On Some Novel Generalizations of Weak Fuzzy Complex Numbers

Zahraa Hasan, Rama Asad Nadweh

[1]University Of Applied Science, Faculty of Literature and Science, Manama, Bahrain

[2]Online Islamic University, Department Of Science and Information Technology, Doha, Qatar
Emails: Zahraamathscience1243@gmail.com; ramaanadwehh@gmail.com

Abstract
The ring of weak fuzzy complex numbers is an extension of real numbers ring by using an algebraic element with fuzzy property. In this paper, we present two novel generalizations of weak fuzzy complex numbers, where the concepts of strong fuzzy complex numbers and split-complex weak fuzzy complex numbers will be defined for the first time with a general study of their elementary properties and special elements. On the other hand, we provide an algorithm to compute the dempotent elements in the ring of split-complex weak fuzzy complex numbers with many related examples that clarify the validity of our work.

Keywords: weak fuzzy complex number; strong fuzzy complex number; split-complex weak fuzzy complex number; idempotent element

1. Introduction

The generalizations of real numbers are considered interesting research areas, where It attracted the attention of many researchers worldwide due to its great importance in the study of the theory of both spaces and matrices [2-3,9].

One of the most recent generalizations of real numbers was the concept of weak fuzzy complex numbers presented in [1] as follows:

The set of *Weak Fuzzy Complex numbers* was defined as follows, where 'J' is the Weak Fuzzy Complex operator ($J \notin \boldsymbol{R}$):
$\boldsymbol{F}_J = \{x_0 + x_1 J \,;\, x_0, x_1 \in \boldsymbol{R}, J^2 = t \in]0,1[\,\}$.
This novel generalization played a central role in the theory of Diophantine equations [6-8], and the classification of A-curves generated from the solutions of some vectorial equations [4-5].

In this work, we build some novel generalizations of weak fuzzy complex numbers, and these new algebraic classes of numbers will be very similar to the original structure, with many possible future applications.

Main Discussion

Definition:
The set of strong fuzzy complex numbers is defined as follows:
$S_f = \{a + bJ + ci + diJ \;; a.b.c.d \in \mathbb{R}.\ i^2 = -1.J^2 = t \in]0.1[\}$,
Addition on S_f is defined as follows:

Doi: https://doi.org/10.54216/JNFS.080201
Received: November 22, 2023 Revised: January 19, 2024 Accepted: April 12, 2024

$(+): S_f \times S_f \to S_f \ : (a_0 + b_0 J + c_0 i + d_0 iJ) + (a_1 + b_1 J + c_1 i + d_1 iJ) = (a_0 + a_1) + (b_0 + b_1)J + (c_0 + c_1)i + (d_0 + d_1)iJ.$

Multiplication on S_f is defined as follows:

$(\times): S_f \times S_f \to S_f \ : (a_0 + b_0 J + c_0 i + d_0 iJ) \times (a_1 + b_1 J + c_1 i + d_1 iJ) = a_0 a_1 + a_0 b_1 J + a_0 c_1 i + a_0 d_1 iJ$

$$+b_0 a_1 J + b_0 b_1 J^2 + b_0 c_1 iJ + b_0 d_1 iJ^2 + c_0 a_1 i + c_0 b_1 iJ + c_0 c_1 i^2 + c_0 d_1 i^2 J + d_0 a_1 iJ + d_0 b_1 iJ^2 + d_0 c_1 i^2 J + d_0 d_1 i^2 J^2$$

$= (a_0 a_1 + b_0 b_1 t - c_0 c_1 - t d_0 d_1) + J(a_0 b_1 + b_0 a_1 - c_0 d_1 - d_0 c_1) + i(a_0 c_1 + b_0 d_1 t + c_0 a_1 - d_0 b_1 t) + iJ(a_0 d_1 + b_0 c_1 + c_0 b_1 + d_0 a_1).$

Remark:

If $X = a + bJ + ci + diJ \ \in S_f$, then $X = (a + ci) + J(b + di)$.

Definition:

Consider the mapping: $\varphi: S_f \to \mathbb{C} \times \mathbb{C}$ such that:

$\varphi[(a + ci) + J(b + di)] = (a + ci - \sqrt{+}(b + di) \,.\, a + ci + \sqrt{+}(b + di)\,)$

Theorem:

The mapping (φ) is a ring isomorphism.

