



New algebraic approach towards interval-valued neutrosophic cubic vague set based on subbisemiring over bisemiring

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Abstract

We introduce the concept of an interval-valued neutrosophic cubic vague subbisemiring (IVNCVSBS) and level set of IVNCVSBS of a bisemiring. An IVNCVSBS is the new extension of neutrosophic subbisemirings and SBS over bisemirings. Let \mathbb{N} be a neutrosophic vague subset in Ξ , we show that $\square = ([\neg_{\mathbb{N}}^-, \neg_{\mathbb{N}}^+], [\exists_{\mathbb{N}}^-, \exists_{\mathbb{N}}^+], [\exists_{\mathbb{N}}^-, \exists_{\mathbb{N}}^+])$ is a IVNCVSBS of Ξ if and only if all non empty level set $\square^{(\ell_1, \ell_2, s)}$ is a SBS of Ξ for all $\ell_1, \ell_2, s \in [0, 1]$. Let \mathbb{N} be the IVNCVSBS of Ξ and Υ be the strongest cubic neutrosophic vague relation of Ξ . To prove that \mathbb{N} is a IVNCVSBS of $\Xi \times \Xi$. Let \mathbb{N} be any IVNCVSBS of Ξ , prove that pseudo cubic neutrosophic vague coset $(\varsigma \mathbb{N})^p$ is a IVNCVSBS of Ξ , for all $\varsigma \in \Xi$. Let $\mathbb{N}_1, \mathbb{N}_2, \dots, \mathbb{N}_n$ be the family of $IVNCVSBS^s$ of $\Xi_1, \Xi_2, \dots, \Xi_n$ respectively. To prove that $\mathbb{N}_1 \times \mathbb{N}_2 \times \dots \times \mathbb{N}_n$ is a IVNCVSBS of $\Xi_1 \times \Xi_2 \times \dots \times \Xi_n$. The homomorphic image of every IVNCVSBS is a IVNCVSBS. The homomorphic pre-image of every IVNCVSBS is a IVNCVSBS. Examples are provided to strengthen our results.

Keywords: Subbisemiring; cubic neutrosophic subbisemiring; vague bisemiring; homomorphism.

1 Introduction

Due to the limitations of classical mathematics, such as fuzzy set (FS)¹ and vague set (VS),² mathematical theories have been developed to address uncertainty and fuzziness. In the case of uncertain or vague situations, FS

introduced by Zadeh¹ is the most appropriate technique. In recent years, many hybrid fuzzy models have been developed based on FS. A generalization of FS, intuitionistic fuzzy set (IFS) incorporate hesitation levels into the notion of FS, which were first proposed by Attanasov³ in 1983. The neutrosophic set (NSS) was proposed in 1999 by Smarandache.⁴ In NSS, each proposition is estimated to have a degree of truth, an indeterminacy degree, and a falsity degree. As a result of Smarandache,⁵ he further generalised and expanded the theory of IFSs to include the neutrosophic model as well. A study of fuzzy semirings was initiated by Ahsan et al.⁶ Recently many researchers discussed the various ideal structures of SBS and its applications⁷⁻¹⁰. In 2004, Sen et al.¹¹ extended the study of semirings and proposed the concept of bisemiring to further develop them. The study of vague algebra was initiated by Biswas¹² through the introduction of vague groups, vague cuts and vague normal groups. A semiring $(S, +, \cdot)$ is a non-empty set in which $(S, +)$ and (S, \cdot) are semigroups such that “.” is distributive over “+”.¹³ In 1993, Ahsan et al.⁶ introduced the notion of fuzzy semirings. An introduction to bisemirings was made in 2001 by Sen et al.¹⁴ A bisemiring $(\Xi, \diamond, \odot, \boxtimes)$ is an algebraic structure in which (Ξ, \diamond, \odot) and (Ξ, \odot, \boxtimes) are semirings in which (Ξ, \diamond) , (Ξ, \odot) and (Ξ, \boxtimes) are semigroups such that (a) $\zeta \odot (\partial \diamond \varsigma) = (\varphi \odot \partial) \diamond (\varphi \odot \varsigma)$, (b) $(\partial \diamond \varsigma) \odot \varphi = (\partial \odot \varphi) \diamond (\varsigma \odot \varphi)$, (c) $\varphi \boxtimes (\partial \odot \varsigma) = (\varphi \boxtimes \partial) \odot (\varphi \boxtimes \varsigma)$ and (d) $(\partial \odot \varsigma) \boxtimes \varphi = (\partial \boxtimes \varphi) \odot (\varsigma \boxtimes \varphi)$ for all $\varphi, \partial, \varsigma \in \Xi$.¹¹ A non-empty subset \mathbb{N} of a bisemiring $(\Xi, \diamond, \odot, \boxtimes)$ is a subbisemiring (SBS) if and only if $\varphi \diamond \partial \in \mathbb{N}$, $\varphi \odot \partial \in \mathbb{N}$ and $\varphi \boxtimes \partial \in \mathbb{N}$ for all $\varphi, \partial \in \mathbb{N}$.¹⁴ However, numerous algebraic concepts had been generalized using FS theory. Fuzzy algebraic structures of semirings have been extensively investigated by Vandiver.¹⁵ These are generalizations of rings requiring only a monoid, rather than a group, to achieve a particular additive structure and have been shown to be useful for a wide range of problems. Golan¹³ and Glazek¹⁶ have both extensively studied the application of semirings.

Bipolar fuzzy information has been applied to various algebraic structures over the past few years, like semi-groups and BCK/BCI algebras.^{17,18,21} An application of bipolar fuzzy metric spaces was discussed by Zararsz et al.²² A vague soft hyperring and a vague soft hyper ideal were introduced by Selvachandran.²³ The bipolar fuzzy translation was introduced by Jun et al.²⁴ and BCK/BCI-algebra and its properties were investigated. A bipolar fuzzy regularity, bipolar fuzzy regular sub-algebra, a bipolar fuzzy filter, and a bipolar fuzzy closed quasi filter have been introduced into BCH algebras in.²⁵ In 2004, Sen et al.¹¹ contributed to the field of semirings by proposing bisemiring as a concept. Hussain et al.²⁶ defined the congruence relation between bisemirings and bisemiring homomorphisms. In addition to bisemiring, Hussain et al.^{14,26} described an algebraic structure called semiring and congruence relations between homomorphisms and n-semirings based on this algebraic structure. We discuss the concept of interval-valued neutrosophic cubic vague subbisemiring (IVNCVSBS) and level sets. The IVNCVSBS is a extension of subbisemiring. A number of illustrative examples are provided to illustrate. Following is an outline of the preliminary definitions and results presented in Section 2. The concept of a IVNCVSBS is introduced in Section 3.

2 Preliminaries

For our future studies, we will quickly review some fundamental terms in this section.

Definition 2.1.⁴ A neutrosophic set (NSS) \mathbb{N} in a universal set Γ is $\mathbb{N} = \{(\varphi, \neg_{\mathbb{N}}(\varphi), \exists_{\mathbb{N}}(\varphi), \dashv_{\mathbb{N}}(\varphi)) : \varphi \in \Gamma\}$, where $\neg_{\mathbb{N}}, \exists_{\mathbb{N}}, \dashv_{\mathbb{N}} : \Gamma \rightarrow [0, 1]$ denotes the truth, indeterminacy and the falsity membership function, respectively. For $\langle \neg_{\mathbb{N}}, \exists_{\mathbb{N}}, \dashv_{\mathbb{N}} \rangle$ is used for the NSS $\mathbb{N} = \{(\varphi, \neg_{\mathbb{N}}(\varphi), \exists_{\mathbb{N}}(\varphi), \dashv_{\mathbb{N}}(\varphi)) : \varphi \in \Gamma\}$.

Definition 2.2.⁴ Let $\mathbb{N} = \langle \neg_{\mathbb{N}}, \exists_{\mathbb{N}}, \dashv_{\mathbb{N}} \rangle$ and $\mathbb{h} = \langle \neg_{\mathbb{h}}, \exists_{\mathbb{h}}, \dashv_{\mathbb{h}} \rangle$ be the two NSS of Γ . Then

1. $\mathbb{N} \wedge \mathbb{h} = \{(\varphi, \min\{\neg_{\mathbb{N}}(\varphi), \neg_{\mathbb{h}}(\varphi)\}, \min\{\exists_{\mathbb{N}}(\varphi), \exists_{\mathbb{h}}(\varphi)\}, \max\{\dashv_{\mathbb{N}}(\varphi), \dashv_{\mathbb{h}}(\varphi)\}) : \varphi \in \Gamma\}$,
2. $\mathbb{N} \vee \mathbb{h} = \{(\varphi, \max\{\neg_{\mathbb{N}}(\varphi), \neg_{\mathbb{h}}(\varphi)\}, \max\{\exists_{\mathbb{N}}(\varphi), \exists_{\mathbb{h}}(\varphi)\}, \min\{\dashv_{\mathbb{N}}(\varphi), \dashv_{\mathbb{h}}(\varphi)\}) : \varphi \in \Gamma\}$.

Definition 2.3.⁴ For any NSS $\mathbb{N} = \langle \neg_{\mathbb{N}}, \exists_{\mathbb{N}}, \dashv_{\mathbb{N}} \rangle$ of Γ , we defined a (ℓ, s) -cut of as the crisp subset $\{\varphi \in \Gamma : \neg_{\mathbb{N}}(\varphi) \geq \ell, \exists_{\mathbb{N}}(\varphi) \geq \ell, \dashv_{\mathbb{N}}(\varphi) \leq s\}$ of Γ .

Definition 2.4.⁴ Let \mathbb{N} and \mathbb{h} be two neutrosophic subsets of S . The Cartesian product of \mathbb{N} and \mathbb{h} is defined as $\mathbb{N} \times \mathbb{h} = \{((\varphi, \partial), \neg_{\mathbb{N} \times \mathbb{h}}(\varphi, \partial), \exists_{\mathbb{N} \times \mathbb{h}}(\varphi, \partial), \dashv_{\mathbb{N} \times \mathbb{h}}(\varphi, \partial)) : \varphi, \partial \in S\}$, where $\neg_{\mathbb{N} \times \mathbb{h}}(\varphi, \partial) = \min\{\neg_{\mathbb{N}}(\varphi), \neg_{\mathbb{h}}(\partial)\}$, $\exists_{\mathbb{N} \times \mathbb{h}}(\varphi, \partial) = \frac{\exists_{\mathbb{N}}(\varphi) + \exists_{\mathbb{h}}(\partial)}{2}$ and $\dashv_{\mathbb{N} \times \mathbb{h}}(\varphi, \partial) = \max\{\dashv_{\mathbb{N}}(\varphi), \dashv_{\mathbb{h}}(\partial)\}$.

Definition 2.5.¹² A vague set (VS) $\mathbb{N} = \langle \neg_{\mathbb{N}}, \dashv_{\mathbb{N}} \rangle$ of Ξ is said to be vague semiring if

$$\left\{ \begin{array}{l} \neg_{\mathbb{N}}(b_1 + b_2) \geq \min\{\neg_{\mathbb{N}}(b_1), \neg_{\mathbb{N}}(b_2)\} \\ \neg_{\mathbb{N}}(b_1 \cdot b_2) \geq \min\{\neg_{\mathbb{N}}(b_1), \neg_{\mathbb{N}}(b_2)\} \end{array} \right\}$$

and

$$\begin{cases} 1 - \exists_N(\varphi_1 + \varphi_2) \geq \min\{1 - \exists_N(\varphi_1), 1 - \exists_N(\varphi_2)\} \\ 1 - \exists_N(\varphi_1 \cdot \varphi_2) \geq \min\{1 - \exists_N(\varphi_1), 1 - \exists_N(\varphi_2)\} \end{cases}.$$

for all $\varphi_1, \varphi_2 \in \Xi$.

Definition 2.6. ¹² A VS N in Γ . Then

1. A VS $N = (\top_N, \exists_N)$, where $\top_N : \Gamma \rightarrow [0, 1]$, $\exists_N : \Gamma \rightarrow [0, 1]$ are mappings such that $\top_N(\varphi) + \exists_N(\varphi) \leq 1$, for all $\varphi \in \Gamma$ where \top_N and \exists_N are called true and false membership function, respectively.
2. The interval $[\top_N(\varphi), 1 - \exists_N(\varphi)]$ is called the vague value of φ in N and it is denoted by $V_N(\varphi)$, i.e., $V_N(\varphi) = [\top_N(\varphi), 1 - \exists_N(\varphi)]$.

Definition 2.7. ¹² Let N and \hbar be the two VSs of Γ . Then

1. N is contained in \hbar as $N \subseteq \hbar$ if and only if $V_N(\varphi) \leq V_\hbar(\varphi)$, i.e. $\top_N(\varphi) \leq \top_\hbar(\varphi)$ and $1 - \exists_N(\varphi) \leq 1 - \exists_\hbar(\varphi)$ for all $\varphi \in \Gamma$,
2. the union of N and \hbar as $N \vee \hbar = N \cup \hbar$, $\top_{N \vee \hbar} = \max\{\top_N, \top_\hbar\}$ and $1 - \exists_{N \vee \hbar} = \min\{1 - \exists_N, 1 - \exists_\hbar\} = 1 - \min\{\exists_N, \exists_\hbar\}$,
3. the intersection of N and \hbar as $N \wedge \hbar = N \cap \hbar$, $\top_{N \wedge \hbar} = \min\{\top_N, \top_\hbar\}$ and $1 - \exists_{N \wedge \hbar} = \max\{1 - \exists_N, 1 - \exists_\hbar\} = 1 - \max\{\exists_N, \exists_\hbar\}$.

Definition 2.8. ¹² Let N be a VS of Γ . Then

1. $\top_N(\varphi) = 0$ and $\exists_N(\varphi) = 1$ is called zero VS of Γ ,
2. $\top_N(\varphi) = 1$ and $\exists_N(\varphi) = 0$ is called unit VS of Γ .

for all $\varphi \in U$.

Definition 2.9. ¹² Let N be a VS of Γ with true membership function \top_N and false membership function \exists_N . For $\alpha, \beta \in [0, 1]$ with $\alpha \leq \beta$, the (α, β) - cut or vague cut of a VS N is the crisp subset of Γ is given by $N_{(\alpha, \beta)} = \{\varphi \in \Gamma : V_N(\varphi) \geq [\alpha, \beta]\}$. That is, $N_{(\alpha, \beta)} = \{\varphi \in \Gamma : \top_N(\varphi) \geq \alpha, 1 - \exists_N(\varphi) \geq \beta\}$.

Definition 2.10. ¹² Let N and \hbar be any two VSs in Γ . Then

1. $N \wedge \hbar = \{(\varphi, \min\{\top_N(\varphi), \top_\hbar(\varphi)\}, \min\{1 - \exists_N(\varphi), 1 - \exists_\hbar(\varphi)\}) : \varphi \in \Gamma\}$,
2. $N \vee \hbar = \{(\varphi, \max\{\top_N(\varphi), \top_\hbar(\varphi)\}, \max\{1 - \exists_N(\varphi), 1 - \exists_\hbar(\varphi)\}) : \varphi \in \Gamma\}$,
3. $\square N = \{(\varphi, \top_N(\varphi), 1 - \top_N(\varphi)) : \varphi \in \Gamma\}$,
4. $\diamond N = \{(\varphi, 1 - \exists_N(\varphi), \exists_N(\varphi)) : \varphi \in \Gamma\}$.

3 Interval valued neutrosophic cubic vague subbisemirings

In all cases, assume that Ξ represents a bisemiring. Unless otherwise specified.

Definition 3.1. A interval-valued neutrosophic cubic VS N of Ξ is represent a IVNCVSBS of Ξ if

$$\left\{ \begin{array}{l} \widehat{\top}_N(\varphi \triangle_1 \partial) \geq \min\{\widehat{\top}_N(\varphi), \widehat{\top}_N(\partial)\} \\ \widehat{\top}_N(\varphi \triangle_2 \partial) \geq \min\{\widehat{\top}_N(\varphi), \widehat{\top}_N(\partial)\} \\ \widehat{\top}_N(\varphi \triangle_3 \partial) \geq \min\{\widehat{\top}_N(\varphi), \widehat{\top}_N(\partial)\} \end{array} \right\} \quad \left\{ \begin{array}{l} \widehat{\exists}_N(\varphi \triangle_1 \partial) \geq \frac{\widehat{\exists}_N(\varphi) + \widehat{\exists}_N(\partial)}{2} \\ OR \\ \widehat{\exists}_N(\varphi \triangle_2 \partial) \geq \frac{\widehat{\exists}_N(\varphi) + \widehat{\exists}_N(\partial)}{2} \\ OR \\ \widehat{\exists}_N(\varphi \triangle_3 \partial) \geq \frac{\widehat{\exists}_N(\varphi) + \widehat{\exists}_N(\partial)}{2} \end{array} \right\}$$

$$\begin{cases} \widehat{\mathbb{B}}_{\mathcal{R}}^{\pm}(\wp \triangle_1 \partial) \leq \max\{\widehat{\mathbb{B}}_{\mathcal{R}}^{\pm}(\wp), \widehat{\mathbb{B}}_{\mathcal{R}}^{\pm}(\partial)\} \\ \widehat{\mathbb{B}}_{\mathcal{R}}^{\pm}(\wp \triangle_2 \partial) \leq \max\{\widehat{\mathbb{B}}_{\mathcal{R}}^{\pm}(\wp), \widehat{\mathbb{B}}_{\mathcal{R}}^{\pm}(\partial)\} \\ \widehat{\mathbb{B}}_{\mathcal{R}}^{\pm}(\wp \triangle_3 \partial) \leq \max\{\widehat{\mathbb{B}}_{\mathcal{R}}^{\pm}(\wp), \widehat{\mathbb{B}}_{\mathcal{R}}^{\pm}(\partial)\} \end{cases}.$$

$$\left\{ \begin{array}{l} \mathfrak{B}_{\mathbb{R}}(\varphi \triangle_1 \partial) \geq \min\{\mathfrak{B}_{\mathbb{R}}(\varphi), \mathfrak{B}_{\mathbb{R}}(\partial)\} \\ \mathfrak{B}_{\mathbb{R}}(\varphi \triangle_2 \partial) \geq \min\{\mathfrak{B}_{\mathbb{R}}(\varphi), \mathfrak{B}_{\mathbb{R}}(\partial)\} \\ \mathfrak{B}_{\mathbb{R}}(\varphi \triangle_3 \partial) \geq \min\{\mathfrak{B}_{\mathbb{R}}(\varphi), \mathfrak{B}_{\mathbb{R}}(\partial)\} \end{array} \right\}$$

$$\left\{ \begin{array}{l} \beth_{\aleph_0}^{\downarrow}(\wp \triangleleft_1 \partial) \leq \max\{\beth_{\aleph_0}^{\downarrow}(\wp), \beth_{\aleph_0}^{\downarrow}(\partial)\} \\ \beth_{\aleph_0}^{\downarrow}(\wp \triangleleft_2 \partial) \leq \max\{\beth_{\aleph_0}^{\downarrow}(\wp), \beth_{\aleph_0}^{\downarrow}(\partial)\} \\ \beth_{\aleph_0}^{\downarrow}(\wp \triangleleft_3 \partial) \leq \max\{\beth_{\aleph_0}^{\downarrow}(\wp), \beth_{\aleph_0}^{\downarrow}(\partial)\} \end{array} \right\}.$$

That is,

$$\left\{ \begin{array}{l} \left(1 - \widehat{\exists}_{\aleph}^{-}(\wp \triangle_1 \partial) \geq \min\{\widehat{\exists}_{\aleph}^{-}(\wp), \widehat{\exists}_{\aleph}^{-}(\partial)\}, \right. \\ \left. 1 - \widehat{\exists}_{\aleph}^{-}(\wp \triangle_1 \partial) \geq \min\{1 - \widehat{\exists}_{\aleph}^{-}(\wp), 1 - \widehat{\exists}_{\aleph}^{-}(\partial)\} \right) \\ \left\{ \begin{array}{l} \left(\widehat{\exists}_{\aleph}^{-}(\wp \triangle_2 \partial) \geq \min\{\widehat{\exists}_{\aleph}^{-}(\wp), \widehat{\exists}_{\aleph}^{-}(\partial)\}, \right. \\ \left. 1 - \widehat{\exists}_{\aleph}^{-}(\wp \triangle_2 \partial) \geq \min\{1 - \widehat{\exists}_{\aleph}^{-}(\wp), 1 - \widehat{\exists}_{\aleph}^{-}(\partial)\} \right) \\ \left\{ \begin{array}{l} \left(\widehat{\exists}_{\aleph}^{-}(\wp \triangle_3 \partial) \geq \min\{\widehat{\exists}_{\aleph}^{-}(\wp), \widehat{\exists}_{\aleph}^{-}(\partial)\}, \right. \\ \left. 1 - \widehat{\exists}_{\aleph}^{-}(\wp \triangle_3 \partial) \geq \min\{1 - \widehat{\exists}_{\aleph}^{-}(\wp), 1 - \widehat{\exists}_{\aleph}^{-}(\partial)\} \right) \end{array} \right. \end{array} \right. \right\}$$

$$\left\{ \begin{array}{l} \widehat{\exists}_{\mathcal{R}}^-(\wp \Delta_1 \theta) \leq \max\{\widehat{\exists}_{\mathcal{R}}^-(\wp), \widehat{\exists}_{\mathcal{R}}^-(\theta)\}, \\ 1 - \widehat{\exists}_{\mathcal{R}}^-(\wp \Delta_1 \theta) \leq \max\{1 - \widehat{\exists}_{\mathcal{R}}^-(\wp), 1 - \widehat{\exists}_{\mathcal{R}}^-(\theta)\} \end{array} \right\}$$

$$\left\{ \begin{array}{l} \widehat{\exists}_{\mathcal{R}}^-(\wp \Delta_2 \theta) \leq \max\{\widehat{\exists}_{\mathcal{R}}^-(\wp), \widehat{\exists}_{\mathcal{R}}^-(\theta)\}, \\ 1 - \widehat{\exists}_{\mathcal{R}}^-(\wp \Delta_2 \theta) \leq \max\{1 - \widehat{\exists}_{\mathcal{R}}^-(\wp), 1 - \widehat{\exists}_{\mathcal{R}}^-(\theta)\} \end{array} \right\}$$

$$\left\{ \begin{array}{l} \widehat{\exists}_{\mathcal{R}}^-(\wp \Delta_3 \theta) \leq \max\{\widehat{\exists}_{\mathcal{R}}^-(\wp), \widehat{\exists}_{\mathcal{R}}^-(\theta)\}, \\ 1 - \widehat{\exists}_{\mathcal{R}}^-(\wp \Delta_3 \theta) \leq \max\{1 - \widehat{\exists}_{\mathcal{R}}^-(\wp), 1 - \widehat{\exists}_{\mathcal{R}}^-(\theta)\} \end{array} \right\}$$

