,n+1,s} = 0;\ s = 1, 2, \\ &\cdots, q, q+1, \cdots, Q,\ C < n < N \end{aligned} \tag{9}$$

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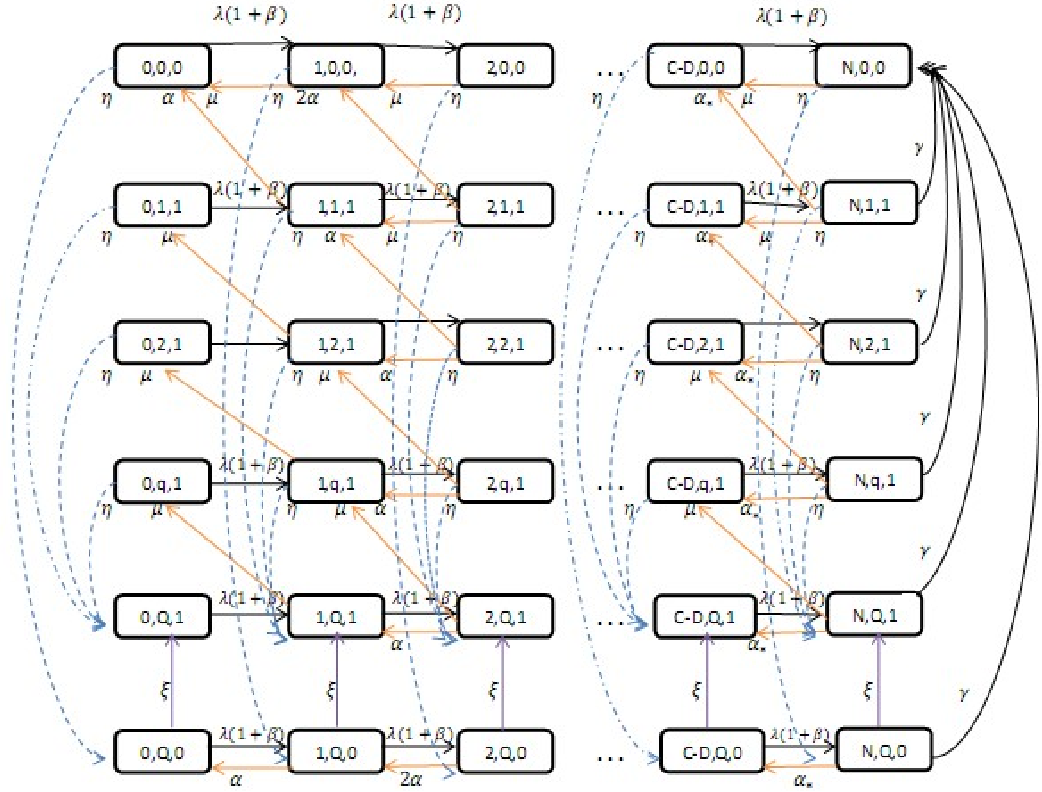

Figure 1: State-transition diagram of the QIS model

$$\xi p_{0,N,s} + \lambda(1+\beta)b_{N-1,s}p_{1,N-1,s} - [(C\mu+\eta+\kappa)+(N-C)\alpha(1-r)]p_{1,N,s} = 0;\ s = 1,2,\cdots,q,q+1,\cdots,Q \tag{10}$$

Normalization condition:

$$\sum_{s=1}^{Q}\left[\sum_{n=0}^{C-D} p_{0,n,s} + \sum_{n=C-D+1}^{N} p_{1,n,s}\right] = 1 \tag{11}$$

4.1 Steady-State Solution

To obtain neat and closed-form solutions, a recursive technique is employed to find all probabilities $p_{0,n,s}$ and $p_{1,n,s}$ in terms of $p_{0,0,0}$, λ, μ, β, b_n, r, α, ξ, η, and κ. The solution approach for the system is stated in the form of Theorem 4.1 as follows:

Theorem 4.1. *If the steady-state recurrence equations of a finite-capacity multi-server MQIS with attraction-retention mechanisms for impatient customers and catastrophes in the warehouse under the* $(0,q,Q)$ *inventory policy are as in (1 - 11), then the probabilities of the system size are given by:*

$$p_{0,0,0} = \left[L + \sum_{n=C-D}^{N-1}\left[\frac{(\lambda(1+\beta))^n}{\mu+\eta+\kappa+\xi}\prod_{i=C-D+1}^{N-1} b_i\right] + p_{1,N,s}\right]^{-1}, \quad s = 0,1,2,\cdots,q,\cdots,Q \tag{12}$$

$$p_{0,1,s} = \rho p_{0,0,0}, \quad s = 0,1,2,\cdots,q,\cdots,Q \tag{13}$$

$$p_{0,n,s} = \frac{1}{n!}\rho^n p_{0,0,0}, \quad 2 \le n \le C-D,\ s = 0,1,2,\cdots,q,\cdots,Q \tag{14}$$

$$p_{1,1,s} = \frac{\lambda(1+\beta)}{\mu+\eta+\kappa+\xi}\prod_{i=2}^{N-1}[b_i]\,p_{0,0,0}, \quad s = 0,1,2,\cdots,q,\cdots,Q \tag{15}$$

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$$p_{1,n,s} = \frac{(\lambda\,(1+\beta))^n}{\mu+\eta+\kappa+\xi} \prod_{i=C-D+1}^{N-1} [b_i]\, p_{0,0,0}, \qquad C-D+1 \le n < N,\ s=0,1,2,\cdots,q,\cdots,Q \tag{16}$$

$$p_{1,N,s} = \frac{(\lambda\,(1+\beta))^N}{\mu+\eta+\kappa+\xi} \prod_{i=C-D+1}^{N-1} [b_i]\, p_{0,0,0}, \quad s=0,1,2,\cdots,q,\cdots,Q \tag{17}$$