Proof:

First, we prove that $(S_f \,.+.\times)$ is a commutative ring.

It is clear that $(S_f.+)$ is a belian group.

Also, the multiplication $(\times)$ is commutative, associative, and distributive over (+), thus $(S_f \,.+.\times)$ is a commutative ring.

The mapping (φ) is well defined.

If $X = (x_0 + x_1 i) + J(x_2 + x_3 i) = Y = (y_0 + y_1 i) + J(y_2 + y_3 i)$,

Then $x_i = y_i$ for all $0 \le i \le 3$, thus:

$$\begin{cases}(x_0 + x_1 i) - \sqrt{+}(x_2 + x_3 i) = (y_0 + y_1 i) - \sqrt{+}(y_2 + y_3 i)\\(x_0 + x_1 i) + \sqrt{+}(x_2 + x_3 i) = (y_0 + y_1 i) + \sqrt{+}(y_2 + y_3 i)\end{cases}$$

Hence $\varphi(X) = \varphi(Y)$.

The mapping (φ) preserves addition and multiplication:

$X + Y = [(x_0 + y_0) + (x_1 + y_1)i] + J[(x_2 + y_2) + (x_3 + y_3)i]$,

$\varphi(X + Y) = (A.B)$, where:

$$A = (x_0 + y_0) + (x_1 + y_1)i - \sqrt{+}[(x_2 + y_2) + (x_3 + y_3)i] = \left[(x_0 + x_1 i) - \sqrt{+}(x_2 + x_3 i)\right] + [(y_0 + y_1 i) - \sqrt{+}(y_2 + y_3 i)]$$

$$B = (x_0 + y_0) + (x_1 + y_1)i + \sqrt{+}[(x_2 + y_2) + (x_3 + y_3)i] = \left[(x_0 + x_1 i) + \sqrt{+}(x_2 + x_3 i)\right] + [(y_0 + y_1 i) + \sqrt{+}(y_2 + y_3 i)]$$

Thus $\varphi(X + Y) = \varphi(X) + \varphi(Y)$.

Also, $X \cdot Y = (x_0 + x_1 i)(y_0 + y_1 i) + t(x_2 + x_3 i)(y_2 + y_3 i) + J[(x_0 + x_1 i)(y_2 + y_3 i) + (y_0 + y_1 i)(x_2 + x_3 i)] = A + B\,J.$

$\varphi(X \cdot Y) = (C.D)$, where:

$$C = A - \sqrt{+}B = (x_0 + x_1 i)(y_0 + y_1 i) + t(x_2 + x_3 i)(y_2 + y_3 i) - \sqrt{+}(x_0 + x_1 i)(y_2 + y_3 i) - \sqrt{+}\,(y_0 + y_1 i)(x_2 + x_3 i)$$

$= \left[(x_0 + x_1 i) - \sqrt{+}(x_2 + x_3 i)\right] \times [(y_0 + y_1 i) - \sqrt{+}(y_2 + y_3 i)].$

$D = A + \sqrt{+}B = \left[(x_0 + x_1 i) + \sqrt{+}(x_2 + x_3 i)\right] \times [(y_0 + y_1 i) + \sqrt{+}(y_2 + y_3 i)].$

Thus $\varphi(X \cdot Y) = \varphi(X) \cdot \varphi(Y)$.

If $\varphi(X) = (0.0)$, then: $\begin{cases}(x_0 + x_1 i) - \sqrt{+}(x_2 + x_3 i) = 0\\(x_0 + x_1 i) + \sqrt{+}(x_2 + x_3 i) = 0\end{cases}$

Thus: $\begin{cases}x_0 + x_1 i = 0\\x_2 + x_3 i = 0\end{cases} \implies x_0 = x_1 = x_2 = x_3 = 0$. and $X = 0$

So that $k_{er}(\varphi) = \{0\}$.

On the other hand, for every $(a + bi \,.\, c + di) \ \in \mathbb{C} \times \mathbb{C}$, we have $X = \frac{1}{2}[a + c + (b + d)i] + \frac{1}{2\sqrt{+}} J[c - a + (d - b)i] \in S_f$ such that $\varphi(X) = (a + bi \,.\, c + di)$.Thus (φ) is a ring isomorphism.

