New algebraic approach towards interval-valued neutrosophic cubic vague set based on subbisemiring over bisemiring

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Abstract

We introduce the concept of an interval-valued neutrosophic cubic vague subbisemiring (IVNCVSBS) and level set of IVNCVSBS of a bisemiring. An IVNCVSBS is the new extension of neutrosophic subbisemirings and SBS over bisemirings. Let $\aleph$ be a neutrosophic vague subset in Ξ, we show that $\sqsupset = ([\daleth^-_\aleph, \daleth^+_\aleph], [\beth^-_\aleph, \beth^+_\aleph], [\dashv^-_\aleph, \dashv^+_\aleph])$ is a IVNCVSBS of Ξ if and only if all non empty level set $\sqsupset^{(\ell_1,\ell_2,s)}$ is a SBS of Ξ for all $\ell_1, \ell_2, s \in [0,1]$. Let $\aleph$ be the IVNCVSBS of Ξ and Υ be the strongest cubic neutrosophic vague relation of Ξ. To prove that $\aleph$ is a IVNCVSBS of $\Xi \ltimes \Xi$. Let $\aleph$ be any IVNCVSBS of Ξ, prove that pseudo cubic neutrosophic vague coset $(\varsigma\aleph)^p$ is a IVNCVSBS of Ξ, for all $\varsigma \in \Xi$. Let $\aleph_1, \aleph_2, ..., \aleph_n$ be the family of $IVNCVSBS^s$ of $\Xi_1, \Xi_2, ..., \Xi_n$ respectively. To prove that $\aleph_1 \ltimes \aleph_2 \ltimes ... \ltimes \aleph_n$ is a IVNCVSBS of $\Xi_1 \ltimes \Xi_2 \ltimes ... \ltimes \Xi_n$. The homomorphic image of every IVNCVSBS is a IVNCVSBS. The homomorphic pre-image of every IVNCVSBS is a IVNCVSBS. Examples are provided to strengthen our results.

Keywords: Subbisemiring; cubic neutrosophic subbisemiring; vague bisemiring; homomorphism.

1 Introduction

Due to the limitations of classical mathematics, such as fuzzy set (FS)[1] and vague set (VS),[2] mathematical theories have been developed to address uncertainty and fuzziness. In the case of uncertain or vague situations, FS

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introduced by Zadeh[1] is the most appropriate technique. In recent years, many hybrid fuzzy models have been developed based on FS. A generalization of FS, intuitionistic fuzzy set (IFS) incorporate hesitation levels into the notion of FS, which were first proposed by Attanasov[3] in 1983. The neutrosophic set (NSS) was proposed in 1999 by Smarandache.[4] In NSS, each proposition is estimated to have a degree of truth, an indeterminacy degree, and a falsity degree. As a result of Smarandache,[5] he further generalised and expanded the theory of IFSs to include the neutrosophic model as well. A study of fuzzy semirings was initiated by Ahsan et al.[6] Recently many researchers discussed the various ideal structures of SBS and its applications[7]-.[10] In 2004, Sen et al.[11] extended the study of semirings and proposed the concept of bisemiring to further develop them. The study of vague algebra was initiated by Biswas[12] through the introduction of vague groups, vague cuts and vague normal groups. A semiring $(S, +, \cdot)$ is a non-empty set in which $(S, +)$ and $(S, \cdot)$ are semigroups such that "$\cdot$" is distributive over "$+$".[13] In 1993, Ahsan et al.[6] introduced the notion of fuzzy semirings. An introduction to bisemirings was made in 2001 by Sen et al.[14] A bisemiring $(\Xi, \diamondsuit, \circledcirc, \boxtimes)$ is an algebraic structure in which $(\Xi, \diamondsuit, \circledcirc)$ and $(\Xi, \circledcirc, \boxtimes)$ are semirings in which $(\Xi, \diamondsuit)$, $(\Xi, \circledcirc)$ and $(\Xi, \boxtimes)$ are semigroups such that (a) $\zeta \circledcirc (\partial \diamondsuit \varsigma) = (\wp \circledcirc \partial)\diamondsuit(\wp \circledcirc \varsigma)$, (b) $(\partial \diamondsuit \varsigma) \circledcirc \wp = (\partial \circledcirc \wp)\diamondsuit(\varsigma \circledcirc \wp)$, (c) $\wp \boxtimes (\partial \circledcirc \varsigma) = (\wp \boxtimes \partial) \circledcirc (\wp \boxtimes \varsigma)$ and (d) $(\partial \circledcirc \varsigma) \boxtimes \wp = (\partial \boxtimes \wp) \circledcirc (\varsigma \boxtimes \wp)$ for all $\wp, \partial, \varsigma \in \Xi$.[11] A non-empty subset $\aleph$ of a bisemiring $(\Xi, \diamondsuit, \circledcirc, \boxtimes)$ is a subbisemiring (SBS) if and only if $\wp \diamondsuit \partial \in \aleph$, $\wp \circledcirc \partial \in \aleph$ and $\wp \boxtimes \partial \in \aleph$ for all $\wp, \partial \in \aleph$.[14] However, numerous algebraic concepts had been generalized using FS theory. Fuzzy algebraic structures of semirings have been extensively investigated by Vandiver.[15] These are generalizations of rings requiring only a monoid, rather than a group, to achieve a particular additive structure and have been shown to be useful for a wide range of problems. Golan[13] and Glazek[16] have both extensively studied the application of semirings.

Bipolar fuzzy information has been applied to various algebraic structures over the past few years, like semigroups and BCK/BCI algebras.[17,18,21] An application of bipolar fuzzy metric spaces was discussed by Zararsz et al.[22] A vague soft hyperring and a vague soft hyper ideal were introduced by Selvachandran.[23] The bipolar fuzzy translation was introduced by Jun et al.[24] and BCK/BCI-algebra and its properties were investigated. A bipolar fuzzy regularity, bipolar fuzzy regular sub-algebra, a bipolar fuzzy filter, and a bipolar fuzzy closed quasi filter have been introduced into BCH algebras in.[25] In 2004, Sen et al.[11] contributed to the field of semirings by proposing bisemiring as a concept. Hussain et al.[26] defined the congruence relation between bisemirings and bisemiring homomorphisms. In addition to bisemiring, Hussain et al.[14,26] described an algebraic structure called semiring and congruence relations between homomorphisms and n-semirings based on this algebraic structure. We discuss the concept of interval-valued neutosophic cubic vague subbisemiring (IVNCVSBS) and level sets. The IVNCVSBS is a extension of subbisemiring. A number of illustrative examples are provided to illustrate. Following is an outline of the preliminary definitions and results presented in Section 2. The concept of a IVNCVSBS is introduced in Section 3.

2 Preliminaries

For our future studies, we will quickly review some fundamental terms in this section.

Definition 2.1. [4] A neutrosophic set (NSS) $\aleph$ in a universal set Γ is $\aleph = \{(\wp, \daleth_\aleph(\wp), \beth_\aleph(\wp), \exists_\aleph(\wp)) : \wp \in \Gamma\}$, where $\daleth_\aleph, \beth_\aleph, \exists_\aleph : \Gamma \to [0, 1]$ denotes the truth, indeterminacy and the falsity membership function, respectively. For $\langle \daleth_\aleph, \beth_\aleph, \exists_\aleph \rangle$ is used for the NSS $\aleph = \{(\wp, \daleth_\aleph(\wp), \beth_\aleph(\wp), \exists_\aleph(\wp)) : \wp \in \Gamma\}$.

Definition 2.2. [4] Let $\aleph = \langle \daleth_\aleph, \beth_\aleph, \exists_\aleph \rangle$ and $\hbar = \langle \daleth_\hbar, \beth_\hbar, \exists_\hbar \rangle$ be the two NSS of Γ. Then

1. $\aleph \bigwedge \hbar = \{(\wp, \min\{\daleth_\aleph(\wp), \daleth_\hbar(\wp)\}, \min\{\beth_\aleph(\wp), \beth_\hbar(\wp)\}, \max\{\exists_\aleph(\wp), \exists_\hbar(\wp)\}) : \wp \in \Gamma\}$,
2. $\aleph \bigvee \hbar = \{(\wp, \max\{\daleth_\aleph(\wp), \daleth_\hbar(\wp)\}, \max\{\beth_\aleph(\wp), \beth_\hbar(\wp)\}, \min\{\exists_\aleph(\wp), \exists_\hbar(\wp)\}) : \wp \in \Gamma\}$.

Definition 2.3. [4] For any NSS $\aleph = \langle \daleth_\aleph, \beth_\aleph, \exists_\aleph \rangle$ of Γ, we defined a (ℓ, s)-cut of as the crisp subset $\{\wp \in \Gamma : \daleth_\aleph(\wp) \geqq \ell, \beth_\aleph(\wp) \geqq \ell, \exists_\aleph(\wp) \leqq s\}$ of Γ.

Definition 2.4. [4] Let $\aleph$ and $\hbar$ be two neutrosophic subsets of S. The Cartesian product of $\aleph$ and $\hbar$ is defined as $\aleph \ltimes \hbar = \{((\wp, \partial), \daleth_{\aleph\ltimes\hbar}(\wp, \partial), \beth_{\aleph\ltimes\hbar}(\wp, \partial), \exists_{\aleph\ltimes\hbar}(\wp, \partial)) : \wp, \partial \in S\}$, where $\daleth_{\aleph\ltimes\hbar}(\wp, \partial) = \min\{\daleth_\aleph(\wp), \daleth_\hbar(\partial)\}$, $\beth_{\aleph\ltimes\hbar}(\wp, \partial) = \frac{\beth_\aleph(\wp)+\beth_\hbar(\partial)}{2}$ and $\exists_{\aleph\ltimes\hbar}(\wp, \partial) = \max\{\exists_\aleph(\wp), \exists_\hbar(\partial)\}$.

Definition 2.5. [12] A vague set (VS) $\aleph = (\daleth_\aleph, \exists_\aleph)$ of Ξ is said to be vague semiring if

$$\left\{ \begin{array}{l} \daleth_\aleph(\flat_1 + \flat_2) \geqq \min\{\daleth_\aleph(\flat_1), \daleth_\aleph(\flat_2)\} \\ \daleth_\aleph(\flat_1 \cdot \flat_2) \geqq \min\{\daleth_\aleph(\flat_1), \daleth_\aleph(\flat_2)\} \end{array} \right\}$$

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and

$$\left.\begin{cases} 1-\gimel_{\aleph}(\flat_1+\flat_2) \geqq \min\{1-\gimel_{\aleph}(\flat_1), 1-\gimel_{\aleph}(\flat_2)\} \\ 1-\gimel_{\aleph}(\flat_1\cdot\flat_2) \geqq \min\{1-\gimel_{\aleph}(\flat_1), 1-\gimel_{\aleph}(\flat_2)\} \end{cases}\right\}.$$

for all $\flat_1, \flat_2 \in \Xi$.

Definition 2.6. [12] A VS $\aleph$ in Γ. Then

1. A VS $\aleph = (\daleth_{\aleph}, \gimel_{\aleph})$, where $\daleth_{\aleph} : \Gamma \to [0,1]$, $\gimel_{\aleph} : \Gamma \to [0,1]$ are mappings such that $\daleth_{\aleph}(\wp)+\gimel_{\aleph}(\wp) \leqq 1$, for all $\wp \in \Gamma$ where $\daleth_{\aleph}$ and $\gimel_{\aleph}$ are called true and false membership function, respectively.
2. The interval $[\daleth_{\aleph}(\wp), 1-\gimel_{\aleph}(\wp)]$ is called the vague value of $\wp$ in $\aleph$ and it is denoted by $V_{\aleph}(\wp)$, i.e., $V_{\aleph}(\wp) = [\daleth_{\aleph}(\wp), 1-\gimel_{\aleph}(\wp)]$.

Definition 2.7. [12] Let $\aleph$ and $\hbar$ be the two VSs of Γ. Then

1. $\aleph$ is contained in $\hbar$ as $\aleph \subseteq \hbar$ if and only if $V_{\aleph}(\wp) \leqq V_{\hbar}(\wp)$, i.e. $\daleth_{\aleph}(\wp) \leqq \daleth_{\hbar}(\wp)$ and $1-\gimel_{\aleph}(\wp) \leqq 1-\gimel_{\hbar}(\wp)$ for all $\wp \in \Gamma$,
2. the union of $\aleph$ and $\hbar$ as $N = \aleph \bigvee \hbar$, $\daleth_N = \max\{\daleth_{\aleph}, \daleth_{\hbar}\}$ and $1-\gimel_N = \max\{1-\gimel_{\aleph}, 1-\gimel_{\hbar}\} = 1-\min\{\gimel_{\aleph}, \gimel_{\hbar}\}$,
3. the intersection of $\aleph$ and $\hbar$ as $N = \aleph \bigwedge \hbar$, $\daleth_N = \min\{\daleth_{\aleph}, \daleth_{\hbar}\}$ and $1-\gimel_N = \min\{1-\gimel_{\aleph}, 1-\gimel_{\hbar}\} = 1-\max\{\gimel_{\aleph}, \gimel_{\hbar}\}$.

Definition 2.8. [12] Let $\aleph$ be a VS of Γ. Then

1. $\daleth_{\aleph}(\wp) = 0$ and $\gimel_{\aleph}(\wp) = 1$ is called zero VS of Γ,
2. $\daleth_{\aleph}(\wp) = 1$ and $\gimel_{\aleph}(\wp) = 0$ is called unit VS of Γ.

for all $\wp \in U$.

Definition 2.9. [12] Let $\aleph$ be a VS of Γ with true membership function $\daleth_{\aleph}$ and false membership function $\gimel_{\aleph}$. For $\alpha, \beta \in [0,1]$ with $\alpha \leqq \beta$, the (α,β)- cut or vague cut of a VS $\aleph$ is the crisp subset of Γ is given by $\aleph_{(\alpha,\beta)} = \{\wp \in \Gamma : V_{\aleph}(\wp) \geqq [\alpha,\beta]\}$. That is, $\aleph_{(\alpha,\beta)} = \{\wp \in \Gamma : \daleth_{\aleph}(\wp) \geqq \alpha,\ 1-\gimel_{\aleph}(\wp) \geqq \beta]\}$.

Definition 2.10. [12] Let $\aleph$ and $\hbar$ be any two VSs in Γ. Then

1. $\aleph \bigwedge \hbar = \big\{(\wp, \min\{\daleth_{\aleph}(\wp), \daleth_{\hbar}(\wp)\}, \min\{1-\gimel_{\aleph}(\wp), 1-\gimel_{\hbar}(\wp)\}) : \wp \in \Gamma\big\}$,
2. $\aleph \bigvee \hbar = \big\{(\wp, \max\{\daleth_{\aleph}(\wp), \daleth_{\hbar}(\wp)\}, \max\{1-\gimel_{\aleph}(\wp), 1-\gimel_{\hbar}(\wp)\}) : \wp \in \Gamma\big\}$,
3. $\square\aleph = \big\{(\wp, \daleth_{\aleph}(\wp), 1-\daleth_{\aleph}(\wp)) : \wp \in \Gamma\big\}$,
4. $\diamond\aleph = \big\{(\wp, 1-\gimel_{\aleph}(\wp), \gimel_{\aleph}(\wp)) : \wp \in \Gamma\big\}$.

3 Interval valued neutrosophic cubic vague subbisemirings

In all cases, assume that Ξ represents a bisemiring. Onless otherwise specified.

Definition 3.1. A interval-valued neutrosophic cubic VS $\aleph$ of Ξ is represent a IVNCVSBS of Ξ if

$$\left\{\begin{array}{l} \widehat{\beth}^{\daleth}_{\aleph}(\wp\triangle_1\partial) \geqq \min\{\widehat{\beth}^{\daleth}_{\aleph}(\wp), \widehat{\beth}^{\daleth}_{\aleph}(\partial)\} \\ \widehat{\beth}^{\daleth}_{\aleph}(\wp\triangle_2\partial) \geqq \min\{\widehat{\beth}^{\daleth}_{\aleph}(\wp), \widehat{\beth}^{\daleth}_{\aleph}(\partial)\} \\ \widehat{\beth}^{\daleth}_{\aleph}(\wp\triangle_3\partial) \geqq \min\{\widehat{\beth}^{\daleth}_{\aleph}(\wp), \widehat{\beth}^{\daleth}_{\aleph}(\partial)\} \end{array}\right\} \quad \left\{\begin{array}{c} \widehat{\beth}^{\gimel}_{\aleph}(\wp\triangle_1\partial) \geqq \frac{\widehat{\beth}^{\gimel}_{\aleph}(\wp)+\widehat{\beth}^{\gimel}_{\aleph}(\partial)}{2} \\ OR \\ \widehat{\beth}^{\gimel}_{\aleph}(\wp\triangle_2\partial) \geqq \frac{\widehat{\beth}^{\gimel}_{\aleph}(\wp)+\widehat{\beth}^{\gimel}_{\aleph}(\partial)}{2} \\ OR \\ \widehat{\beth}^{\gimel}_{\aleph}(\wp\triangle_3\partial) \geqq \frac{\widehat{\beth}^{\gimel}_{\aleph}(\wp)+\widehat{\beth}^{\gimel}_{\aleph}(\partial)}{2} \end{array}\right\}$$

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$$\begin{Bmatrix} \widehat{\sqsupset}^{\Finv}_{\aleph}(\wp\triangle_1\partial) \leqq \max\{\widehat{\sqsupset}^{\Finv}_{\aleph}(\wp), \widehat{\sqsupset}^{\Finv}_{\aleph}(\partial)\} \\ \widehat{\sqsupset}^{\Finv}_{\aleph}(\wp\triangle_2\partial) \leqq \max\{\widehat{\sqsupset}^{\Finv}_{\aleph}(\wp), \widehat{\sqsupset}^{\Finv}_{\aleph}(\partial)\} \\ \widehat{\sqsupset}^{\Finv}_{\aleph}(\wp\triangle_3\partial) \leqq \max\{\widehat{\sqsupset}^{\Finv}_{\aleph}(\wp), \widehat{\sqsupset}^{\Finv}_{\aleph}(\partial)\} \end{Bmatrix}.$$

$$\begin{Bmatrix} \sqsupset^{\daleth}_{\aleph}(\wp\triangle_1\partial) \geqq \min\{\sqsupset^{\daleth}_{\aleph}(\wp), \sqsupset^{\daleth}_{\aleph}(\partial)\} \\ \sqsupset^{\daleth}_{\aleph}(\wp\triangle_2\partial) \geqq \min\{\sqsupset^{\daleth}_{\aleph}(\wp), \sqsupset^{\daleth}_{\aleph}(\partial)\} \\ \sqsupset^{\daleth}_{\aleph}(\wp\triangle_3\partial) \geqq \min\{\sqsupset^{\daleth}_{\aleph}(\wp), \sqsupset^{\daleth}_{\aleph}(\partial)\} \end{Bmatrix} \quad \begin{Bmatrix} \sqsupset^{\gimel}_{\aleph}(\wp\triangle_1\partial) \geqq \frac{\sqsupset^{\gimel}_{\aleph}(\wp)+\sqsupset^{\gimel}_{\aleph}(\partial)}{2} \\ OR \\ \sqsupset^{\gimel}_{\aleph}(\wp\triangle_2\partial) \geqq \frac{\sqsupset^{\gimel}_{\aleph}(\wp)+\sqsupset^{\gimel}_{\aleph}(\partial)}{2} \\ OR \\ \sqsupset^{\gimel}_{\aleph}(\wp\triangle_3\partial) \geqq \frac{\sqsupset^{\gimel}_{\aleph}(\wp)+\sqsupset^{\gimel}_{\aleph}(\partial)}{2} \end{Bmatrix}$$

$$\begin{Bmatrix} \sqsupset^{\Finv}_{\aleph}(\wp\triangle_1\partial) \leqq \max\{\sqsupset^{\Finv}_{\aleph}(\wp), \sqsupset^{\Finv}_{\aleph}(\partial)\} \\ \sqsupset^{\Finv}_{\aleph}(\wp\triangle_2\partial) \leqq \max\{\sqsupset^{\Finv}_{\aleph}(\wp), \sqsupset^{\Finv}_{\aleph}(\partial)\} \\ \sqsupset^{\Finv}_{\aleph}(\wp\triangle_3\partial) \leqq \max\{\sqsupset^{\Finv}_{\aleph}(\wp), \sqsupset^{\Finv}_{\aleph}(\partial)\} \end{Bmatrix}.$$

That is,

$$\begin{Bmatrix} \begin{pmatrix} \widehat{\daleth}^{-}_{\aleph}(\wp\triangle_1\partial) \geqq \min\{\widehat{\daleth}^{-}_{\aleph}(\wp), \widehat{\daleth}^{-}_{\aleph}(\partial)\}, \\ 1-\widehat{\Finv}^{-}_{\aleph}(\wp\triangle_1\partial) \geqq \min\{1-\widehat{\Finv}^{-}_{\aleph}(\wp), 1-\widehat{\Finv}^{-}_{\aleph}(\partial)\} \end{pmatrix} \\ \begin{pmatrix} \widehat{\daleth}^{-}_{\aleph}(\wp\triangle_2\partial) \geqq \min\{\widehat{\daleth}^{-}_{\aleph}(\wp), \widehat{\daleth}^{-}_{\aleph}(\partial)\}, \\ 1-\widehat{\Finv}^{-}_{\aleph}(\wp\triangle_2\partial) \geqq \min\{1-\widehat{\Finv}^{-}_{\aleph}(\wp), 1-\widehat{\Finv}^{-}_{\aleph}(\partial)\} \end{pmatrix} \\ \begin{pmatrix} \widehat{\daleth}^{-}_{\aleph}(\wp\triangle_3\partial) \geqq \min\{\widehat{\daleth}^{-}_{\aleph}(\wp), \widehat{\daleth}^{-}_{\aleph}(\partial)\}, \\ 1-\widehat{\Finv}^{-}_{\aleph}(\wp\triangle_3\partial) \geqq \min\{1-\widehat{\Finv}^{-}_{\aleph}(\wp), 1-\widehat{\Finv}^{-}_{\aleph}(\partial)\} \end{pmatrix} \end{Bmatrix} \quad \begin{Bmatrix} \begin{pmatrix} \widehat{\gimel}^{+}_{\aleph}(\wp\triangle_1\partial) \geqq \frac{\widehat{\gimel}^{+}_{\aleph}(\wp)+\widehat{\gimel}^{+}_{\aleph}(\partial)}{2}, \\ \widehat{\gimel}^{-}_{\aleph}(\wp\triangle_1\partial) \geqq \frac{\widehat{\gimel}^{-}_{\aleph}(\wp)-\widehat{\gimel}^{-}_{\aleph}(\partial)}{2} \end{pmatrix} \\ OR \\ \begin{pmatrix} \widehat{\gimel}^{+}_{\aleph}(\wp\triangle_2\partial) \geqq \frac{\widehat{\gimel}^{+}_{\aleph}(\wp)+\widehat{\gimel}^{+}_{\aleph}(\partial)}{2}, \\ \widehat{\gimel}^{-}_{\aleph}(\wp\triangle_2\partial) \geqq \frac{\widehat{\gimel}^{-}_{\aleph}(\wp)-\widehat{\gimel}^{-}_{\aleph}(\partial)}{2} \end{pmatrix} \\ OR \\ \begin{pmatrix} \widehat{\gimel}^{+}_{\aleph}(\wp\triangle_3\partial) \geqq \frac{\widehat{\gimel}^{+}_{\aleph}(\wp)+\widehat{\gimel}^{+}_{\aleph}(\partial)}{2}, \\ \widehat{\gimel}^{-}_{\aleph}(\wp\triangle_3\partial) \geqq \frac{\widehat{\gimel}^{-}_{\aleph}(\wp)-\widehat{\gimel}^{-}_{\aleph}(\partial)}{2} \end{pmatrix} \end{Bmatrix}$$

$$\begin{Bmatrix} \begin{pmatrix} \widehat{\Finv}^{-}_{\aleph}(\wp\triangle_1\partial) \leqq \max\{\widehat{\Finv}^{-}_{\aleph}(\wp), \widehat{\Finv}^{-}_{\aleph}(\partial)\}, \\ 1-\widehat{\daleth}^{-}_{\aleph}(\wp\triangle_1\partial) \leqq \max\{1-\widehat{\daleth}^{-}_{\aleph}(\wp), 1-\widehat{\daleth}^{-}_{\aleph}(\partial)\} \end{pmatrix} \\ \begin{pmatrix} \widehat{\Finv}^{-}_{\aleph}(\wp\triangle_2\partial) \leqq \max\{\widehat{\Finv}^{-}_{\aleph}(\wp), \widehat{\Finv}^{-}_{\aleph}(\partial)\}, \\ 1-\widehat{\daleth}^{-}_{\aleph}(\wp\triangle_2\partial) \leqq \max\{1-\widehat{\daleth}^{-}_{\aleph}(\wp), 1-\widehat{\daleth}^{-}_{\aleph}(\partial)\} \end{pmatrix} \\ \begin{pmatrix} \widehat{\Finv}^{-}_{\aleph}(\wp\triangle_3\partial) \leqq \max\{\widehat{\Finv}^{-}_{\aleph}(\wp), \widehat{\Finv}^{-}_{\aleph}(\partial)\}, \\ 1-\widehat{\daleth}^{-}_{\aleph}(\wp\triangle_3\partial) \leqq \max\{1-\widehat{\daleth}^{-}_{\aleph}(\wp), 1-\widehat{\daleth}^{-}_{\aleph}(\partial)\} \end{pmatrix} \end{Bmatrix}$$

$$\begin{Bmatrix} \begin{pmatrix} \daleth^{-}_{\aleph}(\wp\triangle_1\partial) \geqq \min\{\daleth^{-}_{\aleph}(\wp), \daleth^{-}_{\aleph}(\partial)\}, \\ 1-\Finv^{-}_{\aleph}(\wp\triangle_1\partial) \geqq \min\{1-\Finv^{-}_{\aleph}(\wp), 1-\Finv^{-}_{\aleph}(\partial)\} \end{pmatrix} \\ \begin{pmatrix} \daleth^{-}_{\aleph}(\wp\triangle_2\partial) \geqq \min\{\daleth^{-}_{\aleph}(\wp), \daleth^{-}_{\aleph}(\partial)\}, \\ 1-\Finv^{-}_{\aleph}(\wp\triangle_2\partial) \geqq \min\{1-\Finv^{-}_{\aleph}(\wp), 1-\Finv^{-}_{\aleph}(\partial)\} \end{pmatrix} \\ \begin{pmatrix} \daleth^{-}_{\aleph}(\wp\triangle_3\partial) \geqq \min\{\daleth^{-}_{\aleph}(\wp), \daleth^{-}_{\aleph}(\partial)\}, \\ 1-\Finv^{-}_{\aleph}(\wp\triangle_3\partial) \geqq \min\{1-\Finv^{-}_{\aleph}(\wp), 1-\Finv^{-}_{\aleph}(\partial)\} \end{pmatrix} \end{Bmatrix} \quad \begin{Bmatrix} \begin{pmatrix} \gimel^{+}_{\aleph}(\wp\triangle_1\partial) \geqq \frac{\gimel^{+}_{\aleph}(\wp)+\gimel^{+}_{\aleph}(\partial)}{2}, \\ \gimel^{-}_{\aleph}(\wp\triangle_1\partial) \geqq \frac{\gimel^{-}_{\aleph}(\wp)-\gimel^{-}_{\aleph}(\partial)}{2} \end{pmatrix} \\ OR \\ \begin{pmatrix} \gimel^{+}_{\aleph}(\wp\triangle_2\partial) \geqq \frac{\gimel^{+}_{\aleph}(\wp)+\gimel^{+}_{\aleph}(\partial)}{2}, \\ \gimel^{-}_{\aleph}(\wp\triangle_2\partial) \geqq \frac{\gimel^{-}_{\aleph}(\wp)-\gimel^{-}_{\aleph}(\partial)}{2} \end{pmatrix} \\ OR \\ \begin{pmatrix} \gimel^{+}_{\aleph}(\wp\triangle_3\partial) \geqq \frac{\gimel^{+}_{\aleph}(\wp)+\gimel^{+}_{\aleph}(\partial)}{2}, \\ \gimel^{-}_{\aleph}(\wp\triangle_3\partial) \geqq \frac{\gimel^{-}_{\aleph}(\wp)-\gimel^{-}_{\aleph}(\partial)}{2} \end{pmatrix} \end{Bmatrix}$$

$$\begin{Bmatrix} \begin{pmatrix} \Finv^{-}_{\aleph}(\wp\triangle_1\partial) \leqq \max\{\Finv^{-}_{\aleph}(\wp), \Finv^{-}_{\aleph}(\partial)\}, \\ 1-\daleth^{-}_{\aleph}(\wp\triangle_1\partial) \leqq \max\{1-\daleth^{-}_{\aleph}(\wp), 1-\daleth^{-}_{\aleph}(\partial)\} \end{pmatrix} \\ \begin{pmatrix} \Finv^{-}_{\aleph}(\wp\triangle_2\partial) \leqq \max\{\Finv^{-}_{\aleph}(\wp), \Finv^{-}_{\aleph}(\partial)\}, \\ 1-\daleth^{-}_{\aleph}(\wp\triangle_2\partial) \leqq \max\{1-\daleth^{-}_{\aleph}(\wp), 1-\daleth^{-}_{\aleph}(\partial)\} \end{pmatrix} \\ \begin{pmatrix} \Finv^{-}_{\aleph}(\wp\triangle_3\partial) \leqq \max\{\Finv^{-}_{\aleph}(\wp), \Finv^{-}_{\aleph}(\partial)\}, \\ 1-\daleth^{-}_{\aleph}(\wp\triangle_3\partial) \leqq \max\{1-\daleth^{-}_{\aleph}(\wp), 1-\daleth^{-}_{\aleph}(\partial)\} \end{pmatrix} \end{Bmatrix}$$

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for all $\wp, \partial \in \Xi$.