Where

$$L = 1+\rho+\sum_{n=2}^{C-D-1} \left[(n!)^{-1}(\rho)^n\right]$$

$$\rho = \left(\frac{\lambda\,(1+\beta)}{\mu+\eta+\kappa}\right)$$

$$b_i = \frac{b_{i-1}}{\mu+\eta+\kappa+(i-(C-D))\alpha(1-r)+\xi}, \quad i=C-D+1, C-D+2, \cdots, N-1$$

Proof. We obtain the steady-state probabilities using a recursive method. Rearranging (1), we get the value of $p_{0,1,s}$ as

$$p_{0,1,s} = \rho p_{0,0,0}, \quad s=0,1,2,\cdots,q,\cdots,Q$$

which is (13). From (2),

$$\lambda(1+\beta)p_{0,0,s} - (\lambda(1+\beta)+\mu+\eta+\kappa)p_{0,1,s} + 2(\mu+\eta+\kappa)p_{0,2,s} = 0, \quad n=1$$

Using (1), we have

$$-\lambda(1+\beta)p_{0,1,s} + 2(\mu+\eta+\kappa)p_{0,2,s} = 0, \quad s=0,1,2,\cdots,q,q+1,\cdots,Q$$

This leads to

$$p_{0,2,s} = \left(\frac{(\lambda(1+\beta))^2}{2(\mu+\eta+\kappa)^2}\right)p_{0,0,0} = \frac{1}{2}\rho^2 p_{0,0,0}, \quad s=0,1,2,\cdots,Q$$

where

$$\rho = \frac{\lambda(1+\beta)}{\mu+\eta+\kappa}$$

Similarly, for $n=2$, the above procedure gives

$$p_{0,3,s} = \frac{1}{6}\rho^3 p_{0,0,0}, \quad s=0,1,2,\cdots,Q$$

Therefore, by the principle of induction, we conclude for any n such that $3 < n \le C-D$:

$$p_{0,n,s} = \frac{1}{n!}\rho^n p_{0,0,0}, \quad s=0,1,2,\cdots,Q$$

which is (14). From (3) for $n=C-D+1$, we obtain

$$p_{1,C-D+1} = \frac{(\lambda(1+\beta+\kappa))^{C-D+1}}{\mu+\eta+\kappa+\xi}\left(\frac{b_{C-D}}{\mu+\eta+\kappa+\alpha(1-r)+\xi}\right)\left(\frac{b_{C-D+1}}{\mu+\eta+\kappa+2(\alpha(1-r))+\xi}\right) \cdots \left(\frac{b_{N-2}}{\mu+\eta+\kappa+(N-1-(C-D))\,\alpha(1-r)+\xi}\right)p_{0,0,0};\ s=0,1,2,\cdots,q.$$

Therefore, by the principle of induction, we conclude for any n such that $C-D < n < N$:

$$p_{1,n} = \frac{(\lambda(1+\beta))^n}{\mu+\eta+\kappa+\xi}\prod_{i=C-D+1}^{N-1}[b_i]\,p_{0,0,0}$$

where

$$b_i = \frac{b_{i-1}}{\mu+\eta+\kappa+(i-(C-D))\alpha(1-r)+\xi}, \quad i=C-D+1, C-D+2, \cdots, N-1$$

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which is (16). It is not difficult to obtain (15) and (17) from (16) for $n = 1$ and $n = N$, respectively. Hence, equations (14) and (16) completely determine all the steady-state probabilities.

Now, it only remains to determine $p_{0,0,0}$. Applying the normalization condition, we have

$$p_{0,0,0} + \sum_{s=0}^{Q} \left[\sum_{n=0}^{C-D} p_{0,n,s} + \sum_{n=C-D+1}^{N} p_{1,n,s} \right] = 1.$$

This leads to

$$p_{0,0,0} + Lp_{0,0,0} + \sum_{n=C-D}^{N-1} \left[\frac{(\lambda(1+\beta))^n}{\mu+\eta+\kappa+\xi} \prod_{i=C-D+1}^{N-1} b_i \right] p_{0,0,0} + p_{1,N,s} = 1, \quad s = 0, 1, 2, \cdots, q, \cdots, Q$$

where

$$b_i = \frac{b_{i-1}}{\mu+\eta+\kappa+(i-(C-D))\alpha(1-r)+\xi}, \quad i = C-D+1, C-D+2, \cdots, N-1$$

and

$$p_{1,N,s} = \frac{(\lambda(1+\beta))^N}{\mu+\eta+\kappa+\xi} \prod_{i=C-D+1}^{N-1} [b_i]\, p_{0,0,0}.$$

Thus,

$$p_{0,0,0} = \left[L + \sum_{n=C-D}^{N-1} \left[\frac{(\lambda(1+\beta))^n}{\mu+\eta+\kappa+\xi} \prod_{i=C-D+1}^{N-1} b_i \right] + p_{1,N,s} \right]^{-1}, \quad s = 0, 1, 2, \cdots, q, \cdots, Q$$

where

$$L = 1 + \rho + \sum_{n=2}^{C-D-1} \left[(n!)^{-1} (\rho)^n \right],$$

$$\rho = \frac{\lambda(1+\beta)}{\mu+\eta+\kappa},$$

$$b_i = \frac{b_{i-1}}{\mu+\eta+\kappa+(i-(C-D))\alpha(1-r)+\xi}, \quad i = C-D+1, C-D+2, \cdots, N-1$$

which is (12). The system is stable for any value of the utilization factor ρ.