$$\left\{ \begin{array}{l} \left(\begin{array}{l} \mathsf{T}_{\mathcal{R}}^-(\wp \triangle_1 \partial) \geq \min\{\mathsf{T}_{\mathcal{R}}^-(\wp), \mathsf{T}_{\mathcal{R}}^-(\partial)\}, \\ 1 - \mathsf{J}_{\mathcal{R}}^-(\wp \triangle_1 \partial) \geq \min\{1 - \mathsf{J}_{\mathcal{R}}^-(\wp), 1 - \mathsf{J}_{\mathcal{R}}^-(\partial)\} \end{array} \right) \\ \left(\begin{array}{l} \mathsf{T}_{\mathcal{R}}^-(\wp \triangle_2 \partial) \geq \min\{\mathsf{T}_{\mathcal{R}}^-(\wp), \mathsf{T}_{\mathcal{R}}^-(\partial)\}, \\ 1 - \mathsf{J}_{\mathcal{R}}^-(\wp \triangle_2 \partial) \geq \min\{1 - \mathsf{J}_{\mathcal{R}}^-(\wp), 1 - \mathsf{J}_{\mathcal{R}}^-(\partial)\} \end{array} \right) \\ \left(\begin{array}{l} \mathsf{T}_{\mathcal{R}}^-(\wp \triangle_3 \partial) \geq \min\{\mathsf{T}_{\mathcal{R}}^-(\wp), \mathsf{T}_{\mathcal{R}}^-(\partial)\}, \\ 1 - \mathsf{J}_{\mathcal{R}}^-(\wp \triangle_3 \partial) \geq \min\{1 - \mathsf{J}_{\mathcal{R}}^-(\wp), 1 - \mathsf{J}_{\mathcal{R}}^-(\partial)\} \end{array} \right) \end{array} \right\} \quad \left\{ \begin{array}{l} \left(\begin{array}{l} \mathsf{J}_{\mathcal{R}}^+(\wp \triangle_1 \partial) \geq \frac{\mathsf{J}_{\mathcal{R}}^+(\wp) + \mathsf{J}_{\mathcal{R}}^+(\partial)}{2}, \\ \mathsf{J}_{\mathcal{R}}^-(\wp \triangle_1 \partial) \geq \frac{\mathsf{J}_{\mathcal{R}}^-(\wp) - \mathsf{J}_{\mathcal{R}}^-(\partial)}{2} \end{array} \right) \\ OR \\ \left(\begin{array}{l} \mathsf{J}_{\mathcal{R}}^+(\wp \triangle_2 \partial) \geq \frac{\mathsf{J}_{\mathcal{R}}^+(\wp) + \mathsf{J}_{\mathcal{R}}^+(\partial)}{2}, \\ \mathsf{J}_{\mathcal{R}}^-(\wp \triangle_2 \partial) \geq \frac{\mathsf{J}_{\mathcal{R}}^-(\wp) - \mathsf{J}_{\mathcal{R}}^-(\partial)}{2} \end{array} \right) \\ OR \\ \left(\begin{array}{l} \mathsf{J}_{\mathcal{R}}^+(\wp \triangle_3 \partial) \geq \frac{\mathsf{J}_{\mathcal{R}}^+(\wp) + \mathsf{J}_{\mathcal{R}}^+(\partial)}{2}, \\ \mathsf{J}_{\mathcal{R}}^-(\wp \triangle_3 \partial) \geq \frac{\mathsf{J}_{\mathcal{R}}^-(\wp) - \mathsf{J}_{\mathcal{R}}^-(\partial)}{2} \end{array} \right) \end{array} \right\}$$

$$\left\{ \begin{array}{l} \left(\begin{array}{l} \exists_{\mathcal{R}}^-(\phi \triangle_1 \theta) \leq \max\{\exists_{\mathcal{R}}^-(\phi), \exists_{\mathcal{R}}^-(\theta)\}, \\ 1 - \neg_{\mathcal{R}}^-(\phi \triangle_1 \theta) \leq \max\{1 - \neg_{\mathcal{R}}^-(\phi), 1 - \neg_{\mathcal{R}}^-(\theta)\} \end{array} \right) \\ \left(\begin{array}{l} \exists_{\mathcal{R}}^-(\phi \triangle_2 \theta) \leq \max\{\exists_{\mathcal{R}}^-(\phi), \exists_{\mathcal{R}}^-(\theta)\}, \\ 1 - \neg_{\mathcal{R}}^-(\phi \triangle_2 \theta) \leq \max\{1 - \neg_{\mathcal{R}}^-(\phi), 1 - \neg_{\mathcal{R}}^-(\theta)\} \end{array} \right) \\ \left(\begin{array}{l} \exists_{\mathcal{R}}^-(\phi \triangle_3 \theta) \leq \max\{\exists_{\mathcal{R}}^-(\phi), \exists_{\mathcal{R}}^-(\theta)\}, \\ 1 - \neg_{\mathcal{R}}^-(\phi \triangle_3 \theta) \leq \max\{1 - \neg_{\mathcal{R}}^-(\phi), 1 - \neg_{\mathcal{R}}^-(\theta)\} \end{array} \right) \end{array} \right.$$

for all $\wp, \partial \in \Xi$.

Example 3.2. Let $\Xi = \{\zeta_a, \zeta_b, \zeta_c, \zeta_d\}$ be the bisemiring.

Δ_1	ζ_a	ζ_b	ζ_c	ζ_d	Δ_2	ζ_a	ζ_b	ζ_c	ζ_d	Δ_3	ζ_a	ζ_b	ζ_c	ζ_d
ζ_a	ζ_a	ζ_a	ζ_a	ζ_a	ζ_a	ζ_a	ζ_b	ζ_c	ζ_d	ζ_a	ζ_a	ζ_a	ζ_a	ζ_a
ζ_b	ζ_a	ζ_b	ζ_a	ζ_b	ζ_b	ζ_b	ζ_b	ζ_d	ζ_d	ζ_b	ζ_a	ζ_b	ζ_c	ζ_d
ζ_c	ζ_a	ζ_a	ζ_c	ζ_c	ζ_c	ζ_c	ζ_d	ζ_c	ζ_d	ζ_c	ζ_d	ζ_d	ζ_d	ζ_d
ζ_d	ζ_a	ζ_b	ζ_c	ζ_d	ζ_d	ζ_d	ζ_d	ζ_d	ζ_d	ζ_d	ζ_d	ζ_d	ζ_d	ζ_d

	$[\widehat{\sqcap}_{\mathbb{N}}^-(\beta), \widehat{\sqcap}_{\mathbb{N}}^+(\beta)]$	$[\widehat{\sqcup}_{\mathbb{N}}^-(\beta), \widehat{\sqcup}_{\mathbb{N}}^+(\beta)]$	$[\widehat{\sqcup}_{\mathbb{N}}^-(\beta), \widehat{\sqcup}_{\mathbb{N}}^+(\beta)]$
$\beta = \zeta_a$	$[0.75, 0.8], [0.85, 0.9]$	$[0.65, 0.7], [0.75, 0.85]$	$[0.2, 0.25], [0.1, 0.15]$
$\beta = \zeta_b$	$[0.65, 0.7], [0.75, 0.8]$	$[0.55, 0.6], [0.65, 0.7]$	$[0.3, 0.35], [0.2, 0.25]$
$\beta = \zeta_c$	$[0.45, 0.5], [0.65, 0.7]$	$[0.35, 0.4], [0.45, 0.5]$	$[0.5, 0.55], [0.3, 0.35]$
$\beta = \zeta_d$	$[0.6, 0.65], [0.7, 0.75]$	$[0.4, 0.45], [0.55, 0.6]$	$[0.35, 0.4], [0.25, 0.3]$

	$[\widehat{\sqcap}_{\mathbb{N}}^-(\beta), \widehat{\sqcap}_{\mathbb{N}}^+(\beta)]$	$[\widehat{\sqcup}_{\mathbb{N}}^-(\beta), \widehat{\sqcup}_{\mathbb{N}}^+(\beta)]$	$[\widehat{\sqcup}_{\mathbb{N}}^-(\beta), \widehat{\sqcup}_{\mathbb{N}}^+(\beta)]$
$\beta = \zeta_a$	$[0.65, 0.7]$	$[0.75, 0.8]$	$[0.3, 0.35]$
$\beta = \zeta_b$	$[0.55, 0.65]$	$[0.7, 0.75]$	$[0.35, 0.45]$
$\beta = \zeta_c$	$[0.40, 0.45]$	$[0.55, 0.60]$	$[0.55, 0.60]$
$\beta = \zeta_d$	$[0.45, 0.55]$	$[0.65, 0.70]$	$[0.45, 0.55]$

Clearly, \mathbb{N} is a IVNCVSBS of Ξ .

Theorem 3.3. The intersection of a family of every IVNCVSBS^s of Ξ is a IVNCVSBS of Ξ .

Proof. Let $\{\Xi_i : i \in I\}$ be a collection of IVNCVSBS^s of Ξ and $\mathbb{N} = \bigcap_{i \in I} \Xi_i$.

Let \wp, ∂ in Ξ . Then

$$\begin{aligned} \widehat{\sqcap}_{\mathbb{N}}^-(\wp \Delta_1 \partial) &= \inf_{i \in I} \widehat{\sqcap}_{\Xi_i}^-(\wp \Delta_1 \partial) \\ &\geq \inf_{i \in I} \min\{\widehat{\sqcap}_{\Xi_i}^-(\wp), \widehat{\sqcap}_{\Xi_i}^-(\partial)\} \\ &= \min\left\{\inf_{i \in I} \widehat{\sqcap}_{\Xi_i}^-(\wp), \inf_{i \in I} \widehat{\sqcap}_{\Xi_i}^-(\partial)\right\} \\ &= \min\{\widehat{\sqcap}_{\mathbb{N}}^-(\wp), \widehat{\sqcap}_{\mathbb{N}}^-(\partial)\}. \end{aligned}$$

$$\begin{aligned} 1 - \widehat{\sqcup}_{\mathbb{N}}^-(\wp \Delta_1 \partial) &= \inf_{i \in I} 1 - \widehat{\sqcup}_{\Xi_i}^-(\wp \Delta_1 \partial) \\ &\geq \inf_{i \in I} \min\{1 - \widehat{\sqcup}_{\Xi_i}^-(\wp), 1 - \widehat{\sqcup}_{\Xi_i}^-(\partial)\} \\ &= \min\left\{1 - \widehat{\sqcup}_{\Xi_i}^-(\wp), 1 - \widehat{\sqcup}_{\Xi_i}^-(\partial)\right\} \\ &= \min\{1 - \widehat{\sqcup}_{\mathbb{N}}^-(\wp), 1 - \widehat{\sqcup}_{\mathbb{N}}^-(\partial)\}. \end{aligned}$$

Thus $\widehat{\sqcap}_{\mathbb{N}}^-(\wp \Delta_1 \partial) \geq \min\{\widehat{\sqcap}_{\mathbb{N}}^-(\wp), \widehat{\sqcap}_{\mathbb{N}}^-(\partial)\}$. Similarly, $\widehat{\sqcup}_{\mathbb{N}}^-(\wp \Delta_2 \partial) \geq \min\{\widehat{\sqcup}_{\mathbb{N}}^-(\wp), \widehat{\sqcup}_{\mathbb{N}}^-(\partial)\}$ and $\widehat{\sqcup}_{\mathbb{N}}^-(\wp \Delta_3 \partial) \geq \min\{\widehat{\sqcup}_{\mathbb{N}}^-(\wp), \widehat{\sqcup}_{\mathbb{N}}^-(\partial)\}$. Now,

$$\begin{aligned} \widehat{\sqcap}_{\mathbb{N}}^-(\wp \Delta_1 \partial) &= \inf_{i \in I} \widehat{\sqcap}_{\Xi_i}^-(\wp \Delta_1 \partial) \\ &\geq \inf_{i \in I} \frac{\widehat{\sqcap}_{\Xi_i}^-(\wp) + \widehat{\sqcap}_{\Xi_i}^-(\partial)}{2} \\ &= \frac{\inf_{i \in I} \widehat{\sqcap}_{\Xi_i}^-(\wp) + \inf_{i \in I} \widehat{\sqcap}_{\Xi_i}^-(\partial)}{2} \\ &= \frac{\widehat{\sqcap}_{\mathbb{N}}^-(\wp) + \widehat{\sqcap}_{\mathbb{N}}^-(\partial)}{2}. \end{aligned}$$

$$\begin{aligned}
\widehat{\exists}_{\mathbb{N}}^+(\wp \Delta_1 \partial) &= \inf_{i \in I^+} \widehat{\exists}_{\mathbb{B}_i}^+(\wp \Delta_1 \partial) \\
&\leq \inf_{i \in I^+} \frac{\widehat{\exists}_{\mathbb{B}_i}^+(\wp) + \widehat{\exists}_{\mathbb{B}_i}^+(\partial)}{2} \\
&= \frac{\inf_{i \in I^+} \widehat{\exists}_{\mathbb{B}_i}^+(\wp) + \inf_{i \in I^+} \widehat{\exists}_{\mathbb{B}_i}^+(\partial)}{2} \\
&= \frac{\widehat{\exists}_{\mathbb{N}}^+(\wp) + \widehat{\exists}_{\mathbb{N}}^+(\partial)}{2}.
\end{aligned}$$

Thus $\widehat{\exists}_{\mathbb{N}}^+(\wp \Delta_1 \partial) \geq \min\{\widehat{\exists}_{\mathbb{N}}(\wp), \widehat{\exists}_{\mathbb{N}}(\partial)\}$. Similarly, $\widehat{\exists}_{\mathbb{N}}^+(\wp \Delta_2 \partial) \geq \min\{\widehat{\exists}_{\mathbb{N}}(\wp), \widehat{\exists}_{\mathbb{N}}(\partial)\}$ and $\widehat{\exists}_{\mathbb{N}}^+(\wp \Delta_3 \partial) \geq \min\{\widehat{\exists}_{\mathbb{N}}(\wp), \widehat{\exists}_{\mathbb{N}}(\partial)\}$.

Now,

$$\begin{aligned}
\widehat{\exists}_{\mathbb{N}}^-(\wp \Delta_1 \partial) &= \sup_{i \in I} \widehat{\exists}_{\mathbb{B}_i}^-(\wp \Delta_1 \partial) \\
&\leq \sup_{i \in I} \max\{\widehat{\exists}_{\mathbb{B}_i}^-(\wp), \widehat{\exists}_{\mathbb{B}_i}^-(\partial)\} \\
&= \max\left\{\sup_{i \in I} \widehat{\exists}_{\mathbb{B}_i}^-(\wp), \sup_{i \in I} \widehat{\exists}_{\mathbb{B}_i}^-(\partial)\right\} \\
&= \max\{\widehat{\exists}_{\mathbb{N}}^-(\wp), \widehat{\exists}_{\mathbb{N}}^-(\partial)\}.
\end{aligned}$$

$$\begin{aligned}
1 - \widehat{\exists}_{\mathbb{N}}^-(\wp \Delta_1 \partial) &= \sup_{i \in I} 1 - \widehat{\exists}_{\mathbb{B}_i}^-(\wp \Delta_1 \partial) \\
&\leq \sup_{i \in I} \max\{1 - \widehat{\exists}_{\mathbb{B}_i}^-(\wp), 1 - \widehat{\exists}_{\mathbb{B}_i}^-(\partial)\} \\
&= \max\left\{\sup_{i \in I} 1 - \widehat{\exists}_{\mathbb{B}_i}^-(\wp), \sup_{i \in I} 1 - \widehat{\exists}_{\mathbb{B}_i}^-(\partial)\right\} \\
&= \max\{1 - \widehat{\exists}_{\mathbb{N}}^-(\wp), 1 - \widehat{\exists}_{\mathbb{N}}^-(\partial)\}.
\end{aligned}$$

Thus $\widehat{\exists}_{\mathbb{N}}^-(\wp \Delta_1 \partial) \leq \max\{\widehat{\exists}_{\mathbb{N}}(\wp), \widehat{\exists}_{\mathbb{N}}(\partial)\}$. Similarly, $\widehat{\exists}_{\mathbb{N}}^-(\wp \Delta_2 \partial) \leq \max\{\widehat{\exists}_{\mathbb{N}}(\wp), \widehat{\exists}_{\mathbb{N}}(\partial)\}$ and $\widehat{\exists}_{\mathbb{N}}^-(\wp \Delta_3 \partial) \leq \max\{\widehat{\exists}_{\mathbb{N}}(\wp), \widehat{\exists}_{\mathbb{N}}(\partial)\}$.

Now,

$$\begin{aligned}
\widehat{\exists}_{\mathbb{N}}^-(\wp \Delta_1 \partial) &= \inf_{i \in I} \widehat{\exists}_{\mathbb{B}_i}^-(\wp \Delta_1 \partial) \\
&\geq \inf_{i \in I} \min\{\widehat{\exists}_{\mathbb{B}_i}^-(\wp), \widehat{\exists}_{\mathbb{B}_i}^-(\partial)\} \\
&= \min\left\{\inf_{i \in I} \widehat{\exists}_{\mathbb{B}_i}^-(\wp), \inf_{i \in I} \widehat{\exists}_{\mathbb{B}_i}^-(\partial)\right\} \\
&= \min\{\widehat{\exists}_{\mathbb{N}}^-(\wp), \widehat{\exists}_{\mathbb{N}}^-(\partial)\}.
\end{aligned}$$

$$\begin{aligned}
1 - \widehat{\exists}_{\mathbb{N}}^-(\wp \Delta_1 \partial) &= \inf_{i \in I} 1 - \widehat{\exists}_{\mathbb{B}_i}^-(\wp \Delta_1 \partial) \\
&\geq \inf_{i \in I} \min\{1 - \widehat{\exists}_{\mathbb{B}_i}^-(\wp), 1 - \widehat{\exists}_{\mathbb{B}_i}^-(\partial)\} \\
&= \min\left\{\inf_{i \in I} 1 - \widehat{\exists}_{\mathbb{B}_i}^-(\wp), \inf_{i \in I} 1 - \widehat{\exists}_{\mathbb{B}_i}^-(\partial)\right\} \\
&= \min\{1 - \widehat{\exists}_{\mathbb{N}}^-(\wp), 1 - \widehat{\exists}_{\mathbb{N}}^-(\partial)\}.
\end{aligned}$$

Thus $\widehat{\exists}_{\mathbb{N}}^-(\wp \Delta_1 \partial) \geq \min\{\widehat{\exists}_{\mathbb{N}}(\wp), \widehat{\exists}_{\mathbb{N}}(\partial)\}$. Similarly, $\widehat{\exists}_{\mathbb{N}}^-(\wp \Delta_2 \partial) \geq \min\{\widehat{\exists}_{\mathbb{N}}(\wp), \widehat{\exists}_{\mathbb{N}}(\partial)\}$ and $\widehat{\exists}_{\mathbb{N}}^-(\wp \Delta_3 \partial) \geq \min\{\widehat{\exists}_{\mathbb{N}}(\wp), \widehat{\exists}_{\mathbb{N}}(\partial)\}$. Now,

$$\begin{aligned}
\widehat{\exists}_{\mathbb{N}}^-(\wp \Delta_1 \partial) &= \inf_{i \in I^-} \widehat{\exists}_{\mathbb{B}_i}^-(\wp \Delta_1 \partial) \\
&\geq \inf_{i \in I^-} \frac{\widehat{\exists}_{\mathbb{B}_i}^-(\wp) + \widehat{\exists}_{\mathbb{B}_i}^-(\partial)}{2} \\
&= \frac{\inf_{i \in I^-} \widehat{\exists}_{\mathbb{B}_i}^-(\wp) + \inf_{i \in I^-} \widehat{\exists}_{\mathbb{B}_i}^-(\partial)}{2} \\
&= \frac{\widehat{\exists}_{\mathbb{N}}^-(\wp) + \widehat{\exists}_{\mathbb{N}}^-(\partial)}{2}.
\end{aligned}$$

$$\begin{aligned}
\mathbb{J}_{\mathfrak{N}}^+(\wp \Delta_1 \partial) &= \inf_{i \in I^+} \mathbb{J}_{\mathfrak{B}_i}^+(\wp \Delta_1 \partial) \\
&\geq \inf_{i \in I^+} \frac{\mathbb{J}_{\mathfrak{B}_i}^+(\wp) + \mathbb{J}_{\mathfrak{B}_i}^+(\partial)}{2} \\
&= \frac{\inf_{i \in I^+} \mathbb{J}_{\mathfrak{B}_i}^+(\wp) + \inf_{i \in I^+} \mathbb{J}_{\mathfrak{B}_i}^+(\partial)}{2} \\
&= \frac{\mathbb{J}_{\mathfrak{N}}^+(\wp) + \mathbb{J}_{\mathfrak{N}}^+(\partial)}{2}.
\end{aligned}$$

Thus $\mathbb{D}_{\mathfrak{N}}^1(\wp \Delta_1 \partial) \geq \min\{\mathbb{D}_{\mathfrak{N}}(\wp), \mathbb{D}_{\mathfrak{N}}(\partial)\}$. Similarly, $\mathbb{D}_{\mathfrak{N}}^2(\wp \Delta_2 \partial) \geq \min\{\mathbb{D}_{\mathfrak{N}}(\wp), \mathbb{D}_{\mathfrak{N}}(\partial)\}$ and $\mathbb{D}_{\mathfrak{N}}^3(\wp \Delta_3 \partial) \geq \min\{\mathbb{D}_{\mathfrak{N}}(\wp), \mathbb{D}_{\mathfrak{N}}(\partial)\}$.

Now,

$$\begin{aligned}
\mathbb{J}_{\mathfrak{N}}^-(\wp \Delta_1 \partial) &= \sup_{i \in I} \mathbb{J}_{\mathfrak{B}_i}^-(\wp \Delta_1 \partial) \\
&\leq \sup_{i \in I} \max\{\mathbb{J}_{\mathfrak{B}_i}^-(\wp), \mathbb{J}_{\mathfrak{B}_i}^-(\partial)\} \\
&= \max \left\{ \sup_{i \in I} \mathbb{J}_{\mathfrak{B}_i}^-(\wp), \sup_{i \in I} \mathbb{J}_{\mathfrak{B}_i}^-(\partial) \right\} \\
&= \max\{\mathbb{J}_{\mathfrak{N}}^-(\wp), \mathbb{J}_{\mathfrak{N}}^-(\partial)\}.
\end{aligned}$$

$$\begin{aligned}
1 - \mathbb{T}_{\mathfrak{N}}^-(\wp \Delta_1 \partial) &= \sup_{i \in I} 1 - \mathbb{T}_{\mathfrak{B}_i}^-(\wp \Delta_1 \partial) \\
&\leq \sup_{i \in I} \max\{1 - \mathbb{T}_{\mathfrak{B}_i}^-(\wp), 1 - \mathbb{T}_{\mathfrak{B}_i}^-(\partial)\} \\
&= \max \left\{ \sup_{i \in I} 1 - \mathbb{T}_{\mathfrak{B}_i}^-(\wp), \sup_{i \in I} 1 - \mathbb{T}_{\mathfrak{B}_i}^-(\partial) \right\} \\
&= \max\{1 - \mathbb{T}_{\mathfrak{N}}^-(\wp), 1 - \mathbb{T}_{\mathfrak{N}}^-(\partial)\}.
\end{aligned}$$

Thus $\mathbb{D}_{\mathfrak{N}}^d(\wp \Delta_1 \partial) \leq \max\{\mathbb{D}_{\mathfrak{N}}(\wp), \mathbb{D}_{\mathfrak{N}}(\partial)\}$. Similarly, $\mathbb{D}_{\mathfrak{N}}^d(\wp \Delta_2 \partial) \leq \max\{\mathbb{D}_{\mathfrak{N}}(\wp), \mathbb{D}_{\mathfrak{N}}(\partial)\}$ and $\mathbb{D}_{\mathfrak{N}}^d(\wp \Delta_3 \partial) \leq \max\{\mathbb{D}_{\mathfrak{N}}(\wp), \mathbb{D}_{\mathfrak{N}}(\partial)\}$. Hence, \mathfrak{N} is a IVNCVSBS of Ξ .