Example 3.2. Let $\Xi = \{\zeta_a, \zeta_b, \zeta_c, \zeta_d\}$ be the bisemiring.

$\triangle_1$	ζ_a	ζ_b	ζ_c	ζ_d
ζ_a	ζ_a	ζ_a	ζ_a	ζ_a
ζ_b	ζ_a	ζ_b	ζ_a	ζ_b
ζ_c	ζ_a	ζ_a	ζ_c	ζ_c
ζ_d	ζ_a	ζ_b	ζ_c	ζ_d

$\triangle_2$	ζ_a	ζ_b	ζ_c	ζ_d
ζ_a	ζ_a	ζ_b	ζ_c	ζ_d
ζ_b	ζ_b	ζ_b	ζ_d	ζ_d
ζ_c	ζ_c	ζ_d	ζ_c	ζ_d
ζ_d	ζ_d	ζ_d	ζ_d	ζ_d

$\triangle_3$	ζ_a	ζ_b	ζ_c	ζ_d
ζ_a	ζ_a	ζ_a	ζ_a	ζ_a
ζ_b	ζ_a	ζ_b	ζ_c	ζ_d
ζ_c	ζ_d	ζ_d	ζ_d	ζ_d
ζ_d	ζ_d	ζ_d	ζ_d	ζ_d

	$[\widehat{\daleth}^-_{\aleph}(\beta), \daleth^+_{\aleph}(\beta)]$	$[\beth^-_{\aleph}(\beta), \beth^+_{\aleph}(\beta)]$	$[\dashv^-_{\aleph}(\beta), \dashv^+_{\aleph}(\beta)]$
$\beta = \zeta_a$	$\left[[0.75, 0.8], [0.85, 0.9]\right]$	$\left[[0.65, 0.7], [0.75, 0.85]\right]$	$\left[[0.2, 0.25], [0.1, 0.15]\right]$
$\beta = \zeta_b$	$\left[[0.65, 0.7], [0.75, 0.8]\right]$	$\left[[0.55, 0.6], [0.65, 0.7]\right]$	$\left[[0.3, 0.35], [0.2, 0.25]\right]$
$\beta = \zeta_c$	$\left[[0.45, 0.5], [0.65, 0.7]\right]$	$\left[[0.35, 0.4], [0.45, 0.5]\right]$	$\left[[0.5, 0.55], [0.3, 0.35]\right]$
$\beta = \zeta_d$	$\left[[0.6, 0.65], [0.7, 0.75]\right]$	$\left[[0.4, 0.45], [0.55, 0.6]\right]$	$\left[[0.35, 0.4], [0.25, 0.3]\right]$

	$[\daleth^-_{\aleph}(\beta), \daleth^+_{\aleph}(\beta)]$	$[\beth^-_{\aleph}(\beta), \beth^+_{\aleph}(\beta)]$	$[\dashv^-_{\aleph}(\beta), \dashv^+_{\aleph}(\beta)]$
$\beta = \zeta_a$	$[0.65, 0.7]$	$[0.75, 0.8]$	$[0.3, 0.35]$
$\beta = \zeta_b$	$[0.55, 0.65]$	$[0.7, 0.75]$	$[0.35, 0.45]$
$\beta = \zeta_c$	$[0.40, 0.45]$	$[0.55, 0.60]$	$[0.55, 0.60]$
$\beta = \zeta_d$	$[0.45, 0.55]$	$[0.65, 0.70]$	$[0.45, 0.55]$

Clearly, $\aleph$ is a IVNCVSBS of Ξ.

Theorem 3.3. *The intersection of a family of every $IVNCVSBS^s$ of Ξ is a IVNCVSBS of Ξ.*

Proof. Let $\{\beth_i : i \in I\}$ be a collection of $IVNCVSBS^s$ of Ξ and $\aleph = \bigwedge_{i \in I} \beth_i$.

Let $\wp, \partial$ in Ξ. Then

$$\begin{aligned}
\widehat{\daleth}^-_{\aleph}(\wp \triangle_1 \partial) &= \inf_{i \in I} \ \widehat{\daleth}^-_{\beth_i}(\wp \triangle_1 \partial) \\
&\geqq \inf_{i \in I} \ \min\{\widehat{\daleth}^-_{\beth_i}(\wp), \widehat{\daleth}^-_{\beth_i}(\partial)\} \\
&= \min\left\{\inf_{i \in I} \ \widehat{\daleth}^-_{\beth_i}(\wp), \inf_{i \in I} \ \widehat{\daleth}^-_{\beth_i}(\partial)\right\} \\
&= \min\{\widehat{\daleth}^-_{\aleph}(\wp), \widehat{\daleth}^-_{\aleph}(\partial)\}. \\
1 - \widehat{\dashv}^-_{\aleph}(\wp \triangle_1 \partial) &= \inf_{i \in I} \ 1 - \widehat{\dashv}^-_{\beth_i}(\wp \triangle_1 \partial) \\
&\geqq \inf_{i \in I} \ \min\{1 - \widehat{\dashv}^-_{\beth_i}(\wp), 1 - \widehat{\dashv}^-_{\beth_i}(\partial)\} \\
&= \min\left\{\inf_{i \in I} \ 1 - \widehat{\dashv}^-_{\beth_i}(\wp), \inf_{i \in I} \ 1 - \widehat{\dashv}^-_{\beth_i}(\partial)\right\} \\
&= \min\{1 - \widehat{\dashv}^-_{\aleph}(\wp), 1 - \widehat{\dashv}^-_{\aleph}(\partial)\}.
\end{aligned}$$

Thus $\widehat{\beth\daleth}_{\aleph}(\wp \triangle_1 \partial) \geqq \min\{\widehat{\beth}_{\aleph}(\wp), \widehat{\beth}_{\aleph}(\partial)\}$. Similarly, $\widehat{\beth\daleth}_{\aleph}(\wp \triangle_2 \partial) \geqq \min\{\widehat{\beth}_{\aleph}(\wp), \widehat{\beth}_{\aleph}(\partial)\}$ and $\widehat{\beth\daleth}_{\aleph}(\wp \triangle_3 \partial) \geqq \min\{\widehat{\beth}_{\aleph}(\wp), \widehat{\beth}_{\aleph}(\partial)\}$. Now,

$$\begin{aligned}
\widehat{\beth}^-_{\aleph}(\wp \triangle_1 \partial) &= \inf_{i \in I-} \ \widehat{\beth}^-_{\beth_i}(\wp \triangle_1 \partial) \\
&\geqq \inf_{i \in I-} \ \frac{\widehat{\beth}^-_{\beth_i}(\wp) + \widehat{\beth}^-_{\beth_i}(\partial)}{2} \\
&= \frac{\inf_{i \in I-} \ \widehat{\beth}^-_{\beth_i}(\wp) + \inf_{i \in I-} \ \widehat{\beth}^-_{\beth_i}(\partial)}{2} \\
&= \frac{\widehat{\beth}^-_{\aleph}(\wp) + \widehat{\beth}^-_{\aleph}(\partial)}{2}.
\end{aligned}$$

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$$\begin{aligned}
\widehat{\gimel}^{+}_{\aleph}(\wp\triangle_1\partial) &= \inf_{i\in I+}\ \widehat{\gimel}^{+}_{\beth_i}(\wp\triangle_1\partial)\\
&\geqq \inf_{i\in I+}\ \frac{\widehat{\gimel}^{+}_{\beth_i}(\wp)+\widehat{\gimel}^{+}_{\beth_i}(\partial)}{2}\\
&= \frac{\inf_{i\in I+}\ \widehat{\gimel}^{+}_{\beth_i}(\wp)+\inf_{i\in I+}\ \widehat{\gimel}^{+}_{\beth_i}(\partial)}{2}\\
&= \frac{\widehat{\gimel}^{+}_{\aleph}(\wp)+\widehat{\gimel}^{+}_{\aleph}(\partial)}{2}.
\end{aligned}$$

Thus $\widehat{\beth}^{\gimel}_{\aleph}(\wp\triangle_1\partial) \geqq \min\{\widehat{\beth}_{\aleph}(\wp), \widehat{\beth}_{\aleph}(\partial)\}$. Similarly, $\widehat{\beth}^{\gimel}_{\aleph}(\wp\triangle_2\partial) \geqq \min\{\widehat{\beth}_{\aleph}(\wp), \widehat{\beth}_{\aleph}(\partial)\}$ and $\widehat{\beth}^{\gimel}_{\aleph}(\wp\triangle_3\partial) \geqq \min\{\widehat{\beth}_{\aleph}(\wp), \widehat{\beth}_{\aleph}(\partial)\}$.

Now,

$$\begin{aligned}
\widehat{\Finv}^{-}_{\aleph}(\wp\triangle_1\partial) &= \sup_{i\in I}\ \widehat{\Finv}^{-}_{\beth_i}(\wp\triangle_1\partial)\\
&\leqq \sup_{i\in I}\ \max\{\widehat{\Finv}^{-}_{\beth_i}(\wp), \widehat{\Finv}^{-}_{\beth_i}(\partial)\}\\
&= \max\left\{\sup_{i\in I}\ \widehat{\Finv}^{-}_{\beth_i}(\wp), \sup_{i\in I}\ \widehat{\Finv}^{-}_{\beth_i}(\partial)\right\}\\
&= \max\{\widehat{\Finv}^{-}_{\aleph}(\wp), \widehat{\Finv}^{-}_{\aleph}(\partial)\}.
\end{aligned}$$

$$\begin{aligned}
1-\widehat{\daleth}^{-}_{\aleph}(\wp\triangle_1\partial) &= \sup_{i\in I}\ 1-\widehat{\daleth}^{-}_{\beth_i}(\wp\triangle_1\partial)\\
&\leqq \sup_{i\in I}\ \max\{1-\widehat{\daleth}^{-}_{\beth_i}(\wp), 1-\widehat{\daleth}^{-}_{\beth_i}(\partial)\}\\
&= \max\left\{\sup_{i\in I}\ 1-\widehat{\daleth}^{-}_{\beth_i}(\wp), \sup_{i\in I}\ 1-\widehat{\daleth}^{-}_{\beth_i}(\partial)\right\}\\
&= \max\{1-\widehat{\daleth}^{-}_{\aleph}(\wp), 1-\widehat{\daleth}^{-}_{\aleph}(\partial)\}.
\end{aligned}$$

Thus $\widehat{\beth}^{\Finv}_{\aleph}(\wp\triangle_1\partial) \leqq \max\{\widehat{\beth}_{\aleph}(\wp), \widehat{\beth}_{\aleph}(\partial)\}$. Similarly, $\widehat{\beth}^{\Finv}_{\aleph}(\wp\triangle_2\partial) \leqq \max\{\widehat{\beth}_{\aleph}(\wp), \widehat{\beth}_{\aleph}(\partial)\}$ and $\widehat{\beth}^{\Finv}_{\aleph}(\wp\triangle_3\partial) \leqq \max\{\widehat{\beth}_{\aleph}(\wp), \widehat{\beth}_{\aleph}(\partial)\}$.

Now,

$$\begin{aligned}
\daleth^{-}_{\aleph}(\wp\triangle_1\partial) &= \inf_{i\in I}\ \daleth^{-}_{\beth_i}(\wp\triangle_1\partial)\\
&\geqq \inf_{i\in I}\ \min\{\daleth^{-}_{\beth_i}(\wp), \daleth^{-}_{\beth_i}(\partial)\}\\
&= \min\left\{\inf_{i\in I}\ \daleth^{-}_{\beth_i}(\wp), \inf_{i\in I}\ \daleth^{-}_{\beth_i}(\partial)\right\}\\
&= \min\{\daleth^{-}_{\aleph}(\wp), \daleth^{-}_{\aleph}(\partial)\}.
\end{aligned}$$

$$\begin{aligned}
1-\Finv^{-}_{\aleph}(\wp\triangle_1\partial) &= \inf_{i\in I}\ 1-\Finv^{-}_{\beth_i}(\wp\triangle_1\partial)\\
&\geqq \inf_{i\in I}\ \min\{1-\Finv^{-}_{\beth_i}(\wp), 1-\Finv^{-}_{\beth_i}(\partial)\}\\
&= \min\left\{\inf_{i\in I}\ 1-\Finv^{-}_{\beth_i}(\wp), \inf_{i\in I}\ 1-\Finv^{-}_{\beth_i}(\partial)\right\}\\
&= \min\{1-\Finv^{-}_{\aleph}(\wp), 1-\Finv^{-}_{\aleph}(\partial)\}.
\end{aligned}$$

Thus $\beth^{\daleth}_{\aleph}(\wp\triangle_1\partial) \geqq \min\{\beth_{\aleph}(\wp), \beth_{\aleph}(\partial)\}$. Similarly, $\beth^{\daleth}_{\aleph}(\wp\triangle_2\partial) \geqq \min\{\beth_{\aleph}(\wp), \beth_{\aleph}(\partial)\}$ and $\beth^{\daleth}_{\aleph}(\wp\triangle_3\partial) \geqq \min\{\beth_{\aleph}(\wp), \beth_{\aleph}(\partial)\}$. Now,

$$\begin{aligned}
\gimel^{-}_{\aleph}(\wp\triangle_1\partial) &= \inf_{i\in I-}\ \gimel^{-}_{\beth_i}(\wp\triangle_1\partial)\\
&\geqq \inf_{i\in I-}\ \frac{\gimel^{-}_{\beth_i}(\wp)+\gimel^{-}_{\beth_i}(\partial)}{2}\\
&= \frac{\inf_{i\in I-}\ \gimel^{-}_{\beth_i}(\wp)+\inf_{i\in I-}\ \gimel^{-}_{\beth_i}(\partial)}{2}\\
&= \frac{\gimel^{-}_{\aleph}(\wp)+\gimel^{-}_{\aleph}(\partial)}{2}.
\end{aligned}$$

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$$\begin{aligned}
\gimel^{+}_{\aleph}(\wp\triangle_1\partial) &= \inf_{i\in I+}\ \gimel^{+}_{\beth_i}(\wp\triangle_1\partial)\\
&\geqq \inf_{i\in I+}\ \frac{\gimel^{+}_{\beth_i}(\wp)+\gimel^{+}_{\beth_i}(\partial)}{2}\\
&= \frac{\inf\limits_{i\in I+}\ \gimel^{+}_{\beth_i}(\wp)+\inf\limits_{i\in I+}\ \gimel^{+}_{\beth_i}(\partial)}{2}\\
&= \frac{\gimel^{+}_{\aleph}(\wp)+\gimel^{+}_{\aleph}(\partial)}{2}.
\end{aligned}$$

Thus $\beth^{\gimel}_{\aleph}(\wp\triangle_1\partial) \geqq \min\{\beth_{\aleph}(\wp),\beth_{\aleph}(\partial)\}$. Similarly, $\beth^{\gimel}_{\aleph}(\wp\triangle_2\partial) \geqq \min\{\beth_{\aleph}(\wp),\beth_{\aleph}(\partial)\}$ and $\beth^{\gimel}_{\aleph}(\wp\triangle_3\partial) \geqq \min\{\beth_{\aleph}(\wp),\beth_{\aleph}(\partial)\}$.
Now,

$$\begin{aligned}
\dashv^{-}_{\aleph}(\wp\triangle_1\partial) &= \sup_{i\in I}\ \dashv^{-}_{\beth_i}(\wp\triangle_1\partial)\\
&\leqq \sup_{i\in I}\ \max\{\dashv^{-}_{\beth_i}(\wp),\dashv^{-}_{\beth_i}(\partial)\}\\
&= \max\left\{\sup_{i\in I}\ \dashv^{-}_{\beth_i}(\wp),\sup_{i\in I}\ \dashv^{-}_{\beth_i}(\partial)\right\}\\
&= \max\{\dashv^{-}_{\aleph}(\wp),\dashv^{-}_{\aleph}(\partial)\}.
\end{aligned}$$

$$\begin{aligned}
1-\daleth^{-}_{\aleph}(\wp\triangle_1\partial) &= \sup_{i\in I}\ 1-\daleth^{-}_{\beth_i}(\wp\triangle_1\partial)\\
&\leqq \sup_{i\in I}\ \max\{1-\daleth^{-}_{\beth_i}(\wp),1-\daleth^{-}_{\beth_i}(\partial)\}\\
&= \max\left\{\sup_{i\in I}\ 1-\daleth^{-}_{\beth_i}(\wp),\sup_{i\in I}\ 1-\daleth^{-}_{\beth_i}(\partial)\right\}\\
&= \max\{1-\daleth^{-}_{\aleph}(\wp),1-\daleth^{-}_{\aleph}(\partial)\}.
\end{aligned}$$

Thus $\beth^{\dashv}_{\aleph}(\wp\triangle_1\partial) \leqq \max\{\beth_{\aleph}(\wp),\beth_{\aleph}(\partial)\}$. Similarly, $\beth^{\dashv}_{\aleph}(\wp\triangle_2\partial) \leqq \max\{\beth_{\aleph}(\wp),\beth_{\aleph}(\partial)\}$ and $\beth^{\dashv}_{\aleph}(\wp\triangle_3\partial) \leqq \max\{\beth_{\aleph}(\wp),\beth_{\aleph}(\partial)\}$. Hence, $\aleph$ is a IVNCVSBS of Ξ.

Theorem 3.4. *If $\aleph$ and $\hbar$ are the $IVNCVSBS^s$ of Ξ_1 and Ξ_2 respectively, then $\aleph\ltimes\hbar$ is a IVNCVSBS of $\Xi_1\ltimes\Xi_2$.*

Proof. Let $\aleph$ and $\hbar$ be the $IVNCVSBS^s$ of Ξ_1 and Ξ_2 respectively. Let $\wp_1,\wp_2\in\Xi_1$ and $\partial_1,\partial_2\in\Xi_2$. Then $(\wp_1,\partial_1),(\wp_2,\partial_2)$ belong to $\Xi_1\ltimes\Xi_2$. Now

$$\begin{aligned}
\widehat{\daleth}^{-}_{\aleph\ltimes\hbar}[(\wp_1,\partial_1)\triangle_1(\wp_2,\partial_2)] &= \widehat{\daleth}^{-}_{\aleph\ltimes\hbar}(\wp_1\triangle_1\wp_2,\partial_1\triangle_1\partial_2)\\
&= \min\{\widehat{\daleth}^{-}_{\aleph}(\wp_1\triangle_1\wp_2),\widehat{\daleth}^{-}_{\hbar}(\partial_1\triangle_1\partial_2)\}\\
&\geqq \min\{\min\{\widehat{\daleth}^{-}_{\aleph}(\wp_1),\widehat{\daleth}^{-}_{\aleph}(\wp_2)\},\min\{\widehat{\daleth}^{-}_{\hbar}(\partial_1),\widehat{\daleth}^{-}_{\hbar}(\partial_2)\}\}\\
&= \min\{\min\{\widehat{\daleth}^{-}_{\aleph}(\wp_1),\widehat{\daleth}^{-}_{\hbar}(\partial_1)\},\min\{\widehat{\daleth}^{-}_{\aleph}(\wp_2),\widehat{\daleth}^{-}_{\hbar}(\partial_2)\}\}\\
&= \min\{\widehat{\daleth}^{-}_{\aleph\ltimes\hbar}(\wp_1,\partial_1),\widehat{\daleth}^{-}_{\aleph\ltimes\hbar}(\wp_2,\partial_2)\}.
\end{aligned}$$

$$\begin{aligned}
1-\widehat{\dashv}^{-}_{\aleph\ltimes\hbar}[(\wp_1,\partial_1)\triangle_1(\wp_2,\partial_2)] &= 1-\widehat{\dashv}^{-}_{\aleph\ltimes\hbar}(\wp_1\triangle_1\wp_2,\partial_1\triangle_1\partial_2)\\
&= \min\{1-\widehat{\dashv}^{-}_{\aleph}(\wp_1\triangle_1\wp_2),1-\widehat{\dashv}^{-}_{\hbar}(\partial_1\triangle_1\partial_2)\}\\
&\geqq \min\{\min\{1-\widehat{\dashv}^{-}_{\aleph}(\wp_1),1-\widehat{\dashv}^{-}_{\aleph}(\wp_2)\},\min\{1-\widehat{\dashv}^{-}_{\hbar}(\partial_1),1-\widehat{\dashv}^{-}_{\hbar}(\partial_2)\}\}\\
&= \min\{\min\{1-\widehat{\dashv}^{-}_{\aleph}(\wp_1),1-\widehat{\dashv}^{-}_{\hbar}(\partial_1)\},\min\{1-\widehat{\dashv}^{-}_{\aleph}(\wp_2),1-\widehat{\dashv}^{-}_{\hbar}(\partial_2)\}\}\\
&= \min\{1-\widehat{\dashv}^{-}_{\aleph\ltimes\hbar}(\wp_1,\partial_1),1-\widehat{\dashv}^{-}_{\aleph\ltimes\hbar}(\wp_2,\partial_2)\}.
\end{aligned}$$

Thus $\widehat{\beth}^{\daleth}_{\aleph\ltimes\hbar}(\wp\triangle_1\partial) \geqq \min\{\widehat{\beth}^{\daleth}_{\aleph\ltimes\hbar}(\wp),\widehat{\beth}^{\daleth}_{\aleph\ltimes\hbar}(\partial)\}$. Similarly, $\widehat{\beth}^{\daleth}_{\aleph\ltimes\hbar}(\wp\triangle_2\partial) \geqq \min\{\widehat{\beth}^{\daleth}_{\aleph\ltimes\hbar}(\wp),\widehat{\beth}^{\daleth}_{\aleph\ltimes\hbar}(\partial)\}$ and

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$\widehat{\beth\daleth}_{\aleph\ltimes\hbar}(\wp\triangle_3\partial) \geqq \min\{\widehat{\beth\daleth}_{\aleph\ltimes\hbar}(\wp), \widehat{\beth\daleth}_{\aleph\ltimes\hbar}(\partial)\}$. Now,

$$\begin{aligned}
\widehat{\gimel}^{-}_{\aleph\ltimes\hbar}[(\wp_1,\partial_1)\triangle_1(\wp_2,\partial_2)] &= \widehat{\gimel}^{-}_{\aleph\ltimes\hbar}(\wp_1\triangle_1\wp_2, \partial_1\triangle_1\partial_2)\\
&= \frac{\widehat{\gimel}^{-}_{\aleph}(\wp_1\triangle_1\wp_2) + \widehat{\gimel}^{-}_{\hbar}(\partial_1\triangle_1\partial_2)}{2}\\
&\geqq \frac{1}{2}\left[\frac{\widehat{\gimel}^{-}_{\aleph}(\wp_1) + \widehat{\gimel}^{-}_{\aleph}(\wp_2)}{2} + \frac{\widehat{\gimel}^{-}_{\hbar}(\partial_1) + \widehat{\gimel}^{-}_{\hbar}(\partial_2)}{2}\right]\\
&= \frac{1}{2}\left[\frac{\widehat{\gimel}^{-}_{\aleph}(\wp_1) + \widehat{\gimel}^{-}_{\hbar}(\partial_1)}{2} + \frac{\widehat{\gimel}^{-}_{\aleph}(\wp_2) + \widehat{\gimel}^{-}_{\hbar}(\partial_2)}{2}\right]\\
&= \frac{1}{2}\Big[\widehat{\gimel}^{-}_{\aleph\ltimes\hbar}(\wp_1,\partial_1) + \widehat{\gimel}^{-}_{\aleph\ltimes\hbar}(\wp_2,\partial_2)\Big].
\end{aligned}$$

$$\begin{aligned}
\widehat{\gimel}^{+}_{\aleph\ltimes\hbar}[(\wp_1,\partial_1)\triangle_1(\wp_2,\partial_2)] &= \widehat{\gimel}^{+}_{\aleph\ltimes\hbar}(\wp_1\triangle_1\wp_2, \partial_1\triangle_1\partial_2)\\
&= \frac{\widehat{\gimel}^{+}_{\aleph}(\wp_1\triangle_1\wp_2) + \widehat{\gimel}^{+}_{\hbar}(\partial_1\triangle_1\partial_2)}{2}\\
&\geqq \frac{1}{2}\left[\frac{\widehat{\gimel}^{+}_{\aleph}(\wp_1) + \widehat{\gimel}^{+}_{\aleph}(\wp_2)}{2} + \frac{\widehat{\gimel}^{+}_{\hbar}(\partial_1) + \widehat{\gimel}^{+}_{\hbar}(\partial_2)}{2}\right]\\
&= \frac{1}{2}\left[\frac{\widehat{\gimel}^{+}_{\aleph}(\wp_1) + \widehat{\gimel}^{+}_{\hbar}(\partial_1)}{2} + \frac{\widehat{\gimel}^{+}_{\aleph}(\wp_2) + \widehat{\gimel}^{+}_{\hbar}(\partial_2)}{2}\right]\\
&= \frac{1}{2}\Big[\widehat{\gimel}^{+}_{\aleph\ltimes\hbar}(\wp_1,\partial_1) + \widehat{\gimel}^{+}_{\aleph\ltimes\hbar}(\wp_2,\partial_2)\Big].
\end{aligned}$$