This completes the proof. □

4.2 Special Cases

In this section, we derive the results of some models found in the existing literature by taking specific values of the parameters $b_n, \alpha, \xi, \beta, r$, and γ.

Case 1: In the absence of catastrophes, balking, multiple servers, server vacation, and attraction-retention mechanisms for impatient customers (i.e., for $\gamma = 0$, $b_n = 1, \forall n = 0, 1, 2, ..., N$, $C = 1$, $\xi = \beta = r = 0$), the model reduces to an $M/M/1/N$ queuing scheme with inventory for impatient customers under a random order size as studied by Alnowibet et al.[17]

Case 2: In the absence of server vacation, multiple servers, and attraction-retention mechanisms for impatient customers (i.e., for $\gamma = 0$, $\xi = 0$, $C = 1$, $\beta = r = 0$), the model reduces to a single-server queuing inventory system (QIS) with a finite waiting room under catastrophes in the warehouse as studied by Melikove et al.[12]

Case 3: For a fixed balking rate (i.e., $b_n = b, \forall n = 0, 1, 2, \cdots, N$), in the absence of catastrophes, reneging, and attraction-retention mechanisms for impatient customers (i.e., $\gamma = \alpha = \beta = r = 0$), the study reduces to a multi-server retrial QIS with asynchronous vacations as examined by Jeganathan et al.[10]

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4.3 Performance Measures

This section explores key performance metrics related to both inventory and queuing systems, following the determination of steady-state probabilities. The derivation of these metrics is organized as follows:

1. **Mean Number of Items in Inventory**:
$$E_I = \sum_{s=0}^{Q} \sum_{n=0}^{C-D} s p_{0,n,s} + \sum_{s=0}^{Q} \sum_{n=C-D+1}^{N} s p_{1,n,s} \tag{18}$$

2. **Mean Reorder Rate**:
$$E_r = \eta \sum_{s=0}^{q} \sum_{n=0}^{C-D} p_{0,n,s} + \eta \sum_{s=0}^{q} \sum_{n=C-D+1}^{N} p_{1,n,s} \tag{19}$$
Under the $(0, q, Q)$ policy, a reorder is initiated when the inventory level falls to s $(0 \leq s \leq q)$, triggering an order of size $(Q - s)$ at the end of service.

3. **Mean Order Size**:
$$E_0 = \sum_{n=0}^{C-D} Q p_{0,n,0} + \sum_{s=0}^{q} \sum_{n=C-D+1}^{N} (Q - s) p_{1,n,s} \tag{20}$$
Reorders are triggered when inventory levels drop to s $(0 \leq s \leq q)$, resulting in an order size of $(Q-s)$.

4. **Mean Loss Rate of Customers**:
$$LS = \lambda(1+\beta) \left[\sum_{n=0}^{C-D} p_{0,n,0} + \sum_{s=1}^{Q} p_{1,N,s} \right] \tag{21}$$
Lost sales occur when customers either find no available inventory despite waiting space or discover full waiting space.

5. **Quality of Service Measure (β_1-Service Level)**:
$$\beta_1 = \frac{\lambda(1+\beta) - LS}{\lambda(1+\beta)} \tag{22}$$
A service quality closer to one indicates better service, whereas a value closer to zero suggests poorer service.

6. **Effective Arrival Rate**:
$$\lambda_{eff} = \lambda(1+\beta) - LS = \lambda(1+\beta)\beta_1 \tag{23}$$
This measure provides insights into customer experience without detailing waiting times.

7. **Mean Number of Customers in the System**:
$$L_s = \sum_{s=0}^{Q} \sum_{n=1}^{C-D} n p_{0,n,s} + \sum_{s=1}^{Q} \sum_{n=C-D+1}^{N} n p_{1,n,s} \tag{24}$$

8. **Mean Number of Customers in the Queue**:
$$L_q = \sum_{s=0}^{Q} \sum_{n=C-D+1}^{N} \left[n - (C-D) \right] p_{1,n,s} \tag{25}$$

9. **Mean Waiting Times**: The mean waiting time in the system (W_s) and in the queue (W_q) can be derived using effective arrival rates and Little's formula:
$$W_s = \frac{L_s}{\lambda_{eff}} \tag{26}$$
$$W_q = \frac{L_q}{\lambda_{eff}} \tag{27}$$

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Received: November 16, 2024 Revised: 27 December, 2024 Accepted: January 20, 2025

10. **Probability of All Servers Being Busy**:

$$P_B = \sum_{s=1}^{Q} \sum_{n=C-D}^{N} p_{1,n,s} \tag{28}$$

11. **Probability of Idle Servers**:

$$P_I = \sum_{s=0}^{Q} \sum_{n=0}^{C-D-1} p_{0,n,s} + \sum_{n=C-D}^{N} p_{1,n,0} \tag{29}$$

This indicates the presence of idle servers when customer numbers are fewer than active servers.

12. **Mean Number of Customers Not Entering the Queue** (BR) and **Mean Number of Customers Leaving Without Service** (RR):

$$BR = \lambda(1+\beta) \sum_{s=0}^{Q} \sum_{n=C-D}^{N} (1-b_n) p_{1,n,s} \tag{30}$$

$$RR = \alpha(1-r) \sum_{s=0}^{Q} \sum_{n=C-D+1}^{N} (n-C+D) p_{1,n,s} \tag{31}$$

In (31), $(n-C+D)(1-r)\alpha$ represents the instantaneous reneging rate of customers.

$$LR = BR + RR \tag{32}$$

13. **Mean Number of Customers Joining for Service and Leaving After Being Served**:

$$GR = L_s - LR \tag{33}$$

14. **Mean Number of Customers Lost Due to Server Vacations**:

$$VL_r = \xi \sum_{s=0}^{Q} \sum_{n=0}^{C-D} n p_{0,n,s} \tag{34}$$

where ξ denotes the reneging rate triggered by server vacations.