Theorem 3.4. If \mathfrak{N} and \mathfrak{h} are the IVNCVSBS^s of Ξ_1 and Ξ_2 respectively, then $\mathfrak{N} \times \mathfrak{h}$ is a IVNCVSBS of $\Xi_1 \times \Xi_2$.

Proof. Let \mathfrak{N} and \mathfrak{h} be the IVNCVSBS^s of Ξ_1 and Ξ_2 respectively. Let $\wp_1, \wp_2 \in \Xi_1$ and $\partial_1, \partial_2 \in \Xi_2$. Then $(\wp_1, \partial_1), (\wp_2, \partial_2)$ belong to $\Xi_1 \times \Xi_2$. Now

$$\begin{aligned}
\widehat{\mathbb{T}}_{\mathfrak{N} \times \mathfrak{h}}^-[(\wp_1, \partial_1) \Delta_1 (\wp_2, \partial_2)] &= \widehat{\mathbb{T}}_{\mathfrak{N} \times \mathfrak{h}}^-(\wp_1 \Delta_1 \wp_2, \partial_1 \Delta_1 \partial_2) \\
&= \min\{\widehat{\mathbb{T}}_{\mathfrak{N}}^-(\wp_1 \Delta_1 \wp_2), \widehat{\mathbb{T}}_{\mathfrak{h}}^-(\partial_1 \Delta_1 \partial_2)\} \\
&\geq \min\{\min\{\widehat{\mathbb{T}}_{\mathfrak{N}}^-(\wp_1), \widehat{\mathbb{T}}_{\mathfrak{N}}^-(\wp_2)\}, \min\{\widehat{\mathbb{T}}_{\mathfrak{h}}^-(\partial_1), \widehat{\mathbb{T}}_{\mathfrak{h}}^-(\partial_2)\}\} \\
&= \min\{\min\{\widehat{\mathbb{T}}_{\mathfrak{N}}^-(\wp_1), \widehat{\mathbb{T}}_{\mathfrak{h}}^-(\partial_1)\}, \min\{\widehat{\mathbb{T}}_{\mathfrak{N}}^-(\wp_2), \widehat{\mathbb{T}}_{\mathfrak{h}}^-(\partial_2)\}\} \\
&= \min\{\widehat{\mathbb{T}}_{\mathfrak{N} \times \mathfrak{h}}^-(\wp_1, \partial_1), \widehat{\mathbb{T}}_{\mathfrak{N} \times \mathfrak{h}}^-(\wp_2, \partial_2)\}.
\end{aligned}$$

$$\begin{aligned}
1 - \widehat{\mathbb{E}}_{\mathfrak{N} \times \mathfrak{h}}^-[(\wp_1, \partial_1) \Delta_1 (\wp_2, \partial_2)] &= 1 - \widehat{\mathbb{E}}_{\mathfrak{N} \times \mathfrak{h}}^-(\wp_1 \Delta_1 \wp_2, \partial_1 \Delta_1 \partial_2) \\
&= \min\{1 - \widehat{\mathbb{E}}_{\mathfrak{N}}^-(\wp_1 \Delta_1 \wp_2), 1 - \widehat{\mathbb{E}}_{\mathfrak{h}}^-(\partial_1 \Delta_1 \partial_2)\} \\
&\geq \min\{\min\{1 - \widehat{\mathbb{E}}_{\mathfrak{N}}^-(\wp_1), 1 - \widehat{\mathbb{E}}_{\mathfrak{N}}^-(\wp_2)\}, \min\{1 - \widehat{\mathbb{E}}_{\mathfrak{h}}^-(\partial_1), 1 - \widehat{\mathbb{E}}_{\mathfrak{h}}^-(\partial_2)\}\} \\
&= \min\{\min\{1 - \widehat{\mathbb{E}}_{\mathfrak{N}}^-(\wp_1), 1 - \widehat{\mathbb{E}}_{\mathfrak{h}}^-(\partial_1)\}, \min\{1 - \widehat{\mathbb{E}}_{\mathfrak{N}}^-(\wp_2), 1 - \widehat{\mathbb{E}}_{\mathfrak{h}}^-(\partial_2)\}\} \\
&= \min\{1 - \widehat{\mathbb{E}}_{\mathfrak{N} \times \mathfrak{h}}^-(\wp_1, \partial_1), 1 - \widehat{\mathbb{E}}_{\mathfrak{N} \times \mathfrak{h}}^-(\wp_2, \partial_2)\}.
\end{aligned}$$

Thus $\widehat{\mathbb{D}}_{\mathfrak{N} \times \mathfrak{h}}^1(\wp \Delta_1 \partial) \geq \min\{\widehat{\mathbb{D}}_{\mathfrak{N} \times \mathfrak{h}}^1(\wp), \widehat{\mathbb{D}}_{\mathfrak{N} \times \mathfrak{h}}^1(\partial)\}$. Similarly, $\widehat{\mathbb{D}}_{\mathfrak{N} \times \mathfrak{h}}^2(\wp \Delta_2 \partial) \geq \min\{\widehat{\mathbb{D}}_{\mathfrak{N} \times \mathfrak{h}}^2(\wp), \widehat{\mathbb{D}}_{\mathfrak{N} \times \mathfrak{h}}^2(\partial)\}$ and

$\widehat{\exists}_{\mathbb{N} \times \mathbb{H}}(\wp \Delta_3 \partial) \geq \min\{\widehat{\exists}_{\mathbb{N} \times \mathbb{H}}(\wp), \widehat{\exists}_{\mathbb{N} \times \mathbb{H}}(\partial)\}$. Now,

$$\begin{aligned}\widehat{\exists}_{\mathbb{N} \times \mathbb{H}}[(\wp_1, \partial_1) \Delta_1 (\wp_2, \partial_2)] &= \widehat{\exists}_{\mathbb{N} \times \mathbb{H}}^-(\wp_1 \Delta_1 \wp_2, \partial_1 \Delta_1 \partial_2) \\ &= \frac{\widehat{\exists}_{\mathbb{N}}^-(\wp_1 \Delta_1 \wp_2) + \widehat{\exists}_{\mathbb{H}}^-(\partial_1 \Delta_1 \partial_2)}{2} \\ &\geq \frac{1}{2} \left[\frac{\widehat{\exists}_{\mathbb{N}}^-(\wp_1) + \widehat{\exists}_{\mathbb{N}}^-(\wp_2)}{2} + \frac{\widehat{\exists}_{\mathbb{H}}^-(\partial_1) + \widehat{\exists}_{\mathbb{H}}^-(\partial_2)}{2} \right] \\ &= \frac{1}{2} \left[\frac{\widehat{\exists}_{\mathbb{N}}^-(\wp_1) + \widehat{\exists}_{\mathbb{H}}^-(\partial_1)}{2} + \frac{\widehat{\exists}_{\mathbb{N}}^-(\wp_2) + \widehat{\exists}_{\mathbb{H}}^-(\partial_2)}{2} \right] \\ &= \frac{1}{2} [\widehat{\exists}_{\mathbb{N} \times \mathbb{H}}^-(\wp_1, \partial_1) + \widehat{\exists}_{\mathbb{N} \times \mathbb{H}}^-(\wp_2, \partial_2)].\end{aligned}$$

$$\begin{aligned}\widehat{\exists}_{\mathbb{N} \times \mathbb{H}}^+[(\wp_1, \partial_1) \Delta_1 (\wp_2, \partial_2)] &= \widehat{\exists}_{\mathbb{N} \times \mathbb{H}}^+(\wp_1 \Delta_1 \wp_2, \partial_1 \Delta_1 \partial_2) \\ &= \frac{\widehat{\exists}_{\mathbb{N}}^+(\wp_1 \Delta_1 \wp_2) + \widehat{\exists}_{\mathbb{H}}^+(\partial_1 \Delta_1 \partial_2)}{2} \\ &\geq \frac{1}{2} \left[\frac{\widehat{\exists}_{\mathbb{N}}^+(\wp_1) + \widehat{\exists}_{\mathbb{N}}^+(\wp_2)}{2} + \frac{\widehat{\exists}_{\mathbb{H}}^+(\partial_1) + \widehat{\exists}_{\mathbb{H}}^+(\partial_2)}{2} \right] \\ &= \frac{1}{2} \left[\frac{\widehat{\exists}_{\mathbb{N}}^+(\wp_1) + \widehat{\exists}_{\mathbb{H}}^+(\partial_1)}{2} + \frac{\widehat{\exists}_{\mathbb{N}}^+(\wp_2) + \widehat{\exists}_{\mathbb{H}}^+(\partial_2)}{2} \right] \\ &= \frac{1}{2} [\widehat{\exists}_{\mathbb{N} \times \mathbb{H}}^+(\wp_1, \partial_1) + \widehat{\exists}_{\mathbb{N} \times \mathbb{H}}^+(\wp_2, \partial_2)].\end{aligned}$$

Thus $\widehat{\exists}_{\mathbb{N} \times \mathbb{H}}(\wp \Delta_1 \partial) \geq \frac{1}{2} [\widehat{\exists}_{\mathbb{N} \times \mathbb{H}}(\wp_1, \partial_1) + \widehat{\exists}_{\mathbb{N} \times \mathbb{H}}(\wp_2, \partial_2)]$. Similarly, $\widehat{\exists}_{\mathbb{N} \times \mathbb{H}}(\wp \Delta_2 \partial) \geq \frac{1}{2} [\widehat{\exists}_{\mathbb{N} \times \mathbb{H}}(\wp_1, \partial_1) + \widehat{\exists}_{\mathbb{N} \times \mathbb{H}}(\wp_2, \partial_2)]$ and $\widehat{\exists}_{\mathbb{N} \times \mathbb{H}}(\wp \Delta_3 \partial) \geq \frac{1}{2} [\widehat{\exists}_{\mathbb{N} \times \mathbb{H}}(\wp_1, \partial_1) + \widehat{\exists}_{\mathbb{N} \times \mathbb{H}}(\wp_2, \partial_2)]$. Now

$$\begin{aligned}\widehat{\exists}_{\mathbb{N} \times \mathbb{H}}^-[[(\wp_1, \partial_1) \Delta_1 (\wp_2, \partial_2)]] &= \widehat{\exists}_{\mathbb{N} \times \mathbb{H}}^-(\wp_1 \Delta_1 \wp_2, \partial_1 \Delta_1 \partial_2) \\ &= \max\{\widehat{\exists}_{\mathbb{N}}^-(\wp_1 \Delta_1 \wp_2), \widehat{\exists}_{\mathbb{H}}^-(\partial_1 \Delta_1 \partial_2)\} \\ &\leq \max\{\max\{\widehat{\exists}_{\mathbb{N}}^-(\wp_1), \widehat{\exists}_{\mathbb{N}}^-(\wp_2)\}, \max\{\widehat{\exists}_{\mathbb{H}}^-(\partial_1), \widehat{\exists}_{\mathbb{H}}^-(\partial_2)\}\} \\ &= \max\{\max\{\widehat{\exists}_{\mathbb{N}}^-(\wp_1), \widehat{\exists}_{\mathbb{H}}^-(\partial_1)\}, \max\{\widehat{\exists}_{\mathbb{N}}^-(\wp_2), \widehat{\exists}_{\mathbb{H}}^-(\partial_2)\}\} \\ &= \max\{\widehat{\exists}_{\mathbb{N} \times \mathbb{H}}^-(\wp_1, \partial_1), \widehat{\exists}_{\mathbb{N} \times \mathbb{H}}^-(\wp_2, \partial_2)\}.\end{aligned}$$

$$\begin{aligned}1 - \widehat{\exists}_{\mathbb{N} \times \mathbb{H}}^-[[(\wp_1, \partial_1) \Delta_1 (\wp_2, \partial_2)]] &= 1 - \widehat{\exists}_{\mathbb{N} \times \mathbb{H}}^-(\wp_1 \Delta_1 \wp_2, \partial_1 \Delta_1 \partial_2) \\ &= \max\{1 - \widehat{\exists}_{\mathbb{N}}^-(\wp_1 \Delta_1 \wp_2), 1 - \widehat{\exists}_{\mathbb{H}}^-(\partial_1 \Delta_1 \partial_2)\} \\ &\leq \max\{\max\{1 - \widehat{\exists}_{\mathbb{N}}^-(\wp_1), 1 - \widehat{\exists}_{\mathbb{N}}^-(\wp_2)\}, \max\{1 - \widehat{\exists}_{\mathbb{H}}^-(\partial_1), 1 - \widehat{\exists}_{\mathbb{H}}^-(\partial_2)\}\} \\ &= \max\{\max\{1 - \widehat{\exists}_{\mathbb{N}}^-(\wp_1), 1 - \widehat{\exists}_{\mathbb{H}}^-(\partial_1)\}, \max\{1 - \widehat{\exists}_{\mathbb{N}}^-(\wp_2), 1 - \widehat{\exists}_{\mathbb{H}}^-(\partial_2)\}\} \\ &= \max\{1 - \widehat{\exists}_{\mathbb{N} \times \mathbb{H}}^-(\wp_1, \partial_1), 1 - \widehat{\exists}_{\mathbb{N} \times \mathbb{H}}^-(\wp_2, \partial_2)\}.\end{aligned}$$

Thus $\widehat{\exists}_{\mathbb{N} \times \mathbb{H}}^d(\wp \Delta_1 \partial) \leq \max\{\widehat{\exists}_{\mathbb{N} \times \mathbb{H}}^d(\wp), \widehat{\exists}_{\mathbb{N} \times \mathbb{H}}^d(\partial)\}$. Similarly, $\widehat{\exists}_{\mathbb{N} \times \mathbb{H}}^d(\wp \Delta_2 \partial) \leq \max\{\widehat{\exists}_{\mathbb{N} \times \mathbb{H}}^d(\wp), \widehat{\exists}_{\mathbb{N} \times \mathbb{H}}^d(\partial)\}$ and $\widehat{\exists}_{\mathbb{N} \times \mathbb{H}}^d(\wp \Delta_3 \partial) \leq \max\{\widehat{\exists}_{\mathbb{N} \times \mathbb{H}}^d(\wp), \widehat{\exists}_{\mathbb{N} \times \mathbb{H}}^d(\partial)\}$. Now

$$\begin{aligned}\widehat{\exists}_{\mathbb{N} \times \mathbb{H}}^d[(\wp_1, \partial_1) \Delta_1 (\wp_2, \partial_2)] &= \widehat{\exists}_{\mathbb{N} \times \mathbb{H}}^d(\wp_1 \Delta_1 \wp_2, \partial_1 \Delta_1 \partial_2) \\ &= \min\{\widehat{\exists}_{\mathbb{N}}^d(\wp_1 \Delta_1 \wp_2), \widehat{\exists}_{\mathbb{H}}^d(\partial_1 \Delta_1 \partial_2)\} \\ &\geq \min\{\min\{\widehat{\exists}_{\mathbb{N}}^d(\wp_1), \widehat{\exists}_{\mathbb{N}}^d(\wp_2)\}, \min\{\widehat{\exists}_{\mathbb{H}}^d(\partial_1), \widehat{\exists}_{\mathbb{H}}^d(\partial_2)\}\} \\ &= \min\{\min\{\widehat{\exists}_{\mathbb{N}}^d(\wp_1), \widehat{\exists}_{\mathbb{H}}^d(\partial_1)\}, \min\{\widehat{\exists}_{\mathbb{N}}^d(\wp_2), \widehat{\exists}_{\mathbb{H}}^d(\partial_2)\}\} \\ &= \min\{\widehat{\exists}_{\mathbb{N} \times \mathbb{H}}^d(\wp_1, \partial_1), \widehat{\exists}_{\mathbb{N} \times \mathbb{H}}^d(\wp_2, \partial_2)\}.\end{aligned}$$

$$\begin{aligned}
1 - \exists_{N \times h}^-[(\wp_1, \partial_1) \Delta_1 (\wp_2, \partial_2)] &= 1 - \exists_{N \times h}^- (\wp_1 \Delta_1 \wp_2, \partial_1 \Delta_1 \partial_2) \\
&= \min \{1 - \exists_N^- (\wp_1 \Delta_1 \wp_2), 1 - \exists_h^- (\partial_1 \Delta_1 \partial_2)\} \\
&\geq \min \{\min \{1 - \exists_N^- (\wp_1), 1 - \exists_N^- (\wp_2)\}, \min \{1 - \exists_h^- (\partial_1), 1 - \exists_h^- (\partial_2)\}\} \\
&= \min \{\min \{1 - \exists_N^- (\wp_1), 1 - \exists_h^- (\partial_1)\}, \min \{1 - \exists_N^- (\wp_2), 1 - \exists_h^- (\partial_2)\}\} \\
&= \min \{1 - \exists_{N \times h}^- (\wp_1, \partial_1), 1 - \exists_{N \times h}^- (\wp_2, \partial_2)\}.
\end{aligned}$$

Thus $\exists_{N \times h}^+(\wp \Delta_1 \partial) \geq \min \{\exists_{N \times h}^+(\wp), \exists_{N \times h}^+(\partial)\}$. Similarly, $\exists_{N \times h}^-(\wp \Delta_2 \partial) \geq \min \{\exists_{N \times h}^-(\wp), \exists_{N \times h}^-(\partial)\}$ and $\exists_{N \times h}^-(\wp \Delta_3 \partial) \geq \min \{\exists_{N \times h}^-(\wp), \exists_{N \times h}^-(\partial)\}$. Now,

$$\begin{aligned}
\exists_{N \times h}^-[(\wp_1, \partial_1) \Delta_1 (\wp_2, \partial_2)] &= \exists_{N \times h}^- (\wp_1 \Delta_1 \wp_2, \partial_1 \Delta_1 \partial_2) \\
&= \frac{\exists_N^- (\wp_1 \Delta_1 \wp_2) + \exists_h^- (\partial_1 \Delta_1 \partial_2)}{2} \\
&\geq \frac{1}{2} \left[\frac{\exists_N^- (\wp_1) + \exists_N^- (\wp_2)}{2} + \frac{\exists_h^- (\partial_1) + \exists_h^- (\partial_2)}{2} \right] \\
&= \frac{1}{2} \left[\frac{\exists_N^- (\wp_1) + \exists_h^- (\partial_1)}{2} + \frac{\exists_N^- (\wp_2) + \exists_h^- (\partial_2)}{2} \right] \\
&= \frac{1}{2} [\exists_{N \times h}^- (\wp_1, \partial_1) + \exists_{N \times h}^- (\wp_2, \partial_2)].
\end{aligned}$$

$$\begin{aligned}
\exists_{N \times h}^+[(\wp_1, \partial_1) \Delta_1 (\wp_2, \partial_2)] &= \exists_{N \times h}^+ (\wp_1 \Delta_1 \wp_2, \partial_1 \Delta_1 \partial_2) \\
&= \frac{\exists_N^+ (\wp_1 \Delta_1 \wp_2) + \exists_h^+ (\partial_1 \Delta_1 \partial_2)}{2} \\
&\geq \frac{1}{2} \left[\frac{\exists_N^+ (\wp_1) + \exists_N^+ (\wp_2)}{2} + \frac{\exists_h^+ (\partial_1) + \exists_h^+ (\partial_2)}{2} \right] \\
&= \frac{1}{2} \left[\frac{\exists_N^+ (\wp_1) + \exists_h^+ (\partial_1)}{2} + \frac{\exists_N^+ (\wp_2) + \exists_h^+ (\partial_2)}{2} \right] \\
&= \frac{1}{2} [\exists_{N \times h}^+ (\wp_1, \partial_1) + \exists_{N \times h}^+ (\wp_2, \partial_2)].
\end{aligned}$$

Thus $\exists_{N \times h}^1(\wp \Delta_1 \partial) \geq \frac{1}{2} [\exists_{N \times h}^1(\wp_1, \partial_1) + \exists_{N \times h}^1(\wp_2, \partial_2)]$. Similarly, $\exists_{N \times h}^1(\wp \Delta_2 \partial) \geq \frac{1}{2} [\exists_{N \times h}^1(\wp_1, \partial_1) + \exists_{N \times h}^1(\wp_2, \partial_2)]$ and $\exists_{N \times h}^1(\wp \Delta_3 \partial) \geq \frac{1}{2} [\exists_{N \times h}^1(\wp_1, \partial_1) + \exists_{N \times h}^1(\wp_2, \partial_2)]$. Now

$$\begin{aligned}
\exists_{N \times h}^-[(\wp_1, \partial_1) \Delta_1 (\wp_2, \partial_2)] &= \exists_{N \times h}^- (\wp_1 \Delta_1 \wp_2, \partial_1 \Delta_1 \partial_2) \\
&= \max \{\exists_N^- (\wp_1 \Delta_1 \wp_2), \exists_h^- (\partial_1 \Delta_1 \partial_2)\} \\
&\leq \max \{\max \{\exists_N^- (\wp_1), \exists_N^- (\wp_2)\}, \max \{\exists_h^- (\partial_1), \exists_h^- (\partial_2)\}\} \\
&= \max \{\max \{\exists_N^- (\wp_1), \exists_h^- (\partial_1)\}, \max \{\exists_N^- (\wp_2), \exists_h^- (\partial_2)\}\} \\
&= \max \{\exists_{N \times h}^- (\wp_1, \partial_1), \exists_{N \times h}^- (\wp_2, \partial_2)\}.
\end{aligned}$$

$$\begin{aligned}
1 - \neg_{N \times h}^-[(\wp_1, \partial_1) \Delta_1 (\wp_2, \partial_2)] &= 1 - \neg_{N \times h}^- (\wp_1 \Delta_1 \wp_2, \partial_1 \Delta_1 \partial_2) \\
&= \max \{1 - \neg_N^- (\wp_1 \Delta_1 \wp_2), 1 - \neg_h^- (\partial_1 \Delta_1 \partial_2)\} \\
&\leq \max \{\max \{1 - \neg_N^- (\wp_1), 1 - \neg_N^- (\wp_2)\}, \max \{1 - \neg_h^- (\partial_1), 1 - \neg_h^- (\partial_2)\}\} \\
&= \max \{\max \{1 - \neg_N^- (\wp_1), 1 - \neg_h^- (\partial_1)\}, \max \{1 - \neg_N^- (\wp_2), 1 - \neg_h^- (\partial_2)\}\} \\
&= \max \{1 - \neg_{N \times h}^- (\wp_1, \partial_1), 1 - \neg_{N \times h}^- (\wp_2, \partial_2)\}.
\end{aligned}$$

Thus $\exists_{N \times h}^d(\wp \Delta_1 \partial) \leq \max \{\exists_{N \times h}^d(\wp), \exists_{N \times h}^d(\partial)\}$. Similarly, $\exists_{N \times h}^d(\wp \Delta_2 \partial) \leq \max \{\exists_{N \times h}^d(\wp), \exists_{N \times h}^d(\partial)\}$ and $\exists_{N \times h}^d(\wp \Delta_3 \partial) \leq \max \{\exists_{N \times h}^d(\wp), \exists_{N \times h}^d(\partial)\}$. Hence, $N \times h$ is a IVNCVSBS of Ξ .

Corollary 3.5. If N_1, N_2, \dots, N_n are the families of IVNCVSBS^s of $\Xi_1, \Xi_2, \dots, \Xi_n$ respectively, then $N_1 \times N_2 \times \dots \times N_n$ is a IVNCVSBS of $\Xi_1 \times \Xi_2 \times \dots \times \Xi_n$.