Thus $\widehat{\beth\gimel}_{\aleph\ltimes\hbar}(\wp\triangle_1\partial) \geqq \frac{1}{2}\Big[\widehat{\beth\gimel}_{\aleph\ltimes\hbar}(\wp_1,\partial_1) + \widehat{\beth\gimel}_{\aleph\ltimes\hbar}(\wp_2,\partial_2)\Big]$. Similarly, $\widehat{\beth\gimel}_{\aleph\ltimes\hbar}(\wp\triangle_2\partial) \geqq \frac{1}{2}\Big[\widehat{\beth\gimel}_{\aleph\ltimes\hbar}(\wp_1,\partial_1) + \widehat{\beth\gimel}_{\aleph\ltimes\hbar}(\wp_2,\partial_2)\Big]$ and $\widehat{\beth\gimel}_{\aleph\ltimes\hbar}(\wp\triangle_3\partial) \geqq \frac{1}{2}\Big[\widehat{\beth\gimel}_{\aleph\ltimes\hbar}(\wp_1,\partial_1) + \widehat{\beth\gimel}_{\aleph\ltimes\hbar}(\wp_2,\partial_2)\Big]$. Now

$$\begin{aligned}
\widehat{\dashv}^{-}_{\aleph\ltimes\hbar}[(\wp_1,\partial_1)\triangle_1(\wp_2,\partial_2)] &= \widehat{\dashv}^{-}_{\aleph\ltimes\hbar}(\wp_1\triangle_1\wp_2, \partial_1\triangle_1\partial_2)\\
&= \max\{\widehat{\dashv}^{-}_{\aleph}(\wp_1\triangle_1\wp_2), \widehat{\dashv}^{-}_{\hbar}(\partial_1\triangle_1\partial_2)\}\\
&\leqq \max\{\max\{\widehat{\dashv}^{-}_{\aleph}(\wp_1), \widehat{\dashv}^{-}_{\aleph}(\wp_2)\}, \max\{\widehat{\dashv}^{-}_{\hbar}(\partial_1), \widehat{\dashv}^{-}_{\hbar}(\partial_2)\}\}\\
&= \max\{\max\{\widehat{\dashv}^{-}_{\aleph}(\wp_1), \widehat{\dashv}^{-}_{\hbar}(\partial_1)\}, \max\{\widehat{\dashv}^{-}_{\aleph}(\wp_2), \widehat{\dashv}^{-}_{\hbar}(\partial_2)\}\}\\
&= \max\{\widehat{\dashv}^{-}_{\aleph\ltimes\hbar}(\wp_1,\partial_1), \widehat{\dashv}^{-}_{\aleph\ltimes\hbar}(\wp_2,\partial_2)\}.
\end{aligned}$$

$$\begin{aligned}
1-\widehat{\daleth}^{-}_{\aleph\ltimes\hbar}[(\wp_1,\partial_1)\triangle_1(\wp_2,\partial_2)] &= 1-\widehat{\daleth}^{-}_{\aleph\ltimes\hbar}(\wp_1\triangle_1\wp_2, \partial_1\triangle_1\partial_2)\\
&= \max\{1-\widehat{\daleth}^{-}_{\aleph}(\wp_1\triangle_1\wp_2), 1-\widehat{\daleth}^{-}_{\hbar}(\partial_1\triangle_1\partial_2)\}\\
&\leqq \max\{\max\{1-\widehat{\daleth}^{-}_{\aleph}(\wp_1), 1-\widehat{\daleth}^{-}_{\aleph}(\wp_2)\}, \max\{1-\widehat{\daleth}^{-}_{\hbar}(\partial_1), 1-\widehat{\daleth}^{-}_{\hbar}(\partial_2)\}\}\\
&= \max\{\max\{1-\widehat{\daleth}^{-}_{\aleph}(\wp_1), 1-\widehat{\daleth}^{-}_{\hbar}(\partial_1)\}, \max\{1-\widehat{\daleth}^{-}_{\aleph}(\wp_2), 1-\widehat{\daleth}^{-}_{\hbar}(\partial_2)\}\}\\
&= \max\{1-\widehat{\daleth}^{-}_{\aleph\ltimes\hbar}(\wp_1,\partial_1), 1-\widehat{\daleth}^{-}_{\aleph\ltimes\hbar}(\wp_2,\partial_2)\}.
\end{aligned}$$

Thus $\widehat{\beth\dashv}_{\aleph\ltimes\hbar}(\wp\triangle_1\partial) \leqq \max\{\widehat{\beth\dashv}_{\aleph\ltimes\hbar}(\wp), \widehat{\beth\dashv}_{\aleph\ltimes\hbar}(\partial)\}$. Similarly, $\widehat{\beth\dashv}_{\aleph\ltimes\hbar}(\wp\triangle_2\partial) \leqq \max\{\widehat{\beth\dashv}_{\aleph\ltimes\hbar}(\wp), \widehat{\beth\dashv}_{\aleph\ltimes\hbar}(\partial)\}$ and $\widehat{\beth\dashv}_{\aleph\ltimes\hbar}(\wp\triangle_3\partial) \leqq \max\{\widehat{\beth\dashv}_{\aleph\ltimes\hbar}(\wp), \widehat{\beth\dashv}_{\aleph\ltimes\hbar}(\partial)\}$.
Now

$$\begin{aligned}
\daleth^{-}_{\aleph\ltimes\hbar}[(\wp_1,\partial_1)\triangle_1(\wp_2,\partial_2)] &= \daleth^{-}_{\aleph\ltimes\hbar}(\wp_1\triangle_1\wp_2, \partial_1\triangle_1\partial_2)\\
&= \min\{\daleth^{-}_{\aleph}(\wp_1\triangle_1\wp_2), \daleth^{-}_{\hbar}(\partial_1\triangle_1\partial_2)\}\\
&\geqq \min\{\min\{\daleth^{-}_{\aleph}(\wp_1), \daleth^{-}_{\aleph}(\wp_2)\}, \min\{\daleth^{-}_{\hbar}(\partial_1), \daleth^{-}_{\hbar}(\partial_2)\}\}\\
&= \min\{\min\{\daleth^{-}_{\aleph}(\wp_1), \daleth^{-}_{\hbar}(\partial_1)\}, \min\{\daleth^{-}_{\aleph}(\wp_2), \daleth^{-}_{\hbar}(\partial_2)\}\}\\
&= \min\{\daleth^{-}_{\aleph\ltimes\hbar}(\wp_1,\partial_1), \daleth^{-}_{\aleph\ltimes\hbar}(\wp_2,\partial_2)\}.
\end{aligned}$$

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$$
\begin{aligned}
1-\dashv^{-}_{\aleph\ltimes\hbar}[(\wp_1,\partial_1)\triangle_1(\wp_2,\partial_2)] &= 1-\dashv^{-}_{\aleph\ltimes\hbar}(\wp_1\triangle_1\wp_2,\partial_1\triangle_1\partial_2)\\
&=\min\{1-\dashv^{-}_{\aleph}(\wp_1\triangle_1\wp_2),1-\dashv^{-}_{\hbar}(\partial_1\triangle_1\partial_2)\}\\
&\geqq \min\{\min\{1-\dashv^{-}_{\aleph}(\wp_1),1-\dashv^{-}_{\aleph}(\wp_2)\},\min\{1-\dashv^{-}_{\hbar}(\partial_1),1-\dashv^{-}_{\hbar}(\partial_2)\}\}\\
&=\min\{\min\{1-\dashv^{-}_{\aleph}(\wp_1),1-\dashv^{-}_{\hbar}(\partial_1)\},\min\{1-\dashv^{-}_{\aleph}(\wp_2),1-\dashv^{-}_{\hbar}(\partial_2)\}\}\\
&=\min\{1-\dashv^{-}_{\aleph\ltimes\hbar}(\wp_1,\partial_1),1-\dashv^{-}_{\aleph\ltimes\hbar}(\wp_2,\partial_2)\}.
\end{aligned}
$$

Thus $\sqsupset^{\daleth}_{\aleph\ltimes\hbar}(\wp\triangle_1\partial)\geqq\min\{\sqsupset^{\daleth}_{\aleph\ltimes\hbar}(\wp),\sqsupset^{\daleth}_{\aleph\ltimes\hbar}(\partial)\}$. Similarly, $\sqsupset^{\daleth}_{\aleph\ltimes\hbar}(\wp\triangle_2\partial)\geqq\min\{\sqsupset^{\daleth}_{\aleph\ltimes\hbar}(\wp),\sqsupset^{\daleth}_{\aleph\ltimes\hbar}(\partial)\}$ and $\sqsupset^{\daleth}_{\aleph\ltimes\hbar}(\wp\triangle_3\partial)\geqq\min\{\sqsupset^{\daleth}_{\aleph\ltimes\hbar}(\wp),\sqsupset^{\daleth}_{\aleph\ltimes\hbar}(\partial)\}$. Now,

$$
\begin{aligned}
\gimel^{-}_{\aleph\ltimes\hbar}[(\wp_1,\partial_1)\triangle_1(\wp_2,\partial_2)] &= \gimel^{-}_{\aleph\ltimes\hbar}(\wp_1\triangle_1\wp_2,\partial_1\triangle_1\partial_2)\\
&=\frac{\gimel^{-}_{\aleph}(\wp_1\triangle_1\wp_2)+\gimel^{-}_{\hbar}(\partial_1\triangle_1\partial_2)}{2}\\
&\geqq\frac{1}{2}\left[\frac{\gimel^{-}_{\aleph}(\wp_1)+\gimel^{-}_{\aleph}(\wp_2)}{2}+\frac{\gimel^{-}_{\hbar}(\partial_1)+\gimel^{-}_{\hbar}(\partial_2)}{2}\right]\\
&=\frac{1}{2}\left[\frac{\gimel^{-}_{\aleph}(\wp_1)+\gimel^{-}_{\hbar}(\partial_1)}{2}+\frac{\gimel^{-}_{\aleph}(\wp_2)+\gimel^{-}_{\hbar}(\partial_2)}{2}\right]\\
&=\frac{1}{2}\Big[\gimel^{-}_{\aleph\ltimes\hbar}(\wp_1,\partial_1)+\gimel^{-}_{\aleph\ltimes\hbar}(\wp_2,\partial_2)\Big].
\end{aligned}
$$

$$
\begin{aligned}
\gimel^{+}_{\aleph\ltimes\hbar}[(\wp_1,\partial_1)\triangle_1(\wp_2,\partial_2)] &= \gimel^{+}_{\aleph\ltimes\hbar}(\wp_1\triangle_1\wp_2,\partial_1\triangle_1\partial_2)\\
&=\frac{\gimel^{+}_{\aleph}(\wp_1\triangle_1\wp_2)+\gimel^{+}_{\hbar}(\partial_1\triangle_1\partial_2)}{2}\\
&\geqq\frac{1}{2}\left[\frac{\gimel^{+}_{\aleph}(\wp_1)+\gimel^{+}_{\aleph}(\wp_2)}{2}+\frac{\gimel^{+}_{\hbar}(\partial_1)+\gimel^{+}_{\hbar}(\partial_2)}{2}\right]\\
&=\frac{1}{2}\left[\frac{\gimel^{+}_{\aleph}(\wp_1)+\gimel^{+}_{\hbar}(\partial_1)}{2}+\frac{\gimel^{+}_{\aleph}(\wp_2)+\gimel^{+}_{\hbar}(\partial_2)}{2}\right]\\
&=\frac{1}{2}\Big[\gimel^{+}_{\aleph\ltimes\hbar}(\wp_1,\partial_1)+\gimel^{+}_{\aleph\ltimes\hbar}(\wp_2,\partial_2)\Big].
\end{aligned}
$$

Thus $\sqsupset^{\gimel}_{\aleph\ltimes\hbar}(\wp\triangle_1\partial)\geqq\frac{1}{2}\Big[\sqsupset^{\gimel}_{\aleph\ltimes\hbar}(\wp_1,\partial_1)+\sqsupset^{\gimel}_{\aleph\ltimes\hbar}(\wp_2,\partial_2)\Big]$. Similarly, $\sqsupset^{\gimel}_{\aleph\ltimes\hbar}(\wp\triangle_2\partial)\geqq\frac{1}{2}\Big[\sqsupset^{\gimel}_{\aleph\ltimes\hbar}(\wp_1,\partial_1)+\sqsupset^{\gimel}_{\aleph\ltimes\hbar}(\wp_2,\partial_2)\Big]$ and $\sqsupset^{\gimel}_{\aleph\ltimes\hbar}(\wp\triangle_3\partial)\geqq\frac{1}{2}\Big[\sqsupset^{\gimel}_{\aleph\ltimes\hbar}(\wp_1,\partial_1)+\sqsupset^{\gimel}_{\aleph\ltimes\hbar}(\wp_2,\partial_2)\Big]$. Now

$$
\begin{aligned}
\dashv^{-}_{\aleph\ltimes\hbar}[(\wp_1,\partial_1)\triangle_1(\wp_2,\partial_2)] &= \dashv^{-}_{\aleph\ltimes\hbar}(\wp_1\triangle_1\wp_2,\partial_1\triangle_1\partial_2)\\
&=\max\{\dashv^{-}_{\aleph}(\wp_1\triangle_1\wp_2),\dashv^{-}_{\hbar}(\partial_1\triangle_1\partial_2)\}\\
&\leqq\max\{\max\{\dashv^{-}_{\aleph}(\wp_1),\dashv^{-}_{\aleph}(\wp_2)\},\max\{\dashv^{-}_{\hbar}(\partial_1),\dashv^{-}_{\hbar}(\partial_2)\}\}\\
&=\max\{\max\{\dashv^{-}_{\aleph}(\wp_1),\dashv^{-}_{\hbar}(\partial_1)\},\max\{\dashv^{-}_{\aleph}(\wp_2),\dashv^{-}_{\hbar}(\partial_2)\}\}\\
&=\max\{\dashv^{-}_{\aleph\ltimes\hbar}(\wp_1,\partial_1),\dashv^{-}_{\aleph\ltimes\hbar}(\wp_2,\partial_2)\}.
\end{aligned}
$$

$$
\begin{aligned}
1-\daleth^{-}_{\aleph\ltimes\hbar}[(\wp_1,\partial_1)\triangle_1(\wp_2,\partial_2)] &= 1-\daleth^{-}_{\aleph\ltimes\hbar}(\wp_1\triangle_1\wp_2,\partial_1\triangle_1\partial_2)\\
&=\max\{1-\daleth^{-}_{\aleph}(\wp_1\triangle_1\wp_2),1-\daleth^{-}_{\hbar}(\partial_1\triangle_1\partial_2)\}\\
&\leqq\max\{\max\{1-\daleth^{-}_{\aleph}(\wp_1),1-\daleth^{-}_{\aleph}(\wp_2)\},\max\{1-\daleth^{-}_{\hbar}(\partial_1),1-\daleth^{-}_{\hbar}(\partial_2)\}\}\\
&=\max\{\max\{1-\daleth^{-}_{\aleph}(\wp_1),1-\daleth^{-}_{\hbar}(\partial_1)\},\max\{1-\daleth^{-}_{\aleph}(\wp_2),1-\daleth^{-}_{\hbar}(\partial_2)\}\}\\
&=\max\{1-\daleth^{-}_{\aleph\ltimes\hbar}(\wp_1,\partial_1),1-\daleth^{-}_{\aleph\ltimes\hbar}(\wp_2,\partial_2)\}.
\end{aligned}
$$

Thus $\sqsupset^{\dashv}_{\aleph\ltimes\hbar}(\wp\triangle_1\partial)\leqq\max\{\sqsupset^{\dashv}_{\aleph\ltimes\hbar}(\wp),\sqsupset^{\dashv}_{\aleph\ltimes\hbar}(\partial)\}$. Similarly, $\sqsupset^{\dashv}_{\aleph\ltimes\hbar}(\wp\triangle_2\partial)\leqq\max\{\sqsupset^{\dashv}_{\aleph\ltimes\hbar}(\wp),\sqsupset^{\dashv}_{\aleph\ltimes\hbar}(\partial)\}$ and $\sqsupset^{\dashv}_{\aleph\ltimes\hbar}(\wp\triangle_3\partial)\leqq\max\{\sqsupset^{\dashv}_{\aleph\ltimes\hbar}(\wp),\sqsupset^{\dashv}_{\aleph\ltimes\hbar}(\partial)\}$. Hence, $\aleph\ltimes\hbar$ is a IVNCVSBS of Ξ.

Corollary 3.5. *If $\aleph_1,\aleph_2,...,\aleph_n$ are the families of $IVNCVSBS^s$ of $\Xi_1,\Xi_2,...,\Xi_n$ respectively, then $\aleph_1\ltimes\aleph_2\ltimes...\ltimes\aleph_n$ is a IVNCVSBS of $\Xi_1\ltimes\Xi_2\ltimes...\ltimes\Xi_n$.*

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Definition 3.6. Let $\aleph$ be a neutrosophic VS in Ξ, the strongest interval-valued neutrosophic cubic vague relation (SIVNCVR) on Ξ is defined as

$$\left\{\begin{array}{l} \widehat{\beth}^{\daleth}_{\Upsilon}(\wp,\partial)=\min\{\widehat{\beth}^{\daleth}_{\aleph}(\wp),\widehat{\beth}^{\daleth}_{\aleph}(\partial)\} \\ \widehat{\beth}^{\gimel}_{\Upsilon}(\wp,\partial)=\frac{\widehat{\beth}^{\gimel}_{\aleph}(\wp)+\widehat{\beth}^{\gimel}_{\aleph}(\partial)}{2} \\ \widehat{\beth}^{\exists}_{\Upsilon}(\wp,\partial)=\max\{\widehat{\beth}^{\exists}_{\aleph}(\wp),\widehat{\beth}^{\exists}_{\aleph}(\partial)\} \end{array}\right\}.$$

$$\left\{\begin{array}{l} \beth^{\daleth}_{\Upsilon}(\wp,\partial)=\min\{\beth^{\daleth}_{\aleph}(\wp),\beth^{\daleth}_{\aleph}(\partial)\} \\ \beth^{\gimel}_{\Upsilon}(\wp,\partial)=\frac{\beth^{\gimel}_{\aleph}(\wp)+\beth^{\gimel}_{\aleph}(\partial)}{2} \\ \beth^{\exists}_{\Upsilon}(\wp,\partial)=\max\{\beth^{\exists}_{\aleph}(\wp),\beth^{\exists}_{\aleph}(\partial)\} \end{array}\right\}.$$

Theorem 3.7. *Let $\aleph$ be the IVNCVSBS of Ξ and Υ be the SNSVR of Ξ. Then $\aleph$ is a IVNCVSBS of Ξ if and only if Υ is a IVNCVSBS of $\Xi \ltimes \Xi$.*

Proof. Let $\aleph$ be the IVNCVSBS of Ξ and Υ be the SNSVR of Ξ. Then for any $\wp=(\wp_1,\wp_2)$ and $\partial=(\partial_1,\partial_2)$ are in $\Xi \ltimes \Xi$. Now,

$$\begin{aligned} \widehat{\daleth}^{-}_{\Upsilon}(\wp\triangle_1\partial) &= \widehat{\daleth}^{-}_{\Upsilon}[((\wp_1,\wp_2)\triangle_1(\partial_1,\partial_2)] \\ &= \widehat{\daleth}^{-}_{\Upsilon}(\wp_1\triangle_1\partial_1,\wp_2\triangle_1\partial_2) \\ &= \min\{\widehat{\daleth}^{-}_{\aleph}(\wp_1\triangle_1\partial_1),\widehat{\daleth}^{-}_{\aleph}(\wp_2\triangle_1\partial_2)\} \\ &\geqq \min\{\min\{\widehat{\daleth}^{-}_{\aleph}(\wp_1),\widehat{\daleth}^{-}_{\aleph}(\partial_1)\},\min\{\widehat{\daleth}^{-}_{\aleph}(\wp_2),\widehat{\daleth}^{-}_{\aleph}(\partial_2)\}\} \\ &= \min\{\min\{\widehat{\daleth}^{-}_{\aleph}(\wp_1),\widehat{\daleth}^{-}_{\aleph}(\wp_2)\},\min\{\widehat{\daleth}^{-}_{\aleph}(\partial_1),\widehat{\daleth}^{-}_{\aleph}(\partial_2)\}\} \\ &= \min\{\widehat{\daleth}^{-}_{\Upsilon}(\wp_1,\wp_2),\widehat{\daleth}^{-}_{\Upsilon}(\partial_1,\partial_2)\} \\ &= \min\{\widehat{\daleth}^{-}_{\Upsilon}(\wp),\widehat{\daleth}^{-}_{\Upsilon}(\partial)\}. \end{aligned}$$

$$\begin{aligned} 1-\widehat{\exists}^{-}_{\Upsilon}(\wp\triangle_1\partial) &= 1-\widehat{\exists}^{-}_{\Upsilon}[((\wp_1,\wp_2)\triangle_1(\partial_1,\partial_2)] \\ &= 1-\widehat{\exists}^{-}_{\Upsilon}(\wp_1\triangle_1\partial_1,\wp_2\triangle_1\partial_2) \\ &= \min\{1-\widehat{\exists}^{-}_{\aleph}(\wp_1\triangle_1\partial_1),1-\widehat{\exists}^{-}_{\aleph}(\wp_2\triangle_1\partial_2)\} \\ &\geqq \min\{\min\{1-\widehat{\exists}^{-}_{\aleph}(\wp_1),1-\widehat{\exists}^{-}_{\aleph}(\partial_1)\},\min\{1-\widehat{\exists}^{-}_{\aleph}(\wp_2),1-\widehat{\exists}^{-}_{\aleph}(\partial_2)\}\} \\ &= \min\{\min\{1-\widehat{\exists}^{-}_{\aleph}(\wp_1),1-\widehat{\exists}^{-}_{\aleph}(\wp_2)\},\min\{1-\widehat{\exists}^{-}_{\aleph}(\partial_1),1-\widehat{\exists}^{-}_{\aleph}(\partial_2)\}\} \\ &= \min\{1-\widehat{\exists}^{-}_{\Upsilon}(\wp_1,\wp_2),1-\widehat{\exists}^{-}_{\Upsilon}(\partial_1,\partial_2)\} \\ &= \min\{1-\widehat{\exists}^{-}_{\Upsilon}(\wp),1-\widehat{\exists}^{-}_{\Upsilon}(\partial)\}. \end{aligned}$$

Thus $\widehat{\beth}^{\daleth}_{\Upsilon}(\wp\triangle_1\partial)\geqq\min\{\widehat{\beth}^{\daleth}_{\Upsilon}(\wp),\widehat{\beth}^{\daleth}_{\Upsilon}(\partial)\}$. Similarly, $\widehat{\beth}^{\daleth}_{\Upsilon}(\wp\triangle_2\partial)\geqq\min\{\widehat{\beth}^{\daleth}_{\Upsilon}(\wp),\widehat{\beth}^{\daleth}_{\Upsilon}(\partial)\}$ and $\widehat{\beth}^{\daleth}_{\Upsilon}(\wp\triangle_3\partial)\geqq\min\{\widehat{\beth}^{\daleth}_{\Upsilon}(\wp),\widehat{\beth}^{\daleth}_{\Upsilon}(\partial)\}$. Now,

$$\begin{aligned} \widehat{\gimel}^{-}_{\Upsilon}(\wp\triangle_1\partial) &= \widehat{\gimel}^{-}_{\Upsilon}[((\wp_1,\wp_2)\triangle_1(\partial_1,\partial_2)] \\ &= \widehat{\gimel}^{-}_{\Upsilon}(\wp_1\triangle_1\partial_1,\wp_2\triangle_1\partial_2) \\ &= \frac{\widehat{\gimel}^{-}_{\aleph}(\wp_1\triangle_1\partial_1)+\widehat{\gimel}^{-}_{\aleph}(\wp_2\triangle_1\partial_2)}{2} \\ &\geqq \frac{1}{2}\left[\frac{\widehat{\gimel}^{-}_{\aleph}(\wp_1)+\widehat{\gimel}^{-}_{\aleph}(\partial_1)}{2}+\frac{\widehat{\gimel}^{-}_{\aleph}(\wp_2)+\widehat{\gimel}^{-}_{\aleph}(\partial_2)}{2}\right] \\ &= \frac{1}{2}\left[\frac{\widehat{\gimel}^{-}_{\aleph}(\wp_1)+\widehat{\gimel}^{-}_{\aleph}(\wp_2)}{2}+\frac{\widehat{\gimel}^{-}_{\aleph}(\partial_1)+\widehat{\gimel}^{-}_{\aleph}(\partial_2)}{2}\right] \\ &= \frac{\widehat{\gimel}^{-}_{\Upsilon}(\wp_1,\wp_2)+\widehat{\gimel}^{-}_{\Upsilon}(\partial_1,\partial_2)}{2} \\ &= \frac{\widehat{\gimel}^{-}_{\Upsilon}(\wp)+\widehat{\gimel}^{-}_{\Upsilon}(\partial)}{2}. \end{aligned}$$

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$$\begin{aligned}
\widehat{\gimel}^{+}_{\Upsilon}(\wp\triangle_1\partial) &= \widehat{\gimel}^{+}_{\Upsilon}[((\wp_1,\wp_2)\triangle_1(\partial_1,\partial_2)] \\
&= \widehat{\gimel}^{+}_{\Upsilon}(\wp_1\triangle_1\partial_1,\wp_2\triangle_1\partial_2) \\
&= \frac{\widehat{\gimel}^{+}_{\aleph}(\wp_1\triangle_1\partial_1)+\widehat{\gimel}^{+}_{\aleph}(\wp_2\triangle_1\partial_2)}{2} \\
&\geqq \frac{1}{2}\left[\frac{\widehat{\gimel}^{+}_{\aleph}(\wp_1)+\widehat{\gimel}^{+}_{\aleph}(\partial_1)}{2}+\frac{\widehat{\gimel}^{+}_{\aleph}(\wp_2)+\widehat{\gimel}^{+}_{\aleph}(\partial_2)}{2}\right] \\
&= \frac{1}{2}\left[\frac{\widehat{\gimel}^{+}_{\aleph}(\wp_1)+\widehat{\gimel}^{+}_{\aleph}(\wp_2)}{2}+\frac{\widehat{\gimel}^{+}_{\aleph}(\partial_1)+\widehat{\gimel}^{+}_{\aleph}(\partial_2)}{2}\right] \\
&= \frac{\widehat{\gimel}^{+}_{\Upsilon}(\wp_1,\wp_2)+\widehat{\gimel}^{+}_{\Upsilon}(\partial_1,\partial_2)}{2} \\
&= \frac{\widehat{\gimel}^{+}_{\Upsilon}(\wp)+\widehat{\gimel}^{+}_{\Upsilon}(\partial)}{2}.
\end{aligned}$$