15. **Mean Number of Items Destroyed Due to Catastrophes**:

$$E_c = \sum_{s=0}^{Q} \sum_{n=0}^{C-D} s\gamma_{kn} p_{0,n,s} + \sum_{s=0}^{Q} \sum_{n=C-D+1}^{N} s\gamma_{kn} p_{1,n,s} \tag{35}$$

where

$$\gamma_{kn} = \begin{cases} \gamma_{0n}, & \text{if no catastrophe occurs at state } n \\ \gamma_{1n}, & \text{if a catastrophe occurs at state } n \end{cases}$$

Finally, an optimization study will be conducted, focusing on minimizing expected total costs with the warehouse capacity as the control parameter.

4.4 Cost-Loss Model

In this section, we develop a cost model associated with catastrophes, which encompasses the expected costs related to operating the system, arising from both customer queuing and inventory holding. Following the establishment of performance measures, we formulate the total expected cost function per unit of time, where the number of servers on vacation D and the service rate μ are treated as decision variables. Our primary objective is to determine the optimal number of servers to go on vacation D^* and to identify the optimal service rate μ^* that minimizes the expected total cost (ETC) loss.

We define the cost elements as follows:

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- C_1: Holding cost of inventory per unit time.
- C_2: Fixed cost for placing an order.
- C_3: Replenishment cost per item.
- C_4: Cost per unit time when a server is on vacation.
- C_5: Cost per unit time when a server is busy.
- C_6: Cost per unit time when a server is idle.
- C_7: Cost per unit time when a customer joins the queue and waits for service (cost of waiting).
- C_8: Cost per unit time when a customer is served (cost of service).
- C_9: Cost per unit time when a customer balks or reneges.
- C_{10}: Cost per unit time when inventory is destroyed due to a catastrophe.

Using the definitions of these cost elements along with their corresponding system characteristics, the ETC loss function per unit time is expressed as:

$$F(\mu, D) = C_1E_I + C_2E_r + C_3E_0E_r + C_4VL_r + C_5P_B + C_6P_I + C_7L_q + \\ C_8GR + C_9LR + C_{10}E_c. \tag{36}$$

In equation (36), each term represents the following costs:

- C_1E_I: Inventory holding cost.
- C_2E_r: Order cost incurred when placing an order.
- $C_3E_0E_r$: Cost incurred during replenishment.
- C_4VL_r: Cost incurred when servers go on vacation.
- C_5P_B: Cost incurred when servers are busy.
- C_6P_I: Cost incurred by idle servers.
- C_7L_q: Cost incurred by customers waiting in line for service.
- C_8GR: Cost incurred when customers are served.
- C_9LR: Cost incurred when customers are lost.
- $C_{10}E_c$: Cost incurred when items are destroyed due to catastrophes.

Thus, the optimization problem can be formulated as:

$$\text{Minimize } F(\mu, D)$$

Subject to the constraints:

$$0 \leq \mu \leq M_1$$

$$0 \leq D \leq M_2$$

where M_1 is a real number and M_2 is an integer. The cost function $F(\mu, D)$ is nonlinear in both μ and D, complicating the analysis of its convexity. To tackle this problem, meta-heuristic algorithms, such as genetic algorithms, are employed to find solutions using computer software like MATLAB. The steps of the genetic algorithm used to derive the optimal solution are outlined below.

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Received: November 16, 2024 Revised: 27 December, 2024 Accepted: January 20, 2025

Genetic Algorithm
begin
Set
cost function, $F(m)$
population size, n
maximum generation, $MaxGen$
length of chromosome, l_c
minimum value of m, mm
maximum value of m, mM
for $mm \leq m \leq mM$
randomly generate an initial population of n chromosomes, $m_1, m_2, \cdots, m_n$
end for
set an iteration counter $i = 0$
compute the fitness values of each chromosome, $F(m_1), F(m_2), \cdots, F(m_n)$
while ($i \leq MaxGen$)
select a pair of chromosomes from the initial population based on fitness,
Apply crossover operation on selected pair with crossover probability, p_c,
Apply mutation on the offspring with mutation probability, p_m,
Replace old population with newly generated population,
Iteration increment, $i = i + 1$
end while
return the best solution, $m - best$ and $F - best$
end

5 Data Presentation

The primary data for this analysis were collected at the Ethio Telecom Service Center in Arba Minch City, Ethiopia. The results are summarized in Tables 1 and 2.