Definition 3.6. Let \aleph be a neutrosophic VS in Ξ , the strongest interval-valued neutrosophic cubic vague relation (SIVNCVR) on Ξ is defined as

$$\left\{ \begin{array}{l} \widehat{\exists}_{\Upsilon}^{\exists}(\varphi, \partial) = \min\{\widehat{\exists}_{\aleph}^{\exists}(\varphi), \widehat{\exists}_{\aleph}^{\exists}(\partial)\} \\ \widehat{\exists}_{\Upsilon}^{\exists}(\varphi, \partial) = \frac{\widehat{\exists}_{\aleph}^{\exists}(\varphi) + \widehat{\exists}_{\aleph}^{\exists}(\partial)}{2} \\ \widehat{\exists}_{\Upsilon}^{\exists}(\varphi, \partial) = \max\{\widehat{\exists}_{\aleph}^{\exists}(\varphi), \widehat{\exists}_{\aleph}^{\exists}(\partial)\} \end{array} \right\}.$$

$$\left\{ \begin{array}{l} \widehat{\exists}_{\Upsilon}^{\exists}(\varphi, \partial) = \min\{\widehat{\exists}_{\aleph}^{\exists}(\varphi), \widehat{\exists}_{\aleph}^{\exists}(\partial)\} \\ \widehat{\exists}_{\Upsilon}^{\exists}(\varphi, \partial) = \frac{\widehat{\exists}_{\aleph}^{\exists}(\varphi) + \widehat{\exists}_{\aleph}^{\exists}(\partial)}{2} \\ \widehat{\exists}_{\Upsilon}^{\exists}(\varphi, \partial) = \max\{\widehat{\exists}_{\aleph}^{\exists}(\varphi), \widehat{\exists}_{\aleph}^{\exists}(\partial)\} \end{array} \right\}.$$

Theorem 3.7. Let \aleph be the IVNCVSBS of Ξ and Υ be the SNSVR of Ξ . Then \aleph is a IVNCVSBS of Ξ if and only if Υ is a IVNCVSBS of $\Xi \times \Xi$.

Proof. Let \aleph be the IVNCVSBS of Ξ and Υ be the SNSVR of Ξ . Then for any $\varphi = (\varphi_1, \varphi_2)$ and $\partial = (\partial_1, \partial_2)$ are in $\Xi \times \Xi$. Now,

$$\begin{aligned} \widehat{\exists}_{\Upsilon}^{\exists}(\varphi \Delta_1 \partial) &= \widehat{\exists}_{\Upsilon}^{\exists}[((\varphi_1, \varphi_2) \Delta_1 (\partial_1, \partial_2))] \\ &= \widehat{\exists}_{\Upsilon}^{\exists}(\varphi_1 \Delta_1 \partial_1, \varphi_2 \Delta_1 \partial_2) \\ &= \min\{\widehat{\exists}_{\aleph}^{\exists}(\varphi_1 \Delta_1 \partial_1), \widehat{\exists}_{\aleph}^{\exists}(\varphi_2 \Delta_1 \partial_2)\} \\ &\geq \min\{\min\{\widehat{\exists}_{\aleph}^{\exists}(\varphi_1), \widehat{\exists}_{\aleph}^{\exists}(\partial_1)\}, \min\{\widehat{\exists}_{\aleph}^{\exists}(\varphi_2), \widehat{\exists}_{\aleph}^{\exists}(\partial_2)\}\} \\ &= \min\{\min\{\widehat{\exists}_{\aleph}^{\exists}(\varphi_1), \widehat{\exists}_{\aleph}^{\exists}(\varphi_2)\}, \min\{\widehat{\exists}_{\aleph}^{\exists}(\partial_1), \widehat{\exists}_{\aleph}^{\exists}(\partial_2)\}\} \\ &= \min\{\widehat{\exists}_{\Upsilon}^{\exists}(\varphi_1, \varphi_2), \widehat{\exists}_{\Upsilon}^{\exists}(\partial_1, \partial_2)\} \\ &= \min\{\widehat{\exists}_{\Upsilon}^{\exists}(\varphi), \widehat{\exists}_{\Upsilon}^{\exists}(\partial)\}. \end{aligned}$$

$$\begin{aligned} 1 - \widehat{\exists}_{\Upsilon}^{\exists}(\varphi \Delta_1 \partial) &= 1 - \widehat{\exists}_{\Upsilon}^{\exists}[((\varphi_1, \varphi_2) \Delta_1 (\partial_1, \partial_2))] \\ &= 1 - \widehat{\exists}_{\Upsilon}^{\exists}(\varphi_1 \Delta_1 \partial_1, \varphi_2 \Delta_1 \partial_2) \\ &= \min\{1 - \widehat{\exists}_{\aleph}^{\exists}(\varphi_1 \Delta_1 \partial_1), 1 - \widehat{\exists}_{\aleph}^{\exists}(\varphi_2 \Delta_1 \partial_2)\} \\ &\geq \min\{\min\{1 - \widehat{\exists}_{\aleph}^{\exists}(\varphi_1), 1 - \widehat{\exists}_{\aleph}^{\exists}(\partial_1)\}, \min\{1 - \widehat{\exists}_{\aleph}^{\exists}(\varphi_2), 1 - \widehat{\exists}_{\aleph}^{\exists}(\partial_2)\}\} \\ &= \min\{\min\{1 - \widehat{\exists}_{\aleph}^{\exists}(\varphi_1), 1 - \widehat{\exists}_{\aleph}^{\exists}(\varphi_2)\}, \min\{1 - \widehat{\exists}_{\aleph}^{\exists}(\partial_1), 1 - \widehat{\exists}_{\aleph}^{\exists}(\partial_2)\}\} \\ &= \min\{1 - \widehat{\exists}_{\Upsilon}^{\exists}(\varphi_1, \varphi_2), 1 - \widehat{\exists}_{\Upsilon}^{\exists}(\partial_1, \partial_2)\} \\ &= \min\{1 - \widehat{\exists}_{\Upsilon}^{\exists}(\varphi), 1 - \widehat{\exists}_{\Upsilon}^{\exists}(\partial)\}. \end{aligned}$$

Thus $\widehat{\exists}_{\Upsilon}^{\exists}(\varphi \Delta_1 \partial) \geq \min\{\widehat{\exists}_{\Upsilon}^{\exists}(\varphi), \widehat{\exists}_{\Upsilon}^{\exists}(\partial)\}$. Similarly, $\widehat{\exists}_{\Upsilon}^{\exists}(\varphi \Delta_2 \partial) \geq \min\{\widehat{\exists}_{\Upsilon}^{\exists}(\varphi), \widehat{\exists}_{\Upsilon}^{\exists}(\partial)\}$ and $\widehat{\exists}_{\Upsilon}^{\exists}(\varphi \Delta_3 \partial) \geq \min\{\widehat{\exists}_{\Upsilon}^{\exists}(\varphi), \widehat{\exists}_{\Upsilon}^{\exists}(\partial)\}$. Now,

$$\begin{aligned} \widehat{\exists}_{\Upsilon}^{\exists}(\varphi \Delta_1 \partial) &= \widehat{\exists}_{\Upsilon}^{\exists}[((\varphi_1, \varphi_2) \Delta_1 (\partial_1, \partial_2))] \\ &= \widehat{\exists}_{\Upsilon}^{\exists}(\varphi_1 \Delta_1 \partial_1, \varphi_2 \Delta_1 \partial_2) \\ &= \frac{\widehat{\exists}_{\aleph}^{\exists}(\varphi_1 \Delta_1 \partial_1) + \widehat{\exists}_{\aleph}^{\exists}(\varphi_2 \Delta_1 \partial_2)}{2} \\ &\geq \frac{1}{2} \left[\frac{\widehat{\exists}_{\aleph}^{\exists}(\varphi_1) + \widehat{\exists}_{\aleph}^{\exists}(\partial_1)}{2} + \frac{\widehat{\exists}_{\aleph}^{\exists}(\varphi_2) + \widehat{\exists}_{\aleph}^{\exists}(\partial_2)}{2} \right] \\ &= \frac{1}{2} \left[\frac{\widehat{\exists}_{\aleph}^{\exists}(\varphi_1) + \widehat{\exists}_{\aleph}^{\exists}(\varphi_2)}{2} + \frac{\widehat{\exists}_{\aleph}^{\exists}(\partial_1) + \widehat{\exists}_{\aleph}^{\exists}(\partial_2)}{2} \right] \\ &= \frac{\widehat{\exists}_{\Upsilon}^{\exists}(\varphi_1, \varphi_2) + \widehat{\exists}_{\Upsilon}^{\exists}(\partial_1, \partial_2)}{2} \\ &= \frac{\widehat{\exists}_{\Upsilon}^{\exists}(\varphi) + \widehat{\exists}_{\Upsilon}^{\exists}(\partial)}{2}. \end{aligned}$$

$$\begin{aligned}
\widehat{\mathfrak{I}}_{\Upsilon}^{+}(\wp \triangle_1 \partial) &= \widehat{\mathfrak{I}}_{\Upsilon}^{+}[((\wp_1, \wp_2) \triangle_1 (\partial_1, \partial_2))] \\
&= \widehat{\mathfrak{I}}_{\Upsilon}^{+}(\wp_1 \triangle_1 \partial_1, \wp_2 \triangle_1 \partial_2) \\
&= \frac{\widehat{\mathfrak{I}}_{\mathbb{N}}^{+}(\wp_1 \triangle_1 \partial_1) + \widehat{\mathfrak{I}}_{\mathbb{N}}^{+}(\wp_2 \triangle_1 \partial_2)}{2} \\
&\geq \frac{1}{2} \left[\frac{\widehat{\mathfrak{I}}_{\mathbb{N}}^{+}(\wp_1) + \widehat{\mathfrak{I}}_{\mathbb{N}}^{+}(\partial_1)}{2} + \frac{\widehat{\mathfrak{I}}_{\mathbb{N}}^{+}(\wp_2) + \widehat{\mathfrak{I}}_{\mathbb{N}}^{+}(\partial_2)}{2} \right] \\
&= \frac{1}{2} \left[\frac{\widehat{\mathfrak{I}}_{\mathbb{N}}^{+}(\wp_1) + \widehat{\mathfrak{I}}_{\mathbb{N}}^{+}(\wp_2)}{2} + \frac{\widehat{\mathfrak{I}}_{\mathbb{N}}^{+}(\partial_1) + \widehat{\mathfrak{I}}_{\mathbb{N}}^{+}(\partial_2)}{2} \right] \\
&= \frac{\widehat{\mathfrak{I}}_{\Upsilon}^{+}(\wp_1, \wp_2) + \widehat{\mathfrak{I}}_{\Upsilon}^{+}(\partial_1, \partial_2)}{2} \\
&= \frac{\widehat{\mathfrak{I}}_{\Upsilon}^{+}(\wp) + \widehat{\mathfrak{I}}_{\Upsilon}^{+}(\partial)}{2}.
\end{aligned}$$

Thus $\widehat{\mathfrak{D}}_{\Upsilon}^{+}(\wp \triangle_1 \partial) \geq \frac{\widehat{\mathfrak{D}}_{\Upsilon}^{+}(\wp) + \widehat{\mathfrak{D}}_{\Upsilon}^{+}(\partial)}{2}$. Similarly, $\widehat{\mathfrak{D}}_{\Upsilon}^{+}(\wp \triangle_2 \partial) \geq \frac{\widehat{\mathfrak{D}}_{\Upsilon}^{+}(\wp) + \widehat{\mathfrak{D}}_{\Upsilon}^{+}(\partial)}{2}$ and $\widehat{\mathfrak{D}}_{\Upsilon}^{+}(\wp \triangle_3 \partial) \geq \frac{\widehat{\mathfrak{D}}_{\Upsilon}^{+}(\wp) + \widehat{\mathfrak{D}}_{\Upsilon}^{+}(\partial)}{2}$. Similarly, $\widehat{\mathfrak{D}}_{\Upsilon}^{\pm}(\wp \triangle_1 \partial) \leq \max\{\widehat{\mathfrak{D}}_{\Upsilon}^{\pm}(\wp), \widehat{\mathfrak{D}}_{\Upsilon}^{\pm}(\partial)\}$, $\widehat{\mathfrak{D}}_{\Upsilon}^{\pm}(\wp \triangle_2 \partial) \leq \max\{\widehat{\mathfrak{D}}_{\Upsilon}^{\pm}(\wp), \widehat{\mathfrak{D}}_{\Upsilon}^{\pm}(\partial)\}$ and $\widehat{\mathfrak{D}}_{\Upsilon}^{\pm}(\wp \triangle_3 \partial) \leq \max\{\widehat{\mathfrak{D}}_{\Upsilon}^{\pm}(\wp), \widehat{\mathfrak{D}}_{\Upsilon}^{\pm}(\partial)\}$.

Now,

$$\begin{aligned}
\mathfrak{T}_{\Upsilon}^{-}(\wp \triangle_1 \partial) &= \mathfrak{T}_{\Upsilon}^{-}[((\wp_1, \wp_2) \triangle_1 (\partial_1, \partial_2))] \\
&= \mathfrak{T}_{\Upsilon}^{-}(\wp_1 \triangle_1 \partial_1, \wp_2 \triangle_1 \partial_2) \\
&= \min\{\mathfrak{T}_{\mathbb{N}}^{-}(\wp_1 \triangle_1 \partial_1), \mathfrak{T}_{\mathbb{N}}^{-}(\wp_2 \triangle_1 \partial_2)\} \\
&\geq \min\{\min\{\mathfrak{T}_{\mathbb{N}}^{-}(\wp_1), \mathfrak{T}_{\mathbb{N}}^{-}(\partial_1)\}, \min\{\mathfrak{T}_{\mathbb{N}}^{-}(\wp_2), \mathfrak{T}_{\mathbb{N}}^{-}(\partial_2)\}\} \\
&= \min\{\min\{\mathfrak{T}_{\mathbb{N}}^{-}(\wp_1), \mathfrak{T}_{\mathbb{N}}^{-}(\wp_2)\}, \min\{\mathfrak{T}_{\mathbb{N}}^{-}(\partial_1), \mathfrak{T}_{\mathbb{N}}^{-}(\partial_2)\}\} \\
&= \min\{\mathfrak{T}_{\Upsilon}^{-}(\wp_1, \wp_2), \mathfrak{T}_{\Upsilon}^{-}(\partial_1, \partial_2)\} \\
&= \min\{\mathfrak{T}_{\Upsilon}^{-}(\wp), \mathfrak{T}_{\Upsilon}^{-}(\partial)\}.
\end{aligned}$$

$$\begin{aligned}
1 - \mathfrak{D}_{\Upsilon}^{-}(\wp \triangle_1 \partial) &= 1 - \mathfrak{D}_{\Upsilon}^{-}[((\wp_1, \wp_2) \triangle_1 (\partial_1, \partial_2))] \\
&= 1 - \mathfrak{D}_{\Upsilon}^{-}(\wp_1 \triangle_1 \partial_1, \wp_2 \triangle_1 \partial_2) \\
&= \min\{1 - \mathfrak{D}_{\mathbb{N}}^{-}(\wp_1 \triangle_1 \partial_1), 1 - \mathfrak{D}_{\mathbb{N}}^{-}(\wp_2 \triangle_1 \partial_2)\} \\
&\geq \min\{\min\{1 - \mathfrak{D}_{\mathbb{N}}^{-}(\wp_1), 1 - \mathfrak{D}_{\mathbb{N}}^{-}(\partial_1)\}, \min\{1 - \mathfrak{D}_{\mathbb{N}}^{-}(\wp_2), 1 - \mathfrak{D}_{\mathbb{N}}^{-}(\partial_2)\}\} \\
&= \min\{\min\{1 - \mathfrak{D}_{\mathbb{N}}^{-}(\wp_1), 1 - \mathfrak{D}_{\mathbb{N}}^{-}(\wp_2)\}, \min\{1 - \mathfrak{D}_{\mathbb{N}}^{-}(\partial_1), 1 - \mathfrak{D}_{\mathbb{N}}^{-}(\partial_2)\}\} \\
&= \min\{1 - \mathfrak{D}_{\Upsilon}^{-}(\wp_1, \wp_2), 1 - \mathfrak{D}_{\Upsilon}^{-}(\partial_1, \partial_2)\} \\
&= \min\{1 - \mathfrak{D}_{\Upsilon}^{-}(\wp), 1 - \mathfrak{D}_{\Upsilon}^{-}(\partial)\}.
\end{aligned}$$

Thus $\mathfrak{D}_{\Upsilon}^{-}(\wp \triangle_1 \partial) \geq \min\{\mathfrak{D}_{\Upsilon}^{-}(\wp), \mathfrak{D}_{\Upsilon}^{-}(\partial)\}$. Similarly, $\mathfrak{D}_{\Upsilon}^{-}(\wp \triangle_2 \partial) \geq \min\{\mathfrak{D}_{\Upsilon}^{-}(\wp), \mathfrak{D}_{\Upsilon}^{-}(\partial)\}$ and $\mathfrak{D}_{\Upsilon}^{-}(\wp \triangle_3 \partial) \geq \min\{\mathfrak{D}_{\Upsilon}^{-}(\wp), \mathfrak{D}_{\Upsilon}^{-}(\partial)\}$. Now,

$$\begin{aligned}
\mathfrak{I}_{\Upsilon}^{-}(\wp \triangle_1 \partial) &= \mathfrak{I}_{\Upsilon}^{-}[((\wp_1, \wp_2) \triangle_1 (\partial_1, \partial_2))] \\
&= \mathfrak{I}_{\Upsilon}^{-}(\wp_1 \triangle_1 \partial_1, \wp_2 \triangle_1 \partial_2) \\
&= \frac{\mathfrak{I}_{\mathbb{N}}^{-}(\wp_1 \triangle_1 \partial_1) + \mathfrak{I}_{\mathbb{N}}^{-}(\wp_2 \triangle_1 \partial_2)}{2} \\
&\geq \frac{1}{2} \left[\frac{\mathfrak{I}_{\mathbb{N}}^{-}(\wp_1) + \mathfrak{I}_{\mathbb{N}}^{-}(\partial_1)}{2} + \frac{\mathfrak{I}_{\mathbb{N}}^{-}(\wp_2) + \mathfrak{I}_{\mathbb{N}}^{-}(\partial_2)}{2} \right] \\
&= \frac{1}{2} \left[\frac{\mathfrak{I}_{\mathbb{N}}^{-}(\wp_1) + \mathfrak{I}_{\mathbb{N}}^{-}(\wp_2)}{2} + \frac{\mathfrak{I}_{\mathbb{N}}^{-}(\partial_1) + \mathfrak{I}_{\mathbb{N}}^{-}(\partial_2)}{2} \right] \\
&= \frac{\mathfrak{I}_{\Upsilon}^{-}(\wp_1, \wp_2) + \mathfrak{I}_{\Upsilon}^{-}(\partial_1, \partial_2)}{2} \\
&= \frac{\mathfrak{I}_{\Upsilon}^{-}(\wp) + \mathfrak{I}_{\Upsilon}^{-}(\partial)}{2}.
\end{aligned}$$

$$\begin{aligned}
J_Y^+(\varphi \Delta_1 \partial) &= J_Y^+[(\varphi_1, \varphi_2) \Delta_1 (\partial_1, \partial_2)] \\
&= J_Y^+(\varphi_1 \Delta_1 \partial_1, \varphi_2 \Delta_1 \partial_2) \\
&= \frac{J_N^+(\varphi_1 \Delta_1 \partial_1) + J_N^+(\varphi_2 \Delta_1 \partial_2)}{2} \\
&\geq \frac{1}{2} \left[\frac{J_N^+(\varphi_1) + J_N^+(\partial_1)}{2} + \frac{J_N^+(\varphi_2) + J_N^+(\partial_2)}{2} \right] \\
&= \frac{1}{2} \left[\frac{J_N^+(\varphi_1) + J_N^+(\varphi_2)}{2} + \frac{J_N^+(\partial_1) + J_N^+(\partial_2)}{2} \right] \\
&= \frac{J_Y^+(\varphi_1, \varphi_2) + J_Y^+(\partial_1, \partial_2)}{2} \\
&= \frac{J_Y^+(\varphi) + J_Y^+(\partial)}{2}.
\end{aligned}$$

Thus $J_Y^+(\varphi \Delta_1 \partial) \geq \frac{J_Y(\varphi) + J_Y(\partial)}{2}$. Similarly, $J_Y^+(\varphi \Delta_2 \partial) \geq \frac{J_Y(\varphi) + J_Y(\partial)}{2}$ and $J_Y^+(\varphi \Delta_3 \partial) \geq \frac{J_Y(\varphi) + J_Y(\partial)}{2}$. Similarly, $J_Y^+(\varphi \Delta_1 \partial) \leq \max\{J_Y^+(\varphi), J_Y^+(\partial)\}$, $J_Y^+(\varphi \Delta_2 \partial) \leq \max\{J_Y^+(\varphi), J_Y^+(\partial)\}$ and $J_Y^+(\varphi \Delta_3 \partial) \leq \max\{J_Y^+(\varphi), J_Y^+(\partial)\}$. Thus, Υ is a IVNCVSBS of $\Xi \bowtie \Xi$.