Doi: https://doi.org/10.54216/JNFS.080201
Received: November 22, 2023 Revised: January 19, 2024 Accepted: April 12, 2024

Remark:

The inverse isomorphism is: $\varphi^{-1}: \mathbb{C} \times \mathbb{C} \to S_f$ such that: $\varphi^{-1}(a + bi \,.\, c + di) = \frac{1}{2}[a + c + (b + d)i] + \frac{1}{2\sqrt{t}} J[c - a + (d - b)i]$.

Special properties of S_f:

1] for $X = (x_0 + x_1 i) + J(x_2 + x_3 i) \in S_f \,.\, n \in \mathbb{N}$. we have:

$X^n = \varphi^{-1}(\varphi(x))^n = \varphi^{-1}(x_0 + x_1 i - \sqrt{t}(x_2 + x_3 i).\, x_0 + x_1 i + \sqrt{t}(x_2 + x_3 i))^n = \varphi^{-1}(T^n . S^n) = \frac{1}{2}(T^n + S^n) + \frac{1}{2\sqrt{t}} J\,(S^n - T^n)$, where

$$\begin{cases} S = x_0 + x_1 i + \sqrt{t}(x_2 + x_3 i) \\ T = x_0 + x_1 i - \sqrt{t}(x_2 + x_3 i) \end{cases}$$

2] X is invertible in S_f if and only if $S \neq 0 \,.\, T \neq 0$ and $X^{-1} = \frac{1}{X} = \frac{1}{2}\left(\frac{1}{S} + \frac{1}{T}\right) + \frac{1}{2\sqrt{t}} J\left(\frac{1}{S} - \frac{1}{T}\right)$, where

$$\begin{cases} S = x_0 + x_1 i + \sqrt{t}(x_2 + x_3 i) \\ T = x_0 + x_1 i - \sqrt{t}(x_2 + x_3 i) \end{cases}$$

3] The n-th root:

The n-th root of X is $Y = (y_0 + y_1 i) + J(y_2 + y_3 i)$ such that $Y^n = X$.

Hence $\varphi(Y^n) = \varphi(X) \quad \Rightarrow \varphi(Y) = [\varphi(X)]^{\frac{1}{n}}$

$$\Rightarrow \begin{cases} (y_0 + y_1 i) - \sqrt{t}(y_2 + y_3 i) = \left[x_0 + x_1 i - \sqrt{t}(x_2 + x_3 i)\right]^{\frac{1}{n}} = T^{\frac{1}{n}} \\ (y_0 + y_1 i) + \sqrt{t}(y_2 + y_3 i) = \left[x_0 + x_1 i + \sqrt{t}(x_2 + x_3 i)\right]^{\frac{1}{n}} = S^{\frac{1}{n}} \end{cases}$$

So that: $\begin{cases} y_0 + y_1 i = \frac{1}{2}(T^{\frac{1}{n}} + S^{\frac{1}{n}}) \\ (y_2 + y_3 i) = \frac{1}{2\sqrt{t}}(S^{\frac{1}{n}} - T^{\frac{1}{n}}) \end{cases}$

Example:

Let's find the square roots of $X = -1 + 0 \cdot J = -1$. for $J^2 = t = \frac{1}{2}$.

$\varphi(X) = (-1.-1)$, the square roots of -1 are $\{i.-i\}$.

Thus, the square roots of $\varphi(X)$ are: $\{(i.i).(i.-i).(-i.i).(-i.-i)\}$

Hence, the square roots of X are:

$Y_1 = \varphi^{-1}(i.i) = i \;.\; Y_2 = \varphi^{-1}(-i.-i) = -i,$

$Y_3 = \varphi^{-1}(i.-i) = \frac{1}{2}(i - i) + \frac{1}{2\sqrt{t}} J\,[-i - i] = -\sqrt{2}iJ$

$Y_4 = \varphi^{-1}(-i.i) = \sqrt{2}iJ.$

4] X is idempotent if and only if $X^2 = X$, thus:

$[\varphi(X)]^2 = \varphi(X)$. hence: $\begin{cases} T^2 = T \\ S^2 = S \end{cases} \Rightarrow T.S \in \{0.1\}$

For $T = S = 0$. then $X = 0$. for $T = S = 1$. then $X = 1$.