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Received: November 16, 2024 Revised: 27 December, 2024 Accepted: January 20, 2025

Table 1: **Summary of Customer Arrivals** $(A(t))$**, Served** $(\mu(t))$**, Reneges** $(R(t))$**, and Servers on Vacation** $(D(t))$ **at Ethio Telecom, Arba Minch District Head Office from February 4-9, 2024, and February 11-16, 2024, at any given time** (t)

Week	Day	Time (t)	$\mu(t)$	$A(t)$	$D(t)$	$R(t)$
Week (I)	Monday	8 : 30 am - 5 : 30 pm	7.375	15.125	9.875	4.750
	Tuesday	8 : 30 am - 5 : 30 pm	7.125	9.875	7.600	1.750
	Wednesday	8 : 30 am - 5 : 30 pm	7.250	10.500	4.760	2.250
	Thursday	8 : 30 am - 5 : 30 pm	7.50	5.500	7.940	0.500
	Friday	8 : 30 am - 5 : 30 pm	7.625	17.500	1.780	4.000
	Saturday	8 : 30 am - 5 : 30 pm	8.375	8.500	4.750	2.250
Week (II)	Monday	8 : 30 am - 5 : 30 pm	7.250	11.125	5.450	1.250
	Tuesday	8 : 30 am - 5 : 30 pm	7.375	13.875	9.250	5.250
	Wednesday	8 : 30 am - 5 : 30 pm	7.500	7.500	4.850	2.250
	Thursday	8 : 30 am - 5 : 30 pm	7.250	6.500	7.950	1.500
	Friday	8 : 30 am - 5 : 30 pm	7.000	18.500	5.650	3.000
	Saturday	8 : 30 am - 5 : 30 pm	9.000	9.500	1.750	2.250

Table 2: **Ordering and Receiving of Items at Ethio Telecom, Arba Minch District Head Office from February 4-9, 2024, and February 11-16, 2024, at any given time** (t)

Week	Day	Time (t)	Ordering Time	Receiving Time
Week (I)	Monday	8 : 30 am - 5 : 30 pm	9 : 10 am	–
	Tuesday	8 : 30 am - 5 : 30 pm	–	–
	Wednesday	8 : 30 am - 5 : 30 pm	–	10 : 20 am
	Thursday	8 : 30 am - 5 : 30 pm	–	–
	Friday	8 : 30 am - 5 : 30 pm	–	–
	Saturday	8 : 30 am - 5 : 30 pm	–	–
Week (II)	Monday	8 : 30 am - 5 : 30 pm	–	–
	Tuesday	8 : 30 am - 5 : 30 pm	–	–
	Wednesday	8 : 30 am - 5 : 30 pm	9 : 30 am	–
	Thursday	8 : 30 am - 5 : 30 pm	–	–
	Friday	8 : 30 am - 5 : 30 pm	–	11 : 00 am
	Saturday	8 : 30 am - 5 : 30 pm	–	–

Note: Data collection was not conducted from 12:30 pm to 1:30 pm due to the lunch break.

6 Numerical Results and Discussion

This section presents a numerical experiment designed to underscore the significance of the theoretical results. The experiment has three primary objectives: (1) to evaluate the accuracy of the derived formulas for steady-state probabilities, (2) to analyze the behavior of various performance metrics as key parameters change, and (3) to tackle an optimization problem aimed at minimizing the Expected Total Cost (ETC), as computed through Equation (36).

We implemented a Design of Experiments (DoE) framework to ensure fair comparisons among algorithms and to identify optimal parameter values through structured experimentation. Our statistical analysis, particularly using ANOVA, revealed statistically significant differences across weekdays in both the service rate ($F(5,6) = 7.597, p = 0.014$) and the arrival rate ($F(5,6) = 9.503, p = 0.008$). These results indicate that specific algorithm settings significantly impact performance outcomes. Although the vacation rate and reneging rate did not yield significant results ($p > 0.05$), they still provide insights into the variability within our data.

To better understand the weekday variations, we conducted a post-hoc Tukey HSD test, which highlighted significant differences in service rate performance, particularly between Saturdays and other weekdays. For example, the mean difference in service rate between Monday and Saturday was significant, with Mean Difference $= -1.375$, $p = 0.022$. This suggests that Saturday's service rate is notably lower, which could be due to fewer

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vendors or a reduction in operational efficiency. The consistently significant mean differences between Saturday and the rest of the week (Tuesday: Mean Difference $= -1.4375$, $p = 0.018$) imply that operational adjustments might be necessary to enhance Saturday's service performance.

Similarly, the arrival rate analysis revealed significant mean differences between specific weekdays, particularly between Friday and both Wednesday and Saturday. For instance, the arrival rate on Friday differed significantly from that on Wednesday (Mean Difference $= 9.0, p = 0.024$), and Saturday (Mean Difference $= 9.0, p = 0.024$). This variation could reflect changing customer behavior towards the end of the week, potentially due to limited weekend operational hours or specific promotional events that attract more customers.

These insights into service and arrival rates across different days are valuable for scheduling and staffing optimizations. Adjustments based on these patterns can help balance the load, reduce customer wait times, and maintain consistent service quality throughout the week. Future studies might explore additional operational factors affecting Saturday service rates and weekday customer arrival patterns.

For the first goal, determining the accuracy of the developed formulas analytically is challenging. With finite system capacity, steady-state probabilities exist for any utilization factor ρ, calculated exactly through recursive methods using MATLAB. Since probabilities are non-negative real numbers that sum to one, this confirms the validity of the theoretical results in Section 4. These steady-state probabilities enable the calculation of key performance measures that characterize system efficiency and help predict future system dynamics with greater precision.

For the second goal, we analyze the impact of parameter changes on performance metrics (18 - 35). Empirical data highlights the dynamic balance between customer demand and service capacity at Ethio Telecom's Arba Minch center, where 13 vendors serve an average of 89 customers daily. On average, 6.06 vendors are on vacation, with a customer reneging rate of 0.029, potentially increasing to 0.044 during server vacations. Customer arrival and service rates are 11.17 and 7.25 per hour, respectively. We assume occasional catastrophes in inventory, which reduce the service rate—an important factor in managing inventory-related disruptions. The center's objective is to minimize wait times and system costs by optimizing the number of available vendors. Unlike infinite-capacity systems, this model does not require ρ to be below 1.