Thus $\widehat{\beth}^{\gimel}_{\Upsilon}(\wp\triangle_1\partial) \geqq \frac{\widehat{\beth}_{\Upsilon}(\wp)+\widehat{\beth}_{\Upsilon}(\partial)}{2}$. Similarly, $\widehat{\beth}^{\gimel}_{\Upsilon}(\wp\triangle_2\partial) \geqq \frac{\widehat{\beth}_{\Upsilon}(\wp)+\widehat{\beth}_{\Upsilon}(\partial)}{2}$ and $\widehat{\beth}^{\gimel}_{\Upsilon}(\wp\triangle_3\partial) \geqq \frac{\widehat{\beth}_{\Upsilon}(\wp)+\widehat{\beth}_{\Upsilon}(\partial)}{2}$. Similarly, $\widehat{\beth}^{\dashv}_{\Upsilon}(\wp\triangle_1\partial) \leqq \max\{\widehat{\beth}^{\dashv}_{\Upsilon}(\wp),\widehat{\beth}^{\dashv}_{\Upsilon}(\partial)\}$, $\widehat{\beth}^{\dashv}_{\Upsilon}(\wp\triangle_2\partial) \leqq \max\{\widehat{\beth}^{\dashv}_{\Upsilon}(\wp),\widehat{\beth}^{\dashv}_{\Upsilon}(\partial)\}$ and $\widehat{\beth}^{\dashv}_{\Upsilon}(\wp\triangle_3\partial) \leqq \max\{\widehat{\beth}^{\dashv}_{\Upsilon}(\wp),\widehat{\beth}^{\dashv}_{\Upsilon}(\partial)\}$.
Now,

$$\begin{aligned}
\daleth^{-}_{\Upsilon}(\wp\triangle_1\partial) &= \daleth^{-}_{\Upsilon}[((\wp_1,\wp_2)\triangle_1(\partial_1,\partial_2)] \\
&= \daleth^{-}_{\Upsilon}(\wp_1\triangle_1\partial_1,\wp_2\triangle_1\partial_2) \\
&= \min\{\daleth^{-}_{\aleph}(\wp_1\triangle_1\partial_1),\daleth^{-}_{\aleph}(\wp_2\triangle_1\partial_2)\} \\
&\geqq \min\{\min\{\daleth^{-}_{\aleph}(\wp_1),\daleth^{-}_{\aleph}(\partial_1)\},\min\{\daleth^{-}_{\aleph}(\wp_2),\daleth^{-}_{\aleph}(\partial_2)\}\} \\
&= \min\{\min\{\daleth^{-}_{\aleph}(\wp_1),\daleth^{-}_{\aleph}(\wp_2)\},\min\{\daleth^{-}_{\aleph}(\partial_1),\daleth^{-}_{\aleph}(\partial_2)\}\} \\
&= \min\{\daleth^{-}_{\Upsilon}(\wp_1,\wp_2),\daleth^{-}_{\Upsilon}(\partial_1,\partial_2)\} \\
&= \min\{\daleth^{-}_{\Upsilon}(\wp),\daleth^{-}_{\Upsilon}(\partial)\}.
\end{aligned}$$

$$\begin{aligned}
1-\dashv^{-}_{\Upsilon}(\wp\triangle_1\partial) &= 1-\dashv^{-}_{\Upsilon}[((\wp_1,\wp_2)\triangle_1(\partial_1,\partial_2)] \\
&= 1-\dashv^{-}_{\Upsilon}(\wp_1\triangle_1\partial_1,\wp_2\triangle_1\partial_2) \\
&= \min\{1-\dashv^{-}_{\aleph}(\wp_1\triangle_1\partial_1),1-\dashv^{-}_{\aleph}(\wp_2\triangle_1\partial_2)\} \\
&\geqq \min\{\min\{1-\dashv^{-}_{\aleph}(\wp_1),1-\dashv^{-}_{\aleph}(\partial_1)\},\min\{1-\dashv^{-}_{\aleph}(\wp_2),1-\dashv^{-}_{\aleph}(\partial_2)\}\} \\
&= \min\{\min\{1-\dashv^{-}_{\aleph}(\wp_1),1-\dashv^{-}_{\aleph}(\wp_2)\},\min\{1-\dashv^{-}_{\aleph}(\partial_1),1-\dashv^{-}_{\aleph}(\partial_2)\}\} \\
&= \min\{1-\dashv^{-}_{\Upsilon}(\wp_1,\wp_2),1-\dashv^{-}_{\Upsilon}(\partial_1,\partial_2)\} \\
&= \min\{1-\dashv^{-}_{\Upsilon}(\wp),1-\dashv^{-}_{\Upsilon}(\partial)\}.
\end{aligned}$$

Thus $\beth^{\daleth}_{\Upsilon}(\wp\triangle_1\partial) \geqq \min\{\beth^{\daleth}_{\Upsilon}(\wp),\beth^{\daleth}_{\Upsilon}(\partial)\}$. Similarly, $\beth^{\daleth}_{\Upsilon}(\wp\triangle_2\partial) \geqq \min\{\beth^{\daleth}_{\Upsilon}(\wp),\beth^{\daleth}_{\Upsilon}(\partial)\}$ and $\beth^{\daleth}_{\Upsilon}(\wp\triangle_3\partial) \geqq \min\{\beth^{\daleth}_{\Upsilon}(\wp),\beth^{\daleth}_{\Upsilon}(\partial)\}$. Now,

$$\begin{aligned}
\gimel^{-}_{\Upsilon}(\wp\triangle_1\partial) &= \gimel^{-}_{\Upsilon}[((\wp_1,\wp_2)\triangle_1(\partial_1,\partial_2)] \\
&= \gimel^{-}_{\Upsilon}(\wp_1\triangle_1\partial_1,\wp_2\triangle_1\partial_2) \\
&= \frac{\gimel^{-}_{\aleph}(\wp_1\triangle_1\partial_1)+\gimel^{-}_{\aleph}(\wp_2\triangle_1\partial_2)}{2} \\
&\geqq \frac{1}{2}\left[\frac{\gimel^{-}_{\aleph}(\wp_1)+\gimel^{-}_{\aleph}(\partial_1)}{2}+\frac{\gimel^{-}_{\aleph}(\wp_2)+\gimel^{-}_{\aleph}(\partial_2)}{2}\right] \\
&= \frac{1}{2}\left[\frac{\gimel^{-}_{\aleph}(\wp_1)+\gimel^{-}_{\aleph}(\wp_2)}{2}+\frac{\gimel^{-}_{\aleph}(\partial_1)+\gimel^{-}_{\aleph}(\partial_2)}{2}\right] \\
&= \frac{\gimel^{-}_{\Upsilon}(\wp_1,\wp_2)+\gimel^{-}_{\Upsilon}(\partial_1,\partial_2)}{2} \\
&= \frac{\gimel^{-}_{\Upsilon}(\wp)+\gimel^{-}_{\Upsilon}(\partial)}{2}.
\end{aligned}$$

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$$
\begin{aligned}
\beth^+_\Upsilon(\wp\triangle_1\partial) &= \beth^+_\Upsilon[((\wp_1,\wp_2)\triangle_1(\partial_1,\partial_2)]\\
&= \beth^+_\Upsilon(\wp_1\triangle_1\partial_1,\wp_2\triangle_1\partial_2)\\
&= \frac{\beth^+_\aleph(\wp_1\triangle_1\partial_1)+\beth^+_\aleph(\wp_2\triangle_1\partial_2)}{2}\\
&\geqq \frac{1}{2}\left[\frac{\beth^+_\aleph(\wp_1)+\beth^+_\aleph(\partial_1)}{2}+\frac{\beth^+_\aleph(\wp_2)+\beth^+_\aleph(\partial_2)}{2}\right]\\
&= \frac{1}{2}\left[\frac{\beth^+_\aleph(\wp_1)+\beth^+_\aleph(\wp_2)}{2}+\frac{\beth^+_\aleph(\partial_1)+\beth^+_\aleph(\partial_2)}{2}\right]\\
&= \frac{\beth^+_\Upsilon(\wp_1,\wp_2)+\beth^+_\Upsilon(\partial_1,\partial_2)}{2}\\
&= \frac{\beth^+_\Upsilon(\wp)+\beth^+_\Upsilon(\partial)}{2}.
\end{aligned}
$$

Thus $\sqsupset^{\beth}_\Upsilon(\wp\triangle_1\partial) \geqq \frac{\sqsupset_\Upsilon(\wp)+\sqsupset_\Upsilon(\partial)}{2}$. Similarly, $\sqsupset^{\beth}_\Upsilon(\wp\triangle_2\partial) \geqq \frac{\sqsupset_\Upsilon(\wp)+\sqsupset_\Upsilon(\partial)}{2}$ and $\sqsupset^{\beth}_\Upsilon(\wp\triangle_3\partial) \geqq \frac{\sqsupset_\Upsilon(\wp)+\sqsupset_\Upsilon(\partial)}{2}$. Similarly, $\sqsupset^{\dashv}_\Upsilon(\wp\triangle_1\partial) \leqq \max\{\sqsupset^{\dashv}_\Upsilon(\wp),\sqsupset^{\dashv}_\Upsilon(\partial)\}$, $\sqsupset^{\dashv}_\Upsilon(\wp\triangle_2\partial) \leqq \max\{\sqsupset^{\dashv}_\Upsilon(\wp),\sqsupset^{\dashv}_\Upsilon(\partial)\}$ and $\sqsupset^{\dashv}_\Upsilon(\wp\triangle_3\partial) \leqq \max\{\sqsupset^{\dashv}_\Upsilon(\wp),\sqsupset^{\dashv}_\Upsilon(\partial)\}$. Thus, Υ is a IVNCVSBS of $\Xi\ltimes\Xi$.
Conversely let us assume that Υ is a IVNCVSBS of $\Xi\ltimes\Xi$, then for any $\wp=(\wp_1,\wp_2)$ and $\partial=(\partial_1,\partial_2)$ are in $\Xi\ltimes\Xi$. Now,

$$
\begin{aligned}
\min\{\widehat{\daleth}^-_\aleph(\wp_1\triangle_1\partial_1),\widehat{\daleth}^-_\aleph(\wp_2\triangle_1\partial_2)\} &= \widehat{\daleth}^-_\Upsilon(\wp_1\triangle_1\partial_1,\wp_2\triangle_1\partial_2)\\
&= \widehat{\daleth}^-_\Upsilon[(\wp_1,\wp_2)\triangle_1(\partial_1,\partial_2)]\\
&= \widehat{\daleth}^-_\Upsilon(\wp\triangle_1\partial)\\
&\geqq \min\{\widehat{\daleth}^-_\Upsilon(\wp),\widehat{\daleth}^-_\Upsilon(\partial)\}\\
&= \min\{\widehat{\daleth}^-_\Upsilon(\wp_1,\wp_2)\},\widehat{\daleth}^-_\Upsilon(\partial_1,\partial_2)\}\\
&= \min\{\min\{\widehat{\daleth}^-_\aleph(\wp_1),\widehat{\daleth}^-_\aleph(\wp_2)\},\min\{\widehat{\daleth}^-_\aleph(\partial_1),\widehat{\daleth}^-_\aleph(\partial_2)\}\}.
\end{aligned}
$$

If $\widehat{\daleth}^-_\aleph(\wp_1\triangle_1\partial_1) \leqq \widehat{\daleth}^-_\aleph(\wp_2\triangle_1\partial_2)$, then $\widehat{\daleth}^-_\aleph(\wp_1) \leqq \widehat{\daleth}^-_\aleph(\wp_2)$ and $\widehat{\daleth}^-_\aleph(\partial_1) \leqq \widehat{\daleth}^-_\aleph(\partial_2)$. We get $\widehat{\daleth}^-_\aleph(\wp_1\triangle_1\partial_1) \geqq \min\{\widehat{\daleth}^-_\aleph(\wp_1),\widehat{\daleth}^-_\aleph(\partial_1)\}$ for all $\wp_1,\partial_1\in\Xi$, and
$\min\{\widehat{\daleth}^-_\aleph(\wp_1\triangle_2\partial_1),\widehat{\daleth}^-_\aleph(\wp_2\triangle_2\partial_2)\} \geqq \min\{\min\{\widehat{\daleth}^-_\aleph(\wp_1),\widehat{\daleth}^-_\aleph(\wp_2)\},\min\{\widehat{\daleth}^-_\aleph(\partial_1),\widehat{\daleth}^-_\aleph(\partial_2)\}\}$
If $\widehat{\daleth}^-_\aleph(\wp_1\triangle_2\partial_1) \leqq \widehat{\daleth}^-_\aleph(\wp_2\triangle_2\partial_2)$, then $\widehat{\daleth}^-_\aleph(\wp_1\triangle_2\partial_1) \geqq \min\{\widehat{\daleth}^-_\aleph(\wp_1),\widehat{\daleth}^-_\aleph(\partial_1)\}$.
$\min\{\widehat{\daleth}^-_\aleph(\wp_1\triangle_3\partial_1),\widehat{\daleth}^-_\aleph(\wp_2\triangle_3\partial_2)\} \geqq \min\{\min\{\widehat{\daleth}^-_\aleph(\wp_1),\widehat{\daleth}^-_\aleph(\wp_2)\},\min\{\widehat{\daleth}^-_\aleph(\partial_1),\widehat{\daleth}^-_\aleph(\partial_2)\}\}$.
If $\widehat{\daleth}^-_\aleph(\wp_1\triangle_3\partial_1) \leqq \widehat{\daleth}^-_\aleph(\wp_2\triangle_3\partial_2)$, then $\widehat{\daleth}^-_\aleph(\wp_1\triangle_3\partial_1) \geqq \min\{\widehat{\daleth}^-_\aleph(\wp_1),\widehat{\daleth}^-_\aleph(\partial_1)\}$.

$$
\begin{aligned}
&\min\{1-\widehat{\exists}^-_\aleph(\wp_1\triangle_1\partial_1),1-\widehat{\exists}^-_\aleph(\wp_2\triangle_1\partial_2)\}\\
&= 1-\widehat{\exists}^-_\Upsilon(\wp_1\triangle_1\partial_1,\wp_2\triangle_1\partial_2)\\
&= 1-\widehat{\exists}^-_\Upsilon[(\wp_1,\wp_2)\triangle_1(\partial_1,\partial_2)]\\
&= 1-\widehat{\exists}^-_\Upsilon(\wp\triangle_1\partial)\\
&\geqq \min\{1-\widehat{\exists}^-_\Upsilon(\wp),1-\widehat{\exists}^-_\Upsilon(\partial)\}\\
&= \min\{1-\widehat{\exists}^-_\Upsilon(\wp_1,\wp_2)\},1-\widehat{\exists}^-_\Upsilon(\partial_1,\partial_2)\}\\
&= \min\{\min\{1-\widehat{\exists}^-_\aleph(\wp_1),1-\widehat{\exists}^-_\aleph(\wp_2)\},\min\{1-\widehat{\exists}^-_\aleph(\partial_1),1-\widehat{\exists}^-_\aleph(\partial_2)\}\}.
\end{aligned}
$$

If $1-\widehat{\exists}^-_\aleph(\wp_1\triangle_1\partial_1) \leqq 1-\widehat{\exists}^-_\aleph(\wp_2\triangle_1\partial_2)$, then $1-\widehat{\exists}^-_\aleph(\wp_1) \leqq 1-\widehat{\exists}^-_\aleph(\wp_2)$ and $1-\widehat{\exists}^-_\aleph(\partial_1) \leqq 1-\widehat{\exists}^-_\aleph(\partial_2)$. We get $1-\widehat{\exists}^-_\aleph(\wp_1\triangle_1\partial_1) \geqq \min\{1-\widehat{\exists}^-_\aleph(\wp_1),1-\widehat{\exists}^-_\aleph(\partial_1)\}$ for all $\wp_1,\partial_1\in\Xi$, and $\min\{1-\widehat{\exists}^-_\aleph(\wp_1\triangle_2\partial_1),1-\widehat{\exists}^-_\aleph(\wp_2\triangle_2\partial_2)\} \geqq \min\{\min\{1-\widehat{\exists}^-_\aleph(\wp_1),1-\widehat{\exists}^-_\aleph(\wp_2)\},\min\{1-\widehat{\exists}^-_\aleph(\partial_1),1-\widehat{\exists}^-_\aleph(\partial_2)\}\}$.
If $1-\widehat{\exists}^-_\aleph(\wp_1\triangle_2\partial_1) \leqq 1-\widehat{\exists}^-_\aleph(\wp_2\triangle_2\partial_2)$, then $1-\widehat{\exists}^-_\aleph(\wp_1\triangle_2\partial_1) \geqq \min\{1-\widehat{\exists}^-_\aleph(\wp_1),1-\widehat{\exists}^-_\aleph(\partial_1)\}$.
$\min\{1-\widehat{\exists}^-_\aleph(\wp_1\triangle_3\partial_1),1-\widehat{\exists}^-_\aleph(\wp_2\triangle_3\partial_2)\} \geqq \min\{\min\{1-\widehat{\exists}^-_\aleph(\wp_1),1-\widehat{\exists}^-_\aleph(\wp_2)\},\min\{1-\widehat{\exists}^-_\aleph(\partial_1),1-\widehat{\exists}^-_\aleph(\partial_2)\}\}$. If $1-\widehat{\exists}^-_\aleph(\wp_1\triangle_3\partial_1) \leqq 1-\widehat{\exists}^-_\aleph(\wp_2\triangle_3\partial_2)$, then $1-\widehat{\exists}^-_\aleph(\wp_1\triangle_3\partial_1) \geqq \min\{1-\widehat{\exists}^-_\aleph(\wp_1),1-\widehat{\exists}^-_\aleph(\partial_1)\}$.
Thus $\widehat{\sqsupset\daleth}_\Upsilon(\wp\triangle_1\partial) \geqq \min\{\widehat{\sqsupset\daleth}_\Upsilon(\wp),\widehat{\sqsupset\daleth}_\Upsilon(\partial)\}$. Similarly, $\widehat{\sqsupset\daleth}_\Upsilon(\wp\triangle_2\partial) \geqq \min\{\widehat{\sqsupset\daleth}_\Upsilon(\wp),\widehat{\sqsupset\daleth}_\Upsilon(\partial)\}$ and $\widehat{\sqsupset\daleth}_\Upsilon(\wp\triangle_3\partial) \geqq$

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$\min\{\widehat{\beth}^{\daleth}_{\Upsilon}(\wp), \widehat{\beth}^{\daleth}_{\Upsilon}(\partial)\}$. Now,

$$\begin{aligned}\frac{1}{2}\Big[\widehat{\gimel}^{-}_{\aleph}(\wp_1\triangle_1\partial_1)+\widehat{\gimel}^{-}_{\aleph}(\wp_2\triangle_1\partial_2)\Big] &= \widehat{\gimel}^{-}_{\Upsilon}(\wp_1\triangle_1\partial_1, \wp_2\triangle_1\partial_2)\\ &= \widehat{\gimel}^{-}_{\Upsilon}[(\wp_1,\wp_2)\triangle_1(\partial_1,\partial_2)]\\ &= \widehat{\gimel}^{-}_{\Upsilon}(\wp\triangle_1\partial)\\ &\geqq \frac{\widehat{\gimel}^{-}_{\Upsilon}(\wp)+\widehat{\gimel}^{-}_{\Upsilon}(\partial)}{2}\\ &= \frac{\widehat{\gimel}^{-}_{\Upsilon}(\wp_1,\wp_2)+\widehat{\gimel}^{-}_{\Upsilon}(\partial_1,\partial_2)}{2}\\ &= \frac{1}{2}\left[\frac{\widehat{\gimel}^{-}_{\aleph}(\wp_1)+\widehat{\gimel}^{-}_{\aleph}(\wp_2)}{2}+\frac{\widehat{\gimel}^{-}_{\aleph}(\partial_1)+\widehat{\gimel}^{-}_{\aleph}(\partial_2)}{2}\right].\end{aligned}$$

If $\widehat{\gimel}^{-}_{\aleph}(\wp_1\triangle_1\partial_1)\leqq\widehat{\gimel}^{-}_{\aleph}(\wp_2\triangle_1\partial_2)$, then $\widehat{\gimel}^{-}_{\aleph}(\wp_1)\leqq\widehat{\gimel}^{-}_{\aleph}(\wp_2)$ and $\widehat{\gimel}^{-}_{\aleph}(\partial_1)\leqq\widehat{\gimel}^{-}_{\aleph}(\partial_2)$.
We get $\widehat{\gimel}^{-}_{\aleph}(\wp_1\triangle_1\partial_1)\geqq\frac{\widehat{\gimel}^{-}_{\aleph}(\wp_1)+\widehat{\gimel}^{-}_{\aleph}(\partial_1)}{2}$. Similarly, $\widehat{\gimel}^{-}_{\aleph}(\wp_1\triangle_2\partial_1)\geqq\frac{\widehat{\gimel}^{-}_{\aleph}(\wp_1)+\widehat{\gimel}^{-}_{\aleph}(\partial_1)}{2}$ and $\widehat{\gimel}^{-}_{\aleph}(\wp_1\triangle_3\partial_1)\geqq\frac{\widehat{\gimel}^{-}_{\aleph}(\wp_1)+\widehat{\gimel}^{-}_{\aleph}(\partial_1)}{2}$.
Also, $\frac{1}{2}\Big[\widehat{\gimel}^{+}_{\aleph}(\wp_1\triangle_1\partial_1)+\widehat{\gimel}^{+}_{\aleph}(\wp_2\triangle_1\partial_2)\Big]\geqq\frac{1}{2}\left[\frac{\widehat{\gimel}^{+}_{\aleph}(\wp_1)+\widehat{\gimel}^{+}_{\aleph}(\wp_2)}{2}+\frac{\widehat{\gimel}^{+}_{\aleph}(\partial_1)+\widehat{\gimel}^{+}_{\aleph}(\partial_2)}{2}\right]$.
If $\widehat{\gimel}^{+}_{\aleph}(\wp_1\triangle_1\partial_1)\leqq\widehat{\gimel}^{+}_{\aleph}(\wp_2\triangle_1\partial_2)$, then $\widehat{\gimel}^{+}_{\aleph}(\wp_1)\leqq\widehat{\gimel}^{+}_{\aleph}(\wp_2)$ and $\widehat{\gimel}^{+}_{\aleph}(\partial_1)\leqq\widehat{\gimel}^{+}_{\aleph}(\partial_2)$.
We get $\widehat{\gimel}^{+}_{\aleph}(\wp_1\triangle_1\partial_1)\geqq\frac{\widehat{\gimel}^{+}_{\aleph}(\wp_1)+\widehat{\gimel}^{+}_{\aleph}(\partial_1)}{2}$ and $\widehat{\gimel}^{+}_{\aleph}(\wp_1\triangle_2\partial_1)\geqq\frac{\widehat{\gimel}^{+}_{\aleph}(\wp_1)+\widehat{\gimel}^{+}_{\aleph}(\partial_1)}{2}$ and $\widehat{\gimel}^{+}_{\aleph}(\wp_1\triangle_3\partial_1)\geqq\frac{\widehat{\gimel}^{+}_{\aleph}(\wp_1)+\widehat{\gimel}^{+}_{\aleph}(\partial_1)}{2}$.
Thus $\widehat{\beth}^{\gimel}_{\Upsilon}(\wp\triangle_1\partial)\geqq\frac{\widehat{\beth}_{\Upsilon}(\wp)+\widehat{\beth}_{\Upsilon}(\partial)}{2}$. Similarly, $\widehat{\beth}^{\gimel}_{\Upsilon}(\wp\triangle_2\partial)\geqq\frac{\widehat{\beth}_{\Upsilon}(\wp)+\widehat{\beth}_{\Upsilon}(\partial)}{2}$ and $\widehat{\beth}^{\gimel}_{\Upsilon}(\wp\triangle_3\partial)\geqq\frac{\widehat{\beth}_{\Upsilon}(\wp)+\widehat{\beth}_{\Upsilon}(\partial)}{2}$.
Similarly, $\max\{\widehat{\exists}^{-}_{\aleph}(\wp_1\triangle_1\partial_1),\widehat{\exists}^{-}_{\aleph}(\wp_2\triangle_1\partial_2)\}\leqq\max\{\max\{\widehat{\exists}^{-}_{\aleph}(\wp_1),\widehat{\exists}^{-}_{\aleph}(\wp_2)\},\max\{\widehat{\exists}^{-}_{\aleph}(\partial_1),\widehat{\exists}^{-}_{\aleph}(\partial_2)\}\}$.
If $\widehat{\exists}^{-}_{\aleph}(\wp_1\triangle_1\partial_1)\geqq\widehat{\exists}^{-}_{\aleph}(\wp_2\triangle_1\partial_2)$, then $\widehat{\exists}^{-}_{\aleph}(\wp_1)\geqq\widehat{\exists}^{-}_{\aleph}(\wp_2)$ and $\widehat{\exists}^{-}_{\aleph}(\partial_1)\geqq\widehat{\exists}^{-}_{\aleph}(\partial_2)$.
We get $\widehat{\exists}^{-}_{\aleph}(\wp_1\triangle_1\partial_1)\leqq\max\{\widehat{\exists}^{-}_{\aleph}(\wp_1),\widehat{\exists}^{-}_{\aleph}(\partial_1)\}$.
$\max\{\widehat{\exists}^{-}_{\aleph}(\wp_1\triangle_2\partial_1),\widehat{\exists}^{-}_{\aleph}(\wp_2\triangle_2\partial_2)\}\leqq\max\{\max\{\widehat{\exists}^{-}_{\aleph}(\wp_1),\widehat{\exists}^{-}_{\aleph}(\wp_2)\},\max\{\widehat{\exists}^{-}_{\aleph}(\partial_1),\widehat{\exists}^{-}_{\aleph}(\partial_2)\}\}$.
If $\widehat{\exists}^{-}_{\aleph}(\wp_1\triangle_2\partial_1)\geqq\widehat{\exists}^{-}_{\aleph}(\wp_2\triangle_2\partial_2)$, then $\widehat{\exists}^{-}_{\aleph}(\wp_1\triangle_2\partial_1)\leqq\max\{\widehat{\exists}^{-}_{\aleph}(\wp_1),\widehat{\exists}^{-}_{\aleph}(\partial_1)\}$.
$\max\{\widehat{\exists}^{-}_{\aleph}(\wp_1\triangle_3\partial_1),\widehat{\exists}^{-}_{\aleph}(\wp_2\triangle_3\partial_2)\}\leqq\max\{\max\{\widehat{\exists}^{-}_{\aleph}(\wp_1),\widehat{\exists}^{-}_{\aleph}(\wp_2)\},\max\{\widehat{\exists}^{-}_{\aleph}(\partial_1),\widehat{\exists}^{-}_{\aleph}(\partial_2)\}\}$
If $\widehat{\exists}^{-}_{\aleph}(\wp_1\triangle_3\partial_1)\geqq\widehat{\exists}^{-}_{\aleph}(\wp_2\triangle_3\partial_2)$, then $\widehat{\exists}^{-}_{\aleph}(\wp_1\triangle_3\partial_1)\leqq\max\{\widehat{\exists}^{-}_{\aleph}(\wp_1),\widehat{\exists}^{-}_{\aleph}(\partial_1)\}$.
Also, Similarly to prove that $\max\{1-\widehat{\daleth}^{-}_{\aleph}(\wp_1\triangle_1\partial_1),1-\widehat{\daleth}^{-}_{\aleph}(\wp_2\triangle_1\partial_2)\}\leqq\max\{\max\{1-\widehat{\daleth}^{-}_{\aleph}(\wp_1),1-\widehat{\daleth}^{-}_{\aleph}(\wp_2)\},\max\{1-\widehat{\daleth}^{-}_{\aleph}(\partial_1),1-\widehat{\daleth}^{-}_{\aleph}(\partial_2)\}\}$.
If $1-\widehat{\daleth}^{-}_{\aleph}(\wp_1\triangle_1\partial_1)\geqq1-\widehat{\daleth}^{-}_{\aleph}(\wp_2\triangle_1\partial_2)$, then $1-\widehat{\daleth}^{-}_{\aleph}(\wp_1)\geqq1-\widehat{\daleth}^{-}_{\aleph}(\wp_2)$ and $1-\widehat{\daleth}^{-}_{\aleph}(\partial_1)\geqq1-\widehat{\daleth}^{-}_{\aleph}(\partial_2)$.
We get $1-\widehat{\daleth}^{-}_{\aleph}(\wp_1\triangle_1\partial_1)\leqq\max\{1-\widehat{\daleth}^{-}_{\aleph}(\wp_1),1-\widehat{\daleth}^{-}_{\aleph}(\partial_1)\}$.
$\max\{1-\widehat{\daleth}^{-}_{\aleph}(\wp_1\triangle_2\partial_1),1-\widehat{\daleth}^{-}_{\aleph}(\wp_2\triangle_2\partial_2)\}\leqq\max\{\max\{1-\widehat{\daleth}^{-}_{\aleph}(\wp_1),1-\widehat{\daleth}^{-}_{\aleph}(\wp_2)\},\max\{1-\widehat{\daleth}^{-}_{\aleph}(\partial_1),1-\widehat{\daleth}^{-}_{\aleph}(\partial_2)\}\}$.
If $1-\widehat{\daleth}^{-}_{\aleph}(\wp_1\triangle_2\partial_1)\geqq1-\widehat{\daleth}^{-}_{\aleph}(\wp_2\triangle_2\partial_2)$, then $1-\widehat{\daleth}^{-}_{\aleph}(\wp_1\triangle_2\partial_1)\leqq\max\{1-\widehat{\daleth}^{-}_{\aleph}(\wp_1),1-\widehat{\daleth}^{-}_{\aleph}(\partial_1)\}$.
$\max\{1-\widehat{\daleth}^{-}_{\aleph}(\wp_1\triangle_3\partial_1),1-\widehat{\daleth}^{-}_{\aleph}(\wp_2\triangle_3\partial_2)\}\leqq\max\{\max\{1-\widehat{\daleth}^{-}_{\aleph}(\wp_1),1-\widehat{\daleth}^{-}_{\aleph}(\wp_2)\},\max\{1-\widehat{\daleth}^{-}_{\aleph}(\partial_1),1-\widehat{\daleth}^{-}_{\aleph}(\partial_2)\}\}$.
If $1-\widehat{\daleth}^{-}_{\aleph}(\wp_1\triangle_3\partial_1)\geqq1-\widehat{\daleth}^{-}_{\aleph}(\wp_2\triangle_3\partial_2)$, then $1-\widehat{\daleth}^{-}_{\aleph}(\wp_1\triangle_3\partial_1)\leqq\max\{1-\widehat{\daleth}^{-}_{\aleph}(\wp_1),1-\widehat{\daleth}^{-}_{\aleph}(\partial_1)\}$.
Hence, $\widehat{\beth}^{\exists}_{\Upsilon}(\wp\triangle_1\partial)\leqq\max\{\widehat{\beth}^{\exists}_{\Upsilon}(\wp),\widehat{\beth}^{\exists}_{\Upsilon}(\partial)\}$, $\widehat{\beth}^{\exists}_{\Upsilon}(\wp\triangle_2\partial)\leqq\max\{\widehat{\beth}^{\exists}_{\Upsilon}(\wp),\widehat{\beth}^{\exists}_{\Upsilon}(\partial)\}$ and
$\widehat{\beth}^{\exists}_{\Upsilon}(\wp\triangle_3\partial)\leqq\max\{\widehat{\beth}^{\exists}_{\Upsilon}(\wp),\widehat{\beth}^{\exists}_{\Upsilon}(\partial)\}$.
Let us assume that Υ is a IVNCVSBS of $\Xi\ltimes\Xi$, then for any $\wp=(\wp_1,\wp_2)$ and $\partial=(\partial_1,\partial_2)$ are in $\Xi\ltimes\Xi$. Now,