For $T = 0 \,.\, S = 1$, we have: $\begin{cases} (x_0 + x_1 i) - \sqrt{t}(x_2 + x_3 i) = 0 \\ (x_0 + x_1 i) + \sqrt{t}(x_2 + x_3 i) = 1 \end{cases}$

$\Rightarrow x_0 = \frac{1}{2} \,.\, x_1 = 0 \,.\, x_2 = \frac{1}{2\sqrt{t}} \,.\, x_3 = 0$, thus $X = \frac{1}{2} + \frac{1}{2\sqrt{t}} J$.

For $T = 1 \,.\, S = 0$, we have: $\begin{cases} (x_0 + x_1 i) - \sqrt{t}(x_2 + x_3 i) = 1 \\ (x_0 + x_1 i) + \sqrt{t}(x_2 + x_3 i) = 0 \end{cases}$

$\Rightarrow x_0 = \frac{1}{2} \,.\, x_1 = 0 \,.\, x_2 = \frac{-1}{2\sqrt{t}} \,.\, x_3 = 0$, hence $X = \frac{1}{2} - \frac{1}{2\sqrt{t}} J$.

Definition:

We define the set of weak fuzzy split-complex numbers as follows: $W_s = \{a + bJ + cK + dJK; a.b.c.d \in \mathbb{R} \,.\, K^2 = 1. J^2 = t \in]0.1[\}$

Addition on W_s is defined as follows:

$$(a_0 + b_0 J + c_0 K + d_0 JK) + (a_1 + b_1 J + c_1 K + d_1 JK) = (a_0 + a_1) + J(b_0 + b_1) + K(c_0 + c_1) + JK(d_0 + d_1)$$

Multiplication on W_s is defined as follows:

$$(a_0 + b_0 J + c_0 K + d_0 JK) \times (a_1 + b_1 J + c_1 K + d_1 JK) = a_0 a_1 + a_0 b_1 J + a_0 c_1 K + a_0 d_1 JK + b_0 a_1 J + b_0 b_1 J^2 + b_0 c_1 JK + b_0 d_1 J^2 K$$

Doi: https://doi.org/10.54216/JNFS.080201
Received: November 22, 2023 Revised: January 19, 2024 Accepted: April 12, 2024

$+c_0a_1K + c_0b_1JK + c_0c_1K^2 + c_0d_1JK^2 + d_0a_1JK + d_0b_1J^2K + d_0c_1JK^2 + d_0d_1J^2K^2 =$
$(a_0a_1 + b_0b_1t + c_0c_1 + d_0d_1t)$
$+J(a_0b_1 + b_0a_1 + c_0d_1 + d_0c_1) + K(a_0c_1 + b_0d_1t + c_0a_1 + d_0b_1t) + JK(a_0d_1 + b_0c_1 + c_0b_1 + d_0a_1).$

Remark:

1] $(W_s\,.+.\times)$ is a commutative ring.

2] For every $X = (x_0 + x_1J + x_2K + x_3JK \in W_s$, we can write X as follows:

$X = (x_0 + x_2K) + J(x_1 + x_3K)\ ;\ x_i \in \mathbb{R}\ \ 0 \le i \le 3.$

Or $X = (x_0 + x_1J) + K(x_2 + x_3J).$

Definition:

We define the mapping $\varphi: W_s \to F_J \times F_J$ such that:

$\varphi[(x_0 + x_1J) + K(x_2 + x_3J)] = (x_0 - x_2 + J(x_1 - x_3)\ .x_0 + x_2 + J(x_1 + x_3))$

Theorem:

$(W_s\,.+.\times) \cong (F_J \times F_J\,.+.\times)$

Proof:

The mapping is (φ) a ring isomorphism, that is because:

If $X = (x_0 + x_1J) + K(x_2 + x_3J) = Y = (y_0 + y_1J) + K(y_2 + y_3J)$, then $x_i = y_i$ for all $0 \le i \le 3\ .and\ \varphi(X) = \varphi(Y)$.