Although rare, catastrophic warehouse events such as fires, floods, or outages can significantly impact service rates by damaging inventory, disabling equipment, or halting operations. Such events may lead to order delays or stock-outs. Recovery times depend on the severity of the event, potentially lasting from days to weeks, which affects service rates until operations normalize. To predict system dynamics at any time t, we define the probability of a customer joining the queue as:

$$b_n = \begin{cases} 1-\gamma, & \text{for } 0 \le n \le C-D-1, \\ \frac{N-n}{N}(1-\gamma), & \text{for } C-D \le n \le N \end{cases}$$

The cost elements are defined as follows: $C_1 = 100$, $C_2 = 110$, $C_3 = 120$, $C_4 = 150$, $C_5 = 130$, $C_6 = 140$, $C_7 = 120$, $C_8 = 56$, $C_9 = 80$, and $C_{10} = 200$.

A genetic algorithm is used to determine the optimal number of servers on vacation, D^*, and the optimal service rate, μ^*, to minimize the Expected Total Cost (ETC). Performance metrics, including the mean inventory level, mean order size, mean queue length, and mean customer loss, are evaluated at (μ^*, D^*).

The sensitivity analysis examines how variations in key parameters—such as the catastrophe rate γ, restoration rate κ, reordering rate η, attraction rate β, retention rate r, arrival rate λ, reneging rate α, and vacation rate ξ impact the optimal vacationing vendor count D^*, service rate μ^*, and F^*. Identifying the optimal values for D^* and μ^* is challenging due to the integer nature of D and the nonlinear complexity of the ETC function. To address this, a heuristic approach is used to obtain D^* and μ^* values that satisfy:

$$F(D^* - 1) > F(D^*) < F(D^* + 1) \tag{37}$$

$$F(\mu^* - d) > F(\mu^*) < F(\mu^* + d) \tag{38}$$

where d is a positive constant. Tables 3 – 5 present the results, with each column showing parameter changes.

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Table 3: **System performance measures for** $D = 6$**,** $\alpha = 0.029$**,** $\xi = 0.015$**,** $\beta = 0.3$**,** $r = 0.4$**,** $\eta = 0.0588$**,** $\gamma = 0.02$**,** $\kappa = 0.8$

λ	11.17	11.20	11.23	11.26	11.29	11.32
μ	7.25	7.00	6.75	6.50	6.25	6.00
E_I	81.757188	81.554349	81.352507	81.151656	80.951788	80.752895
E_r	0.003770	0.003761	0.003752	0.003743	0.003733	0.003724
E_0	78.551024	78.356139	78.162213	77.969238	77.777208	77.586115
LS	13.521000	13.560000	13.599000	13.638000	13.677000	13.716000
L_s	88.513795	88.516212	88.518610	88.520990	88.523351	88.525694
L_q	81.546992	81.549232	81.551454	81.553659	81.555847	81.558018
W_s	0.546992	0.549232	0.551454	0.553659	0.555847	0.558018
W_q	0.513795	0.516212	0.518610	0.520990	0.523351	0.525694
LR	41.552631	41.560621	41.568568	41.576472	41.584335	41.592155
P_B	0.995258	0.995283	0.995308	0.995333	0.995358	0.995382
P_I	0.004742	0.004717	0.004692	0.004667	0.004642	0.004618
E_c	80.122044	79.923262	79.725457	79.528623	79.332752	79.137837
F	42427.691626	42368.529597	42309.657552	42251.073382	42192.774997	42134.760330

Using the parameter values: number of servers on vacation $D = 6$, reneging rate $\alpha = 0.029$, reneging rate due to server vacation $\xi = 0.015$, customer encouragement rate $\beta = 0.3$, retention rate $r = 0.4$, replenishment lead time rate $\eta = 0.0588$, catastrophe rate $\gamma = 0.02$, and restoration rate $\kappa = 0.8$, an analysis is conducted to examine the influence of varying arrival rate λ and service rate μ. The numerical results are presented in Table 3. From this table, we conclude that:

1. Mean inventory level E_I, mean order size E_0, idle probability of servers P_I, and the ETC F all decrease as the arrival rate of customers λ increases. The mean reordering rate E_r and mean number of items destroyed due to catastrophes E_c decline slightly, while the busy probability of the servers P_B increases significantly with an increase in λ.
2. The values of performance measures such as mean lost sales LS, mean number of customers in the system L_s (including the queue L_q), mean waiting time in the system W_s (including the queue W_q), and mean number of customers lost LR all increase as λ grows.

Next, a similar analysis is conducted by fixing specific parameters: setting λ to 11.17, D to 6, η to 0.0588, γ to 0.02, and κ to 0.8, while allowing α, β, r, and μ to vary. The results of this analysis are presented in Table 4, and the implications are discussed below.

1. As ξ decreases, ETC F also decreases with increasing values of α and β, and with a decreasing value of r.
2. The performance measures LS, L_s, L_q, W_s, W_q, LR, and P_B all increase as μ and r decrease, and as α and β increase. Mean inventory level E_I, mean reordering rate E_r, mean order size E_0, mean number of items destroyed due to catastrophes E_c, and idle probability of servers P_I all decrease as μ and r decrease, and as α and β increase.

Finally, by fixing $\mu = 19.9833$, $D = 6$, $\eta = 0.0588$, $\gamma = 0.02$, and $\kappa = 0.8$, we vary the parameters β, λ, α, r, and ξ. The detailed results are presented in Table 5, and the findings are outlined below.