Conversely let us assume that Υ is a IVNCVSBS of $\Xi \bowtie \Xi$, then for any $\varphi = (\varphi_1, \varphi_2)$ and $\partial = (\partial_1, \partial_2)$ are in $\Xi \bowtie \Xi$. Now,

$$\begin{aligned}
\min\{\hat{T}_N^-(\varphi_1 \Delta_1 \partial_1), \hat{T}_N^-(\varphi_2 \Delta_1 \partial_2)\} &= \hat{T}_Y^-(\varphi_1 \Delta_1 \partial_1, \varphi_2 \Delta_1 \partial_2) \\
&= \hat{T}_Y^-[((\varphi_1, \varphi_2) \Delta_1 (\partial_1, \partial_2))] \\
&= \hat{T}_Y^-(\varphi \Delta_1 \partial) \\
&\geq \min\{\hat{T}_Y^-(\varphi), \hat{T}_Y^-(\partial)\} \\
&= \min\{\hat{T}_Y^-(\varphi_1, \varphi_2), \hat{T}_Y^-(\partial_1, \partial_2)\} \\
&= \min\{\min\{\hat{T}_N^-(\varphi_1), \hat{T}_N^-(\varphi_2)\}, \min\{\hat{T}_N^-(\partial_1), \hat{T}_N^-(\partial_2)\}\}.
\end{aligned}$$

If $\hat{T}_N^-(\varphi_1 \Delta_1 \partial_1) \leq \hat{T}_N^-(\varphi_2 \Delta_1 \partial_2)$, then $\hat{T}_N^-(\varphi_1) \leq \hat{T}_N^-(\varphi_2)$ and $\hat{T}_N^-(\partial_1) \leq \hat{T}_N^-(\partial_2)$. We get $\hat{T}_N^-(\varphi_1 \Delta_1 \partial_1) \geq \min\{\hat{T}_N^-(\varphi_1), \hat{T}_N^-(\partial_1)\}$ for all $\varphi_1, \partial_1 \in \Xi$, and

$$\min\{\hat{T}_N^-(\varphi_1 \Delta_2 \partial_1), \hat{T}_N^-(\varphi_2 \Delta_2 \partial_2)\} \geq \min\{\min\{\hat{T}_N^-(\varphi_1), \hat{T}_N^-(\varphi_2)\}, \min\{\hat{T}_N^-(\partial_1), \hat{T}_N^-(\partial_2)\}\}$$

If $\hat{T}_N^-(\varphi_1 \Delta_2 \partial_1) \leq \hat{T}_N^-(\varphi_2 \Delta_2 \partial_2)$, then $\hat{T}_N^-(\varphi_1 \Delta_2 \partial_1) \geq \min\{\hat{T}_N^-(\varphi_1), \hat{T}_N^-(\partial_1)\}$.

$$\min\{\hat{T}_N^-(\varphi_1 \Delta_3 \partial_1), \hat{T}_N^-(\varphi_2 \Delta_3 \partial_2)\} \geq \min\{\min\{\hat{T}_N^-(\varphi_1), \hat{T}_N^-(\varphi_2)\}, \min\{\hat{T}_N^-(\partial_1), \hat{T}_N^-(\partial_2)\}\}$$

If $\hat{T}_N^-(\varphi_1 \Delta_3 \partial_1) \leq \hat{T}_N^-(\varphi_2 \Delta_3 \partial_2)$, then $\hat{T}_N^-(\varphi_1 \Delta_3 \partial_1) \geq \min\{\hat{T}_N^-(\varphi_1), \hat{T}_N^-(\partial_1)\}$.

$$\begin{aligned}
&\min\{1 - \hat{E}_N^-(\varphi_1 \Delta_1 \partial_1), 1 - \hat{E}_N^-(\varphi_2 \Delta_1 \partial_2)\} \\
&= 1 - \hat{E}_Y^-(\varphi_1 \Delta_1 \partial_1, \varphi_2 \Delta_1 \partial_2) \\
&= 1 - \hat{E}_Y^-[((\varphi_1, \varphi_2) \Delta_1 (\partial_1, \partial_2))] \\
&= 1 - \hat{E}_Y^-(\varphi \Delta_1 \partial) \\
&\geq \min\{1 - \hat{E}_Y^-(\varphi), 1 - \hat{E}_Y^-(\partial)\} \\
&= \min\{1 - \hat{E}_Y^-(\varphi_1, \varphi_2), 1 - \hat{E}_Y^-(\partial_1, \partial_2)\} \\
&= \min\{\min\{1 - \hat{E}_N^-(\varphi_1), 1 - \hat{E}_N^-(\varphi_2)\}, \min\{1 - \hat{E}_N^-(\partial_1), 1 - \hat{E}_N^-(\partial_2)\}\}.
\end{aligned}$$

If $1 - \hat{E}_N^-(\varphi_1 \Delta_1 \partial_1) \leq 1 - \hat{E}_N^-(\varphi_2 \Delta_1 \partial_2)$, then $1 - \hat{E}_N^-(\varphi_1) \leq 1 - \hat{E}_N^-(\varphi_2)$ and $1 - \hat{E}_N^-(\partial_1) \leq 1 - \hat{E}_N^-(\partial_2)$. We get $1 - \hat{E}_N^-(\varphi_1 \Delta_1 \partial_1) \geq \min\{1 - \hat{E}_N^-(\varphi_1), 1 - \hat{E}_N^-(\partial_1)\}$ for all $\varphi_1, \partial_1 \in \Xi$, and $\min\{1 - \hat{E}_N^-(\varphi_1 \Delta_2 \partial_1), 1 - \hat{E}_N^-(\varphi_2 \Delta_2 \partial_2)\} \geq \min\{\min\{1 - \hat{E}_N^-(\varphi_1), 1 - \hat{E}_N^-(\varphi_2)\}, \min\{1 - \hat{E}_N^-(\partial_1), 1 - \hat{E}_N^-(\partial_2)\}\}$.

If $1 - \hat{E}_N^-(\varphi_1 \Delta_2 \partial_1) \leq 1 - \hat{E}_N^-(\varphi_2 \Delta_2 \partial_2)$, then $1 - \hat{E}_N^-(\varphi_1 \Delta_2 \partial_1) \geq \min\{1 - \hat{E}_N^-(\varphi_1), 1 - \hat{E}_N^-(\partial_1)\}$.

$$\min\{1 - \hat{E}_N^-(\varphi_1 \Delta_3 \partial_1), 1 - \hat{E}_N^-(\varphi_2 \Delta_3 \partial_2)\} \geq \min\{\min\{1 - \hat{E}_N^-(\varphi_1), 1 - \hat{E}_N^-(\varphi_2)\}, \min\{1 - \hat{E}_N^-(\partial_1), 1 - \hat{E}_N^-(\partial_2)\}\}$$

If $1 - \hat{E}_N^-(\varphi_1 \Delta_3 \partial_1) \leq 1 - \hat{E}_N^-(\varphi_2 \Delta_3 \partial_2)$, then $1 - \hat{E}_N^-(\varphi_1 \Delta_3 \partial_1) \geq \min\{1 - \hat{E}_N^-(\varphi_1), 1 - \hat{E}_N^-(\partial_1)\}$.

Thus $\hat{E}_Y^-(\varphi \Delta_1 \partial) \geq \min\{\hat{E}_Y^-(\varphi), \hat{E}_Y^-(\partial)\}$. Similarly, $\hat{E}_Y^-(\varphi \Delta_2 \partial) \geq \min\{\hat{E}_Y^-(\varphi), \hat{E}_Y^-(\partial)\}$ and $\hat{E}_Y^-(\varphi \Delta_3 \partial) \geq$

$\min\{\widehat{\mathcal{B}}_Y^-(\varphi), \widehat{\mathcal{B}}_Y^+(\partial)\}$. Now,

$$\begin{aligned}
& \frac{1}{2} \left[\widehat{\mathfrak{I}}_{\mathfrak{N}}^-(\varphi_1 \triangle_1 \partial_1) + \widehat{\mathfrak{I}}_{\mathfrak{N}}^-(\varphi_2 \triangle_1 \partial_2) \right] = \widehat{\mathfrak{I}}_{\Upsilon}^-(\varphi_1 \triangle_1 \partial_1, \varphi_2 \triangle_1 \partial_2) \\
& = \widehat{\mathfrak{I}}_{\Upsilon}^-[(\varphi_1, \varphi_2) \triangle_1 (\partial_1, \partial_2)] \\
& = \widehat{\mathfrak{I}}_{\Upsilon}^-(\varphi \triangle_1 \partial) \\
& \geq \frac{\widehat{\mathfrak{I}}_{\Upsilon}^-(\varphi) + \widehat{\mathfrak{I}}_{\Upsilon}^-(\partial)}{2} \\
& = \frac{\widehat{\mathfrak{I}}_{\Upsilon}^-(\varphi_1, \varphi_2) + \widehat{\mathfrak{I}}_{\Upsilon}^-(\partial_1, \partial_2)}{2} \\
& = \frac{1}{2} \left[\frac{\widehat{\mathfrak{I}}_{\mathfrak{N}}^-(\varphi_1) + \widehat{\mathfrak{I}}_{\mathfrak{N}}^-(\varphi_2)}{2} + \frac{\widehat{\mathfrak{I}}_{\mathfrak{N}}^-(\partial_1) + \widehat{\mathfrak{I}}_{\mathfrak{N}}^-(\partial_2)}{2} \right].
\end{aligned}$$

If $\widehat{\mathfrak{I}}_{\aleph}^-(\wp_1 \triangle_1 \partial_1) \leq \widehat{\mathfrak{I}}_{\aleph}^-(\wp_2 \triangle_1 \partial_2)$, then $\widehat{\mathfrak{I}}_{\aleph}^-(\wp_1) \leq \widehat{\mathfrak{I}}_{\aleph}^-(\wp_2)$ and $\widehat{\mathfrak{I}}_{\aleph}^-(\partial_1) \leq \widehat{\mathfrak{I}}_{\aleph}^-(\partial_2)$.

We get $\widehat{\mathfrak{I}}_{\mathcal{N}}^-(\wp_1 \triangle_1 \partial_1) \geq \frac{\widehat{\mathfrak{I}}_{\mathcal{N}}^-(\wp_1) + \widehat{\mathfrak{I}}_{\mathcal{N}}^-(\partial_1)}{2}$. Similarly, $\widehat{\mathfrak{I}}_{\mathcal{N}}^-(\wp_1 \triangle_2 \partial_1) \geq \frac{\widehat{\mathfrak{I}}_{\mathcal{N}}^-(\wp_1) + \widehat{\mathfrak{I}}_{\mathcal{N}}^-(\partial_1)}{2}$ and $\widehat{\mathfrak{I}}_{\mathcal{N}}^-(\wp_1 \triangle_3 \partial_1) \geq \frac{\widehat{\mathfrak{I}}_{\mathcal{N}}^-(\wp_1) + \widehat{\mathfrak{I}}_{\mathcal{N}}^-(\partial_1)}{2}$.

$$\text{Also, } \frac{1}{2} \left[\widehat{\mathfrak{I}}_{\mathcal{R}}^+(\wp_1 \triangle_1 \partial_1) + \widehat{\mathfrak{I}}_{\mathcal{R}}^+(\wp_2 \triangle_1 \partial_2) \right] \geq \frac{1}{2} \left[\frac{\widehat{\mathfrak{I}}_{\mathcal{R}}^+(\wp_1) + \widehat{\mathfrak{I}}_{\mathcal{R}}^+(\wp_2)}{2} + \frac{\widehat{\mathfrak{I}}_{\mathcal{R}}^+(\partial_1) + \widehat{\mathfrak{I}}_{\mathcal{R}}^+(\partial_2)}{2} \right].$$

If $\widehat{\mathbb{J}}_n^+(\varphi_1 \triangle_1 \partial_1) \leq \widehat{\mathbb{J}}_n^+(\varphi_2 \triangle_1 \partial_2)$, then $\widehat{\mathbb{J}}_n^+(\varphi_1) \leq \widehat{\mathbb{J}}_n^+(\varphi_2)$ and $\widehat{\mathbb{J}}_n^+(\partial_1) \leq \widehat{\mathbb{J}}_n^+(\partial_2)$.

We get $\widehat{\mathbb{J}}_N^+(\varphi_1\triangle_1\partial_1) \geq \frac{\widehat{\mathbb{J}}_N^+(\varphi_1) + \widehat{\mathbb{J}}_N^+(\partial_1)}{2}$ and $\widehat{\mathbb{J}}_N^+(\varphi_1\triangle_2\partial_1) \geq \frac{\widehat{\mathbb{J}}_N^+(\varphi_1) + \widehat{\mathbb{J}}_N^+(\partial_1)}{2}$ and $\widehat{\mathbb{J}}_N^+(\varphi_1\triangle_3\partial_1) \geq \frac{\widehat{\mathbb{J}}_N^+(\varphi_1) + \widehat{\mathbb{J}}_N^+(\partial_1)}{2}$.

Thus $\hat{\Delta}_\infty(\phi \triangle_1 \theta) \geq \frac{\hat{\Delta}_Y(\phi) + \hat{\Delta}_Y(\theta)}{2}$. Similarly, $\hat{\Delta}_\infty(\phi \triangle_2 \theta) \geq \frac{\hat{\Delta}_Y(\phi) + \hat{\Delta}_Y(\theta)}{2}$ and $\hat{\Delta}_\infty(\phi \triangle_3 \theta) \geq \frac{\hat{\Delta}_Y(\phi) + \hat{\Delta}_Y(\theta)}{2}$.

Similarly, $\sum_{\Gamma}(\psi \rightarrow \phi) = \frac{1}{2}$. Similarly, $\sum_{\Gamma}(\psi \rightarrow \phi) = \frac{1}{2}$ and $\sum_{\Gamma}(\psi \rightarrow \phi) = \frac{1}{2}$.

If $\widehat{\exists}_{\mathcal{N}}^-(\varphi_1 \triangle_1 \partial_1) \geq \widehat{\exists}_{\mathcal{N}}^-(\varphi_2 \triangle_1 \partial_2)$, then $\widehat{\exists}_{\mathcal{N}}^-(\varphi_1) \geq \widehat{\exists}_{\mathcal{N}}^-(\varphi_2)$ and $\widehat{\exists}_{\mathcal{N}}^-(\partial_1) \geq \widehat{\exists}_{\mathcal{N}}^-(\partial_2)$.

We get $\widehat{\exists}_{\mathcal{R}}^-(\varphi_1 \triangle_1 \vartheta_1) \leq \max\{\widehat{\exists}_{\mathcal{R}}^-(\varphi_1), \widehat{\exists}_{\mathcal{R}}^-(\vartheta_1)\}$.

$$\max\{\widehat{\exists}_{\mathbb{N}}^-(\varphi_1 \triangle_2 \partial_1), \widehat{\exists}_{\mathbb{N}}^-(\varphi_2 \triangle_2 \partial_2)\} \leq \max\{\max$$

If $\widehat{\exists}_{\aleph}^-(\varphi_1 \triangle_2 \vartheta_1) \geq \widehat{\exists}_{\aleph}^-(\varphi_2 \triangle_2 \vartheta_2)$, then $\widehat{\exists}_{\aleph}^-(\varphi_1 \triangle_2 \vartheta_1) \leq \max\{\widehat{\exists}_{\aleph}^-(\varphi_1), \widehat{\exists}_{\aleph}^-(\vartheta_1)\}$.

$$\max\{\widehat{\exists}_{\aleph}^-(\varphi_1 \triangle_3 \partial_1), \widehat{\exists}_{\aleph}^-(\varphi_2 \triangle_3 \partial_2)\} \leqq \max\{\max\{\widehat{\exists}_{\aleph}^-(\varphi_1), \widehat{\exists}_{\aleph}^-(\varphi_2)\}, \max\{\widehat{\exists}_{\aleph}^-($$

If $\widehat{\exists}_{\aleph}^-(\varphi_1 \triangle_3 \vartheta_1) \geq \widehat{\exists}_{\aleph}^-(\varphi_2 \triangle_3 \vartheta_2)$, then $\widehat{\exists}_{\aleph}^-(\varphi_1 \triangle_3 \vartheta_1) \leq \max\{\widehat{\exists}_{\aleph}^-(\varphi_1), \widehat{\exists}_{\aleph}^-(\vartheta_1)\}$.

Also, Similarly to prove that $\max\{1 - \hat{\mathbb{I}}_{\mathcal{K}}^-(\varphi_1 \triangle_1 \partial_1), 1 - \hat{\mathbb{I}}_{\mathcal{K}}^-(\varphi_2 \triangle_1 \partial_2)\} \leq \hat{\mathbb{I}}_{\mathcal{K}}^-(\varphi_1 \triangle_1 \partial_1) + \hat{\mathbb{I}}_{\mathcal{K}}^-(\varphi_2 \triangle_1 \partial_2)$

If $1 - \widehat{\gamma}_n^-(\varphi_1 \triangle_1 \partial_1) \geq 1 - \widehat{\gamma}_n^-(\varphi_2 \triangle_1 \partial_2)$, then $1 - \widehat{\gamma}_n^-(\varphi_1) \geq 1 - \widehat{\gamma}_n^-(\varphi_2)$ and $1 - \widehat{\gamma}_n^-(\partial_1) \geq 1 - \widehat{\gamma}_n^-(\partial_2)$.

We get $1 - \hat{\gamma}_k^-(\varphi_1 \triangle_1 \partial_1) \leq \max\{1 - \hat{\gamma}_k^-(\varphi_1), 1 - \hat{\gamma}_k^-(\partial_1)\}$.

$$\max\{1 - \hat{\mathbb{P}}_{\mathcal{N}}(\phi_1 \triangle \partial_1), 1 - \hat{\mathbb{P}}_{\mathcal{N}}(\phi_2 \triangle \partial_2)\} \leq \max\{\max\{1 -$$

$$\text{If } \sum_{k=1}^n \widehat{\Delta}_k^-(\gamma_k, 2) \geq 1 - \widehat{\Delta}_n^-(\gamma_n, 2), \text{ then } 1 - \widehat{\Delta}_n^-(\gamma_n, 2) \leq \min\{1 - \widehat{\Delta}_n^-(\gamma_1, 1), 1 - \widehat{\Delta}_n^-(\gamma_2, 2)\}.$$

$$\max\left\{1 - \frac{\hat{\gamma}_1}{\gamma_1}(\varphi_1 \wedge \vartheta_1), 1 - \frac{\hat{\gamma}_2}{\gamma_2}(\varphi_2 \wedge \vartheta_2)\right\} \leq \max\left\{\max\left\{1 - \frac{\hat{\gamma}_1}{\gamma_1}(\varphi_1), 1 - \frac{\hat{\gamma}_2}{\gamma_2}(\varphi_2)\right\}, \max\left\{1 - \frac{\hat{\gamma}_1}{\gamma_1}(\vartheta_1), 1 - \frac{\hat{\gamma}_2}{\gamma_2}(\vartheta_2)\right\}\right\}$$

$$\max\{1 - \mathbb{P}_{\mathcal{N}}(\varphi_1 \triangle_3 \partial_1), 1 - \mathbb{P}_{\mathcal{N}}(\varphi_2 \triangle_3 \partial_2)\} \leq \max\{\max\{1 - \mathbb{P}_{\mathcal{N}}(\varphi_1), 1 - \mathbb{P}_{\mathcal{N}}(\varphi_2)\}, \max\{1 - \mathbb{P}_{\mathcal{N}}(\partial_1), 1 - \mathbb{P}_{\mathcal{N}}(\partial_2)\}\}.$$

$$\text{If } 1 - \hat{\mathbb{P}}_N^-(\varphi_1 \Delta_3 \partial_1) \geq 1 - \hat{\mathbb{P}}_N^-(\varphi_2 \Delta_3 \partial_2) \text{ then } \hat{\mathbb{P}}_N^+(\varphi_1 \Delta_3 \partial_1) \leq \hat{\mathbb{P}}_N^+(\varphi_2 \Delta_3 \partial_2).$$

Hence, $\mathbb{B}_Y^{\frac{1}{2}}(\varphi \triangle_1 \partial) \leq \max\{\mathbb{B}_Y^{\frac{1}{2}}(\varphi), \mathbb{B}_Y^{\frac{1}{2}}(\partial)\}$, $\mathbb{B}_Y^{\frac{1}{2}}(\varphi \triangle_2 \partial) \leq \max\{\mathbb{B}_Y^{\frac{1}{2}}(\varphi), \mathbb{B}_Y^{\frac{1}{2}}(\partial)\}$ and $\mathbb{B}_Y^{\frac{1}{2}}(\varphi \triangle_3 \partial) \leq \max\{\mathbb{B}_Y^{\frac{1}{2}}(\varphi), \mathbb{B}_Y^{\frac{1}{2}}(\partial)\}$.

$$\begin{aligned}
\min\{\mathsf{T}_{\mathbb{R}}^-(\wp_1 \triangle_1 \partial_1), \mathsf{T}_{\mathbb{R}}^-(\wp_2 \triangle_1 \partial_2)\} &= \mathsf{T}_{\mathbb{Y}}^-(\wp_1 \triangle_1 \partial_1, \wp_2 \triangle_1 \partial_2) \\
&= \mathsf{T}_{\mathbb{Y}}^-[(\wp_1, \wp_2) \triangle_1 (\partial_1, \partial_2)] \\
&= \mathsf{T}_{\mathbb{Y}}^-(\wp \triangle_1 \partial) \\
&\geq \min\{\mathsf{T}_{\mathbb{Y}}^-(\wp), \mathsf{T}_{\mathbb{Y}}^-(\partial)\} \\
&= \min\{\mathsf{T}_{\mathbb{Y}}^-(\wp_1, \wp_2)\}, \mathsf{T}_{\mathbb{Y}}^-(\partial_1, \partial_2)\} \\
&= \min\{\min\{\mathsf{T}_{\mathbb{Y}}^-(\wp_1), \mathsf{T}_{\mathbb{Y}}^-(\wp_2)\}, \min\{\mathsf{T}_{\mathbb{Y}}^-(\partial_1), \mathsf{T}_{\mathbb{Y}}^-(\partial_2)\}\}.
\end{aligned}$$

If $\mathsf{I}_{\mathbb{V}}^-(\varphi_1 \triangle_1 \partial_1) \leq \mathsf{I}_{\mathbb{V}}^-(\varphi_2 \triangle_1 \partial_2)$, then $\mathsf{I}_{\mathbb{V}}^-(\varphi_1) \leq \mathsf{I}_{\mathbb{V}}^-(\varphi_2)$ and $\mathsf{I}_{\mathbb{V}}^-(\partial_1) \leq \mathsf{I}_{\mathbb{V}}^-(\partial_2)$. We get $\mathsf{I}_{\mathbb{V}}^-(\varphi_1 \triangle_1 \partial_1) \geq$

$\min\{\neg_{\mathbb{N}}^-(\varphi_1), \neg_{\mathbb{N}}^-(\partial_1)\}$ for all $\varphi_1, \partial_1 \in \Xi$, and

$\min\{\neg_{\mathbb{N}}^-(\varphi_1\Delta_2\partial_1), \neg_{\mathbb{N}}^-(\varphi_2\Delta_2\partial_2)\} \geq \min\{\min\{\neg_{\mathbb{N}}^-(\varphi_1), \neg_{\mathbb{N}}^-(\varphi_2)\}, \min\{\neg_{\mathbb{N}}^-(\partial_1), \neg_{\mathbb{N}}^-(\partial_2)\}\}$

If $\neg_{\mathbb{N}}^-(\varphi_1\Delta_2\partial_1) \leq \neg_{\mathbb{N}}^-(\varphi_2\Delta_2\partial_2)$, then $\neg_{\mathbb{N}}^-(\varphi_1\Delta_2\partial_1) \geq \min\{\neg_{\mathbb{N}}^-(\varphi_1), \neg_{\mathbb{N}}^-(\partial_1)\}$.

$\min\{\neg_{\mathbb{N}}^-(\varphi_1\Delta_3\partial_1), \neg_{\mathbb{N}}^-(\varphi_2\Delta_3\partial_2)\} \geq \min\{\min\{\neg_{\mathbb{N}}^-(\varphi_1), \neg_{\mathbb{N}}^-(\varphi_2)\}, \min\{\neg_{\mathbb{N}}^-(\partial_1), \neg_{\mathbb{N}}^-(\partial_2)\}\}$.