It is clear that $\varphi(X + Y) = \varphi(X) + \varphi(Y)$.

$X \times Y = (x_0 + x_1J)(y_0 + y_1J) + (x_2 + x_3J)(y_2 + y_3J) + K[(x_0 + x_1J)(y_2 + y_3J) + (x_2 + x_3J)(y_0 + y_1J)].$

$\varphi(X \times Y) = (A - B.A + B)\ ;\ \begin{cases} A = (x_0 + x_1J)(y_0 + y_1J) + (x_2 + x_3J)(y_2 + y_3J) \\ B = (x_0 + x_1J)(y_2 + y_3J) + (x_2 + x_3J)(y_0 + y_1J) \end{cases}$

$A - B = (x_0 + x_1J)(y_0 - y_2 + J(y_1 - y_3)) + (x_2 + x_3J)(y_2 - y_0 + J(y_3 - y_1))$
$= (x_0 - x_2 + (x_1 - x_3)J) \cdot (y_0 - y_2 + J(y_1 - y_3))$

$A + B = (x_0 + x_2 + J(x_1 + x_3)) \cdot (y_0 + y_2 + J(y_1 + y_3))$, thus $\varphi(XY) = \varphi(X)\varphi(Y)$.

If $\varphi(X) = (0.0)$, then $\begin{cases} x_0 - x_2 + J(x_1 - x_3) = 0 \\ x_0 + x_2 + J(x_1 + x_3) = 0 \end{cases}$

So that $x_i = 0$ .and $X = 0$.

For every $Y = (a + bJ\,.c + dJ) \in F_J \times F_J$, there exists

$X = \frac{1}{2}(a + c + (b + d)J) + \frac{1}{2}\,(c - a + (d - b)J) \in W_s$ such that $\varphi(X) = Y$, thus φ is a ring isomorphism.

Special properties of W_s:

1] $X = (x_0 + x_1J) + K(x_2 + x_3J)$ is invertible in W_s if and only if:

$\begin{cases} x_0 - x_2 + J(x_1 - x_3) \\ x_0 + x_2 + J(x_1 + x_3) \end{cases}$ are invertible in F_J

Also, $X^{-1} = \frac{1}{X} = \varphi^{-1}(\varphi(X))^{-1} = \frac{1}{2}\left[\left((x_0 - x_2) + J(x_1 - x_3)\right)^{-1} + \left(x_0 + x_2 + J(x_1 + x_3)\right)^{-1}\right] +$
$\frac{1}{2}\,K\begin{bmatrix} \left(x_0 + x_2 + J(x_1 + x_3)\right)^{-1} \\ -\left(x_0 - x_2 + J(x_1 - x_3)\right)^{-1} \end{bmatrix}$

2] X is idempotent if and only if: $\varphi(X^2) = \varphi(X)$.

Let $a + bJ \in F_J$ with $(a + bJ)^2 = a + bJ$, then $\begin{cases} (a - \sqrt{+}b)^2 = a - \sqrt{+}b \\ (a + \sqrt{+}b)^2 = a + \sqrt{+}b \end{cases} \Rightarrow \begin{cases} a - \sqrt{+}b \in \{0.1\} \\ a + \sqrt{+}b \in \{0.1\} \end{cases}$

$\Rightarrow a + bJ \in \left\{0.1.\frac{1}{2} + \frac{1}{2\sqrt{+}}J.\frac{1}{2} - \frac{1}{2\sqrt{+}}J\right\}.$

Thus $\begin{cases} x_0 - x_2 + J(x_1 - x_3) \\ x_0 + x_2 + J(x_1 - x_3) \end{cases} \in \left\{0.1.\frac{1}{2} + \frac{1}{2\sqrt{+}}J.\frac{1}{2} - \frac{1}{2\sqrt{+}}J\right\}$

The possible cases are:

Case (1):

If $\begin{cases} x_0 - x_2 + J(x_1 - x_3) = 0 \\ x_0 + x_2 + J(x_1 + x_3) = 0 \end{cases}$ .then $x_0 = x_2 = x_1 = x_3 = 0$ .and $X = 0$

Case (2):

If $\begin{cases} x_0 - x_2 + J(x_1 - x_3) = 1 \\ x_0 + x_2 + J(x_1 + x_3) = 1 \end{cases}$ .then $x_0 = 1\,.x_2 = x_3 = x_1 = 0$ .and $X = 1$