1. Mean inventory level E_I, mean reordering rate E_r, mean order size E_0, idle probability of servers P_I, mean number of items destroyed due to catastrophes E_c, and the ETC F all decrease with increasing values of β, λ, and α, as well as with decreasing values of r and ξ. Simultaneously, mean lost sales LS, mean number of customers in the system L_s, mean queue length L_q, mean waiting time in the system W_s, mean waiting time in the queue W_q, mean number of customers lost LR, and busy probability of servers P_B all increase with increasing values of β, λ, and α, and with decreasing values of r and ξ.

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Table 4: **System performance measures for** $\lambda = 11.17$**,** $D = 6$**,** $\eta = 0.0588$**,** $\gamma = 0.02$**,** $\kappa = 0.8$

α	0.024	0.025	0.026	0.027	0.028	0.029
β	0.00	0.10	0.20	0.30	0.40	0.50
r	0.90	0.80	0.70	0.60	0.50	0.40
ξ	0.019	0.018	0.017	0.016	0.015	0.014
μ	7.25	7.00	6.75	6.50	6.25	6.00
E_I	103.926139	95.322844	88.024406	81.757188	76.318444	71.554957
E_r	0.004793	0.004396	0.004059	0.003770	0.003520	0.003300
E_0	99.850604	91.584694	84.572469	78.551024	73.325564	68.748881
LS	10.170000	11.287000	12.404000	13.521000	14.638000	15.755000
L_s	88.205171	88.335717	88.435601	88.513795	88.576204	88.626848
L_q	81.261274	81.382084	81.474562	81.546992	81.604829	81.651783
W_s	0.261274	0.382084	0.474562	0.546992	0.604829	0.651783
W_q	0.205171	0.335717	0.435601	0.513795	0.576204	0.626848
LR	40.558149	40.934988	41.250332	41.518351	41.749171	41.950221
P_B	0.991985	0.993376	0.994434	0.995258	0.995911	0.996438
P_I	0.008015	0.006624	0.005566	0.004742	0.004089	0.003562
E_c	101.847616	93.416388	86.263918	80.122044	74.792076	70.123858
F	48884.779459	46380.178910	44253.849868	42426.868899	40840.636386	39450.795809

Table 5: **System performance measures for** $\mu^* = 19.9833$**,** $D = 6$**,** $\eta = 0.0588$**,** $\gamma = 0.02$**,** $\kappa = 0.8$

β	0.00	0.10	0.20	0.30	0.40	0.50
λ	11.17	11.2	11.23	11.26	11.29	11.32
α	0.024	0.025	0.026	0.027	0.028	0.029
r	0.90	0.80	0.70	0.60	0.50	0.40
ξ	0.019	0.018	0.017	0.016	0.015	0.014
E_I	103.926139	95.090076	87.591819	81.151656	75.562092	70.666176
E_r	0.004793	0.004385	0.004040	0.003743	0.003485	0.003259
E_0	99.850604	91.361054	84.156846	77.969238	72.598873	67.894953
LS	10.170000	11.320000	12.476000	13.638000	14.806000	15.980000
L_s	88.205171	88.339054	88.441219	88.520990	88.584493	88.635891
L_q	81.261274	81.385172	81.479765	81.553659	81.612512	81.660170
W_s	0.261274	0.385172	0.479765	0.553659	0.612512	0.660170
W_q	0.205171	0.339054	0.441219	0.520990	0.584493	0.635891
LR	40.558149	40.944621	41.267867	41.542446	41.778763	41.984455
P_B	0.991985	0.993412	0.994493	0.995333	0.995997	0.996532
P_I	0.008015	0.006588	0.005507	0.004667	0.004003	0.003468
E_c	101.847616	93.188275	85.839983	79.528623	74.050850	69.252852
F	48884.779459	46312.374607	44127.751191	42250.256748	40619.931873	39191.345366

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2. The mean reordering rate E_r decreases slightly with increasing values of β, λ, and α, as well as with decreasing values of r and ξ.

For the parameter values $\beta = 0.3$, $\lambda = 11.17$, $\alpha = 0.029$, $r = 0.4$, $\eta = 0.0588$, $\xi = 0.015$, $D = 6$, $\gamma = 0.02$, and $\kappa = 0.8$, two graphs are presented: Figure 2 and Figure 3.

Figure 2 illustrates that the waiting time in the queue decreases as the number of customers served, denoted as μ, increases. The figure demonstrates that when the warehouse operates under normal conditions, customers experience the shortest waiting time for service.

Figure 3 depicts that the mean loss of sales decreases as the number of customers leaving the system after receiving service increases. The system's service rate rises, indicating that it offers more services and consumes inventory faster. The lost sales decrease with the number of customers leaving, highlighting that as the number of customers served increases, the mean loss of sales declines. This trend is practically valid because, under uncertain inventory order sizes, an increase in the speed of service in a queuing system leads to a reduction in the mean loss of sales.

From Figure 4, we observe that both mean inventory level E_I and mean inventory destroyed due to catastrophes E_c decrease as the arrival rate λ increases. The figure demonstrates the impact of customer arrival rate λ on the mean inventory level E_I and the mean inventory lost to catastrophes E_c. It first shows that a higher arrival rate reduces the total inventory level E_I. Additionally, it illustrates that the inventory destroyed due to catastrophes E_c decreases with an increasing arrival rate λ.