$$\begin{aligned}\min\{\daleth^{-}_{\aleph}(\wp_1\triangle_1\partial_1),\daleth^{-}_{\aleph}(\wp_2\triangle_1\partial_2)\} &= \daleth^{-}_{\Upsilon}(\wp_1\triangle_1\partial_1,\wp_2\triangle_1\partial_2)\\ &= \daleth^{-}_{\Upsilon}[(\wp_1,\wp_2)\triangle_1(\partial_1,\partial_2)]\\ &= \daleth^{-}_{\Upsilon}(\wp\triangle_1\partial)\\ &\geqq \min\{\daleth^{-}_{\Upsilon}(\wp),\daleth^{-}_{\Upsilon}(\partial)\}\\ &= \min\{\daleth^{-}_{\Upsilon}(\wp_1,\wp_2)\},\daleth^{-}_{\Upsilon}(\partial_1,\partial_2)\}\\ &= \min\{\min\{\daleth^{-}_{\aleph}(\wp_1),\daleth^{-}_{\aleph}(\wp_2)\},\min\{\daleth^{-}_{\aleph}(\partial_1),\daleth^{-}_{\aleph}(\partial_2)\}\}.\end{aligned}$$

If $\daleth^{-}_{\aleph}(\wp_1\triangle_1\partial_1)\leqq\daleth^{-}_{\aleph}(\wp_2\triangle_1\partial_2)$, then $\daleth^{-}_{\aleph}(\wp_1)\leqq\daleth^{-}_{\aleph}(\wp_2)$ and $\daleth^{-}_{\aleph}(\partial_1)\leqq\daleth^{-}_{\aleph}(\partial_2)$. We get $\daleth^{-}_{\aleph}(\wp_1\triangle_1\partial_1)\geqq$

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$\min\{\daleth_{\aleph}^{-}(\wp_1), \daleth_{\aleph}^{-}(\partial_1)\}$ for all $\wp_1, \partial_1 \in \Xi$, and
$\min\{\daleth_{\aleph}^{-}(\wp_1 \triangle_2 \partial_1), \daleth_{\aleph}^{-}(\wp_2 \triangle_2 \partial_2)\} \geqq \min\{\min\{\daleth_{\aleph}^{-}(\wp_1), \daleth_{\aleph}^{-}(\wp_2)\}, \min\{\daleth_{\aleph}^{-}(\partial_1), \daleth_{\aleph}^{-}(\partial_2)\}\}$
If $\daleth_{\aleph}^{-}(\wp_1 \triangle_2 \partial_1) \leqq \daleth_{\aleph}^{-}(\wp_2 \triangle_2 \partial_2)$, then $\daleth_{\aleph}^{-}(\wp_1 \triangle_2 \partial_1) \geqq \min\{\daleth_{\aleph}^{-}(\wp_1), \daleth_{\aleph}^{-}(\partial_1)\}$.
$\min\{\daleth_{\aleph}^{-}(\wp_1 \triangle_3 \partial_1), \daleth_{\aleph}^{-}(\wp_2 \triangle_3 \partial_2)\} \geqq \min\{\min\{\daleth_{\aleph}^{-}(\wp_1), \daleth_{\aleph}^{-}(\wp_2)\}, \min\{\daleth_{\aleph}^{-}(\partial_1), \daleth_{\aleph}^{-}(\partial_2)\}\}$.
If $\daleth_{\aleph}^{-}(\wp_1 \triangle_3 \partial_1) \leqq \daleth_{\aleph}^{-}(\wp_2 \triangle_3 \partial_2)$, then $\daleth_{\aleph}^{-}(\wp_1 \triangle_3 \partial_1) \geqq \min\{\daleth_{\aleph}^{-}(\wp_1), \daleth_{\aleph}^{-}(\partial_1)\}$.

$$
\begin{aligned}
&\min\{1 - \Finv_{\aleph}^{-}(\wp_1 \triangle_1 \partial_1), 1 - \Finv_{\aleph}^{-}(\wp_2 \triangle_1 \partial_2)\} \\
&= 1 - \Finv_{\Upsilon}^{-}(\wp_1 \triangle_1 \partial_1, \wp_2 \triangle_1 \partial_2) \\
&= 1 - \Finv_{\Upsilon}^{-}[(\wp_1, \wp_2) \triangle_1 (\partial_1, \partial_2)] \\
&= 1 - \Finv_{\Upsilon}^{-}(\wp \triangle_1 \partial) \\
&\geqq \min\{1 - \Finv_{\Upsilon}^{-}(\wp), 1 - \Finv_{\Upsilon}^{-}(\partial)\} \\
&= \min\{1 - \Finv_{\Upsilon}^{-}(\wp_1, \wp_2)\}, 1 - \Finv_{\Upsilon}^{-}(\partial_1, \partial_2)\} \\
&= \min\{\min\{1 - \Finv_{\aleph}^{-}(\wp_1), 1 - \Finv_{\aleph}^{-}(\wp_2)\}, \min\{1 - \Finv_{\aleph}^{-}(\partial_1), 1 - \Finv_{\aleph}^{-}(\partial_2)\}\}.
\end{aligned}
$$

If $1 - \Finv_{\aleph}^{-}(\wp_1 \triangle_1 \partial_1) \leqq 1 - \Finv_{\aleph}^{-}(\wp_2 \triangle_1 \partial_2)$, then $1 - \Finv_{\aleph}^{-}(\wp_1) \leqq 1 - \Finv_{\aleph}^{-}(\wp_2)$ and $1 - \Finv_{\aleph}^{-}(\partial_1) \leqq 1 - \Finv_{\aleph}^{-}(\partial_2)$. We get $1 - \Finv_{\aleph}^{-}(\wp_1 \triangle_1 \partial_1) \geqq \min\{1 - \Finv_{\aleph}^{-}(\wp_1), 1 - \Finv_{\aleph}^{-}(\partial_1)\}$ for all $\wp_1, \partial_1 \in \Xi$, and $\min\{1 - \Finv_{\aleph}^{-}(\wp_1 \triangle_2 \partial_1), 1 - \Finv_{\aleph}^{-}(\wp_2 \triangle_2 \partial_2)\} \geqq \min\{\min\{1 - \Finv_{\aleph}^{-}(\wp_1), 1 - \Finv_{\aleph}^{-}(\wp_2)\}, \min\{1 - \Finv_{\aleph}^{-}(\partial_1), 1 - \Finv_{\aleph}^{-}(\partial_2)\}\}$.
If $1 - \Finv_{\aleph}^{-}(\wp_1 \triangle_2 \partial_1) \leqq 1 - \Finv_{\aleph}^{-}(\wp_2 \triangle_2 \partial_2)$, then $1 - \Finv_{\aleph}^{-}(\wp_1 \triangle_2 \partial_1) \geqq \min\{1 - \Finv_{\aleph}^{-}(\wp_1), 1 - \Finv_{\aleph}^{-}(\partial_1)\}$.
$\min\{1 - \Finv_{\aleph}^{-}(\wp_1 \triangle_3 \partial_1), 1 - \Finv_{\aleph}^{-}(\wp_2 \triangle_3 \partial_2)\} \geqq \min\{\min\{1 - \Finv_{\aleph}^{-}(\wp_1), 1 - \Finv_{\aleph}^{-}(\wp_2)\}, \min\{1 - \Finv_{\aleph}^{-}(\partial_1), 1 - \Finv_{\aleph}^{-}(\partial_2)\}\}$. If $1 - \Finv_{\aleph}^{-}(\wp_1 \triangle_3 \partial_1) \leqq 1 - \Finv_{\aleph}^{-}(\wp_2 \triangle_3 \partial_2)$, then $1 - \Finv_{\aleph}^{-}(\wp_1 \triangle_3 \partial_1) \geqq \min\{1 - \Finv_{\aleph}^{-}(\wp_1), 1 - \Finv_{\aleph}^{-}(\partial_1)\}$.
Thus $\beth_{\Upsilon}^{\daleth}(\wp \triangle_1 \partial) \geqq \min\{\beth_{\Upsilon}^{\daleth}(\wp), \beth_{\Upsilon}^{\daleth}(\partial)\}$. Similarly, $\beth_{\Upsilon}^{\daleth}(\wp \triangle_2 \partial) \geqq \min\{\beth_{\Upsilon}^{\daleth}(\wp), \beth_{\Upsilon}^{\daleth}(\partial)\}$ and $\beth_{\Upsilon}^{\daleth}(\wp \triangle_3 \partial) \geqq \min\{\beth_{\Upsilon}^{\daleth}(\wp), \beth_{\Upsilon}^{\daleth}(\partial)\}$. Now,

$$
\begin{aligned}
\frac{1}{2}\Big[\gimel_{\aleph}^{-}(\wp_1 \triangle_1 \partial_1) + \gimel_{\aleph}^{-}(\wp_2 \triangle_1 \partial_2)\Big] &= \gimel_{\Upsilon}^{-}(\wp_1 \triangle_1 \partial_1, \wp_2 \triangle_1 \partial_2) \\
&= \gimel_{\Upsilon}^{-}[(\wp_1, \wp_2) \triangle_1 (\partial_1, \partial_2)] \\
&= \gimel_{\Upsilon}^{-}(\wp \triangle_1 \partial) \\
&\geqq \frac{\gimel_{\Upsilon}^{-}(\wp) + \gimel_{\Upsilon}^{-}(\partial)}{2} \\
&= \frac{\gimel_{\Upsilon}^{-}(\wp_1, \wp_2) + \gimel_{\Upsilon}^{-}(\partial_1, \partial_2)}{2} \\
&= \frac{1}{2}\left[\frac{\gimel_{\aleph}^{-}(\wp_1) + \gimel_{\aleph}^{-}(\wp_2)}{2} + \frac{\gimel_{\aleph}^{-}(\partial_1) + \gimel_{\aleph}^{-}(\partial_2)}{2}\right].
\end{aligned}
$$

If $\gimel_{\aleph}^{-}(\wp_1 \triangle_1 \partial_1) \leqq \gimel_{\aleph}^{-}(\wp_2 \triangle_1 \partial_2)$, then $\gimel_{\aleph}^{-}(\wp_1) \leqq \gimel_{\aleph}^{-}(\wp_2)$ and $\gimel_{\aleph}^{-}(\partial_1) \leqq \gimel_{\aleph}^{-}(\partial_2)$.
We get $\gimel_{\aleph}^{-}(\wp_1 \triangle_1 \partial_1) \geqq \frac{\gimel_{\aleph}^{-}(\wp_1) + \gimel_{\aleph}^{-}(\partial_1)}{2}$. Similarly, $\gimel_{\aleph}^{-}(\wp_1 \triangle_2 \partial_1) \geqq \frac{\gimel_{\aleph}^{-}(\wp_1) + \gimel_{\aleph}^{-}(\partial_1)}{2}$ and $\gimel_{\aleph}^{-}(\wp_1 \triangle_3 \partial_1) \geqq \frac{\gimel_{\aleph}^{-}(\wp_1) + \gimel_{\aleph}^{-}(\partial_1)}{2}$.
Also, $\frac{1}{2}\Big[\gimel_{\aleph}^{+}(\wp_1 \triangle_1 \partial_1) + \gimel_{\aleph}^{+}(\wp_2 \triangle_1 \partial_2)\Big] \geqq \frac{1}{2}\left[\frac{\gimel_{\aleph}^{+}(\wp_1) + \gimel_{\aleph}^{+}(\wp_2)}{2} + \frac{\gimel_{\aleph}^{+}(\partial_1) + \gimel_{\aleph}^{+}(\partial_2)}{2}\right]$.
If $\gimel_{\aleph}^{+}(\wp_1 \triangle_1 \partial_1) \leqq \gimel_{\aleph}^{+}(\wp_2 \triangle_1 \partial_2)$, then $\gimel_{\aleph}^{+}(\wp_1) \leqq \gimel_{\aleph}^{+}(\wp_2)$ and $\gimel_{\aleph}^{+}(\partial_1) \leqq \gimel_{\aleph}^{+}(\partial_2)$.
We get $\gimel_{\aleph}^{+}(\wp_1 \triangle_1 \partial_1) \geqq \frac{\gimel_{\aleph}^{+}(\wp_1) + \gimel_{\aleph}^{+}(\partial_1)}{2}$ and $\gimel_{\aleph}^{+}(\wp_1 \triangle_2 \partial_1) \geqq \frac{\gimel_{\aleph}^{+}(\wp_1) + \gimel_{\aleph}^{+}(\partial_1)}{2}$ and $\gimel_{\aleph}^{+}(\wp_1 \triangle_3 \partial_1) \geqq \frac{\gimel_{\aleph}^{+}(\wp_1) + \gimel_{\aleph}^{+}(\partial_1)}{2}$.
Thus $\beth_{\Upsilon}^{\gimel}(\wp \triangle_1 \partial) \geqq \frac{\beth_{\Upsilon}(\wp) + \beth_{\Upsilon}(\partial)}{2}$. Similarly, $\beth_{\Upsilon}^{\gimel}(\wp \triangle_2 \partial) \geqq \frac{\beth_{\Upsilon}(\wp) + \beth_{\Upsilon}(\partial)}{2}$ and $\beth_{\Upsilon}^{\gimel}(\wp \triangle_3 \partial) \geqq \frac{\beth_{\Upsilon}(\wp) + \beth_{\Upsilon}(\partial)}{2}$.
Similarly, $\max\{\Finv_{\aleph}^{-}(\wp_1 \triangle_1 \partial_1), \Finv_{\aleph}^{-}(\wp_2 \triangle_1 \partial_2)\} \leqq \max\{\max\{\Finv_{\aleph}^{-}(\wp_1), \Finv_{\aleph}^{-}(\wp_2)\}, \max\{\Finv_{\aleph}^{-}(\partial_1), \Finv_{\aleph}^{-}(\partial_2)\}\}$.
If $\Finv_{\aleph}^{-}(\wp_1 \triangle_1 \partial_1) \geqq \Finv_{\aleph}^{-}(\wp_2 \triangle_1 \partial_2)$, then $\Finv_{\aleph}^{-}(\wp_1) \geqq \Finv_{\aleph}^{-}(\wp_2)$ and $\Finv_{\aleph}^{-}(\partial_1) \geqq \Finv_{\aleph}^{-}(\partial_2)$.
We get $\Finv_{\aleph}^{-}(\wp_1 \triangle_1 \partial_1) \leqq \max\{\Finv_{\aleph}^{-}(\wp_1), \Finv_{\aleph}^{-}(\partial_1)\}$.
$\max\{\Finv_{\aleph}^{-}(\wp_1 \triangle_2 \partial_1), \Finv_{\aleph}^{-}(\wp_2 \triangle_2 \partial_2)\} \leqq \max\{\max\{\Finv_{\aleph}^{-}(\wp_1), \Finv_{\aleph}^{-}(\wp_2)\}, \max\{\Finv_{\aleph}^{-}(\partial_1), \Finv_{\aleph}^{-}(\partial_2)\}\}$.
If $\Finv_{\aleph}^{-}(\wp_1 \triangle_2 \partial_1) \geqq \Finv_{\aleph}^{-}(\wp_2 \triangle_2 \partial_2)$, then $\Finv_{\aleph}^{-}(\wp_1 \triangle_2 \partial_1) \leqq \max\{\Finv_{\aleph}^{-}(\wp_1), \Finv_{\aleph}^{-}(\partial_1)\}$.
$\max\{\Finv_{\aleph}^{-}(\wp_1 \triangle_3 \partial_1), \Finv_{\aleph}^{-}(\wp_2 \triangle_3 \partial_2)\} \leqq \max\{\max\{\Finv_{\aleph}^{-}(\wp_1), \Finv_{\aleph}^{-}(\wp_2)\}, \max\{\Finv_{\aleph}^{-}(\partial_1), \Finv_{\aleph}^{-}(\partial_2)\}\}$
If $\Finv_{\aleph}^{-}(\wp_1 \triangle_3 \partial_1) \geqq \Finv_{\aleph}^{-}(\wp_2 \triangle_3 \partial_2)$, then $\Finv_{\aleph}^{-}(\wp_1 \triangle_3 \partial_1) \leqq \max\{\Finv_{\aleph}^{-}(\wp_1), \Finv_{\aleph}^{-}(\partial_1)\}$.
Also, Similarly to prove that $\max\{1 - \daleth_{\aleph}^{-}(\wp_1 \triangle_1 \partial_1), 1 - \daleth_{\aleph}^{-}(\wp_2 \triangle_1 \partial_2)\} \leqq \max\{\max\{1 - \daleth_{\aleph}^{-}(\wp_1), 1 - \daleth_{\aleph}^{-}(\wp_2)\}, \max\{1 - \daleth_{\aleph}^{-}(\partial_1), 1 - \daleth_{\aleph}^{-}(\partial_2)\}\}$.
If $1 - \daleth_{\aleph}^{-}(\wp_1 \triangle_1 \partial_1) \geqq 1 - \daleth_{\aleph}^{-}(\wp_2 \triangle_1 \partial_2)$, then $1 - \daleth_{\aleph}^{-}(\wp_1) \geqq 1 - \daleth_{\aleph}^{-}(\wp_2)$ and $1 - \daleth_{\aleph}^{-}(\partial_1) \geqq 1 - \daleth_{\aleph}^{-}(\partial_2)$.
We get $1 - \daleth_{\aleph}^{-}(\wp_1 \triangle_1 \partial_1) \leqq \max\{1 - \daleth_{\aleph}^{-}(\wp_1), 1 - \daleth_{\aleph}^{-}(\partial_1)\}$.

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$\max\{1-\daleth^{-}_{\aleph}(\wp_1\triangle_2\partial_1), 1-\daleth^{-}_{\aleph}(\wp_2\triangle_2\partial_2)\} \leqq \max\{\max\{1-\daleth^{-}_{\aleph}(\wp_1), 1-\daleth^{-}_{\aleph}(\wp_2)\}, \max\{1-\daleth^{-}_{\aleph}(\partial_1), 1-\daleth^{-}_{\aleph}(\partial_2)\}\}$.
If $1-\daleth^{-}_{\aleph}(\wp_1\triangle_2\partial_1) \geqq 1-\daleth^{-}_{\aleph}(\wp_2\triangle_2\partial_2)$, then $1-\daleth^{-}_{\aleph}(\wp_1\triangle_2\partial_1) \leqq \max\{1-\daleth^{-}_{\aleph}(\wp_1), 1-\daleth^{-}_{\aleph}(\partial_1)\}$.
$\max\{1-\daleth^{-}_{\aleph}(\wp_1\triangle_3\partial_1), 1-\daleth^{-}_{\aleph}(\wp_2\triangle_3\partial_2)\} \leqq \max\{\max\{1-\daleth^{-}_{\aleph}(\wp_1), 1-\daleth^{-}_{\aleph}(\wp_2)\}, \max\{1-\daleth^{-}_{\aleph}(\partial_1), 1-\daleth^{-}_{\aleph}(\partial_2)\}\}$.
If $1-\daleth^{-}_{\aleph}(\wp_1\triangle_3\partial_1) \geqq 1-\daleth^{-}_{\aleph}(\wp_2\triangle_3\partial_2)$, then $1-\daleth^{-}_{\aleph}(\wp_1\triangle_3\partial_1) \leqq \max\{1-\daleth^{-}_{\aleph}(\wp_1), 1-\daleth^{-}_{\aleph}(\partial_1)\}$.
Hence, $\sqsupset^{\gimel}_{\Upsilon}(\wp\triangle_1\partial) \leqq \max\{\sqsupset^{\gimel}_{\Upsilon}(\wp), \sqsupset^{\gimel}_{\Upsilon}(\partial)\}$, $\sqsupset^{\gimel}_{\Upsilon}(\wp\triangle_2\partial) \leqq \max\{\sqsupset^{\gimel}_{\Upsilon}(\wp), \sqsupset^{\gimel}_{\Upsilon}(\partial)\}$ and $\sqsupset^{\gimel}_{\Upsilon}(\wp\triangle_3\partial) \leqq \max\{\sqsupset^{\gimel}_{\Upsilon}(\wp), \sqsupset^{\gimel}_{\Upsilon}(\partial)\}$. Hence, $\aleph$ is a IVNCVSBS of Ξ.