If $\neg_{\mathbb{N}}^-(\varphi_1\Delta_3\partial_1) \leq \neg_{\mathbb{N}}^-(\varphi_2\Delta_3\partial_2)$, then $\neg_{\mathbb{N}}^-(\varphi_1\Delta_3\partial_1) \geq \min\{\neg_{\mathbb{N}}^-(\varphi_1), \neg_{\mathbb{N}}^-(\partial_1)\}$.

$$\begin{aligned} & \min\{1 - \exists_{\mathbb{N}}^-(\varphi_1\Delta_1\partial_1), 1 - \exists_{\mathbb{N}}^-(\varphi_2\Delta_1\partial_2)\} \\ &= 1 - \exists_{\mathbb{N}}^-(\varphi_1\Delta_1\partial_1, \varphi_2\Delta_1\partial_2) \\ &= 1 - \exists_{\mathbb{N}}^-[(\varphi_1, \varphi_2)\Delta_1(\partial_1, \partial_2)] \\ &= 1 - \exists_{\mathbb{N}}^-(\varphi\Delta_1\partial) \\ &\geq \min\{1 - \exists_{\mathbb{N}}^-(\varphi), 1 - \exists_{\mathbb{N}}^-(\partial)\} \\ &= \min\{1 - \exists_{\mathbb{N}}^-(\varphi_1, \varphi_2)\}, 1 - \exists_{\mathbb{N}}^-(\partial_1, \partial_2)\} \\ &= \min\{\min\{1 - \exists_{\mathbb{N}}^-(\varphi_1), 1 - \exists_{\mathbb{N}}^-(\varphi_2)\}, \min\{1 - \exists_{\mathbb{N}}^-(\partial_1), 1 - \exists_{\mathbb{N}}^-(\partial_2)\}\}. \end{aligned}$$

If $1 - \exists_{\mathbb{N}}^-(\varphi_1\Delta_1\partial_1) \leq 1 - \exists_{\mathbb{N}}^-(\varphi_2\Delta_1\partial_2)$, then $1 - \exists_{\mathbb{N}}^-(\varphi_1) \leq 1 - \exists_{\mathbb{N}}^-(\varphi_2)$ and $1 - \exists_{\mathbb{N}}^-(\partial_1) \leq 1 - \exists_{\mathbb{N}}^-(\partial_2)$. We get $1 - \exists_{\mathbb{N}}^-(\varphi_1\Delta_1\partial_1) \geq \min\{1 - \exists_{\mathbb{N}}^-(\varphi_1), 1 - \exists_{\mathbb{N}}^-(\partial_1)\}$ for all $\varphi_1, \partial_1 \in \Xi$, and $\min\{1 - \exists_{\mathbb{N}}^-(\varphi_1\Delta_2\partial_1), 1 - \exists_{\mathbb{N}}^-(\varphi_2\Delta_2\partial_2)\} \geq \min\{\min\{1 - \exists_{\mathbb{N}}^-(\varphi_1), 1 - \exists_{\mathbb{N}}^-(\varphi_2)\}, \min\{1 - \exists_{\mathbb{N}}^-(\partial_1), 1 - \exists_{\mathbb{N}}^-(\partial_2)\}\}$.

If $1 - \exists_{\mathbb{N}}^-(\varphi_1\Delta_2\partial_1) \leq 1 - \exists_{\mathbb{N}}^-(\varphi_2\Delta_2\partial_2)$, then $1 - \exists_{\mathbb{N}}^-(\varphi_1\Delta_2\partial_1) \geq \min\{1 - \exists_{\mathbb{N}}^-(\varphi_1), 1 - \exists_{\mathbb{N}}^-(\partial_1)\}$.

$\min\{1 - \exists_{\mathbb{N}}^-(\varphi_1\Delta_3\partial_1), 1 - \exists_{\mathbb{N}}^-(\varphi_2\Delta_3\partial_2)\} \geq \min\{\min\{1 - \exists_{\mathbb{N}}^-(\varphi_1), 1 - \exists_{\mathbb{N}}^-(\varphi_2)\}, \min\{1 - \exists_{\mathbb{N}}^-(\partial_1), 1 - \exists_{\mathbb{N}}^-(\partial_2)\}\}$. If $1 - \exists_{\mathbb{N}}^-(\varphi_1\Delta_3\partial_1) \leq 1 - \exists_{\mathbb{N}}^-(\varphi_2\Delta_3\partial_2)$, then $1 - \exists_{\mathbb{N}}^-(\varphi_1\Delta_3\partial_1) \geq \min\{1 - \exists_{\mathbb{N}}^-(\varphi_1), 1 - \exists_{\mathbb{N}}^-(\partial_1)\}$.

Thus $\exists_{\mathbb{N}}^+(\varphi\Delta_1\partial) \geq \min\{\exists_{\mathbb{N}}^+(\varphi), \exists_{\mathbb{N}}^+(\partial)\}$. Similarly, $\exists_{\mathbb{N}}^+(\varphi\Delta_2\partial) \geq \min\{\exists_{\mathbb{N}}^+(\varphi), \exists_{\mathbb{N}}^+(\partial)\}$ and $\exists_{\mathbb{N}}^+(\varphi\Delta_3\partial) \geq \min\{\exists_{\mathbb{N}}^+(\varphi), \exists_{\mathbb{N}}^+(\partial)\}$. Now,

$$\begin{aligned} \frac{1}{2} [\exists_{\mathbb{N}}^-(\varphi_1\Delta_1\partial_1) + \exists_{\mathbb{N}}^-(\varphi_2\Delta_1\partial_2)] &= \exists_{\mathbb{N}}^-[((\varphi_1, \varphi_2)\Delta_1(\partial_1, \partial_2))] \\ &= \exists_{\mathbb{N}}^-[(\varphi_1, \varphi_2)\Delta_1(\partial_1, \partial_2)] \\ &= \exists_{\mathbb{N}}^-(\varphi\Delta_1\partial) \\ &\geq \frac{\exists_{\mathbb{N}}^-(\varphi) + \exists_{\mathbb{N}}^-(\partial)}{2} \\ &= \frac{\exists_{\mathbb{N}}^-(\varphi_1, \varphi_2) + \exists_{\mathbb{N}}^-(\partial_1, \partial_2)}{2} \\ &= \frac{1}{2} \left[\frac{\exists_{\mathbb{N}}^-(\varphi_1) + \exists_{\mathbb{N}}^-(\varphi_2)}{2} + \frac{\exists_{\mathbb{N}}^-(\partial_1) + \exists_{\mathbb{N}}^-(\partial_2)}{2} \right]. \end{aligned}$$

If $\exists_{\mathbb{N}}^-(\varphi_1\Delta_1\partial_1) \leq \exists_{\mathbb{N}}^-(\varphi_2\Delta_1\partial_2)$, then $\exists_{\mathbb{N}}^-(\varphi_1) \leq \exists_{\mathbb{N}}^-(\varphi_2)$ and $\exists_{\mathbb{N}}^-(\partial_1) \leq \exists_{\mathbb{N}}^-(\partial_2)$.

We get $\exists_{\mathbb{N}}^-(\varphi_1\Delta_1\partial_1) \geq \frac{\exists_{\mathbb{N}}^-(\varphi_1) + \exists_{\mathbb{N}}^-(\partial_1)}{2}$. Similarly, $\exists_{\mathbb{N}}^-(\varphi_1\Delta_2\partial_1) \geq \frac{\exists_{\mathbb{N}}^-(\varphi_1) + \exists_{\mathbb{N}}^-(\partial_1)}{2}$ and $\exists_{\mathbb{N}}^-(\varphi_1\Delta_3\partial_1) \geq \frac{\exists_{\mathbb{N}}^-(\varphi_1) + \exists_{\mathbb{N}}^-(\partial_1)}{2}$.

Also, $\frac{1}{2} [\exists_{\mathbb{N}}^+(\varphi_1\Delta_1\partial_1) + \exists_{\mathbb{N}}^+(\varphi_2\Delta_1\partial_2)] \geq \frac{1}{2} \left[\frac{\exists_{\mathbb{N}}^+(\varphi_1) + \exists_{\mathbb{N}}^+(\varphi_2)}{2} + \frac{\exists_{\mathbb{N}}^+(\partial_1) + \exists_{\mathbb{N}}^+(\partial_2)}{2} \right]$.

If $\exists_{\mathbb{N}}^+(\varphi_1\Delta_1\partial_1) \leq \exists_{\mathbb{N}}^+(\varphi_2\Delta_1\partial_2)$, then $\exists_{\mathbb{N}}^+(\varphi_1) \leq \exists_{\mathbb{N}}^+(\varphi_2)$ and $\exists_{\mathbb{N}}^+(\partial_1) \leq \exists_{\mathbb{N}}^+(\partial_2)$.

We get $\exists_{\mathbb{N}}^+(\varphi_1\Delta_1\partial_1) \geq \frac{\exists_{\mathbb{N}}^+(\varphi_1) + \exists_{\mathbb{N}}^+(\partial_1)}{2}$ and $\exists_{\mathbb{N}}^+(\varphi_1\Delta_2\partial_1) \geq \frac{\exists_{\mathbb{N}}^+(\varphi_1) + \exists_{\mathbb{N}}^+(\partial_1)}{2}$ and $\exists_{\mathbb{N}}^+(\varphi_1\Delta_3\partial_1) \geq \frac{\exists_{\mathbb{N}}^+(\varphi_1) + \exists_{\mathbb{N}}^+(\partial_1)}{2}$.

Thus $\exists_{\mathbb{N}}^+(\varphi\Delta_1\partial) \geq \frac{\exists_{\mathbb{N}}^+(\varphi) + \exists_{\mathbb{N}}^+(\partial)}{2}$. Similarly, $\exists_{\mathbb{N}}^+(\varphi\Delta_2\partial) \geq \frac{\exists_{\mathbb{N}}^+(\varphi) + \exists_{\mathbb{N}}^+(\partial)}{2}$ and $\exists_{\mathbb{N}}^+(\varphi\Delta_3\partial) \geq \frac{\exists_{\mathbb{N}}^+(\varphi) + \exists_{\mathbb{N}}^+(\partial)}{2}$.

Similarly, $\max\{\exists_{\mathbb{N}}^-(\varphi_1\Delta_1\partial_1), \exists_{\mathbb{N}}^-(\varphi_2\Delta_1\partial_2)\} \leq \max\{\max\{\exists_{\mathbb{N}}^-(\varphi_1), \exists_{\mathbb{N}}^-(\varphi_2)\}, \max\{\exists_{\mathbb{N}}^-(\partial_1), \exists_{\mathbb{N}}^-(\partial_2)\}\}$.

If $\exists_{\mathbb{N}}^-(\varphi_1\Delta_1\partial_1) \geq \exists_{\mathbb{N}}^-(\varphi_2\Delta_1\partial_2)$, then $\exists_{\mathbb{N}}^-(\varphi_1) \geq \exists_{\mathbb{N}}^-(\varphi_2)$ and $\exists_{\mathbb{N}}^-(\partial_1) \geq \exists_{\mathbb{N}}^-(\partial_2)$.

We get $\exists_{\mathbb{N}}^-(\varphi_1\Delta_1\partial_1) \leq \max\{\exists_{\mathbb{N}}^-(\varphi_1), \exists_{\mathbb{N}}^-(\partial_1)\}$.

$\max\{\exists_{\mathbb{N}}^-(\varphi_1\Delta_2\partial_1), \exists_{\mathbb{N}}^-(\varphi_2\Delta_2\partial_2)\} \leq \max\{\max\{\exists_{\mathbb{N}}^-(\varphi_1), \exists_{\mathbb{N}}^-(\varphi_2)\}, \max\{\exists_{\mathbb{N}}^-(\partial_1), \exists_{\mathbb{N}}^-(\partial_2)\}\}$.

If $\exists_{\mathbb{N}}^-(\varphi_1\Delta_2\partial_1) \geq \exists_{\mathbb{N}}^-(\varphi_2\Delta_2\partial_2)$, then $\exists_{\mathbb{N}}^-(\varphi_1\Delta_2\partial_1) \leq \max\{\exists_{\mathbb{N}}^-(\varphi_1), \exists_{\mathbb{N}}^-(\partial_1)\}$.

$\max\{\exists_{\mathbb{N}}^-(\varphi_1\Delta_3\partial_1), \exists_{\mathbb{N}}^-(\varphi_2\Delta_3\partial_2)\} \leq \max\{\max\{\exists_{\mathbb{N}}^-(\varphi_1), \exists_{\mathbb{N}}^-(\varphi_2)\}, \max\{\exists_{\mathbb{N}}^-(\partial_1), \exists_{\mathbb{N}}^-(\partial_2)\}\}$.

If $\exists_{\mathbb{N}}^-(\varphi_1\Delta_3\partial_1) \geq \exists_{\mathbb{N}}^-(\varphi_2\Delta_3\partial_2)$, then $\exists_{\mathbb{N}}^-(\varphi_1\Delta_3\partial_1) \leq \max\{\exists_{\mathbb{N}}^-(\varphi_1), \exists_{\mathbb{N}}^-(\partial_1)\}$.

Also, Similarly to prove that $\max\{1 - \exists_{\mathbb{N}}^-(\varphi_1\Delta_1\partial_1), 1 - \exists_{\mathbb{N}}^-(\varphi_2\Delta_1\partial_2)\} \leq \max\{\max\{1 - \exists_{\mathbb{N}}^-(\varphi_1), 1 - \exists_{\mathbb{N}}^-(\varphi_2)\}, \max\{1 - \exists_{\mathbb{N}}^-(\partial_1), 1 - \exists_{\mathbb{N}}^-(\partial_2)\}\}$.

If $1 - \exists_{\mathbb{N}}^-(\varphi_1\Delta_1\partial_1) \geq 1 - \exists_{\mathbb{N}}^-(\varphi_2\Delta_1\partial_2)$, then $1 - \exists_{\mathbb{N}}^-(\varphi_1) \geq 1 - \exists_{\mathbb{N}}^-(\varphi_2)$ and $1 - \exists_{\mathbb{N}}^-(\partial_1) \geq 1 - \exists_{\mathbb{N}}^-(\partial_2)$.

We get $1 - \exists_{\mathbb{N}}^-(\varphi_1\Delta_1\partial_1) \leq \max\{1 - \exists_{\mathbb{N}}^-(\varphi_1), 1 - \exists_{\mathbb{N}}^-(\partial_1)\}$.

$\max\{1 - \overline{\mathbb{T}}_{\mathbb{N}}(\varphi_1 \Delta_2 \partial_1), 1 - \overline{\mathbb{T}}_{\mathbb{N}}(\varphi_2 \Delta_2 \partial_2)\} \leq \max\{\max\{1 - \overline{\mathbb{T}}_{\mathbb{N}}(\varphi_1), 1 - \overline{\mathbb{T}}_{\mathbb{N}}(\varphi_2)\}, \max\{1 - \overline{\mathbb{T}}_{\mathbb{N}}(\partial_1), 1 - \overline{\mathbb{T}}_{\mathbb{N}}(\partial_2)\}\}$.

If $1 - \overline{\mathbb{T}}_{\mathbb{N}}(\varphi_1 \Delta_2 \partial_1) \geq 1 - \overline{\mathbb{T}}_{\mathbb{N}}(\varphi_2 \Delta_2 \partial_2)$, then $1 - \overline{\mathbb{T}}_{\mathbb{N}}(\varphi_1 \Delta_2 \partial_1) \leq \max\{1 - \overline{\mathbb{T}}_{\mathbb{N}}(\varphi_1), 1 - \overline{\mathbb{T}}_{\mathbb{N}}(\partial_1)\}$.

$\max\{1 - \overline{\mathbb{T}}_{\mathbb{N}}(\varphi_1 \Delta_3 \partial_1), 1 - \overline{\mathbb{T}}_{\mathbb{N}}(\varphi_2 \Delta_3 \partial_2)\} \leq \max\{\max\{1 - \overline{\mathbb{T}}_{\mathbb{N}}(\varphi_1), 1 - \overline{\mathbb{T}}_{\mathbb{N}}(\varphi_2)\}, \max\{1 - \overline{\mathbb{T}}_{\mathbb{N}}(\partial_1), 1 - \overline{\mathbb{T}}_{\mathbb{N}}(\partial_2)\}\}$.

If $1 - \overline{\mathbb{T}}_{\mathbb{N}}(\varphi_1 \Delta_3 \partial_1) \geq 1 - \overline{\mathbb{T}}_{\mathbb{N}}(\varphi_2 \Delta_3 \partial_2)$, then $1 - \overline{\mathbb{T}}_{\mathbb{N}}(\varphi_1 \Delta_3 \partial_1) \leq \max\{1 - \overline{\mathbb{T}}_{\mathbb{N}}(\varphi_1), 1 - \overline{\mathbb{T}}_{\mathbb{N}}(\partial_1)\}$.

Hence, $\overline{\mathbb{D}}_{\mathbb{Y}}^{\pm}(\varphi \Delta_1 \partial) \leq \max\{\overline{\mathbb{D}}_{\mathbb{Y}}^{\pm}(\varphi), \overline{\mathbb{D}}_{\mathbb{Y}}^{\pm}(\partial)\}$, $\overline{\mathbb{D}}_{\mathbb{Y}}^{\pm}(\varphi \Delta_2 \partial) \leq \max\{\overline{\mathbb{D}}_{\mathbb{Y}}^{\pm}(\varphi), \overline{\mathbb{D}}_{\mathbb{Y}}^{\pm}(\partial)\}$ and

$\overline{\mathbb{D}}_{\mathbb{Y}}^{\pm}(\varphi \Delta_3 \partial) \leq \max\{\overline{\mathbb{D}}_{\mathbb{Y}}^{\pm}(\varphi), \overline{\mathbb{D}}_{\mathbb{Y}}^{\pm}(\partial)\}$. Hence, \mathbb{N} is a IVNCVSBS of Ξ .

Theorem 3.8. Let \mathbb{N} be a NSV subset in Ξ . Then $\mathbb{D} = ([\overline{\mathbb{T}}_{\mathbb{N}}, \overline{\mathbb{T}}_{\mathbb{N}}^+], [\underline{\mathbb{T}}_{\mathbb{N}}, \underline{\mathbb{T}}_{\mathbb{N}}^+], [\underline{\mathbb{D}}_{\mathbb{N}}, \underline{\mathbb{D}}_{\mathbb{N}}^+])$ is a IVNCVSBS of Ξ if and only if all non empty level set $\mathbb{D}^{(\ell_1, \ell_2, s)}$ is a SBS of Ξ for $\ell_1, \ell_2, s \in [0, 1]$.

Proof. Assume that $\widehat{\mathbb{D}}$ is a IVNCVSBS of Ξ . For $\ell_1, \ell_2, s \in [0, 1]$ and $j_1, j_2 \in \widehat{\mathbb{D}}^{(\ell_1, \ell_2, s)}$. We have $\widehat{\mathbb{T}}_{\mathbb{N}}(j_1) \geq \ell_1$, $\widehat{\mathbb{T}}_{\mathbb{N}}(j_2) \geq \ell_1$ and $1 - \widehat{\mathbb{E}}_{\mathbb{N}}(j_1) \geq s$, $1 - \widehat{\mathbb{E}}_{\mathbb{N}}(j_2) \geq s$ and $\widehat{\mathbb{J}}_{\mathbb{N}}(j_1) \geq \ell_2$, $\widehat{\mathbb{J}}_{\mathbb{N}}(j_2) \geq \ell_2$ and $\widehat{\mathbb{I}}_{\mathbb{N}}^+(j_1) \geq \ell_2$, $\widehat{\mathbb{I}}_{\mathbb{N}}^+(j_2) \geq \ell_2$, $1 - \widehat{\mathbb{I}}_{\mathbb{N}}^-(j_1) \leq \ell_1$, $1 - \widehat{\mathbb{I}}_{\mathbb{N}}^-(j_2) \leq \ell_1$ and $\widehat{\mathbb{E}}_{\mathbb{N}}^-(j_1) \leq s$, $\widehat{\mathbb{E}}_{\mathbb{N}}^-(j_2) \leq s$. Now, $\widehat{\mathbb{T}}_{\mathbb{N}}^-(j_1 \Delta_1 j_2) \geq \min\{\widehat{\mathbb{T}}_{\mathbb{N}}^-(j_1), \widehat{\mathbb{T}}_{\mathbb{N}}^-(j_2)\} \geq \ell_1$, $1 - \widehat{\mathbb{E}}_{\mathbb{N}}^-(j_1 \Delta_1 j_2) \geq \min\{1 - \widehat{\mathbb{E}}_{\mathbb{N}}^-(j_1), 1 - \widehat{\mathbb{E}}_{\mathbb{N}}^-(j_2)\} \geq s$ and $\widehat{\mathbb{J}}_{\mathbb{N}}^-(j_1 \Delta_1 j_2) \geq \widehat{\mathbb{J}}_{\mathbb{N}}^-(j_1) + \widehat{\mathbb{J}}_{\mathbb{N}}^-(j_2) \geq \frac{\ell_1 + \ell_2}{2} \geq \ell_2$, $\widehat{\mathbb{I}}_{\mathbb{N}}^+(j_1 \Delta_1 j_2) \geq \widehat{\mathbb{I}}_{\mathbb{N}}^+(j_1) + \widehat{\mathbb{I}}_{\mathbb{N}}^+(j_2) \geq \frac{\ell_1 + \ell_2}{2} \geq \ell_2$ and $\widehat{\mathbb{E}}_{\mathbb{N}}^-(j_1 \Delta_1 j_2) \leq \max\{\widehat{\mathbb{E}}_{\mathbb{N}}^-(j_1), \widehat{\mathbb{E}}_{\mathbb{N}}^-(j_2)\} \leq s$ and $1 - \widehat{\mathbb{T}}_{\mathbb{N}}^-(j_1 \Delta_1 j_2) \leq \max\{1 - \widehat{\mathbb{T}}_{\mathbb{N}}^-(j_1), 1 - \widehat{\mathbb{T}}_{\mathbb{N}}^-(j_2)\} \leq \ell_1$. This implies that $j_1 \Delta_1 j_2 \in \widehat{\mathbb{D}}^{(\ell_1, \ell_2, s)}$. Similarly, $j_1 \Delta_2 j_2 \in \widehat{\mathbb{D}}^{(\ell_1, \ell_2, s)}$ and $j_1 \Delta_3 j_2 \in \widehat{\mathbb{D}}^{(\ell_1, \ell_2, s)}$. Therefore $\widehat{\mathbb{D}}^{(\ell_1, \ell_2, s)}$ is a SBS of Ξ , where $\ell_1, \ell_2, s \in [0, 1]$.

We have $\overline{\mathbb{T}}_{\mathbb{N}}(j_1) \geq \ell_1$, $\overline{\mathbb{T}}_{\mathbb{N}}(j_2) \geq \ell_1$ and

$1 - \underline{\mathbb{E}}_{\mathbb{N}}(j_1) \geq s$, $1 - \underline{\mathbb{E}}_{\mathbb{N}}(j_2) \geq s$ and $\underline{\mathbb{T}}_{\mathbb{N}}(j_1) \geq \ell_2$, $\underline{\mathbb{T}}_{\mathbb{N}}(j_2) \geq \ell_2$ and $\underline{\mathbb{J}}_{\mathbb{N}}^+(j_1) \geq \ell_2$, $\underline{\mathbb{J}}_{\mathbb{N}}^+(j_2) \geq \ell_2$, $1 - \overline{\mathbb{T}}_{\mathbb{N}}(j_1) \leq \ell_1$, $1 - \overline{\mathbb{T}}_{\mathbb{N}}(j_2) \leq \ell_1$ and $\underline{\mathbb{E}}_{\mathbb{N}}(j_1) \leq s$, $\underline{\mathbb{E}}_{\mathbb{N}}(j_2) \leq s$. Now, $\overline{\mathbb{T}}_{\mathbb{N}}^-(j_1 \Delta_1 j_2) \geq \min\{\overline{\mathbb{T}}_{\mathbb{N}}(j_1), \overline{\mathbb{T}}_{\mathbb{N}}(j_2)\} \geq \ell_1$, $1 - \underline{\mathbb{E}}_{\mathbb{N}}^-(j_1 \Delta_1 j_2) \geq \min\{1 - \underline{\mathbb{E}}_{\mathbb{N}}(j_1), 1 - \underline{\mathbb{E}}_{\mathbb{N}}(j_2)\} \geq s$ and $\underline{\mathbb{J}}_{\mathbb{N}}^-(j_1 \Delta_1 j_2) \geq \frac{\underline{\mathbb{T}}_{\mathbb{N}}(j_1) + \underline{\mathbb{T}}_{\mathbb{N}}(j_2)}{2} \geq \ell_2$, $\underline{\mathbb{I}}_{\mathbb{N}}^+(j_1 \Delta_1 j_2) \geq \frac{\underline{\mathbb{T}}_{\mathbb{N}}^+(j_1) + \underline{\mathbb{T}}_{\mathbb{N}}^+(j_2)}{2} \geq \ell_2$ and $\underline{\mathbb{E}}_{\mathbb{N}}^-(j_1 \Delta_1 j_2) \leq \max\{\underline{\mathbb{E}}_{\mathbb{N}}(j_1), \underline{\mathbb{E}}_{\mathbb{N}}(j_2)\} \leq s$ and $1 - \overline{\mathbb{T}}_{\mathbb{N}}^-(j_1 \Delta_1 j_2) \leq \max\{1 - \overline{\mathbb{T}}_{\mathbb{N}}(j_1), 1 - \overline{\mathbb{T}}_{\mathbb{N}}(j_2)\} \leq \ell_1$. This implies that $j_1 \Delta_1 j_2 \in \mathbb{D}^{(\ell_1, \ell_2, s)}$. Similarly, $j_1 \Delta_2 j_2 \in \mathbb{D}^{(\ell_1, \ell_2, s)}$ and $j_1 \Delta_3 j_2 \in \mathbb{D}^{(\ell_1, \ell_2, s)}$. Therefore $\mathbb{D}^{(\ell_1, \ell_2, s)}$ is a SBS of Ξ , where $\ell_1, \ell_2, s \in [0, 1]$.