Case (3):

Doi: https://doi.org/10.54216/JNFS.080201
Received: November 22, 2023 Revised: January 19, 2024 Accepted: April 12, 2024

If $\begin{cases} x_0 - x_2 + J(x_1 - x_3) = \frac{1}{2} + \frac{1}{2\sqrt{+}}J \\ x_0 + x_2 + J(x_1 + x_3) = \frac{1}{2} + \frac{1}{2\sqrt{+}}J \end{cases}$. then $\begin{cases} x_0 = \frac{1}{2} \ .x_2 = 0 \\ x_1 = \frac{1}{2\sqrt{+}} \ .x_3 = 0 \end{cases}$. so that X $= \frac{1}{2} + \frac{1}{2\sqrt{+}}J$.

Case (4):

If $\begin{cases} x_0 - x_2 + J(x_1 - x_3) = \frac{1}{2} - \frac{1}{2\sqrt{+}}J \\ x_0 + x_2 + J(x_1 + x_3) = \frac{1}{2} - \frac{1}{2\sqrt{+}}J \end{cases}$. then X $= \frac{1}{2} - \frac{1}{2\sqrt{+}}J$.

Case (5):

If $\begin{cases} x_0 - x_2 + J(x_1 - x_3) = 0 \\ x_0 + x_2 + J(x_1 + x_3) = 1 \end{cases}$. then $\begin{cases} x_0 = \frac{1}{2} \ .x_2 = \frac{1}{2} \\ x_1 = x_3 = 0 \end{cases}$. and X $= \frac{1}{2} + \frac{1}{2}K$.

Case (6):

If $\begin{cases} x_0 - x_2 + J(x_1 - x_3) = 1 \\ x_0 + x_2 + J(x_1 + x_3) = 0 \end{cases}$. then X $= \frac{1}{2} - \frac{1}{2}K$.

Case (7):

If $\begin{cases} x_0 - x_2 + J(x_1 - x_3) = 0 \\ x_0 + x_2 + J(x_1 + x_3) = \frac{1}{2} + \frac{1}{2\sqrt{+}}J \end{cases}$. then $\begin{cases} x_0 = \frac{1}{4} \ .x_2 = \frac{1}{4} \\ x_1 = \frac{1}{4\sqrt{+}} \ .x_3 = \frac{1}{4\sqrt{+}} \end{cases}$. so that X $= \left(\frac{1}{4} + \frac{1}{4\sqrt{+}}J\right) + K(\frac{1}{4} + \frac{1}{4\sqrt{+}}J)$.

Case (8):

If $\begin{cases} x_0 - x_2 + J(x_1 - x_3) = \frac{1}{2} + \frac{1}{2\sqrt{+}}J \\ x_0 + x_2 + J(x_1 + x_3) = 0 \end{cases}$. then $\begin{cases} x_0 = \frac{1}{4} \ .x_2 = -\frac{1}{4} \\ x_1 = \frac{1}{4\sqrt{+}} \ .x_3 = \frac{-1}{4\sqrt{+}} \end{cases}$. so that X $= \left(\frac{1}{4} + \frac{1}{4\sqrt{+}}J\right) + K(-\frac{1}{4} - \frac{1}{4\sqrt{+}}J)$.

Case (9):

If $\begin{cases} x_0 - x_2 + J(x_1 - x_3) = 0 \\ x_0 + x_2 + J(x_1 + x_3) = \frac{1}{2} - \frac{1}{2\sqrt{+}}J \end{cases}$. then $\begin{cases} x_0 = \frac{1}{4} \ .x_2 = \frac{1}{4} \\ x_1 = \frac{-1}{4\sqrt{+}} \ .x_3 = \frac{-1}{4\sqrt{+}} \end{cases}$. so that X $= \left(\frac{1}{4} - \frac{1}{4\sqrt{+}}J\right) + K(\frac{1}{4} - \frac{1}{4\sqrt{+}}J)$.