Theorem 3.8. *Let $\aleph$ be a NSV subset in Ξ. Then $\sqsupset = ([\daleth^{-}_{\aleph}, \daleth^{+}_{\aleph}], [\beth^{-}_{\aleph}, \beth^{+}_{\aleph}], [\gimel^{-}_{\aleph}, \gimel^{+}_{\aleph}])$ is a IVNCVSBS of Ξ if and only if all non empty level set $\sqsupset^{(\ell_1,\ell_2,s)}$ is a SBS of Ξ for $\ell_1, \ell_2, s \in [0,1]$.*

Proof. Assume that $\widehat{\sqsupset}$ is a IVNCVSBS of Ξ. For $\ell_1, \ell_2, s \in [0,1]$ and $\jmath_1, \jmath_2 \in \widehat{\sqsupset}^{(\ell_1,\ell_2,s)}$. We have $\widehat{\daleth}^{-}_{\aleph}(\jmath_1) \geqq \ell_1, \widehat{\daleth}^{-}_{\aleph}(\jmath_2) \geqq \ell_1$ and $1-\widehat{\gimel}^{-}_{\aleph}(\jmath_1) \geqq s, 1-\widehat{\gimel}^{-}_{\aleph}(\jmath_2) \geqq s$ and $\widehat{\beth}^{-}_{\aleph}(\jmath_1) \geqq \ell_2, \widehat{\beth}^{-}_{\aleph}(\jmath_2) \geqq \ell_2$ and $\widehat{\beth}^{+}_{\aleph}(\jmath_1) \geqq \ell_2, \widehat{\beth}^{+}_{\aleph}(\jmath_2) \geqq \ell_2$, $1-\widehat{\daleth}^{-}_{\aleph}(\jmath_1) \leqq \ell_1, 1-\widehat{\daleth}^{-}_{\aleph}(\jmath_2) \leqq \ell_1$ and $\widehat{\gimel}^{-}_{\aleph}(\jmath_1) \leqq s, \widehat{\gimel}^{-}_{\aleph}(\jmath_2) \leqq s$. Now, $\widehat{\daleth}^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) \geqq \min\{\widehat{\daleth}^{-}_{\aleph}(\jmath_1), \widehat{\daleth}^{-}_{\aleph}(\jmath_2)\} \geqq \ell_1$, $1-\widehat{\gimel}^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) \geqq \min\{1-\widehat{\gimel}^{-}_{\aleph}(\jmath_1), 1-\widehat{\gimel}^{-}_{\aleph}(\jmath_2)\} \geqq s$ and $\widehat{\beth}^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) \geqq \frac{\widehat{\beth}^{-}_{\aleph}(\jmath_1)+\widehat{\beth}^{-}_{\aleph}(\jmath_2)}{2} \geqq \ell_2$, $\widehat{\beth}^{+}_{\aleph}(\jmath_1\triangle_1\jmath_2) \geqq \frac{\widehat{\beth}^{+}_{\aleph}(\jmath_1)+\widehat{\beth}^{+}_{\aleph}(\jmath_2)}{2} \geqq \ell_2$ and $\widehat{\gimel}^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) \leqq \max\{\widehat{\gimel}^{-}_{\aleph}(\jmath_1), \widehat{\gimel}^{-}_{\aleph}(\jmath_2)\} \leqq s$ and $1-\widehat{\daleth}^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) \leqq \max\{1-\widehat{\daleth}^{-}_{\aleph}(\jmath_1), 1-\widehat{\daleth}^{-}_{\aleph}(\jmath_2)\} \leqq \ell_1$. This implies that $\jmath_1\triangle_1\jmath_2 \in \widehat{\sqsupset}^{(\ell_1,\ell_2,s)}$. Similarly, $\jmath_1\triangle_2\jmath_2 \in \widehat{\sqsupset}^{(\ell_1,\ell_2,s)}$ and $\jmath_1\triangle_3\jmath_2 \in \widehat{\sqsupset}^{(\ell_1,\ell_2,s)}$. Therefore $\widehat{\sqsupset}^{(\ell_1,\ell_2,s)}$ is a SBS of Ξ, where $\ell_1, \ell_2, s \in [0,1]$.
We have $\daleth^{-}_{\aleph}(\jmath_1) \geqq \ell_1, \daleth^{-}_{\aleph}(\jmath_2) \geqq \ell_1$ and
$1-\gimel^{-}_{\aleph}(\jmath_1) \geqq s, 1-\gimel^{-}_{\aleph}(\jmath_2) \geqq s$ and $\beth^{-}_{\aleph}(\jmath_1) \geqq \ell_2, \beth^{-}_{\aleph}(\jmath_2) \geqq \ell_2$ and $\beth^{+}_{\aleph}(\jmath_1) \geqq \ell_2, \beth^{+}_{\aleph}(\jmath_2) \geqq \ell_2$, $1-\daleth^{-}_{\aleph}(\jmath_1) \leqq \ell_1, 1-\daleth^{-}_{\aleph}(\jmath_2) \leqq \ell_1$ and $\gimel^{-}_{\aleph}(\jmath_1) \leqq s, \gimel^{-}_{\aleph}(\jmath_2) \leqq s$. Now, $\daleth^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) \geqq \min\{\daleth^{-}_{\aleph}(\jmath_1), \daleth^{-}_{\aleph}(\jmath_2)\} \geqq \ell_1$, $1-\gimel^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) \geqq \min\{1-\gimel^{-}_{\aleph}(\jmath_1), 1-\gimel^{-}_{\aleph}(\jmath_2)\} \geqq s$ and $\beth^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) \geqq \frac{\beth^{-}_{\aleph}(\jmath_1)+\beth^{-}_{\aleph}(\jmath_2)}{2} \geqq \ell_2$, $\beth^{+}_{\aleph}(\jmath_1\triangle_1\jmath_2) \geqq \frac{\beth^{+}_{\aleph}(\jmath_1)+\beth^{+}_{\aleph}(\jmath_2)}{2} \geqq \ell_2$ and $\gimel^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) \leqq \max\{\gimel^{-}_{\aleph}(\jmath_1), \gimel^{-}_{\aleph}(\jmath_2)\} \leqq s$ and $1-\daleth^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) \leqq \max\{1-\daleth^{-}_{\aleph}(\jmath_1), 1-\daleth^{-}_{\aleph}(\jmath_2)\} \leqq \ell_1$. This implies that $\jmath_1\triangle_1\jmath_2 \in \sqsupset^{(\ell_1,\ell_2,s)}$. Similarly, $\jmath_1\triangle_2\jmath_2 \in \sqsupset^{(\ell_1,\ell_2,s)}$ and $\jmath_1\triangle_3\jmath_2 \in \sqsupset^{(\ell_1,\ell_2,s)}$. Therefore $\sqsupset^{(\ell_1,\ell_2,s)}$ is a SBS of Ξ, where $\ell_1, \ell_2, s \in [0,1]$.
Conversely, assume that $\widehat{\sqsupset}^{(\ell_1,\ell_2,s)}$ is a SBS of Ξ, where $\ell_1, \ell_2, s \in [0,1]$. Suppose if there exist $\jmath_1, \jmath_2 \in \Xi$ such that $\widehat{\daleth}^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) < \min\{\widehat{\daleth}^{-}_{\aleph}(\jmath_1), \widehat{\daleth}^{-}_{\aleph}(\jmath_2)\}$, $1-\widehat{\gimel}^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) < \min\{1-\widehat{\gimel}^{-}_{\aleph}(\jmath_1), 1-\widehat{\gimel}^{-}_{\aleph}(\jmath_2)\}$, $\widehat{\beth}^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) < \frac{\widehat{\beth}^{-}_{\aleph}(\jmath_1)+\widehat{\beth}^{-}_{\aleph}(\jmath_2)}{2}$, $\widehat{\beth}^{+}_{\aleph}(\jmath_1\triangle_1\jmath_2) < \frac{\widehat{\beth}^{+}_{\aleph}(\jmath_1)+\widehat{\beth}^{+}_{\aleph}(\jmath_2)}{2}$ and $\widehat{\gimel}^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) > \max\{\widehat{\gimel}^{-}_{\aleph}(\jmath_1), \widehat{\gimel}^{-}_{\aleph}(\jmath_2)\}$. $1-\widehat{\daleth}^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) > \max\{1-\widehat{\daleth}^{-}_{\aleph}(\jmath_1), 1-\widehat{\daleth}^{-}_{\aleph}(\jmath_2)\}$. Select $\ell_1, \ell_2, s \in [0,1]$ such that $\widehat{\daleth}^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) < \ell_1 \leqq \min\{\widehat{\daleth}^{-}_{\aleph}(\jmath_1), \widehat{\daleth}^{-}_{\aleph}(\jmath_2)\}$ and $1-\widehat{\gimel}^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) < \ell_1 \leqq \min\{1-\widehat{\gimel}^{-}_{\aleph}(\jmath_1), 1-\widehat{\gimel}^{-}_{\aleph}(\jmath_2)\}$ and $\widehat{\beth}^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) < \ell_2 \leqq \frac{\widehat{\beth}^{-}_{\aleph}(\jmath_1)+\widehat{\beth}^{-}_{\aleph}(\jmath_2)}{2}$ and $\widehat{\beth}^{+}_{\aleph}(\jmath_1\triangle_1\jmath_2) < \ell_2 \leqq \frac{\widehat{\beth}^{+}_{\aleph}(\jmath_1)+\widehat{\beth}^{+}_{\aleph}(\jmath_2)}{2}$ and $\widehat{\gimel}^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) > s \geqq \max\{\widehat{\gimel}^{-}_{\aleph}(\jmath_1), \widehat{\gimel}^{-}_{\aleph}(\jmath_2)\}$, $1-\widehat{\daleth}^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) > s \geqq \max\{1-\widehat{\daleth}^{-}_{\aleph}(\jmath_1), 1-\widehat{\daleth}^{-}_{\aleph}(\jmath_2)\}$. Then $\jmath_1, \jmath_2 \in \widehat{\sqsupset}^{(\ell_1,\ell_2,s)}$, but $\jmath_1\triangle_1\jmath_2 \notin \widehat{\sqsupset}^{(\ell_1,\ell_2,s)}$. This contradicts to that $\widehat{\sqsupset}^{(\ell_1,\ell_2,s)}$ is a SBS of Ξ. Hence, $\widehat{\daleth}^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) \geqq \min\{\widehat{\daleth}^{-}_{\aleph}(\jmath_1), \widehat{\daleth}^{-}_{\aleph}(\jmath_2)\}$, $1-\widehat{\gimel}^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) \geqq \min\{1-\widehat{\gimel}^{-}_{\aleph}(\jmath_1), 1-\widehat{\gimel}^{-}_{\aleph}(\jmath_2)\}$, $\widehat{\beth}^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) \geqq \frac{\widehat{\beth}^{-}_{\aleph}(\jmath_1)+\widehat{\beth}^{-}_{\aleph}(\jmath_2)}{2}$, $\widehat{\beth}^{+}_{\aleph}(\jmath_1\triangle_1\jmath_2) \geqq \frac{\widehat{\beth}^{+}_{\aleph}(\jmath_1)+\widehat{\beth}^{+}_{\aleph}(\jmath_2)}{2}$ and $\widehat{\gimel}^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) \leqq \max\{\widehat{\gimel}^{-}_{\aleph}(\jmath_1), \widehat{\gimel}^{-}_{\aleph}(\jmath_2)\}$ and $1-\widehat{\daleth}^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) \leqq \max\{1-\widehat{\daleth}^{-}_{\aleph}(\jmath_1), 1-\widehat{\daleth}^{-}_{\aleph}(\jmath_2)\}$. Similarly, $\triangle_2$ and $\triangle_3$ cases.
Let assume that $\sqsupset^{(\ell_1,\ell_2,s)}$ is a SBS of Ξ, where $\ell_1, \ell_2, s \in [0,1]$. Suppose if there exist $\jmath_1, \jmath_2 \in \Xi$ such that $\daleth^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) < \min\{\daleth^{-}_{\aleph}(\jmath_1), \daleth^{-}_{\aleph}(\jmath_2)\}$, $1-\gimel^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) < \min\{1-\gimel^{-}_{\aleph}(\jmath_1), 1-\gimel^{-}_{\aleph}(\jmath_2)\}$, $\beth^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) < \frac{\beth^{-}_{\aleph}(\jmath_1)+\beth^{-}_{\aleph}(\jmath_2)}{2}$, $\beth^{+}_{\aleph}(\jmath_1\triangle_1\jmath_2) < \frac{\beth^{+}_{\aleph}(\jmath_1)+\beth^{+}_{\aleph}(\jmath_2)}{2}$ and $\gimel^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) > \max\{\gimel^{-}_{\aleph}(\jmath_1), \gimel^{-}_{\aleph}(\jmath_2)\}$. $1-\daleth^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) > \max\{1-\daleth^{-}_{\aleph}(\jmath_1), 1-\daleth^{-}_{\aleph}(\jmath_2)\}$. Select $\ell_1, \ell_2, s \in [0,1]$ such that $\daleth^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) < \ell_1 \leqq \min\{\daleth^{-}_{\aleph}(\jmath_1), \daleth^{-}_{\aleph}(\jmath_2)\}$ and $1-\gimel^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) < \ell_1 \leqq \min\{1-\gimel^{-}_{\aleph}(\jmath_1), 1-\gimel^{-}_{\aleph}(\jmath_2)\}$ and $\beth^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) < \ell_2 \leqq \frac{\beth^{-}_{\aleph}(\jmath_1)+\beth^{-}_{\aleph}(\jmath_2)}{2}$ and $\beth^{+}_{\aleph}(\jmath_1\triangle_1\jmath_2) < \ell_2 \leqq \frac{\beth^{+}_{\aleph}(\jmath_1)+\beth^{+}_{\aleph}(\jmath_2)}{2}$ and $\gimel^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) > s \geqq \max\{\gimel^{-}_{\aleph}(\jmath_1), \gimel^{-}_{\aleph}(\jmath_2)\}$, $1-\daleth^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) > s \geqq \max\{1-\daleth^{-}_{\aleph}(\jmath_1), 1-\daleth^{-}_{\aleph}(\jmath_2)\}$. Then $\jmath_1, \jmath_2 \in \sqsupset^{(\ell_1,\ell_2,s)}$, but $\jmath_1\triangle_1\jmath_2 \notin \sqsupset^{(\ell_1,\ell_2,s)}$. This contradicts to that $\sqsupset^{(\ell_1,\ell_2,s)}$ is a SBS of Ξ. Hence, $\daleth^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) \geqq \min\{\daleth^{-}_{\aleph}(\jmath_1), \daleth^{-}_{\aleph}(\jmath_2)\}$, $1-\gimel^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) \geqq \min\{1-\gimel^{-}_{\aleph}(\jmath_1), 1-\gimel^{-}_{\aleph}(\jmath_2)\}$, $\beth^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) \geqq \frac{\beth^{-}_{\aleph}(\jmath_1)+\beth^{-}_{\aleph}(\jmath_2)}{2}$, $\beth^{+}_{\aleph}(\jmath_1\triangle_1\jmath_2) \geqq \frac{\beth^{+}_{\aleph}(\jmath_1)+\beth^{+}_{\aleph}(\jmath_2)}{2}$ and $\gimel^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) \leqq \max\{\gimel^{-}_{\aleph}(\jmath_1), \gimel^{-}_{\aleph}(\jmath_2)\}$ and $1-\daleth^{-}_{\aleph}(\jmath_1\triangle_1\jmath_2) \leqq \max\{1-\daleth^{-}_{\aleph}(\jmath_1), 1-\daleth^{-}_{\aleph}(\jmath_2)\}$. Similarly, $\triangle_2$ and $\triangle_3$ cases.
Hence, $\sqsupset = ([\daleth^{-}_{\aleph}, \daleth^{+}_{\aleph}], [\beth^{-}_{\aleph}, \beth^{+}_{\aleph}], [\gimel^{-}_{\aleph}, \gimel^{+}_{\aleph}])$ is a IVNCVSBS of Ξ.

Definition 3.9. Let $\aleph$ be any IVNCVSBS of Ξ and $\varsigma \in \Xi$. Then the pseudo NSV coset $(\varsigma\aleph)^{p}$ is defined by

$$\left\{\begin{array}{l}(\varsigma\widehat{\sqsupset}^{\daleth}_{\aleph})^{p}(\wp) = p(\varsigma)\widehat{\sqsupset}^{\daleth}_{\aleph}(\wp),\\ (\varsigma\widehat{\sqsupset}^{\beth}_{\aleph})^{p}(\wp) = p(\varsigma)\widehat{\sqsupset}^{\beth}_{\aleph}(\wp),\\ (\varsigma\widehat{\sqsupset}^{\gimel}_{\aleph})^{p}(\wp) = p(\varsigma)\widehat{\sqsupset}^{\gimel}_{\aleph}(\wp)\end{array}\right\}.$$

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$$\left\{\begin{array}{l}(\varsigma\beth_{\aleph}^{\daleth})^{p}(\wp)=p(\varsigma)\beth_{\aleph}^{\daleth}(\wp),\\(\varsigma\beth_{\aleph}^{\gimel})^{p}(\wp)=p(\varsigma)\beth_{\aleph}^{\gimel}(\wp),\\(\varsigma\beth_{\aleph}^{\dashv})^{p}(\wp)=p(\varsigma)\beth_{\aleph}^{\dashv}(\wp)\end{array}\right\}.$$

That is,

$$\left\{\begin{array}{l}(\varsigma\widehat{\daleth}_{\aleph}^{-})^{p}(\wp)=p(\varsigma)\widehat{\daleth}_{\aleph}^{-}(\wp),\ 1-(\varsigma\widehat{\dashv}_{\aleph}^{-})^{p}(\wp)=p(\varsigma)(1-\widehat{\dashv}_{\aleph}^{-})(\wp),\\(\varsigma\widehat{\gimel}_{\aleph}^{-})^{p}(\wp)=p(\varsigma)\widehat{\gimel}_{\aleph}^{-}(\wp),\ (\varsigma\widehat{\gimel}_{\aleph}^{+})^{p}(\wp)=p(\varsigma)\widehat{\gimel}_{\aleph}^{+}(\wp),\\(\varsigma\widehat{\dashv}_{\aleph}^{-})^{p}(\wp)=p(\varsigma)\widehat{\dashv}_{\aleph}^{-}(\wp),\ 1-(\varsigma\widehat{\daleth}_{\aleph}^{-})^{p}(\wp)=p(\varsigma)(1-\widehat{\daleth}_{\aleph}^{-})(\wp)\end{array}\right\}$$

$$\left\{\begin{array}{l}(\varsigma\daleth_{\aleph}^{-})^{p}(\wp)=p(\varsigma)\daleth_{\aleph}^{-}(\wp),\ 1-(\varsigma\dashv_{\aleph}^{-})^{p}(\wp)=p(\varsigma)(1-\dashv_{\aleph}^{-})(\wp),\\(\varsigma\gimel_{\aleph}^{-})^{p}(\wp)=p(\varsigma)\gimel_{\aleph}^{-}(\wp),\ (\varsigma\gimel_{\aleph}^{+})^{p}(\wp)=p(\varsigma)\gimel_{\aleph}^{+}(\wp),\\(\varsigma\dashv_{\aleph}^{-})^{p}(\wp)=p(\varsigma)\dashv_{\aleph}^{-}(\wp),\ 1-(\varsigma\daleth_{\aleph}^{-})^{p}(\wp)=p(\varsigma)(1-\daleth_{\aleph}^{-})(\wp)\end{array}\right\}$$

each $\wp\in\Xi$ and for any non-empty set $p\in P$.

Theorem 3.10. *Let $\aleph$ be any IVNCVSBS of Ξ, then the pseudo NSV coset $(\varsigma\aleph)^{p}$ is a IVNCVSBS of Ξ.*

Proof. Let $\aleph$ be any IVNCVSBS of Ξ and for each $\wp,\partial\in\Xi$. Now, $(\varsigma\widehat{\daleth}_{\aleph}^{-})^{p}(\wp\triangle_1\partial)=p(\varsigma)\ \widehat{\daleth}_{\aleph}^{-}(\wp\triangle_1\partial)\geqq p(\varsigma)\ \min\{\widehat{\daleth}_{\aleph}^{-}(\wp),\widehat{\daleth}_{\aleph}^{-}(\partial)\}=\min\{p(\varsigma)\ \widehat{\daleth}_{\aleph}^{-}(\wp),p(\varsigma)\ \widehat{\daleth}_{\aleph}^{-}(\partial)\}=\min\{(\varsigma\widehat{\daleth}_{\aleph}^{-})^{p}(\wp),(\varsigma\widehat{\daleth}_{\aleph}^{-})^{p}(\partial)\}$. Thus $(\varsigma\widehat{\daleth}_{\aleph}^{-})^{p}(\wp\triangle_1\partial)\geqq\min\{(\varsigma\widehat{\daleth}_{\aleph}^{-})^{p}(\wp),(\varsigma\widehat{\daleth}_{\aleph}^{-})^{p}(\partial)\}$ and $1-(\varsigma\widehat{\dashv}_{\aleph}^{-})^{p}(\wp\triangle_1\partial)=p(\varsigma)\ (1-\widehat{\dashv}_{\aleph}^{-}(\wp\triangle_1\partial))\geqq p(\varsigma)\ \min\{1-\widehat{\dashv}_{\aleph}^{-}(\wp),1-\widehat{\dashv}_{\aleph}^{-}(\partial)\}=\min\{p(\varsigma)\ (1-\widehat{\dashv}_{\aleph}^{-}(\wp)),p(\varsigma)\ (1-\widehat{\dashv}_{\aleph}^{-}(\partial))\}=\min\{1-(\varsigma\widehat{\dashv}_{\aleph}^{-})^{p}(\wp),1-(\varsigma\widehat{\dashv}_{\aleph}^{-})^{p}(\partial)\}$. Thus
$1-(\varsigma\widehat{\dashv}_{\aleph}^{-})^{p}(\wp\triangle_1\partial)\geqq\min\{1-(\varsigma\widehat{\dashv}_{\aleph}^{-})^{p}(\wp),1-(\varsigma\widehat{\dashv}_{\aleph}^{-})^{p}(\partial)\}$.
Now, $(\varsigma\widehat{\gimel}_{\aleph}^{-})^{p}(\wp\triangle_1\partial)=p(\varsigma)\ \widehat{\gimel}_{\aleph}^{-}(\wp\triangle_1\partial)\geqq p(\varsigma)\left[\frac{\widehat{\gimel}_{\aleph}^{-}(\wp)+\widehat{\gimel}_{\aleph}^{-}(\partial)}{2}\right]=\frac{p(\varsigma)\ \widehat{\gimel}_{\aleph}^{-}(\wp)+p(\varsigma)\ \widehat{\gimel}_{\aleph}^{-}(\partial)}{2}=\frac{(\varsigma\widehat{\gimel}_{\aleph}^{-})^{p}(\wp)+(\varsigma\widehat{\gimel}_{\aleph}^{-})^{p}(\partial)}{2}$.
Thus $(\varsigma\widehat{\gimel}_{\aleph}^{-})^{p}(\wp\triangle_1\partial)\geqq\frac{(\varsigma\widehat{\gimel}_{\aleph}^{-})^{p}(\wp)+(\varsigma\widehat{\gimel}_{\aleph}^{-})^{p}(\partial)}{2}$ and $(\varsigma\widehat{\gimel}_{\aleph}^{+})^{p}(\wp\triangle_1\partial)=p(\varsigma)\ \widehat{\gimel}_{\aleph}^{+}(\wp\triangle_1\partial)\geqq p(\varsigma)\left[\frac{\widehat{\gimel}_{\aleph}^{+}(\wp)+\widehat{\gimel}_{\aleph}^{+}(\partial)}{2}\right]=\frac{p(\varsigma)\ \widehat{\gimel}_{\aleph}^{+}(\wp)+p(\varsigma)\ \widehat{\gimel}_{\aleph}^{+}(\partial)}{2}=\frac{(\varsigma\widehat{\gimel}_{\aleph}^{+})^{p}(\wp)+(\varsigma\widehat{\gimel}_{\aleph}^{+})^{p}(\partial)}{2}$. Thus $(\varsigma\widehat{\gimel}_{\aleph}^{+})^{p}(\wp\triangle_1\partial)\geqq\frac{(\varsigma\widehat{\gimel}_{\aleph}^{+})^{p}(\wp)+(\varsigma\widehat{\gimel}_{\aleph}^{+})^{p}(\partial)}{2}$. Now, $(\varsigma\widehat{\dashv}_{\aleph}^{-})^{p}(\wp\triangle_1\partial)=p(\varsigma)\ \widehat{\dashv}_{\aleph}^{-}(\wp\triangle_1\partial)\leqq p(\varsigma)\ \max\{\widehat{\dashv}_{\aleph}^{-}(\wp),\widehat{\dashv}_{\aleph}^{-}(\partial)\}=\max\{p(\varsigma)\ \widehat{\dashv}_{\aleph}^{-}(\wp),p(\varsigma)\ \widehat{\dashv}_{\aleph}^{-}(\partial)\}=\max\{(\varsigma\widehat{\dashv}_{\aleph}^{-})^{p}(\wp),(\varsigma\widehat{\dashv}_{\aleph}^{-})^{p}(\partial)\}$. Thus $(\varsigma\widehat{\dashv}_{\aleph}^{-})^{p}(\wp\triangle_1\partial)\leqq\max\{(\varsigma\widehat{\dashv}_{\aleph}^{-})^{p}(\wp),(\varsigma\widehat{\dashv}_{\aleph}^{-})^{p}(\partial)\}$ and $1-(\varsigma\widehat{\daleth}_{\aleph}^{-})^{p}(\wp\triangle_1\partial)=p(\varsigma)\ (1-\widehat{\daleth}_{\aleph}^{-}(\wp\triangle_1\partial))\leqq p(\varsigma)\ \max\{1-\widehat{\daleth}_{\aleph}^{-}(\wp),1-\widehat{\daleth}_{\aleph}^{-}(\partial)\}=\max\{p(\varsigma)\ (1-\widehat{\daleth}_{\aleph}^{-}(\wp)),p(\varsigma)\ (1-\widehat{\daleth}_{\aleph}^{-}(\partial))\}=\max\{1-(\varsigma\widehat{\daleth}_{\aleph}^{-})^{p}(\wp),1-(\varsigma\widehat{\daleth}_{\aleph}^{-})^{p}(\partial)\}$. Thus $1-(\varsigma\widehat{\daleth}_{\aleph}^{-})^{p}(\wp\triangle_1\partial)\leqq\max\{1-(\varsigma\widehat{\daleth}_{\aleph}^{-})^{p}(\wp),1-(\varsigma\widehat{\daleth}_{\aleph}^{-})^{p}(\partial)\}$.
Now, $(\varsigma\daleth_{\aleph}^{-})^{p}(\wp\triangle_1\partial)=p(\varsigma)\ \daleth_{\aleph}^{-}(\wp\triangle_1\partial)\geqq p(\varsigma)\ \min\{\daleth_{\aleph}^{-}(\wp),\daleth_{\aleph}^{-}(\partial)\}=\min\{p(\varsigma)\ \daleth_{\aleph}^{-}(\wp),p(\varsigma)\ \daleth_{\aleph}^{-}(\partial)\}=\min\{(\varsigma\daleth_{\aleph}^{-})^{p}(\wp),(\varsigma\daleth_{\aleph}^{-})^{p}(\partial)\}$. Thus $(\varsigma\daleth_{\aleph}^{-})^{p}(\wp\triangle_1\partial)\geqq\min\{(\varsigma\daleth_{\aleph}^{-})^{p}(\wp),(\varsigma\daleth_{\aleph}^{-})^{p}(\partial)\}$ and $1-(\varsigma\dashv_{\aleph}^{-})^{p}(\wp\triangle_1\partial)=p(\varsigma)\ (1-\dashv_{\aleph}^{-}(\wp\triangle_1\partial))\geqq p(\varsigma)\ \min\{1-\dashv_{\aleph}^{-}(\wp),1-\dashv_{\aleph}^{-}(\partial)\}=\min\{p(\varsigma)\ (1-\dashv_{\aleph}^{-}(\wp)),p(\varsigma)\ (1-\dashv_{\aleph}^{-}(\partial))\}=\min\{1-(\varsigma\dashv_{\aleph}^{-})^{p}(\wp),1-(\varsigma\dashv_{\aleph}^{-})^{p}(\partial)\}$. Thus $1-(\varsigma\dashv_{\aleph}^{-})^{p}(\wp\triangle_1\partial)\geqq\min\{1-(\varsigma\dashv_{\aleph}^{-})^{p}(\wp),1-(\varsigma\dashv_{\aleph}^{-})^{p}(\partial)\}$.
Now, $(\varsigma\gimel_{\aleph}^{-})^{p}(\wp\triangle_1\partial)=p(\varsigma)\ \gimel_{\aleph}^{-}(\wp\triangle_1\partial)\geqq p(\varsigma)\left[\frac{\gimel_{\aleph}^{-}(\wp)+\gimel_{\aleph}^{-}(\partial)}{2}\right]=\frac{p(\varsigma)\ \gimel_{\aleph}^{-}(\wp)+p(\varsigma)\ \gimel_{\aleph}^{-}(\partial)}{2}=\frac{(\varsigma\gimel_{\aleph}^{-})^{p}(\wp)+(\varsigma\gimel_{\aleph}^{-})^{p}(\partial)}{2}$.
Thus $(\varsigma\gimel_{\aleph}^{-})^{p}(\wp\triangle_1\partial)\geqq\frac{(\varsigma\gimel_{\aleph}^{-})^{p}(\wp)+(\varsigma\gimel_{\aleph}^{-})^{p}(\partial)}{2}$ and $(\varsigma\gimel_{\aleph}^{+})^{p}(\wp\triangle_1\partial)=p(\varsigma)\ \gimel_{\aleph}^{+}(\wp\triangle_1\partial)\geqq p(\varsigma)\left[\frac{\gimel_{\aleph}^{+}(\wp)+\gimel_{\aleph}^{+}(\partial)}{2}\right]=\frac{p(\varsigma)\ \gimel_{\aleph}^{+}(\wp)+p(\varsigma)\ \gimel_{\aleph}^{+}(\partial)}{2}=\frac{(\varsigma\gimel_{\aleph}^{+})^{p}(\wp)+(\varsigma\gimel_{\aleph}^{+})^{p}(\partial)}{2}$. Thus $(\varsigma\gimel_{\aleph}^{+})^{p}(\wp\triangle_1\partial)\geqq\frac{(\varsigma\gimel_{\aleph}^{+})^{p}(\wp)+(\varsigma\gimel_{\aleph}^{+})^{p}(\partial)}{2}$. Now, $(\varsigma\dashv_{\aleph}^{-})^{p}(\wp\triangle_1\partial)=p(\varsigma)\ \dashv_{\aleph}^{-}(\wp\triangle_1\partial)\leqq p(\varsigma)\ \max\{\dashv_{\aleph}^{-}(\wp),\dashv_{\aleph}^{-}(\partial)\}=\max\{p(\varsigma)\ \dashv_{\aleph}^{-}(\wp),p(\varsigma)\ \dashv_{\aleph}^{-}(\partial)\}=\max\{(\varsigma\dashv_{\aleph}^{-})^{p}(\wp),(\varsigma\dashv_{\aleph}^{-})^{p}(\partial)\}$. Thus $(\varsigma\dashv_{\aleph}^{-})^{p}(\wp\triangle_1\partial)\leqq\max\{(\varsigma\dashv_{\aleph}^{-})^{p}(\wp),(\varsigma\dashv_{\aleph}^{-})^{p}(\partial)\}$ and $1-(\varsigma\daleth_{\aleph}^{-})^{p}(\wp\triangle_1\partial)=p(\varsigma)\ (1-\daleth_{\aleph}^{-}(\wp\triangle_1\partial))\leqq p(\varsigma)\ \max\{1-\daleth_{\aleph}^{-}(\wp),1-\daleth_{\aleph}^{-}(\partial)\}=\max\{p(\varsigma)\ (1-\daleth_{\aleph}^{-}(\wp)),p(\varsigma)\ (1-\daleth_{\aleph}^{-}(\partial))\}=\max\{1-(\varsigma\daleth_{\aleph}^{-})^{p}(\wp),1-(\varsigma\daleth_{\aleph}^{-})^{p}(\partial)\}$. Thus $1-(\varsigma\daleth_{\aleph}^{-})^{p}(\wp\triangle_1\partial)\leqq\max\{1-(\varsigma\daleth_{\aleph}^{-})^{p}(\wp),1-(\varsigma\daleth_{\aleph}^{-})^{p}(\partial)\}$. Similarly, $\triangle_2$ and $\triangle_3$ cases. Hence, $(\varsigma\aleph)^{p}$ is a IVNCVSBS of Ξ.