Conversely, assume that $\widehat{\mathbb{D}}^{(\ell_1, \ell_2, s)}$ is a SBS of Ξ , where $\ell_1, \ell_2, s \in [0, 1]$. Suppose if there exist $j_1, j_2 \in \Xi$ such that $\widehat{\mathbb{T}}_{\mathbb{N}}^-(j_1 \Delta_1 j_2) < \min\{\widehat{\mathbb{T}}_{\mathbb{N}}^-(j_1), \widehat{\mathbb{T}}_{\mathbb{N}}^-(j_2)\}$, $1 - \widehat{\mathbb{E}}_{\mathbb{N}}^-(j_1 \Delta_1 j_2) < \min\{1 - \widehat{\mathbb{E}}_{\mathbb{N}}^-(j_1), 1 - \widehat{\mathbb{E}}_{\mathbb{N}}^-(j_2)\}$, $\widehat{\mathbb{J}}_{\mathbb{N}}^-(j_1 \Delta_1 j_2) < \widehat{\mathbb{J}}_{\mathbb{N}}^-(j_1) + \widehat{\mathbb{J}}_{\mathbb{N}}^-(j_2) \geq \frac{\ell_1 + \ell_2}{2}$, $\widehat{\mathbb{I}}_{\mathbb{N}}^+(j_1 \Delta_1 j_2) < \widehat{\mathbb{I}}_{\mathbb{N}}^+(j_1) + \widehat{\mathbb{I}}_{\mathbb{N}}^+(j_2) \geq \frac{\ell_1 + \ell_2}{2}$ and $\widehat{\mathbb{E}}_{\mathbb{N}}^-(j_1 \Delta_1 j_2) > \max\{\widehat{\mathbb{E}}_{\mathbb{N}}^-(j_1), \widehat{\mathbb{E}}_{\mathbb{N}}^-(j_2)\}$. Then $j_1, j_2 \in \widehat{\mathbb{D}}^{(\ell_1, \ell_2, s)}$, but $j_1 \Delta_1 j_2 \notin \widehat{\mathbb{D}}^{(\ell_1, \ell_2, s)}$. This contradicts to that $\widehat{\mathbb{D}}^{(\ell_1, \ell_2, s)}$ is a SBS of Ξ . Hence, $\widehat{\mathbb{T}}_{\mathbb{N}}^-(j_1 \Delta_1 j_2) \geq \min\{\widehat{\mathbb{T}}_{\mathbb{N}}^-(j_1), \widehat{\mathbb{T}}_{\mathbb{N}}^-(j_2)\}$, $1 - \widehat{\mathbb{E}}_{\mathbb{N}}^-(j_1 \Delta_1 j_2) \geq \min\{1 - \widehat{\mathbb{E}}_{\mathbb{N}}^-(j_1), 1 - \widehat{\mathbb{E}}_{\mathbb{N}}^-(j_2)\}$ and $\widehat{\mathbb{J}}_{\mathbb{N}}^-(j_1 \Delta_1 j_2) < \ell_2 \leq \widehat{\mathbb{J}}_{\mathbb{N}}^-(j_1) + \widehat{\mathbb{J}}_{\mathbb{N}}^-(j_2) \geq \frac{\ell_1 + \ell_2}{2}$ and $\widehat{\mathbb{I}}_{\mathbb{N}}^+(j_1 \Delta_1 j_2) < \ell_2 \leq \widehat{\mathbb{I}}_{\mathbb{N}}^+(j_1) + \widehat{\mathbb{I}}_{\mathbb{N}}^+(j_2) \geq \frac{\ell_1 + \ell_2}{2}$ and $\widehat{\mathbb{E}}_{\mathbb{N}}^-(j_1 \Delta_1 j_2) > s \geq \max\{\widehat{\mathbb{E}}_{\mathbb{N}}^-(j_1), \widehat{\mathbb{E}}_{\mathbb{N}}^-(j_2)\}$, $1 - \widehat{\mathbb{T}}_{\mathbb{N}}^-(j_1 \Delta_1 j_2) > s \geq \max\{1 - \widehat{\mathbb{T}}_{\mathbb{N}}^-(j_1), 1 - \widehat{\mathbb{T}}_{\mathbb{N}}^-(j_2)\}$. Similarly, Δ_2 and Δ_3 cases. Let assume that $\mathbb{D}^{(\ell_1, \ell_2, s)}$ is a SBS of Ξ , where $\ell_1, \ell_2, s \in [0, 1]$. Suppose if there exist $j_1, j_2 \in \Xi$ such that $\overline{\mathbb{T}}_{\mathbb{N}}^-(j_1 \Delta_1 j_2) < \min\{\overline{\mathbb{T}}_{\mathbb{N}}(j_1), \overline{\mathbb{T}}_{\mathbb{N}}(j_2)\}$, $1 - \underline{\mathbb{E}}_{\mathbb{N}}^-(j_1 \Delta_1 j_2) < \min\{1 - \underline{\mathbb{E}}_{\mathbb{N}}(j_1), 1 - \underline{\mathbb{E}}_{\mathbb{N}}(j_2)\}$, $\underline{\mathbb{J}}_{\mathbb{N}}^-(j_1 \Delta_1 j_2) < \underline{\mathbb{J}}_{\mathbb{N}}^-(j_1) + \underline{\mathbb{J}}_{\mathbb{N}}^-(j_2) \geq \frac{\ell_1 + \ell_2}{2}$, $\underline{\mathbb{I}}_{\mathbb{N}}^+(j_1 \Delta_1 j_2) < \underline{\mathbb{I}}_{\mathbb{N}}^+(j_1) + \underline{\mathbb{I}}_{\mathbb{N}}^+(j_2) \geq \frac{\ell_1 + \ell_2}{2}$ and $\underline{\mathbb{E}}_{\mathbb{N}}^-(j_1 \Delta_1 j_2) > \max\{\underline{\mathbb{E}}_{\mathbb{N}}(j_1), \underline{\mathbb{E}}_{\mathbb{N}}(j_2)\}$. Then $j_1, j_2 \in \mathbb{D}^{(\ell_1, \ell_2, s)}$, but $j_1 \Delta_1 j_2 \notin \mathbb{D}^{(\ell_1, \ell_2, s)}$. This contradicts to that $\mathbb{D}^{(\ell_1, \ell_2, s)}$ is a SBS of Ξ . Hence, $\overline{\mathbb{T}}_{\mathbb{N}}^-(j_1 \Delta_1 j_2) \geq \min\{\overline{\mathbb{T}}_{\mathbb{N}}(j_1), \overline{\mathbb{T}}_{\mathbb{N}}(j_2)\}$, $1 - \underline{\mathbb{E}}_{\mathbb{N}}^-(j_1 \Delta_1 j_2) \geq \min\{1 - \underline{\mathbb{E}}_{\mathbb{N}}(j_1), 1 - \underline{\mathbb{E}}_{\mathbb{N}}(j_2)\}$ and $\underline{\mathbb{J}}_{\mathbb{N}}^-(j_1 \Delta_1 j_2) < \ell_2 \leq \underline{\mathbb{J}}_{\mathbb{N}}^-(j_1) + \underline{\mathbb{J}}_{\mathbb{N}}^-(j_2) \geq \frac{\ell_1 + \ell_2}{2}$ and $\underline{\mathbb{I}}_{\mathbb{N}}^+(j_1 \Delta_1 j_2) < \ell_2 \leq \underline{\mathbb{I}}_{\mathbb{N}}^+(j_1) + \underline{\mathbb{I}}_{\mathbb{N}}^+(j_2) \geq \frac{\ell_1 + \ell_2}{2}$ and $\underline{\mathbb{E}}_{\mathbb{N}}^-(j_1 \Delta_1 j_2) > s \geq \max\{\underline{\mathbb{E}}_{\mathbb{N}}(j_1), \underline{\mathbb{E}}_{\mathbb{N}}(j_2)\}$, $1 - \overline{\mathbb{T}}_{\mathbb{N}}^-(j_1 \Delta_1 j_2) > s \geq \max\{1 - \overline{\mathbb{T}}_{\mathbb{N}}(j_1), 1 - \overline{\mathbb{T}}_{\mathbb{N}}(j_2)\}$. Similarly, Δ_2 and Δ_3 cases. Hence, $\mathbb{D} = ([\overline{\mathbb{T}}_{\mathbb{N}}, \overline{\mathbb{T}}_{\mathbb{N}}^+], [\underline{\mathbb{T}}_{\mathbb{N}}, \underline{\mathbb{T}}_{\mathbb{N}}^+], [\underline{\mathbb{D}}_{\mathbb{N}}, \underline{\mathbb{D}}_{\mathbb{N}}^+])$ is a IVNCVSBS of Ξ .

Definition 3.9. Let \mathbb{N} be any IVNCVSBS of Ξ and $\varsigma \in \Xi$. Then the pseudo NSV coset $(\varsigma \mathbb{N})^p$ is defined by

$$\left\{ \begin{array}{l} (\varsigma \widehat{\mathbb{T}}_{\mathbb{N}})^p(\varphi) = p(\varsigma) \widehat{\mathbb{T}}_{\mathbb{N}}(\varphi), \\ (\varsigma \widehat{\mathbb{D}}_{\mathbb{N}})^p(\varphi) = p(\varsigma) \widehat{\mathbb{D}}_{\mathbb{N}}(\varphi), \\ (\varsigma \widehat{\mathbb{D}}_{\mathbb{Y}}^{\pm})^p(\varphi) = p(\varsigma) \widehat{\mathbb{D}}_{\mathbb{Y}}^{\pm}(\varphi). \end{array} \right.$$

$$\left\{ \begin{array}{l} (\zeta \underline{\Box}_{\mathbb{N}})^p(\varphi) = p(\zeta) \underline{\Box}_{\mathbb{N}}(\varphi), \\ (\zeta \underline{\Box}_{\mathbb{N}}^{\dagger})^p(\varphi) = p(\zeta) \underline{\Box}_{\mathbb{N}}^{\dagger}(\varphi), \\ (\zeta \underline{\Box}_{\mathbb{N}}^{\ddagger})^p(\varphi) = p(\zeta) \underline{\Box}_{\mathbb{N}}^{\ddagger}(\varphi) \end{array} \right\}.$$

That is,

$$\left\{ \begin{array}{l} (\zeta \widehat{\top}_{\mathbb{N}})^p(\varphi) = p(\zeta) \widehat{\top}_{\mathbb{N}}(\varphi), \quad 1 - (\zeta \widehat{\top}_{\mathbb{N}})^p(\varphi) = p(\zeta)(1 - \widehat{\exists}_{\mathbb{N}}^-)(\varphi), \\ (\zeta \widehat{\bot}_{\mathbb{N}})^p(\varphi) = p(\zeta) \widehat{\bot}_{\mathbb{N}}(\varphi), \quad (\zeta \widehat{\top}_{\mathbb{N}}^+)^p(\varphi) = p(\zeta) \widehat{\top}_{\mathbb{N}}^+(\varphi), \\ (\zeta \widehat{\bot}_{\mathbb{N}}^-)^p(\varphi) = p(\zeta) \widehat{\bot}_{\mathbb{N}}^-(\varphi), \quad 1 - (\zeta \widehat{\bot}_{\mathbb{N}}^-)^p(\varphi) = p(\zeta)(1 - \widehat{\top}_{\mathbb{N}}^-)(\varphi) \end{array} \right\}$$

$$\left\{ \begin{array}{l} (\zeta \overline{\top}_{\mathbb{N}})^p(\varphi) = p(\zeta) \overline{\top}_{\mathbb{N}}(\varphi), \quad 1 - (\zeta \overline{\exists}_{\mathbb{N}}^-)^p(\varphi) = p(\zeta)(1 - \overline{\exists}_{\mathbb{N}}^-)(\varphi), \\ (\zeta \overline{\bot}_{\mathbb{N}})^p(\varphi) = p(\zeta) \overline{\bot}_{\mathbb{N}}(\varphi), \quad (\zeta \overline{\top}_{\mathbb{N}}^+)^p(\varphi) = p(\zeta) \overline{\top}_{\mathbb{N}}^+(\varphi), \\ (\zeta \overline{\exists}_{\mathbb{N}}^-)^p(\varphi) = p(\zeta) \overline{\exists}_{\mathbb{N}}^-(\varphi), \quad 1 - (\zeta \overline{\top}_{\mathbb{N}})^p(\varphi) = p(\zeta)(1 - \overline{\top}_{\mathbb{N}})(\varphi) \end{array} \right\}$$

each $\varphi \in \Xi$ and for any non-empty set $p \in P$.

Theorem 3.10. Let \mathbb{N} be any IVNCVSBS of Ξ , then the pseudo NSV coset $(\zeta \mathbb{N})^p$ is a IVNCVSBS of Ξ .

Proof. Let \mathbb{N} be any IVNCVSBS of Ξ and for each $\varphi, \partial \in \Xi$. Now, $(\zeta \widehat{\top}_{\mathbb{N}})^p(\varphi \triangle_1 \partial) = p(\zeta) \widehat{\top}_{\mathbb{N}}(\varphi \triangle_1 \partial) \geq p(\zeta) \min\{\widehat{\top}_{\mathbb{N}}(\varphi), \widehat{\top}_{\mathbb{N}}(\partial)\} = \min\{p(\zeta) \widehat{\top}_{\mathbb{N}}(\varphi), p(\zeta) \widehat{\top}_{\mathbb{N}}(\partial)\} = \min\{(\zeta \widehat{\top}_{\mathbb{N}})^p(\varphi), (\zeta \widehat{\top}_{\mathbb{N}})^p(\partial)\}$. Thus $(\zeta \widehat{\top}_{\mathbb{N}})^p(\varphi \triangle_1 \partial) \geq \min\{(\zeta \widehat{\top}_{\mathbb{N}})^p(\varphi), (\zeta \widehat{\top}_{\mathbb{N}})^p(\partial)\}$ and $1 - (\zeta \widehat{\exists}_{\mathbb{N}}^-)^p(\varphi \triangle_1 \partial) = p(\zeta)(1 - \widehat{\exists}_{\mathbb{N}}^-)(\varphi \triangle_1 \partial) \geq p(\zeta) \min\{1 - \widehat{\exists}_{\mathbb{N}}^-(\varphi), 1 - \widehat{\exists}_{\mathbb{N}}^-(\partial)\} = \min\{p(\zeta)(1 - \widehat{\exists}_{\mathbb{N}}^-(\varphi)), p(\zeta)(1 - \widehat{\exists}_{\mathbb{N}}^-(\partial))\} = \min\{1 - (\zeta \widehat{\exists}_{\mathbb{N}}^-)^p(\varphi), 1 - (\zeta \widehat{\exists}_{\mathbb{N}}^-)^p(\partial)\}$. Thus $1 - (\zeta \widehat{\exists}_{\mathbb{N}}^-)^p(\varphi \triangle_1 \partial) \geq \min\{1 - (\zeta \widehat{\exists}_{\mathbb{N}}^-)^p(\varphi), 1 - (\zeta \widehat{\exists}_{\mathbb{N}}^-)^p(\partial)\}$. Now, $(\zeta \widehat{\top}_{\mathbb{N}})^p(\varphi \triangle_1 \partial) = p(\zeta) \widehat{\top}_{\mathbb{N}}(\varphi \triangle_1 \partial) \geq p(\zeta) \left[\frac{\widehat{\top}_{\mathbb{N}}(\varphi) + \widehat{\top}_{\mathbb{N}}(\partial)}{2} \right] = \frac{p(\zeta) \widehat{\top}_{\mathbb{N}}(\varphi) + p(\zeta) \widehat{\top}_{\mathbb{N}}(\partial)}{2} = \frac{(\zeta \widehat{\top}_{\mathbb{N}})^p(\varphi) + (\zeta \widehat{\top}_{\mathbb{N}})^p(\partial)}{2}$. Thus $(\zeta \widehat{\top}_{\mathbb{N}})^p(\varphi \triangle_1 \partial) \geq \frac{(\zeta \widehat{\top}_{\mathbb{N}})^p(\varphi) + (\zeta \widehat{\top}_{\mathbb{N}})^p(\partial)}{2}$ and $(\zeta \widehat{\top}_{\mathbb{N}}^+)^p(\varphi \triangle_1 \partial) = p(\zeta) \widehat{\top}_{\mathbb{N}}^+(\varphi \triangle_1 \partial) \geq p(\zeta) \left[\frac{\widehat{\top}_{\mathbb{N}}^+(\varphi) + \widehat{\top}_{\mathbb{N}}^+(\partial)}{2} \right] = p(\zeta) \widehat{\top}_{\mathbb{N}}^+(\varphi) + p(\zeta) \widehat{\top}_{\mathbb{N}}^+(\partial) = \frac{(\zeta \widehat{\top}_{\mathbb{N}}^+)^p(\varphi) + (\zeta \widehat{\top}_{\mathbb{N}}^+)^p(\partial)}{2}$. Thus $(\zeta \widehat{\top}_{\mathbb{N}})^p(\varphi \triangle_1 \partial) \geq \frac{(\zeta \widehat{\top}_{\mathbb{N}})^p(\varphi) + (\zeta \widehat{\top}_{\mathbb{N}})^p(\partial)}{2}$. Now, $(\zeta \widehat{\bot}_{\mathbb{N}}^-)^p(\varphi \triangle_1 \partial) = p(\zeta) \widehat{\bot}_{\mathbb{N}}^-(\varphi \triangle_1 \partial) = \min\{p(\zeta) \widehat{\top}_{\mathbb{N}}(\varphi), p(\zeta) \widehat{\top}_{\mathbb{N}}(\partial)\} = \min\{(\zeta \widehat{\top}_{\mathbb{N}})^p(\varphi), (\zeta \widehat{\top}_{\mathbb{N}})^p(\partial)\}$. Thus $(\zeta \widehat{\bot}_{\mathbb{N}}^-)^p(\varphi \triangle_1 \partial) \leq \max\{(\zeta \widehat{\bot}_{\mathbb{N}}^-)^p(\varphi), (\zeta \widehat{\bot}_{\mathbb{N}}^-)^p(\partial)\}$ and $1 - (\zeta \widehat{\top}_{\mathbb{N}})^p(\varphi \triangle_1 \partial) = p(\zeta)(1 - \widehat{\top}_{\mathbb{N}}^-)(\varphi \triangle_1 \partial) \leq p(\zeta) \max\{1 - \widehat{\top}_{\mathbb{N}}^-(\varphi), 1 - \widehat{\top}_{\mathbb{N}}^-(\partial)\} = \max\{p(\zeta)(1 - \widehat{\top}_{\mathbb{N}}^-(\varphi)), p(\zeta)(1 - \widehat{\top}_{\mathbb{N}}^-(\partial))\} = \max\{1 - (\zeta \widehat{\top}_{\mathbb{N}})^p(\varphi), 1 - (\zeta \widehat{\top}_{\mathbb{N}})^p(\partial)\}$. Thus $1 - (\zeta \widehat{\top}_{\mathbb{N}})^p(\varphi \triangle_1 \partial) \leq \max\{1 - (\zeta \widehat{\top}_{\mathbb{N}})^p(\varphi), 1 - (\zeta \widehat{\top}_{\mathbb{N}})^p(\partial)\}$. Now, $(\zeta \widehat{\bot}_{\mathbb{N}}^-)^p(\varphi \triangle_1 \partial) = p(\zeta) \widehat{\bot}_{\mathbb{N}}^-(\varphi \triangle_1 \partial) \geq p(\zeta) \left[\frac{\widehat{\bot}_{\mathbb{N}}^-(\varphi) + \widehat{\bot}_{\mathbb{N}}^-(\partial)}{2} \right] = \frac{p(\zeta) \widehat{\bot}_{\mathbb{N}}^-(\varphi) + p(\zeta) \widehat{\bot}_{\mathbb{N}}^-(\partial)}{2} = \frac{(\zeta \widehat{\bot}_{\mathbb{N}}^-)^p(\varphi) + (\zeta \widehat{\bot}_{\mathbb{N}}^-)^p(\partial)}{2}$. Thus $(\zeta \widehat{\bot}_{\mathbb{N}}^-)^p(\varphi \triangle_1 \partial) \geq \frac{(\zeta \widehat{\bot}_{\mathbb{N}}^-)^p(\varphi) + (\zeta \widehat{\bot}_{\mathbb{N}}^-)^p(\partial)}{2}$ and $(\zeta \widehat{\bot}_{\mathbb{N}}^+)^p(\varphi \triangle_1 \partial) = p(\zeta) \widehat{\bot}_{\mathbb{N}}^+(\varphi \triangle_1 \partial) \geq p(\zeta) \left[\frac{\widehat{\bot}_{\mathbb{N}}^+(\varphi) + \widehat{\bot}_{\mathbb{N}}^+(\partial)}{2} \right] = p(\zeta) \widehat{\bot}_{\mathbb{N}}^+(\varphi) + p(\zeta) \widehat{\bot}_{\mathbb{N}}^+(\partial) = \frac{(\zeta \widehat{\bot}_{\mathbb{N}}^+)^p(\varphi) + (\zeta \widehat{\bot}_{\mathbb{N}}^+)^p(\partial)}{2}$. Thus $(\zeta \widehat{\bot}_{\mathbb{N}}^-)^p(\varphi \triangle_1 \partial) \geq \frac{(\zeta \widehat{\bot}_{\mathbb{N}}^-)^p(\varphi) + (\zeta \widehat{\bot}_{\mathbb{N}}^-)^p(\partial)}{2}$. Now, $(\zeta \widehat{\exists}_{\mathbb{N}}^-)^p(\varphi \triangle_1 \partial) = p(\zeta) \widehat{\exists}_{\mathbb{N}}^-(\varphi \triangle_1 \partial) = \min\{p(\zeta) \widehat{\bot}_{\mathbb{N}}(\varphi), p(\zeta) \widehat{\bot}_{\mathbb{N}}(\partial)\} = \min\{(\zeta \widehat{\bot}_{\mathbb{N}})^p(\varphi), (\zeta \widehat{\bot}_{\mathbb{N}})^p(\partial)\}$. Thus $(\zeta \widehat{\exists}_{\mathbb{N}}^-)^p(\varphi \triangle_1 \partial) \leq \max\{(\zeta \widehat{\exists}_{\mathbb{N}}^-)^p(\varphi), (\zeta \widehat{\exists}_{\mathbb{N}}^-)^p(\partial)\}$ and $1 - (\zeta \widehat{\exists}_{\mathbb{N}}^-)^p(\varphi \triangle_1 \partial) = p(\zeta)(1 - \widehat{\exists}_{\mathbb{N}}^-)(\varphi \triangle_1 \partial) \leq p(\zeta) \max\{1 - \widehat{\exists}_{\mathbb{N}}^-(\varphi), 1 - \widehat{\exists}_{\mathbb{N}}^-(\partial)\} = \max\{p(\zeta)(1 - \widehat{\exists}_{\mathbb{N}}^-(\varphi)), p(\zeta)(1 - \widehat{\exists}_{\mathbb{N}}^-(\partial))\} = \max\{1 - (\zeta \widehat{\exists}_{\mathbb{N}}^-)^p(\varphi), 1 - (\zeta \widehat{\exists}_{\mathbb{N}}^-)^p(\partial)\}$. Thus $1 - (\zeta \widehat{\exists}_{\mathbb{N}}^-)^p(\varphi \triangle_1 \partial) \leq \max\{1 - (\zeta \widehat{\exists}_{\mathbb{N}}^-)^p(\varphi), 1 - (\zeta \widehat{\exists}_{\mathbb{N}}^-)^p(\partial)\}$. Similarly, \triangle_2 and \triangle_3 cases. Hence, $(\zeta \mathbb{N})^p$ is a IVNCVSBS of Ξ .