Case (10):

If $\begin{cases} x_0 - x_2 + J(x_1 - x_3) = \frac{1}{2} - \frac{1}{2\sqrt{+}}J \\ x_0 + x_2 + J(x_1 + x_3) = 0 \end{cases}$. then $\begin{cases} x_0 = \frac{1}{4} \ .x_2 = \frac{-1}{4} \\ x_1 = \frac{-1}{4\sqrt{+}} \ .x_3 = \frac{1}{4\sqrt{+}} \end{cases}$. so that X $= \left(\frac{1}{4} - \frac{1}{4\sqrt{+}}J\right) + K(-\frac{1}{4} + \frac{1}{4\sqrt{+}}J)$.

Case (11):

If $\begin{cases} x_0 - x_2 + J(x_1 - x_3) = 1 \\ x_0 + x_2 + J(x_1 + x_3) = \frac{1}{2} + \frac{1}{2\sqrt{+}}J \end{cases}$. then $\begin{cases} x_0 = \frac{3}{4} \ .x_2 = -\frac{1}{4} \\ x_1 = \frac{1}{4\sqrt{+}} \ .x_3 = \frac{1}{4\sqrt{+}} \end{cases}$. and X $= \left(\frac{3}{4} + \frac{1}{4\sqrt{+}}J\right) + K(-\frac{1}{4} + \frac{1}{4\sqrt{+}}J)$.

Case (12):

If $\begin{cases} x_0 - x_2 + J(x_1 - x_3) = \frac{1}{2} + \frac{1}{2\sqrt{+}}J \\ x_0 + x_2 + J(x_1 + x_3) = 1 \end{cases}$. then $\begin{cases} x_0 = \frac{3}{4} \ .x_2 = \frac{1}{4} \\ x_1 = \frac{1}{4\sqrt{+}} \ .x_3 = \frac{-1}{4\sqrt{+}} \end{cases}$. and X $= \left(\frac{3}{4} + \frac{1}{4\sqrt{+}}J\right) + K(\frac{1}{4} - \frac{1}{4\sqrt{+}}J)$.

Case (13):

Doi: https://doi.org/10.54216/JNFS.080201
Received: November 22, 2023 Revised: January 19, 2024 Accepted: April 12, 2024

If $\begin{cases} x_0 - x_2 + J(x_1 - x_3) = 1 \\ x_0 + x_2 + J(x_1 + x_3) = \frac{1}{2} - \frac{1}{2\sqrt{+}}J \end{cases}$. then $\begin{cases} x_0 = \frac{3}{4} & .x_2 = -\frac{1}{4} \\ x_1 = \frac{-1}{4\sqrt{+}} & .x_3 = \frac{-1}{4\sqrt{+}} \end{cases}$. and $X = \left(\frac{3}{4} - \frac{1}{4\sqrt{+}}J\right) + K(-\frac{1}{4} - \frac{1}{4\sqrt{+}}J)$.

Case (14):

If $\begin{cases} x_0 - x_2 + J(x_1 - x_3) = \frac{1}{2} - \frac{1}{2\sqrt{+}}J \\ x_0 + x_2 + J(x_1 + x_3) = 1 \end{cases}$. then $\begin{cases} x_0 = \frac{3}{4} & .x_2 = \frac{1}{4} \\ x_1 = \frac{-1}{4\sqrt{+}} & .x_3 = \frac{1}{4\sqrt{+}} \end{cases}$. and $X = \left(\frac{3}{4} - \frac{1}{4\sqrt{+}}J\right) + K(\frac{1}{4} + \frac{1}{4\sqrt{+}}J)$.

Case (15):

If $\begin{cases} x_0 - x_2 + J(x_1 - x_3) = \frac{1}{2} + \frac{1}{2\sqrt{+}}J \\ x_0 + x_2 + J(x_1 + x_3) = \frac{1}{2} - \frac{1}{2\sqrt{+}}J \end{cases}$. then $\begin{cases} x_0 = \frac{1}{2} & .x_2 = 0 \\ x_1 = 0 & .x_3 = \frac{-1}{2\sqrt{+}} \end{cases}$. and $X = \frac{1}{2} + KJ(\frac{-1}{2\sqrt{+}})$.