Definition 3.11. Let $(\Xi_1,\heartsuit_1,\heartsuit_2,\heartsuit_3)$ and $(\Xi_2,\diamondsuit_1,\diamondsuit_2,\diamondsuit_3)$ be the bisemirings. Let $\Upsilon:\Xi_1\to\Xi_2$ and $\aleph$ be an IVNCVSBS in Ξ_1, Υ be an IVNCVSBS in $\Upsilon(\Xi_1)=\Xi_2$, the image of VS is defined as $\widehat{\beth}_{\Re(V)}(\ell_2)=[\widehat{\daleth}_{\Re(V)}^{-}(\ell_2),1-\widehat{\dashv}_{\Re(V)}^{-}(\ell_2)],[\widehat{\gimel}_{\Re(V)}^{-}(\ell_2),\widehat{\gimel}_{\Re(V)}^{+}(\ell_2)],[\widehat{\dashv}_{\Re(V)}^{-}(\ell_2),1-\widehat{\daleth}_{\Re(V)}^{-}(\ell_2)]$ where $\widehat{\daleth}_{\Re(V)}^{-}(\ell_2)=\widehat{\daleth}_{\Upsilon}^{-}\Re(\ell_2)$, $\widehat{\gimel}_{\Re(V)}^{-}(\ell_2)=\widehat{\gimel}_{\Upsilon}^{-}\Re(\ell_2)$, $\widehat{\gimel}_{\Re(V)}^{+}(\ell_2)=\widehat{\gimel}_{\Upsilon}^{+}\Re(\ell_2)$ and $\widehat{\dashv}_{\Re(V)}^{-}(\ell_2)=\widehat{\dashv}_{\Upsilon}^{-}\Re(\ell_2)$ and $\beth_{\Re(V)}(\ell_2)=[\daleth_{\Re(V)}^{-}(\ell_2),1-\dashv_{\Re(V)}^{-}(\ell_2)],[\gimel_{\Re(V)}^{-}(\ell_2),\gimel_{\Re(V)}^{+}(\ell_2)],[\dashv_{\Re(V)}^{-}(\ell_2),1-\daleth_{\Re(V)}^{-}(\ell_2)]$ where $\daleth_{\Re(V)}^{-}(\ell_2)=\daleth_{\Upsilon}^{-}\Re(\ell_2)$, $\gimel_{\Re(V)}^{-}(\ell_2)=\gimel_{\Upsilon}^{-}\Re(\ell_2)$, $\gimel_{\Re(V)}^{+}(\ell_2)=\gimel_{\Upsilon}^{+}\Re(\ell_2)$ and $\dashv_{\Re(V)}^{-}(\ell_2)=\dashv_{\Upsilon}^{-}\Re(\ell_2)$.

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Definition 3.12. Let $(\Xi_1, \heartsuit_1, \heartsuit_2, \heartsuit_3)$ and $(\Xi_2, \diamondsuit_1, \diamondsuit_2, \diamondsuit_3)$ be the bisemirings. Let $\Re : \Xi_1 \to \Xi_2$ be any function. Let Υ be a VS in $\Re(\Xi_1) = \Xi_2$. Then the inverse image of Υ, $\Re^{-1}$ is the VS in Ξ_1 by $\beth_{\Re^{-1}(V)}(\ell_1) = [\widehat{\daleth}^-_{\Re^{-1}(V)}(\ell_1), 1 - \widehat{\Finv}^-_{\Re^{-1}(V)}(\ell_1)], [\widehat{\gimel}^-_{\Re^{-1}(V)}(\ell_1), \widehat{\gimel}^+_{\Re^{-1}(V)}(\ell_1)], [\widehat{\Finv}^-_{\Re^{-1}(V)}(\ell_1), 1 - \widehat{\daleth}^-_{\Re^{-1}(V)}(\ell_1)]$, where $\widehat{\daleth}^-_{\Re^{-1}(V)}(\ell_1) = \widehat{\daleth}^-_{\Upsilon}(\Re^{-1}(\ell_1)), \widehat{\gimel}^-_{\Re^{-1}(V)}(\ell_1) = \widehat{\gimel}^-_{\Upsilon}(\Re^{-1}(\ell_1)), \widehat{\gimel}^+_{\Re^{-1}(V)}(\ell_1) = \widehat{\gimel}^+_{\Upsilon}(\Re^{-1}(\ell_1)), \widehat{\Finv}^-_{\Re^{-1}(V)}(\ell_1) = \widehat{\Finv}^-_{\Upsilon}(\Re^{-1}(\ell_1))$ and $\beth_{\Re^{-1}(V)}(\ell_1) = [\daleth^-_{\Re^{-1}(V)}(\ell_1), 1 - \Finv^-_{\Re^{-1}(V)}(\ell_1)], [\gimel^-_{\Re^{-1}(V)}(\ell_1), \gimel^+_{\Re^{-1}(V)}(\ell_1)], [\Finv^-_{\Re^{-1}(V)}(\ell_1), 1 - \daleth^-_{\Re^{-1}(V)}(\ell_1)]$, where $\daleth^-_{\Re^{-1}(V)}(\ell_1) = \daleth^-_{\Upsilon}(\Re^{-1}(\ell_1)), \gimel^-_{\Re^{-1}(V)}(\ell_1) = \gimel^-_{\Upsilon}(\Re^{-1}(\ell_1)), \gimel^+_{\Re^{-1}(V)}(\ell_1) = \gimel^+_{\Upsilon}(\Re^{-1}(\ell_1))$, $\Finv^-_{\Re^{-1}(V)}(\ell_1) = \Finv^-_{\Upsilon}(\Re^{-1}(\ell_1))$.

Theorem 3.13. *Every homomorphic image of IVNCVSBS of Ξ_1 is a IVNCVSBS of Ξ_2.*

Proof. Let $\Re : \Xi_1 \to \Xi_2$ be a homomorphism. Now, $\Re(\ell_1 \heartsuit_1 \ell_2) = \Re(\ell_1) \diamondsuit_1 \Re(\ell_2), \Re(\ell_1 \heartsuit_2 \ell_2) = \Re(\ell_1) \diamondsuit_2 \Re(\ell_2)$ and $\Re(\ell_1 \heartsuit_3 \ell_2) = \Re(\ell_1) \diamondsuit_3 \Re(\ell_2)$ for all $\ell_1, \ell_2 \in \Xi_1$. Let $V = \Re(\aleph)$, $\aleph$ is a IVNCVSBS of Ξ_1. Let $\Re(\ell_1), \Re(\ell_2) \in \Xi_2$, $\widehat{\daleth}^-_{\Upsilon}(\Re(\ell_1) \diamondsuit_1 \Re(\ell_2)) \geqq \widehat{\daleth}^-_{\aleph}(\ell_1 \heartsuit_1 \ell_2) \geqq \min\{\widehat{\daleth}^-_{\aleph}(\ell_1), \widehat{\daleth}^-_{\aleph}(\ell_2)\} = \min\{\widehat{\daleth}^-_{\Upsilon}\Re(\ell_1), \widehat{\daleth}^-_{\Upsilon}\Re(\ell_2)\}$ and $1 - \widehat{\Finv}^-_{\Upsilon}(\Re(\ell_1) \diamondsuit_1 \Re(\ell_2)) \geqq 1 - \widehat{\Finv}^-_{\aleph}(\ell_1 \heartsuit_1 \ell_2) \geqq \min\{1 - \widehat{\Finv}^-_{\aleph}(\ell_1), 1 - \widehat{\Finv}^-_{\aleph}(\ell_2)\} = \min\{1 - \widehat{\Finv}^-_{\Upsilon}\Re(\ell_1), 1 - \widehat{\Finv}^-_{\Upsilon}\Re(\ell_2)\}$. Thus $\beth^{\widehat{\daleth}}_{\Upsilon}(\Re(\ell_1) \diamondsuit_1 \Re(\ell_2)) \geqq \min\{\beth^{\widehat{\daleth}}_{\Upsilon}\Re(\ell_1), \beth^{\widehat{\daleth}}_{\Upsilon}\Re(\ell_2)\}$.
Similarly, $\beth^{\widehat{\daleth}}_{\Upsilon}(\Re(\ell_1) \diamondsuit_2 \Re(\ell_2)) \geqq \min\{\beth^{\widehat{\daleth}}_{\Upsilon}\Re(\ell_1), \beth^{\widehat{\daleth}}_{\Upsilon}\Re(\ell_2)\}$ and $\beth^{\widehat{\daleth}}_{\Upsilon}(\Re(\ell_1) \diamondsuit_3 \Re(\ell_2)) \geqq \min\{\beth^{\widehat{\daleth}}_{\Upsilon}\Re(\ell_1), \beth^{\widehat{\daleth}}_{\Upsilon}\Re(\ell_2)\}$.
Now, $\widehat{\gimel}^-_{\Upsilon}(\Re(\ell_1) \diamondsuit_1 \Re(\ell_2)) \geqq \widehat{\gimel}^-_{\aleph}(\ell_1 \heartsuit_1 \ell_2) \geqq \frac{\widehat{\gimel}^-_{\aleph}(\ell_1) + \widehat{\gimel}^-_{\aleph}(\ell_2)}{2} = \frac{\widehat{\gimel}^-_{\Upsilon}\Re(\ell_1) + \widehat{\gimel}^-_{\Upsilon}\Re(\ell_2)}{2}$ and $\widehat{\gimel}^+_{\Upsilon}(\Re(\ell_1) \diamondsuit_1 \Re(\ell_2)) \geqq \widehat{\gimel}^+_{\aleph}(\ell_1 \heartsuit_1 \ell_2) \geqq \frac{\widehat{\gimel}^+_{\aleph}(\ell_1) + \widehat{\gimel}^+_{\aleph}(\ell_2)}{2} = \frac{\widehat{\gimel}^+_{\Upsilon}\Re(\ell_1) + \widehat{\gimel}^+_{\Upsilon}\Re(\ell_2)}{2}$. Thus $\beth^{\widehat{\gimel}}_{\Upsilon}(\Re(\ell_1) \diamondsuit_1 \Re(\ell_2)) \geqq \frac{\beth^{\widehat{\gimel}}_{\Upsilon}\Re(\ell_1) + \beth^{\widehat{\gimel}}_{\Upsilon}\Re(\ell_2)}{2}$. Similarly, $\beth^{\widehat{\gimel}}_{\Upsilon}(\Re(\ell_1) \diamondsuit_2 \Re(\ell_2)) \geqq \min\{\beth^{\widehat{\gimel}}_{\Upsilon}\Re(\ell_1), \beth^{\widehat{\gimel}}_{\Upsilon}\Re(\ell_2)\}$ and $\beth^{\widehat{\gimel}}_{\Upsilon}(\Re(\ell_1) \diamondsuit_3 \Re(\ell_2)) \geqq \min\{\beth^{\widehat{\gimel}}_{\Upsilon}\Re(\ell_1), \beth^{\widehat{\gimel}}_{\Upsilon}\Re(\ell_2)\}$. Now, $\widehat{\Finv}^-_{\Upsilon}(\Re(\ell_1) \diamondsuit_1 \Re(\ell_2)) \leqq \widehat{\Finv}^-_{\aleph}(\ell_1 \heartsuit_1 \ell_2) \leqq \max\{\widehat{\Finv}^-_{\aleph}(\ell_1), \widehat{\Finv}^-_{\aleph}(\ell_2)\} = \max\{\widehat{\Finv}^-_{\Upsilon}\Re(\ell_1), \widehat{\Finv}^-_{\Upsilon}\Re(\ell_2)\}$ and
$1 - \widehat{\daleth}^-_{\Upsilon}(\Re(\ell_1) \diamondsuit_1 \Re(\ell_2)) \leqq 1 - \widehat{\daleth}^-_{\aleph}(\ell_1 \heartsuit_1 \ell_2) \leqq \max\{1 - \widehat{\daleth}^-_{\aleph}(\ell_1), 1 - \widehat{\daleth}^-_{\aleph}(\ell_2)\} = \max\{1 - \widehat{\daleth}^-_{\Upsilon}\Re(\ell_1), 1 - \widehat{\daleth}^-_{\Upsilon}\Re(\ell_2)\}$. Thus $\beth^{\widehat{\Finv}}_{\Upsilon}(\Re(\ell_1) \diamondsuit_1 \Re(\ell_2)) \leqq \max\{\beth^{\widehat{\Finv}}_{\Upsilon}\Re(\ell_1), \beth^{\widehat{\Finv}}_{\Upsilon}\Re(\ell_2)\}$. Similarly,
$\beth^{\widehat{\Finv}}_{\Upsilon}(\Re(\ell_1) \diamondsuit_2 \Re(\ell_2)) \leqq \max\{\beth^{\widehat{\Finv}}_{\Upsilon}\Re(\ell_1), \beth^{\widehat{\Finv}}_{\Upsilon}\Re(\ell_2)\}$ and $\beth^{\widehat{\Finv}}_{\Upsilon}(\Re(\ell_1) \diamondsuit_3 \Re(\ell_2)) \leqq \max\{\beth^{\widehat{\Finv}}_{\Upsilon}\Re(\ell_1), \beth^{\widehat{\Finv}}_{\Upsilon}\Re(\ell_2)\}$.
Let $V = \Re(\aleph)$, $\aleph$ is a IVNCVSBS of Ξ_1. Let $\Re(\ell_1), \Re(\ell_2) \in \Xi_2$, $\daleth^-_{\Upsilon}(\Re(\ell_1) \diamondsuit_1 \Re(\ell_2)) \geqq \daleth^-_{\aleph}(\ell_1 \heartsuit_1 \ell_2) \geqq \min\{\daleth^-_{\aleph}(\ell_1), \daleth^-_{\aleph}(\ell_2)\} = \min\{\daleth^-_{\Upsilon}\Re(\ell_1), \daleth^-_{\Upsilon}\Re(\ell_2)\}$ and $1 - \Finv^-_{\Upsilon}(\Re(\ell_1) \diamondsuit_1 \Re(\ell_2)) \geqq 1 - \Finv^-_{\aleph}(\ell_1 \heartsuit_1 \ell_2) \geqq \min\{1 - \Finv^-_{\aleph}(\ell_1), 1 - \Finv^-_{\aleph}(\ell_2)\} = \min\{1 - \Finv^-_{\Upsilon}\Re(\ell_1), 1 - \Finv^-_{\Upsilon}\Re(\ell_2)\}$.
Thus $\beth^{\daleth}_{\Upsilon}(\Re(\ell_1) \diamondsuit_1 \Re(\ell_2)) \geqq \min\{\beth^{\daleth}_{\Upsilon}\Re(\ell_1), \beth^{\daleth}_{\Upsilon}\Re(\ell_2)\}$.
Similarly, $\beth^{\daleth}_{\Upsilon}(\Re(\ell_1) \diamondsuit_2 \Re(\ell_2)) \geqq \min\{\beth^{\daleth}_{\Upsilon}\Re(\ell_1), \beth^{\daleth}_{\Upsilon}\Re(\ell_2)\}$ and $\beth^{\daleth}_{\Upsilon}(\Re(\ell_1) \diamondsuit_3 \Re(\ell_2)) \geqq \min\{\beth^{\daleth}_{\Upsilon}\Re(\ell_1), \beth^{\daleth}_{\Upsilon}\Re(\ell_2)\}$.
Now, $\gimel^-_{\Upsilon}(\Re(\ell_1) \diamondsuit_1 \Re(\ell_2)) \geqq \gimel^-_{\aleph}(\ell_1 \heartsuit_1 \ell_2) \geqq \frac{\gimel^-_{\aleph}(\ell_1) + \gimel^-_{\aleph}(\ell_2)}{2} = \frac{\gimel^-_{\Upsilon}\Re(\ell_1) + \gimel^-_{\Upsilon}\Re(\ell_2)}{2}$ and $\gimel^+_{\Upsilon}(\Re(\ell_1) \diamondsuit_1 \Re(\ell_2)) \geqq \gimel^+_{\aleph}(\ell_1 \heartsuit_1 \ell_2) \geqq \frac{\gimel^+_{\aleph}(\ell_1) + \gimel^+_{\aleph}(\ell_2)}{2} = \frac{\gimel^+_{\Upsilon}\Re(\ell_1) + \gimel^+_{\Upsilon}\Re(\ell_2)}{2}$. Thus, $\beth^{\gimel}_{\Upsilon}(\Re(\ell_1) \diamondsuit_1 \Re(\ell_2)) \geqq \frac{\beth^{\gimel}_{\Upsilon}\Re(\ell_1) + \beth^{\gimel}_{\Upsilon}\Re(\ell_2)}{2}$. Similarly, $\beth^{\gimel}_{\Upsilon}(\Re(\ell_1) \diamondsuit_2 \Re(\ell_2)) \geqq \min\{\beth^{\gimel}_{\Upsilon}\Re(\ell_1), \beth^{\gimel}_{\Upsilon}\Re(\ell_2)\}$ and $\beth^{\gimel}_{\Upsilon}(\Re(\ell_1) \diamondsuit_3 \Re(\ell_2)) \geqq \min\{\beth^{\gimel}_{\Upsilon}\Re(\ell_1), \beth^{\gimel}_{\Upsilon}\Re(\ell_2)\}$. Now, $\Finv^-_{\Upsilon}(\Re(\ell_1) \diamondsuit_1 \Re(\ell_2)) \leqq \Finv^-_{\aleph}(\ell_1 \heartsuit_1 \ell_2) \leqq \max\{\Finv^-_{\aleph}(\ell_1), \Finv^-_{\aleph}(\ell_2)\} = \max\{\Finv^-_{\Upsilon}\Re(\ell_1), \Finv^-_{\Upsilon}\Re(\ell_2)\}$ and
$1 - \daleth^-_{\Upsilon}(\Re(\ell_1) \diamondsuit_1 \Re(\ell_2)) \leqq 1 - \daleth^-_{\aleph}(\ell_1 \heartsuit_1 \ell_2) \leqq \max\{1 - \daleth^-_{\aleph}(\ell_1), 1 - \daleth^-_{\aleph}(\ell_2)\} = \max\{1 - \daleth^-_{\Upsilon}\Re(\ell_1), 1 - \daleth^-_{\Upsilon}\Re(\ell_2)\}$. Thus, $\beth^{\Finv}_{\Upsilon}(\Re(\ell_1) \diamondsuit_1 \Re(\ell_2)) \leqq \max\{\beth^{\Finv}_{\Upsilon}\Re(\ell_1), \beth^{\Finv}_{\Upsilon}\Re(\ell_2)\}$.
Similarly, $\beth^{\Finv}_{\Upsilon}(\Re(\ell_1) \diamondsuit_2 \Re(\ell_2)) \leqq \max\{\beth^{\Finv}_{\Upsilon}\Re(\ell_1), \beth^{\Finv}_{\Upsilon}\Re(\ell_2)\}$ and $\beth^{\Finv}_{\Upsilon}(\Re(\ell_1) \diamondsuit_3 \Re(\ell_2)) \leqq \max\{\beth^{\Finv}_{\Upsilon}\Re(\ell_1), \beth^{\Finv}_{\Upsilon}\Re(\ell_2)\}$.
Hence, Υ is a IVNCVSBS of Ξ_2.

Theorem 3.14. *Every homomorphic pre-image of IVNCVSBS of Ξ_2 is a IVNCVSBS of Ξ_1.*

Proof. Let $\Re : \Xi_1 \to \Xi_2$ and $\Re(\wp \heartsuit_1 \partial) = \Re(\wp) \diamondsuit_1 \Re(\partial), \Re(\wp \heartsuit_2 \partial) = \Re(\wp) \diamondsuit_2 \Re(\partial)$ and $\Re(\wp \heartsuit_3 \partial) = \Re(\wp) \diamondsuit_3 \Re(\partial)$ for all $\wp, \partial \in \Xi_1$. Let $V = \Re(\aleph)$, where Υ is any IVNCVSBS of Ξ_2. Let $\wp, \partial \in \Xi_1$. Now, $T^-_{\aleph}(\wp \heartsuit_1 \partial) = T^-_{\Upsilon}(\Re(\wp \heartsuit_1 \partial)) = T^-_{\Upsilon}(\Re(\wp) \diamondsuit_1 \Re(\partial)) \geqq \min\{T^-_{\Upsilon}\Re(\wp), T^-_{\Upsilon}\Re(\partial)\} = \min\{T^-_{\aleph}(\wp), T^-_{\aleph}(\partial)\}$. Thus $T^-_{\aleph}(\wp \heartsuit_1 \partial) \geqq \min\{T^-_{\aleph}(\wp), T^-_{\aleph}(\partial)\}$ and $1 - F^-_{\aleph}(\wp \heartsuit_1 \partial) = 1 - F^-_{\Upsilon}(\Re(\wp \heartsuit_1 \partial)) = 1 - F^-_{\Upsilon}(\Re(\wp) \diamondsuit_1 \Re(\partial)) \geqq \min\{1 - F^-_{\Upsilon}\Re(\wp), 1 - F^-_{\Upsilon}\Re(\partial)\} = \min\{1 - F^-_{\aleph}(\wp), 1 - F^-_{\aleph}(\partial)\}$. Thus $1 - F^-_{\aleph}(\wp \heartsuit_1 \partial) \geqq \min\{1 - F^-_{\aleph}(\wp), 1 - F^-_{\aleph}(\partial)\}$. Hence, $\beth^{\widehat{\daleth}}_{\Upsilon}(\wp \heartsuit_1 \partial) \geqq \min\{\beth^{\widehat{\daleth}}_{\Upsilon}(\wp), \beth^{\widehat{\daleth}}_{\Upsilon}(\partial)\}$. Similarly, $\beth^{\widehat{\daleth}}_{\Upsilon}(\wp \heartsuit_2 \partial) \geqq \min\{\beth^{\widehat{\daleth}}_{\Upsilon}(\wp), \beth^{\widehat{\daleth}}_{\Upsilon}(\partial)\}$ and $\beth^{\widehat{\daleth}}_{\Upsilon}(\wp \heartsuit_3 \partial) \geqq \min\{\beth^{\widehat{\daleth}}_{\Upsilon}(\wp), \beth^{\widehat{\daleth}}_{\Upsilon}(\partial)\}$.
Now, $I^-_{\aleph}(\wp \heartsuit_1 \partial) = I^-_{\Upsilon}(\Re(\wp \heartsuit_1 \partial)) = I^-_{\Upsilon}(\Re(\wp) \diamondsuit_1 \Re(\partial)) \geqq \frac{I^-_{\Upsilon}\Re(\wp) + I^-_{\Upsilon}\Re(\partial)}{2} = \frac{I^-_{\aleph}(\wp) + I^-_{\aleph}(\partial)}{2}$. Thus $I^-_{\aleph}(\wp \heartsuit_1 \partial) \geqq \frac{I^-_{\aleph}(\wp) + I^-_{\aleph}(\partial)}{2}$ and $I^+_{\aleph}(\wp \heartsuit_1 \partial) = I^+_{\Upsilon}(\Re(\wp \heartsuit_1 \partial)) = I^+_{\Upsilon}(\Re(\wp) \diamondsuit_1 \Re(\partial)) \geqq \frac{I^+_{\Upsilon}\Re(\wp) + I^+_{\Upsilon}\Re(\partial)}{2} = \frac{I^+_{\aleph}(\wp) + I^+_{\aleph}(\partial)}{2}$. Thus $I^+_{\aleph}(\wp \heartsuit_1 \partial) \geqq \frac{I^+_{\aleph}(\wp) + I^+_{\aleph}(\partial)}{2}$. Hence, $\beth^{\widehat{\gimel}}_{\Upsilon}(\wp \heartsuit_1 \partial) \geqq \frac{\beth^{\widehat{\gimel}}_{\Upsilon}(\wp) + \beth^{\widehat{\gimel}}_{\Upsilon}(\partial)}{2}$. Similarly, $\beth^{\widehat{\gimel}}_{\Upsilon}(\wp \heartsuit_2 \partial) \geqq \frac{\beth^{\widehat{\gimel}}_{\Upsilon}(\wp) + \beth^{\widehat{\gimel}}_{\Upsilon}(\partial)}{2}$ and $\beth^{\widehat{\gimel}}_{\Upsilon}(\wp \heartsuit_3 \partial) \geqq \frac{\beth^{\widehat{\gimel}}_{\Upsilon}(\wp) + \beth^{\widehat{\gimel}}_{\Upsilon}(\partial)}{2}$.
Now, $F^-_{\aleph}(\wp \heartsuit_1 \partial) = F^-_{\Upsilon}(\Re(\wp \heartsuit_1 \partial)) = F^-_{\Upsilon}(\Re(\wp) \diamondsuit_1 \Re(\partial)) \leqq \max\{F^-_{\Upsilon}\Re(\wp), F^-_{\Upsilon}\Re(\partial)\} = \max\{F^-_{\aleph}(\wp), F^-_{\aleph}(\partial)\}$.