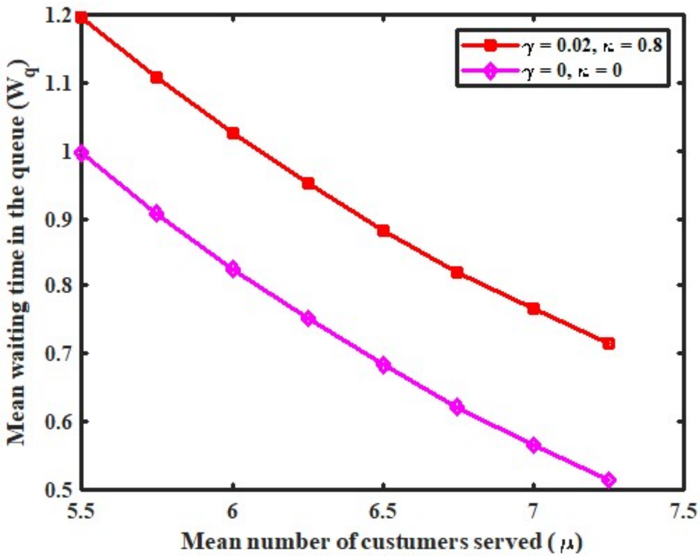

Figure 2: Effect of changing the speed of service μ with respect to waiting time in the queue W_q

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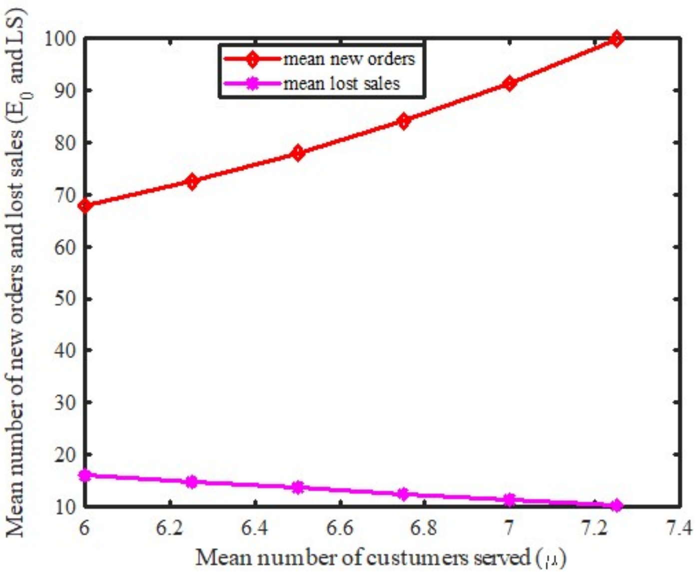

Figure 3: Impact of service rate μ on the mean number of replenishment E_0 and mean lost sales LS

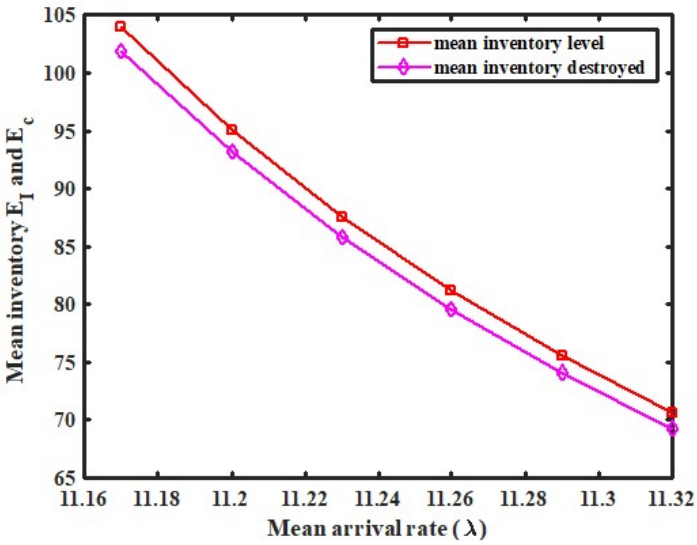

Figure 4: Impact of arrival rate λ on mean inventory level E_I and mean inventory destroyed E_c

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7 Conclusions

This paper examines a multi-server finite-capacity queuing-inventory system. We have incorporated attraction-retention mechanisms for impatient customers and asynchronous server vacations while studying the impact of catastrophes in the warehouse to gain insights through a mathematical model. The study finds that effective management of queues and inventory relies on two key factors: satisfying impatient customers and ensuring quick server availability after breaks.

By applying the Markov process, we derived the governing equations of the system. Steady-state probability distributions for the system size were evaluated using a recursive method, and formulas for calculating various performance measures, including the impact of catastrophes, were also derived. Moreover, several existing models in the literature are shown to be special cases of the model studied here. The impact of disasters on the system characteristics is assessed through numerical illustrations. In contrast to classical queuing-inventory systems without disasters, we observed that in systems with disasters, the expected total cost (ETC) increases with the rate of catastrophes under a given system load due to rising restoration costs.

Building on the performance analysis, we developed a cost-loss model to identify the optimal service rate and the optimal number of servers allowed to go on vacation simultaneously. Furthermore, a genetic algorithm was implemented to search for the optimal values of specific system parameters, aiming to minimize the cost-loss function.

The findings of this paper offer valuable insights for both practitioners and researchers working on real-world problems in computer networks, communication systems, and telecommunication network design. These results can be applied to optimize queue and inventory management, server availability, and system resilience in various applications.

Future Directions: There is ample scope for future extensions of this study, a few examples of which are mentioned below:

- Throughout this paper, we considered the arrival pattern of customers to be single arrivals. Future research could extend the analysis to include customers with batch arrivals and retrials. Additionally, the service pattern could be considered in batches, as it is applicable in transportation and healthcare centers.
- This study focused on warehouse catastrophes as disruptions to the queuing-inventory system (QIS). Future research could explore how the system behaves when outages or disruptions affect the servers.
- Another direction is to investigate these models using a Markov arrival process (MAP) and retrial customers, as well as phase-type (Ph) service time distributions.

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