Case (16):

If $\begin{cases} x_0 - x_2 + J(x_1 - x_3) = \frac{1}{2} - \frac{1}{2\sqrt{+}}J \\ x_0 + x_2 + J(x_1 + x_3) = \frac{1}{2} + \frac{1}{2\sqrt{+}}J \end{cases}$. then $\begin{cases} x_0 = \frac{1}{2} & .x_2 = 0 \\ x_1 = 0 & .x_3 = \frac{1}{2\sqrt{+}} \end{cases}$. and $X = \frac{1}{2} + \frac{1}{2\sqrt{+}}KJ$.

2. Conclusion

In this paper, we presented two novel generalizations of weak fuzzy complex numbers, where the concepts of strong fuzzy complex numbers and split-complex weak fuzzy complex numbers are defined for the first time with a general study of their elementary properties and special elements. On the other hand, we provided an algorithm to compute the dempotent elements in the ring of split-complex weak fuzzy complex numbers with many related examples.

References

[1] Hatip, A., "An Introduction To Weak Fuzzy Complex Numbers ", Galoitica Journal Of Mathematical Structures and Applications, Doi: https://doi.org/10.54216/GJMSA.030101, Vol. 03, No. 01, PP. 08-13, 2023.

[2] Ali, R., "On The Weak Fuzzy Complex Inner Products On Weak Fuzzy Complex Vector Spaces", Neoma Journal Of Mathematics and Computer Science, 2023.

[3] Yaser Ahmad Alhasan, Abuobida Mohammed A. Alfahal, Raja Abdullah Abdulfatah Giorgio Nordo, Musaddak M. Abdul Zahra, "On Some Novel Results About Weak Fuzzy Complex Matrices" ,DOI: https://doi.org/10.54216/IJNS.210112 , 2023.

[4] Alhasan, Y., Xu, L., Abdulfatah, R., & Alfahal, A. (2023). The Geometrical Characterization for The Solutions of a Vectorial Equation By Using Weak Fuzzy Complex Numbers and Other Generalizations Of Real Numbers. *International Journal of Neutrosophic Science (IJNS*), Vol. 21, No. 04, PP. 155-159. https://doi.org/10.54216/IJNS.210415.

[5] Abdallah Shihadeh, Wael Mahmoud M. Salameh, Malik Bataineh, Hassan Al-Tarawneh, Ayman Alahmade, Abdallah Al-Husban. (2024). On the Geometry of Weak Fuzzy Complex Numbers and Applications to the Classification of Some A-Curves. International Journal of Neutrosophic Science, 23 (4), 369-375 (**Doi :** https://doi.org/10.54216/IJNS.230428).

[6] Mayada Abualhomos, Wael Mahmoud M. Salameh, Malik Bataineh, Mowafaq Omar Al-Qadri, Ayman Alahmade, Abdallah Al-Husban. (2024). An Effective Algorithm for Solving Weak Fuzzy Complex Diophantine Equations in Two Variables. International Journal of Neutrosophic Science, 23 (4), 386-394 (**Doi :** https://doi.org/10.54216/IJNS.230431).

[7] Alfahal, A., Abobala, M., Alhasan, Y., & Abdulfatah, R. (2023). Generating Weak Fuzzy Complex and Anti Weak Fuzzy Complex Integer Solutions for Pythagoras Diophantine Equation $X^2 + Y^2 = Z^2$. *International Journal of Neutrosophic Science (IJNS*), Vol. 22, No. 01, PP. 08-14. https://doi.org/10.54216/IJNS.220201

Doi: https://doi.org/10.54216/JNFS.080201
Received: November 22, 2023 Revised: January 19, 2024 Accepted: April 12, 2024

[8] Galarza, F., Flores, M., Rivero, D., & Abobala, M. (2023). On Weak Fuzzy Complex Pythagoras Quadruples. *International Journal of Neutrosophic Science (IJNS*), Vol. 22, No. 02, PP. 108-113. https://doi.org/10.54216/IJNS.220209

[9] Ahmed Hatip. (2023). On The Fuzzy Weak Complex Vector Spaces. Galoitica: Journal of Mathematical Structures and Applications, 3 (1), 29-32
(**Doi :** https://doi.org/10.54216/GJMSA.030105).

Doi: https://doi.org/10.54216/JNFS.080201
Received: November 22, 2023 Revised: January 19, 2024 Accepted: April 12, 2024