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Thus $F^{-}_{\aleph}(\wp\heartsuit_1\partial) \leqq \max\{F^{-}_{\aleph}(\wp), F^{-}_{\aleph}(\partial)\}$ and $1-T^{-}_{\aleph}(\wp\heartsuit_1\partial) = 1-T^{-}_{\Upsilon}(\Re(\wp\heartsuit_1\partial)) = 1-T^{-}_{\Upsilon}(\Re(\wp)\diamondsuit_1\Re(\partial)) \leqq \max\{1-T^{-}_{\Upsilon}\Re(\wp), 1-T^{-}_{\Upsilon}\Re(\partial)\} = \max\{1-T^{-}_{\aleph}(\wp), 1-T^{-}_{\aleph}(\partial)\}$. Thus $1-T^{-}_{\aleph}(\wp\heartsuit_1\partial) \leqq \max\{1-T^{-}_{\aleph}(\wp), 1-T^{-}_{\aleph}(\partial)\}$. Hence, $\sqsupset^{\widehat{\Finv}}_{\Upsilon}(\wp\heartsuit_1\partial) \leqq \max\{\sqsupset^{\widehat{\Finv}}_{\Upsilon}(\wp), \sqsupset^{\widehat{\Finv}}_{\Upsilon}(\partial)\}$. Similarly, $\sqsupset^{\widehat{\Finv}}_{\Upsilon}(\wp\heartsuit_2\partial) \leqq \max\{\sqsupset^{\widehat{\Finv}}_{\Upsilon}(\wp), \sqsupset^{\widehat{\Finv}}_{\Upsilon}(\partial)\}$ and $\sqsupset^{\widehat{\Finv}}_{\Upsilon}(\wp\heartsuit_3\partial) \leqq \max\{\sqsupset^{\widehat{\Finv}}_{\Upsilon}(\wp), \sqsupset^{\widehat{\Finv}}_{\Upsilon}(\partial)\}$.

Now, $\daleth^{-}_{\aleph}(\wp\heartsuit_1\partial) = \daleth^{-}_{\Upsilon}(\Re(\wp\heartsuit_1\partial)) = \daleth^{-}_{\Upsilon}(\Re(\wp)\diamondsuit_1\Re(\partial)) \geqq \min\{\daleth^{-}_{\Upsilon}\Re(\wp), \daleth^{-}_{\Upsilon}\Re(\partial)\} = \min\{\daleth^{-}_{\aleph}(\wp), \daleth^{-}_{\aleph}(\partial)\}$. Thus $\daleth^{-}_{\aleph}(\wp\heartsuit_1\partial) \geqq \min\{\daleth^{-}_{\aleph}(\wp), \daleth^{-}_{\aleph}(\partial)\}$ and $1-\Finv^{-}_{\aleph}(\wp\heartsuit_1\partial) = 1-\Finv^{-}_{\Upsilon}(\Re(\wp\heartsuit_1\partial)) = 1-\Finv^{-}_{\Upsilon}(\Re(\wp)\diamondsuit_1\Re(\partial)) \geqq \min\{1-\Finv^{-}_{\Upsilon}\Re(\wp), 1-\Finv^{-}_{\Upsilon}\Re(\partial)\} = \min\{1-\Finv^{-}_{\aleph}(\wp), 1-\Finv^{-}_{\aleph}(\partial)\}$. Thus $1-\Finv^{-}_{\aleph}(\wp\heartsuit_1\partial) \geqq \min\{1-\Finv^{-}_{\aleph}(\wp), 1-\Finv^{-}_{\aleph}(\partial)\}$. Hence, $\sqsupset^{\widehat{\daleth}}_{\Upsilon}(\wp\heartsuit_1\partial) \geqq \min\{\sqsupset^{\widehat{\daleth}}_{\Upsilon}(\wp), \sqsupset^{\widehat{\daleth}}_{\Upsilon}(\partial)\}$. Similarly, $\sqsupset^{\widehat{\daleth}}_{\Upsilon}(\wp\heartsuit_2\partial) \geqq \min\{\sqsupset^{\widehat{\daleth}}_{\Upsilon}(\wp), \sqsupset^{\widehat{\daleth}}_{\Upsilon}(\partial)\}$ and $\sqsupset^{\widehat{\daleth}}_{\Upsilon}(\wp\heartsuit_3\partial) \geqq \min\{\sqsupset^{\widehat{\daleth}}_{\Upsilon}(\wp), \sqsupset^{\widehat{\daleth}}_{\Upsilon}(\partial)\}$.

Now, $\gimel^{-}_{\aleph}(\wp\heartsuit_1\partial) = \gimel^{-}_{\Upsilon}(\Re(\wp\heartsuit_1\partial)) = \gimel^{-}_{\Upsilon}(\Re(\wp)\diamondsuit_1\Re(\partial)) \geqq \frac{\gimel^{-}_{\Upsilon}\Re(\wp)+\gimel^{-}_{\Upsilon}\Re(\partial)}{2} = \frac{\gimel^{-}_{\aleph}(\wp)+\gimel^{-}_{\aleph}(\partial)}{2}$. Thus $\gimel^{-}_{\aleph}(\wp\heartsuit_1\partial) \geqq \frac{\gimel^{-}_{\aleph}(\wp)+\gimel^{-}_{\aleph}(\partial)}{2}$ and $\gimel^{+}_{\aleph}(\wp\heartsuit_1\partial) = \gimel^{+}_{\Upsilon}(\Re(\wp\heartsuit_1\partial)) = \gimel^{+}_{\Upsilon}(\Re(\wp)\diamondsuit_1\Re(\partial)) \geqq \frac{\gimel^{+}_{\Upsilon}\Re(\wp)+\gimel^{+}_{\Upsilon}\Re(\partial)}{2} = \frac{\gimel^{+}_{\aleph}(\wp)+\gimel^{+}_{\aleph}(\partial)}{2}$. Thus $\gimel^{+}_{\aleph}(\wp\heartsuit_1\partial) \geqq \frac{\gimel^{+}_{\aleph}(\wp)+\gimel^{+}_{\aleph}(\partial)}{2}$. Hence, $\sqsupset^{\widehat{\gimel}}_{\Upsilon}(\wp\heartsuit_1\partial) \geqq \frac{\sqsupset^{\widehat{\gimel}}_{\Upsilon}(\wp)+\sqsupset^{\widehat{\gimel}}_{\Upsilon}(\partial)}{2}$. Similarly, $\sqsupset^{\widehat{\gimel}}_{\Upsilon}(\wp\heartsuit_2\partial) \geqq \frac{\sqsupset^{\widehat{\gimel}}_{\Upsilon}(\wp)+\sqsupset^{\widehat{\gimel}}_{\Upsilon}(\partial)}{2}$ and $\sqsupset^{\widehat{\gimel}}_{\Upsilon}(\wp\heartsuit_3\partial) \geqq \frac{\sqsupset^{\widehat{\gimel}}_{\Upsilon}(\wp)+\sqsupset^{\widehat{\gimel}}_{\Upsilon}(\partial)}{2}$. Now, $\Finv^{-}_{\aleph}(\wp\heartsuit_1\partial) = \Finv^{-}_{\Upsilon}(\Re(\wp\heartsuit_1\partial)) = \Finv^{-}_{\Upsilon}(\Re(\wp)\diamondsuit_1\Re(\partial)) \leqq \max\{\Finv^{-}_{\Upsilon}\Re(\wp), \Finv^{-}_{\Upsilon}\Re(\partial)\} = \max\{\Finv^{-}_{\aleph}(\wp), \Finv^{-}_{\aleph}(\partial)\}$. Thus $\Finv^{-}_{\aleph}(\wp\heartsuit_1\partial) \leqq \max\{\Finv^{-}_{\aleph}(\wp), \Finv^{-}_{\aleph}(\partial)\}$ and $1-\daleth^{-}_{\aleph}(\wp\heartsuit_1\partial) = 1-\daleth^{-}_{\Upsilon}(\Re(\wp\heartsuit_1\partial)) = 1-\daleth^{-}_{\Upsilon}(\Re(\wp)\diamondsuit_1\Re(\partial)) \leqq \max\{1-\daleth^{-}_{\Upsilon}\Re(\wp), 1-\daleth^{-}_{\Upsilon}\Re(\partial)\} = \max\{1-\daleth^{-}_{\aleph}(\wp), 1-\daleth^{-}_{\aleph}(\partial)\}$. Thus $1-\daleth^{-}_{\aleph}(\wp\heartsuit_1\partial) \leqq \max\{1-\daleth^{-}_{\aleph}(\wp), 1-\daleth^{-}_{\aleph}(\partial)\}$. Hence, $\sqsupset^{\widehat{\Finv}}_{\Upsilon}(\wp\heartsuit_1\partial) \leqq \max\{\sqsupset^{\widehat{\Finv}}_{\Upsilon}(\wp), \sqsupset^{\widehat{\Finv}}_{\Upsilon}(\partial)\}$. Similarly, $\sqsupset^{\widehat{\Finv}}_{\Upsilon}(\wp\heartsuit_2\partial) \leqq \max\{\sqsupset^{\widehat{\Finv}}_{\Upsilon}(\wp), \sqsupset^{\widehat{\Finv}}_{\Upsilon}(\partial)\}$ and $\sqsupset^{\widehat{\Finv}}_{\Upsilon}(\wp\heartsuit_3\partial) \leqq \max\{\sqsupset^{\widehat{\Finv}}_{\Upsilon}(\wp), \sqsupset^{\widehat{\Finv}}_{\Upsilon}(\partial)\}$. Hence, $\aleph$ is a IVNCVSBS of Ξ_1.

Theorem 3.15. *If $\Re : \Xi_1 \to \Xi_2$ is a homomorphism, then $\Re(\aleph_{(\ell_1,\ell_2,s)})$ is a level SBS of IVNCVSBS Υ of Ξ_2.*

Proof. Let $\Re : \Xi_1 \to \Xi_2$ be a homomorphism and $\Re(\wp\heartsuit_1\partial) = \Re(\wp)\diamondsuit_1\Re(\partial), \Re(\wp\heartsuit_2\partial) = \Re(\wp)\diamondsuit_2\Re(\partial)$ and $\Re(\wp\heartsuit_3\partial) = \Re(\wp)\diamondsuit_3\Re(\partial)$ for all $\wp, \partial \in \Xi_1$. Let $V = \Re(\aleph)$, $\aleph$ is a IVNCVSBS of Ξ_1. By Theorem 3.13, Υ is a IVNCVSBS of Ξ_2. Let $\aleph_{(\ell_1,\ell_2,s)}$ be any level SBS of $\aleph$. Suppose that $\wp, \partial \in \aleph_{(\ell_1,\ell_2,s)}$. Then $\Re(\wp\heartsuit_1\partial), \Re(\wp\heartsuit_2\partial)$ and $\Re(\wp\heartsuit_3\partial) \in \aleph_{(\ell_1,\ell_2,s)}$. Now, $\widehat{\daleth}^{-}_{\Upsilon}(\Re(\wp)) = \widehat{\daleth}^{-}_{\aleph}(\wp) \geqq \ell_1, \widehat{\daleth}^{-}_{\Upsilon}(\Re(\partial)) = \widehat{\daleth}^{-}_{\aleph}(\partial) \geqq \ell_1$. Thus $\widehat{\daleth}^{-}_{\Upsilon}(\Re(\wp)\diamondsuit_1\Re(\partial)) \geqq \widehat{\daleth}^{-}_{\aleph}(\wp\heartsuit_1\partial) \geqq \ell_1$ and $1-\widehat{\Finv}^{-}_{\Upsilon}(\Re(\wp)) = 1-\widehat{\Finv}^{-}_{\aleph}(\wp) \geqq s, 1-\widehat{\Finv}^{-}_{\Upsilon}(\Re(\partial)) = 1-\widehat{\Finv}^{-}_{\aleph}(\partial) \geqq s$. Thus $1-\widehat{\Finv}^{-}_{\Upsilon}(\Re(\wp)\diamondsuit\Re(\partial)) \geqq 1-\widehat{\Finv}^{-}_{\aleph}(\wp\heartsuit_1\partial) \geqq s$. Now, $\widehat{\gimel}^{-}_{\Upsilon}(\Re(\wp)) = \widehat{\gimel}^{-}_{\aleph}(\wp) \geqq \ell_2, \widehat{\gimel}^{-}_{\Upsilon}(\Re(\partial)) = \widehat{\gimel}^{-}_{\aleph}(\partial) \geqq \ell_2$. Thus $\widehat{\gimel}^{-}_{\Upsilon}(\Re(\wp)\diamondsuit_1\Re(\partial)) \geqq \widehat{\gimel}^{-}_{\aleph}(\wp\heartsuit_1\partial) \geqq \ell_2$ and $\widehat{\gimel}^{+}_{\Upsilon}(\Re(\wp)) = \widehat{\gimel}^{+}_{\aleph}(\wp) \geqq \ell_2, \widehat{\gimel}^{+}_{\Upsilon}(\Re(\partial)) = \widehat{\gimel}^{+}_{\aleph}(\partial) \geqq \ell_2$. Thus $\widehat{\gimel}^{+}_{\Upsilon}(\Re(\wp)\diamondsuit_1\Re(\partial)) \geqq \widehat{\gimel}^{+}_{\aleph}(\wp\heartsuit_1\partial) \geqq \ell_2$. Now, $\widehat{\Finv}^{-}_{\Upsilon}(\Re(\wp)) = \widehat{\Finv}^{-}_{\aleph}(\wp) \leqq s, \widehat{\Finv}^{-}_{\Upsilon}(\Re(\partial)) = \widehat{\Finv}^{-}_{\aleph}(\partial) \leqq s$. Thus $\widehat{\Finv}^{-}_{\Upsilon}(\Re(\wp)\diamondsuit_1\Re(\partial)) \leqq \widehat{\Finv}^{-}_{\aleph}(\wp\heartsuit_1\partial) \leqq s$ and $1-\widehat{\daleth}^{-}_{\Upsilon}(\Re(\wp)) = 1-\widehat{\daleth}^{-}_{\aleph}(\wp) \leqq \ell_1, 1-\widehat{\daleth}^{-}_{\Upsilon}(\Re(\partial)) = 1-\widehat{\daleth}^{-}_{\aleph}(\partial) \leqq \ell_1$. Thus $1-\widehat{\daleth}^{-}_{\Upsilon}(\Re(\wp)\diamondsuit\Re(\partial)) \leqq 1-\widehat{\daleth}^{-}_{\aleph}(\wp\heartsuit_1\partial) \leqq \ell_1$, for all $\Re(\wp), \Re(\partial) \in \Xi_2$.

Now, $\daleth^{-}_{\Upsilon}(\Re(\wp)) = \daleth^{-}_{\aleph}(\wp) \geqq \ell_1, \daleth^{-}_{\Upsilon}(\Re(\partial)) = \daleth^{-}_{\aleph}(\partial) \geqq \ell_1$. Thus $\daleth^{-}_{\Upsilon}(\Re(\wp)\diamondsuit_1\Re(\partial)) \geqq \daleth^{-}_{\aleph}(\wp\heartsuit_1\partial) \geqq \ell_1$ and $1-\Finv^{-}_{\Upsilon}(\Re(\wp)) = 1-\Finv^{-}_{\aleph}(\wp) \geqq s, 1-\Finv^{-}_{\Upsilon}(\Re(\partial)) = 1-\Finv^{-}_{\aleph}(\partial) \geqq s$. Thus $1-\Finv^{-}_{\Upsilon}(\Re(\wp)\diamondsuit\Re(\partial)) \geqq 1-\Finv^{-}_{\aleph}(\wp\heartsuit_1\partial) \geqq s$. Now, $\gimel^{-}_{\Upsilon}(\Re(\wp)) = \gimel^{-}_{\aleph}(\wp) \geqq \ell_2, \gimel^{-}_{\Upsilon}(\Re(\partial)) = \gimel^{-}_{\aleph}(\partial) \geqq \ell_2$. Thus $\gimel^{-}_{\Upsilon}(\Re(\wp)\diamondsuit_1\Re(\partial)) \geqq \gimel^{-}_{\aleph}(\wp\heartsuit_1\partial) \geqq \ell_2$ and $\gimel^{+}_{\Upsilon}(\Re(\wp)) = \gimel^{+}_{\aleph}(\wp) \geqq \ell_2, \gimel^{+}_{\Upsilon}(\Re(\partial)) = \gimel^{+}_{\aleph}(\partial) \geqq \ell_2$. Thus $\gimel^{+}_{\Upsilon}(\Re(\wp)\diamondsuit_1\Re(\partial)) \geqq \gimel^{+}_{\aleph}(\wp\heartsuit_1\partial) \geqq \ell_2$. Now, $\Finv^{-}_{\Upsilon}(\Re(\wp)) = \Finv^{-}_{\aleph}(\wp) \leqq s, \Finv^{-}_{\Upsilon}(\Re(\partial)) = \Finv^{-}_{\aleph}(\partial) \leqq s$. Thus $\Finv^{-}_{\Upsilon}(\Re(\wp)\diamondsuit_1\Re(\partial)) \leqq \Finv^{-}_{\aleph}(\wp\heartsuit_1\partial) \leqq s$ and $1-\daleth^{-}_{\Upsilon}(\Re(\wp)) = 1-\daleth^{-}_{\aleph}(\wp) \leqq \ell_1, 1-\daleth^{-}_{\Upsilon}(\Re(\partial)) = 1-\daleth^{-}_{\aleph}(\partial) \leqq \ell_1$. Thus $1-\daleth^{-}_{\Upsilon}(\Re(\wp)\diamondsuit\Re(\partial)) \leqq 1-\daleth^{-}_{\aleph}(\wp\heartsuit_1\partial) \leqq \ell_1$, for all $\Re(\wp), \Re(\partial) \in \Xi_2$. Similarly to prove other operations. Hence proved.

Theorem 3.16. *If $\Re : \Xi_1 \to \Xi_2$ is any homomorphism, then $\aleph_{(\ell_1,\ell_2,s)}$ is a level SBS of IVNCVSBS $\aleph$ of Ξ_1.*

Proof. Let $\Re : \Xi_1 \to \Xi_2$ be a homomorphism and $\Re(\wp\heartsuit_1\partial) = \Re(\wp)\diamondsuit_1\Re(\partial), \Re(\wp\heartsuit_2\partial) = \Re(\wp)\diamondsuit_2\Re(\partial)$ and $\Re(\wp\heartsuit_3\partial) = \Re(\wp)\diamondsuit_3\Re(\partial)$ for all $\wp, \partial \in \Xi_1$. Let $V = \Re(\aleph)$, Υ is a IVNCVSBS of Ξ_2. By Theorem 3.14, $\aleph$ is a IVNCVSBS of Ξ_1. Let $\Re(\aleph_{(\ell_1,\ell_2,s)})$ be a level SBS of Υ. Suppose that $\Re(\wp), \Re(\partial) \in \Re(\aleph_{(\ell_1,\ell_2,s)})$. Then $\Re(\wp\heartsuit_1\partial), \Re(\wp\heartsuit_2\partial)$ and $\Re(\wp\heartsuit_3\partial) \in \Re(\aleph_{(\ell_1,\ell_2,s)})$. Now, $\widehat{\daleth}^{-}_{\aleph}(\wp) = \widehat{\daleth}^{-}_{\Upsilon}(\Re(\wp)) \geqq \ell_1, \widehat{\daleth}^{-}_{\aleph}(\partial) = \widehat{\daleth}^{-}_{\Upsilon}(\Re(\partial)) \geqq \ell_1$. Thus $\widehat{\daleth}^{-}_{\aleph}(\wp\heartsuit_1\partial) \geqq \min\{\widehat{\daleth}^{-}_{\aleph}(\wp), \widehat{\daleth}^{-}_{\aleph}(\partial)\} \geqq \ell_1$ and $1-\widehat{\Finv}^{-}_{\aleph}(\wp) = 1-\widehat{\Finv}^{-}_{\Upsilon}(\Re(\wp)) \geqq s, 1-\widehat{\Finv}^{-}_{\aleph}(\partial) = 1-\widehat{\Finv}^{-}_{\Upsilon}(\Re(\partial)) \geqq s$. Thus $1-\widehat{\Finv}^{-}_{\aleph}(\wp\heartsuit_1\partial) \geqq \min\{1-\widehat{\Finv}^{-}_{\aleph}(\wp), 1-\widehat{\Finv}^{-}_{\aleph}(\partial)\} \geqq s$. Now, $\widehat{\gimel}^{-}_{\aleph}(\wp) = \widehat{\gimel}^{-}_{\Upsilon}(\Re(\wp)) \geqq \ell_2, \widehat{\gimel}^{-}_{\aleph}(\partial) = \widehat{\gimel}^{-}_{\Upsilon}(\Re(\partial)) \geqq \ell_2$. Thus $\widehat{\gimel}^{-}_{\aleph}(\wp\heartsuit_1\partial) \geqq \frac{\widehat{\gimel}^{-}_{\aleph}(\wp)+\widehat{\gimel}^{-}_{\aleph}(\partial)}{2} \geqq \ell_2$ and $\widehat{\gimel}^{+}_{\aleph}(\wp) = \widehat{\gimel}^{+}_{\Upsilon}(\Re(\wp)) \geqq \ell_2, \widehat{\gimel}^{+}_{\aleph}(\partial) = \widehat{\gimel}^{+}_{\Upsilon}(\Re(\partial)) \geqq \ell_2$. Thus $\widehat{\gimel}^{+}_{\aleph}(\wp\heartsuit_1\partial) \geqq \frac{\widehat{\gimel}^{+}_{\aleph}(\wp)+\widehat{\gimel}^{+}_{\aleph}(\partial)}{2} \geqq \ell_2$. Now, $\widehat{\Finv}^{-}_{\aleph}(\wp) = \widehat{\Finv}^{-}_{\Upsilon}(\Re(\wp)) \leqq s, \widehat{\Finv}^{-}_{\aleph}(\partial) = \widehat{\Finv}^{-}_{\Upsilon}(\Re(\partial)) \leqq s$. Thus $\widehat{\Finv}^{-}_{\aleph}(\wp\heartsuit_1\partial) = \widehat{\Finv}^{-}_{\Upsilon}(\Re(\wp)\diamondsuit_1\Re(\partial)) \leqq \max\{\widehat{\Finv}^{-}_{\aleph}(\wp), \widehat{\Finv}^{-}_{\aleph}(\partial)\} \leqq s$ and $1-\widehat{\daleth}^{-}_{\aleph}(\wp) = 1-\widehat{\daleth}^{-}_{\Upsilon}(\Re(\wp)) \leqq \ell_1, 1-\widehat{\daleth}^{-}_{\aleph}(\partial) = 1-\widehat{\daleth}^{-}_{\Upsilon}(\Re(\partial)) \leqq \ell_1$. Thus $1-\widehat{\daleth}^{-}_{\aleph}(\wp\heartsuit_1\partial) = 1-\widehat{\daleth}^{-}_{\Upsilon}(\Re(\wp)\diamondsuit_1\Re(\partial)) \leqq \max\{1-\widehat{\daleth}^{-}_{\aleph}(\wp), 1-\widehat{\daleth}^{-}_{\aleph}(\partial)\} \leqq \ell_1$, for all $\wp, \partial \in \Xi_1$.

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Now, $\daleth^{-}_{\aleph}(\wp) = \daleth^{-}_{\Upsilon}(\Re(\wp)) \geqq \ell_1, \daleth^{-}_{\aleph}(\partial) = \daleth^{-}_{\Upsilon}(\Re(\partial)) \geqq \ell_1$. Thus $\daleth^{-}_{\aleph}(\wp\heartsuit_1\partial) \geqq \min\{\daleth^{-}_{\aleph}(\wp), \daleth^{-}_{\aleph}(\partial)\} \geqq \ell_1$ and $1 - \Finv^{-}_{\aleph}(\wp) = 1 - \Finv^{-}_{\Upsilon}(\Re(\wp)) \geqq s, 1 - \Finv^{-}_{\aleph}(\partial) = 1 - \Finv^{-}_{\Upsilon}(\Re(\partial)) \geqq s$. Thus $1 - \Finv^{-}_{\aleph}(\wp\heartsuit_1\partial) \geqq \min\{1 - \Finv^{-}_{\aleph}(\wp), 1 - \Finv^{-}_{\aleph}(\partial)\} \geqq s$. Now, $\beth^{-}_{\aleph}(\wp) = \beth^{-}_{\Upsilon}(\Re(\wp)) \geqq \ell_2, \beth^{-}_{\aleph}(\partial) = \beth^{-}_{\Upsilon}(\Re(\partial)) \geqq \ell_2$. Thus $\beth^{-}_{\aleph}(\wp\heartsuit_1\partial) \geqq \frac{\beth^{-}_{\aleph}(\wp)+\beth^{-}_{\aleph}(\partial)}{2} \geqq \ell_2$ and $\beth^{+}_{\aleph}(\wp) = \beth^{+}_{\Upsilon}(\Re(\wp)) \geqq \ell_2, \beth^{+}_{\aleph}(\partial) = \beth^{+}_{\Upsilon}(\Re(\partial)) \geqq \ell_2$. Thus $\beth^{+}_{\aleph}(\wp\heartsuit_1\partial) \geqq \frac{\beth^{+}_{\aleph}(\wp)+\beth^{+}_{\aleph}(\partial)}{2} \geqq \ell_2$. Now, $\Finv^{-}_{\aleph}(\wp) = \Finv^{-}_{\Upsilon}(\Re(\wp)) \leqq s, \Finv^{-}_{\aleph}(\partial) = \Finv^{-}_{\Upsilon}(\Re(\partial)) \leqq s$. Thus $\Finv^{-}_{\aleph}(\wp\heartsuit_1\partial) = \Finv^{-}_{\Upsilon}(\Re(\wp)\diamondsuit_1\Re(\partial)) \leqq \max\{\Finv^{-}_{\aleph}(\wp), \Finv^{-}_{\aleph}(\partial)\} \leqq s$ and $1 - \daleth^{-}_{\aleph}(\wp) = 1 - \daleth^{-}_{\Upsilon}(\Re(\wp)) \leqq \ell_1, 1 - \daleth^{-}_{\aleph}(\partial) = 1 - \daleth^{-}_{\Upsilon}(\Re(\partial)) \leqq \ell_1$. Thus $1 - \daleth^{-}_{\aleph}(\wp\heartsuit_1\partial) = 1 - \daleth^{-}_{\Upsilon}(\Re(\wp)\diamondsuit_1\Re(\partial)) \leqq \max\{1 - \daleth^{-}_{\aleph}(\wp), 1 - \daleth^{-}_{\aleph}(\partial)\} \leqq \ell_1$, for all $\wp, \partial \in \Xi_1$. Similarly to prove other two operations. Hence proved.

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