Definition 3.11. Let $(\Xi_1, \heartsuit_1, \heartsuit_2, \heartsuit_3)$ and $(\Xi_2, \diamondsuit_1, \diamondsuit_2, \diamondsuit_3)$ be the bisemirings. Let $\Upsilon : \Xi_1 \rightarrow \Xi_2$ and \mathbb{N} be an IVNCVSBS in Ξ_1 , Υ be an IVNCVSBS in $\Upsilon(\Xi_1) = \Xi_2$, the image of VS is defined as $\widehat{\Box}_{\mathcal{R}(V)}(\ell_2) = [\widehat{\top}_{\mathcal{R}(V)}(\ell_2), 1 - \widehat{\exists}_{\mathcal{R}(V)}(\ell_2)], [\widehat{\bot}_{\mathcal{R}(V)}(\ell_2), \widehat{\top}_{\mathcal{R}(V)}^+(\ell_2)], [\widehat{\exists}_{\mathcal{R}(V)}^-(\ell_2), 1 - \widehat{\top}_{\mathcal{R}(V)}^-(\ell_2)]$ where $\widehat{\top}_{\mathcal{R}(V)}(\ell_2) = \widehat{\top}_{\mathcal{R}}(\ell_2)$, $\widehat{\bot}_{\mathcal{R}(V)}(\ell_2) = \widehat{\bot}_{\mathcal{R}}(\ell_2)$, $\widehat{\top}_{\mathcal{R}(V)}^+(\ell_2) = \widehat{\top}_{\mathcal{R}}^+(\ell_2)$, $\widehat{\bot}_{\mathcal{R}(V)}^-(\ell_2) = \widehat{\bot}_{\mathcal{R}}^-(\ell_2)$ and $\widehat{\exists}_{\mathcal{R}(V)}^-(\ell_2) = \widehat{\exists}_{\mathcal{R}}^-(\ell_2)$ and $\widehat{\Box}_{\mathcal{R}(V)}(\ell_2) = [\widehat{\top}_{\mathcal{R}(V)}(\ell_2), 1 - \widehat{\exists}_{\mathcal{R}(V)}(\ell_2)], [\widehat{\bot}_{\mathcal{R}(V)}(\ell_2), \widehat{\top}_{\mathcal{R}(V)}^+(\ell_2)], [\widehat{\exists}_{\mathcal{R}(V)}^-(\ell_2), 1 - \widehat{\top}_{\mathcal{R}(V)}^-(\ell_2)]$ where $\widehat{\top}_{\mathcal{R}(V)}(\ell_2) = \widehat{\top}_{\mathcal{R}}(\ell_2)$, $\widehat{\bot}_{\mathcal{R}(V)}(\ell_2) = \widehat{\bot}_{\mathcal{R}}(\ell_2)$, $\widehat{\top}_{\mathcal{R}(V)}^+(\ell_2) = \widehat{\top}_{\mathcal{R}}^+(\ell_2)$ and $\widehat{\exists}_{\mathcal{R}(V)}^-(\ell_2) = \widehat{\exists}_{\mathcal{R}}^-(\ell_2)$.

Definition 3.12. Let $(\Xi_1, \heartsuit_1, \heartsuit_2, \heartsuit_3)$ and $(\Xi_2, \diamondsuit_1, \diamondsuit_2, \diamondsuit_3)$ be the bisemirings. Let $\mathfrak{R} : \Xi_1 \rightarrow \Xi_2$ be any function. Let Υ be a VS in $\mathfrak{R}(\Xi_1) = \Xi_2$. Then the inverse image of Υ , \mathfrak{R}^{-1} is the VS in Ξ_1 by $\beth_{\mathfrak{R}^{-1}(V)}(\ell_1) = [\widehat{\top}_{\mathfrak{R}^{-1}(V)}^-(\ell_1), 1 - \widehat{\exists}_{\mathfrak{R}^{-1}(V)}^-(\ell_1)], [\widehat{\top}_{\mathfrak{R}^{-1}(V)}^+(\ell_1), \widehat{\exists}_{\mathfrak{R}^{-1}(V)}^+(\ell_1)], [\widehat{\exists}_{\mathfrak{R}^{-1}(V)}^-(\ell_1), 1 - \widehat{\top}_{\mathfrak{R}^{-1}(V)}^-(\ell_1)],$ where $\widehat{\top}_{\mathfrak{R}^{-1}(V)}(\ell_1) = \widehat{\top}_\Upsilon(\mathfrak{R}^{-1}(\ell_1)), \widehat{\exists}_{\mathfrak{R}^{-1}(V)}^-(\ell_1) = \widehat{\exists}_\Upsilon(\mathfrak{R}^{-1}(\ell_1)), \widehat{\exists}_{\mathfrak{R}^{-1}(V)}^+(\ell_1) = \widehat{\exists}_\Upsilon^+(\mathfrak{R}^{-1}(\ell_1)), \widehat{\exists}_{\mathfrak{R}^{-1}(V)}(\ell_1) = \widehat{\exists}_\Upsilon^-(\mathfrak{R}^{-1}(\ell_1))$ and $\beth_{\mathfrak{R}^{-1}(V)}(\ell_1) = [\top_{\mathfrak{R}^{-1}(V)}^-(\ell_1), 1 - \exists_{\mathfrak{R}^{-1}(V)}^-(\ell_1)], [\top_{\mathfrak{R}^{-1}(V)}^+(\ell_1), \exists_{\mathfrak{R}^{-1}(V)}^+(\ell_1)], [\exists_{\mathfrak{R}^{-1}(V)}^-(\ell_1), 1 - \top_{\mathfrak{R}^{-1}(V)}^-(\ell_1)],$ where $\top_{\mathfrak{R}^{-1}(V)}(\ell_1) = \top_\Upsilon(\mathfrak{R}^{-1}(\ell_1)), \exists_{\mathfrak{R}^{-1}(V)}^-(\ell_1) = \exists_\Upsilon^-(\mathfrak{R}^{-1}(\ell_1)), \exists_{\mathfrak{R}^{-1}(V)}^+(\ell_1) = \exists_\Upsilon^+(\mathfrak{R}^{-1}(\ell_1)), \exists_{\mathfrak{R}^{-1}(V)}(\ell_1) = \exists_\Upsilon^-(\mathfrak{R}^{-1}(\ell_1)).$

Theorem 3.13. Every homomorphic image of IVNCVSBS of Ξ_1 is a IVNCVSBS of Ξ_2 .

Theorem 3.14. Every homomorphic pre-image of IVNCVSBS of Ξ_2 is a IVNCVSBS of Ξ_1 .

Proof. Let $\Re : \Xi_1 \rightarrow \Xi_2$ and $\Re(\wp\heartsuit_1\partial) = \Re(\wp)\diamondsuit_1\Re(\partial)$, $\Re(\wp\heartsuit_2\partial) = \Re(\wp)\diamondsuit_2\Re(\partial)$ and $\Re(\wp\heartsuit_3\partial) = \Re(\wp)\diamondsuit_3\Re(\partial)$ for all $\wp, \partial \in \Xi_1$. Let $V = \Re(\Upsilon)$, where Υ is any IVNCVSBS of Ξ_2 . Let $\wp, \partial \in \Xi_1$. Now, $T_{\mathfrak{N}}^-(\wp\heartsuit_1\partial) = T_{\Upsilon}^-(\Re(\wp\heartsuit_1\partial)) = T_{\Upsilon}^-(\Re(\wp)\diamondsuit_1\Re(\partial)) \geq \min\{T_{\Upsilon}^-\Re(\wp), T_{\Upsilon}^-\Re(\partial)\} = \min\{T_{\mathfrak{N}}^-(\wp), T_{\mathfrak{N}}^-(\partial)\}$. Thus $T_{\mathfrak{N}}^-(\wp\heartsuit_1\partial) \geq \min\{T_{\mathfrak{N}}^-(\wp), T_{\mathfrak{N}}^-(\partial)\}$ and $1-F_{\mathfrak{N}}^-(\wp\heartsuit_1\partial) = 1-F_{\Upsilon}^-(\Re(\wp\heartsuit_1\partial)) = 1-F_{\Upsilon}^-(\Re(\wp)\diamondsuit_1\Re(\partial)) \geq \min\{1-F_{\Upsilon}^-\Re(\wp), 1-F_{\Upsilon}^-\Re(\partial)\} = \min\{1-F_{\mathfrak{N}}^-(\wp), 1-F_{\mathfrak{N}}^-(\partial)\}$. Thus $1-F_{\mathfrak{N}}^-(\wp\heartsuit_1\partial) \geq \min\{1-F_{\mathfrak{N}}^-(\wp), 1-F_{\mathfrak{N}}^-(\partial)\}$. Hence, $\beth_{\Upsilon}^{\hat{\dagger}}(\wp\heartsuit_1\partial) \geq \min\{\beth_{\Upsilon}^{\hat{\dagger}}(\wp), \beth_{\Upsilon}^{\hat{\dagger}}(\partial)\}$. Similarly, $\beth_{\Upsilon}^{\hat{\dagger}}(\wp\heartsuit_2\partial) \geq \min\{\beth_{\Upsilon}^{\hat{\dagger}}(\wp), \beth_{\Upsilon}^{\hat{\dagger}}(\partial)\}$ and $\beth_{\Upsilon}^{\hat{\dagger}}(\wp\heartsuit_3\partial) \geq \min\{\beth_{\Upsilon}^{\hat{\dagger}}(\wp), \beth_{\Upsilon}^{\hat{\dagger}}(\partial)\}$.

Now, $I_{\mathfrak{N}}^-(\wp\heartsuit_1\partial) = I_{\Upsilon}^-(\Re(\wp\heartsuit_1\partial)) = I_{\Upsilon}^-(\Re(\wp)\diamondsuit_1\Re(\partial)) \geq \frac{I_{\Upsilon}^-\Re(\wp)+I_{\Upsilon}^-\Re(\partial)}{2} = \frac{I_{\mathfrak{N}}^-(\wp)+I_{\mathfrak{N}}^-(\partial)}{2}$. Thus $I_{\mathfrak{N}}^-(\wp\heartsuit_1\partial) \geq \frac{I_{\mathfrak{N}}^-(\wp)+I_{\mathfrak{N}}^-(\partial)}{2}$ and $I_{\mathfrak{N}}^+(\wp\heartsuit_1\partial) = I_{\Upsilon}^+(\Re(\wp\heartsuit_1\partial)) = I_{\Upsilon}^+(\Re(\wp)\diamondsuit_1\Re(\partial)) \geq \frac{I_{\Upsilon}^+\Re(\wp)+I_{\Upsilon}^+\Re(\partial)}{2} = \frac{I_{\mathfrak{N}}^+(\wp)+I_{\mathfrak{N}}^+(\partial)}{2}$. Thus $I_{\mathfrak{N}}^+(\wp\heartsuit_1\partial) \geq \frac{I_{\mathfrak{N}}^+(\wp)+I_{\mathfrak{N}}^+(\partial)}{2}$. Hence, $\beth_{\Upsilon}^{\hat{\ddagger}}(\wp\heartsuit_1\partial) \geq \frac{\beth_{\Upsilon}^{\hat{\dagger}}(\wp)+\beth_{\Upsilon}^{\hat{\dagger}}(\partial)}{2}$. Similarly, $\beth_{\Upsilon}^{\hat{\ddagger}}(\wp\heartsuit_2\partial) \geq \frac{\beth_{\Upsilon}^{\hat{\dagger}}(\wp)+\beth_{\Upsilon}^{\hat{\dagger}}(\partial)}{2}$ and $\beth_{\Upsilon}^{\hat{\ddagger}}(\wp\heartsuit_3\partial) \geq \frac{\beth_{\Upsilon}^{\hat{\dagger}}(\wp)+\beth_{\Upsilon}^{\hat{\dagger}}(\partial)}{2}$.

Now, $F_{\mathfrak{N}}^-(\wp\heartsuit_1\partial) = F_{\Upsilon}^-(\Re(\wp\heartsuit_1\partial)) = F_{\Upsilon}^-(\Re(\wp)\diamondsuit_1\Re(\partial)) \leq \max\{F_{\Upsilon}^-\Re(\wp), F_{\Upsilon}^-\Re(\partial)\} = \max\{F_{\mathfrak{N}}^-(\wp), F_{\mathfrak{N}}^-(\partial)\}$.

Thus $F_N^-(\wp \heartsuit_1 \partial) \leq \max\{F_N^-(\wp), F_N^-(\partial)\}$ and $1 - T_N^-(\wp \heartsuit_1 \partial) = 1 - T_Y^-(\Re(\wp \heartsuit_1 \partial)) = 1 - T_Y^-(\Re(\wp) \diamondsuit_1 \Re(\partial)) \leq \max\{1 - T_Y^-(\Re(\wp)), 1 - T_Y^-(\Re(\partial))\} = \max\{1 - T_N^-(\wp), 1 - T_N^-(\partial)\}$. Thus $1 - T_N^-(\wp \heartsuit_1 \partial) \leq \max\{1 - T_N^-(\wp), 1 - T_N^-(\partial)\}$. Hence, $\beth_N^{\widehat{\exists}}(\wp \heartsuit_1 \partial) \leq \max\{\beth_Y^{\widehat{\exists}}(\wp), \beth_Y^{\widehat{\exists}}(\partial)\}$. Similarly, $\beth_N^{\widehat{\exists}}(\wp \heartsuit_2 \partial) \leq \max\{\beth_Y^{\widehat{\exists}}(\wp), \beth_Y^{\widehat{\exists}}(\partial)\}$ and $\beth_N^{\widehat{\exists}}(\wp \heartsuit_3 \partial) \leq \max\{\beth_Y^{\widehat{\exists}}(\wp), \beth_Y^{\widehat{\exists}}(\partial)\}$.

Now, $\beth_N^-(\wp \heartsuit_1 \partial) = \beth_Y^-(\Re(\wp \heartsuit_1 \partial)) = \beth_Y^-(\Re(\wp) \diamondsuit_1 \Re(\partial)) \geq \min\{\beth_Y^-(\Re(\wp)), \beth_Y^-(\Re(\partial))\} = \min\{\beth_N^-(\wp), \beth_N^-(\partial)\}$.

Thus $\beth_N^-(\wp \heartsuit_1 \partial) \geq \min\{\beth_N^-(\wp), \beth_N^-(\partial)\}$ and $1 - \beth_N^-(\wp \heartsuit_1 \partial) = 1 - \beth_Y^-(\Re(\wp \heartsuit_1 \partial)) = 1 - \beth_Y^-(\Re(\wp) \diamondsuit_1 \Re(\partial)) \geq \min\{1 - \beth_Y^-(\Re(\wp)), 1 - \beth_Y^-(\Re(\partial))\} = \min\{1 - \beth_N^-(\wp), 1 - \beth_N^-(\partial)\}$. Thus $1 - \beth_N^-(\wp \heartsuit_1 \partial) \geq \min\{1 - \beth_N^-(\wp), 1 - \beth_N^-(\partial)\}$. Hence, $\beth_N^-(\wp \heartsuit_1 \partial) \geq \min\{\beth_Y^-(\wp), \beth_Y^-(\partial)\}$. Similarly, $\beth_N^-(\wp \heartsuit_2 \partial) \geq \min\{\beth_Y^-(\wp), \beth_Y^-(\partial)\}$ and $\beth_N^-(\wp \heartsuit_3 \partial) \geq \min\{\beth_Y^-(\wp), \beth_Y^-(\partial)\}$.

Now, $\beth_N^-(\wp \heartsuit_1 \partial) = \beth_Y^-(\Re(\wp \heartsuit_1 \partial)) = \beth_Y^-(\Re(\wp) \diamondsuit_1 \Re(\partial)) \geq \frac{\beth_Y^-(\Re(\wp)) + \beth_Y^-(\Re(\partial))}{2} = \frac{\beth_N^-(\wp) + \beth_N^-(\partial)}{2}$. Thus $\beth_N^-(\wp \heartsuit_1 \partial) \geq \frac{\beth_N^-(\wp) + \beth_N^-(\partial)}{2}$ and $\beth_N^+(\wp \heartsuit_1 \partial) = \beth_Y^+(\Re(\wp \heartsuit_1 \partial)) = \beth_Y^+(\Re(\wp) \diamondsuit_1 \Re(\partial)) \geq \frac{\beth_Y^+(\Re(\wp)) + \beth_Y^+(\Re(\partial))}{2} = \frac{\beth_N^+(\wp) + \beth_N^+(\partial)}{2}$. Thus $\beth_N^+(\wp \heartsuit_1 \partial) \geq \frac{\beth_N^+(\wp) + \beth_N^+(\partial)}{2}$. Hence, $\beth_N^{\widehat{\exists}}(\wp \heartsuit_1 \partial) \geq \frac{\beth_N^{\widehat{\exists}}(\wp) + \beth_N^{\widehat{\exists}}(\partial)}{2}$. Similarly, $\beth_N^{\widehat{\exists}}(\wp \heartsuit_2 \partial) \geq \frac{\beth_N^{\widehat{\exists}}(\wp) + \beth_N^{\widehat{\exists}}(\partial)}{2}$ and $\beth_N^{\widehat{\exists}}(\wp \heartsuit_3 \partial) \geq \frac{\beth_N^{\widehat{\exists}}(\wp) + \beth_N^{\widehat{\exists}}(\partial)}{2}$. Now, $\beth_N^-(\wp \heartsuit_1 \partial) = \beth_Y^-(\Re(\wp \heartsuit_1 \partial)) = \beth_Y^-(\Re(\wp) \diamondsuit_1 \Re(\partial)) \leq \max\{\beth_Y^-(\Re(\wp)), \beth_Y^-(\Re(\partial))\} = \max\{\beth_N^-(\wp), \beth_N^-(\partial)\}$. Thus $\beth_N^-(\wp \heartsuit_1 \partial) \leq \max\{\beth_N^-(\wp), \beth_N^-(\partial)\}$ and $1 - \beth_N^-(\wp \heartsuit_1 \partial) = 1 - \beth_Y^-(\Re(\wp \heartsuit_1 \partial)) = 1 - \beth_Y^-(\Re(\wp) \diamondsuit_1 \Re(\partial)) = 1 - \beth_Y^-(\Re(\wp)) \leq \max\{1 - \beth_Y^-(\Re(\wp)), 1 - \beth_Y^-(\Re(\partial))\} = \max\{1 - \beth_N^-(\wp), 1 - \beth_N^-(\partial)\}$. Thus $1 - \beth_N^-(\wp \heartsuit_1 \partial) \leq \max\{1 - \beth_N^-(\wp), 1 - \beth_N^-(\partial)\}$. Hence, $\beth_N^-(\wp \heartsuit_1 \partial) \leq \max\{\beth_Y^-(\wp), \beth_Y^-(\partial)\}$. Similarly, $\beth_N^-(\wp \heartsuit_2 \partial) \leq \max\{\beth_Y^-(\wp), \beth_Y^-(\partial)\}$ and $\beth_N^-(\wp \heartsuit_3 \partial) \leq \max\{\beth_Y^-(\wp), \beth_Y^-(\partial)\}$. Hence, N is a IVNCVSBS of Ξ_1 .

Theorem 3.15. If $\Re : \Xi_1 \rightarrow \Xi_2$ is a homomorphism, then $\Re(N_{(\ell_1, \ell_2, s)})$ is a level SBS of IVNCVSBS Υ of Ξ_2 .

Proof. Let $\Re : \Xi_1 \rightarrow \Xi_2$ be a homomorphism and $\Re(\wp \heartsuit_1 \partial) = \Re(\wp) \diamondsuit_1 \Re(\partial)$, $\Re(\wp \heartsuit_2 \partial) = \Re(\wp) \diamondsuit_2 \Re(\partial)$ and $\Re(\wp \heartsuit_3 \partial) = \Re(\wp) \diamondsuit_3 \Re(\partial)$ for all $\wp, \partial \in \Xi_1$. Let $V = \Re(N)$, N is a IVNCVSBS of Ξ_1 . By Theorem 3.13, Υ is a IVNCVSBS of Ξ_2 . Let $N_{(\ell_1, \ell_2, s)}$ be any level SBS of N . Suppose that $\wp, \partial \in N_{(\ell_1, \ell_2, s)}$. Then $\Re(\wp \heartsuit_1 \partial), \Re(\wp \heartsuit_2 \partial)$ and $\Re(\wp \heartsuit_3 \partial) \in N_{(\ell_1, \ell_2, s)}$. Now, $\widehat{\beth}_Y^-(\Re(\wp)) = \widehat{\beth}_N^-(\wp) \geq \ell_1$, $\widehat{\beth}_Y^-(\Re(\partial)) = \widehat{\beth}_N^-(\partial) \geq \ell_1$. Thus $\widehat{\beth}_Y^-(\Re(\wp) \diamondsuit_1 \Re(\partial)) \geq \widehat{\beth}_N^-(\wp \heartsuit_1 \partial) \geq \ell_1$ and $1 - \widehat{\beth}_Y^-(\Re(\wp)) = 1 - \widehat{\beth}_N^-(\wp) \geq s$, $1 - \widehat{\beth}_Y^-(\Re(\partial)) = 1 - \widehat{\beth}_N^-(\partial) \geq s$. Thus $1 - \widehat{\beth}_Y^-(\Re(\wp) \diamondsuit_1 \Re(\partial)) \geq 1 - \widehat{\beth}_N^-(\wp \heartsuit_1 \partial) \geq s$. Now, $\widehat{\beth}_Y^-(\Re(\wp)) = \widehat{\beth}_N^-(\wp) \geq \ell_2$, $\widehat{\beth}_Y^-(\Re(\partial)) = \widehat{\beth}_N^-(\partial) \geq \ell_2$. Thus $\widehat{\beth}_Y^-(\Re(\wp) \diamondsuit_1 \Re(\partial)) \geq \widehat{\beth}_N^-(\wp \heartsuit_1 \partial) \geq \ell_2$ and $\widehat{\beth}_Y^+(\Re(\wp)) = \widehat{\beth}_N^+(\wp) \geq \ell_2$, $\widehat{\beth}_Y^+(\Re(\partial)) = \widehat{\beth}_N^+(\partial) \geq \ell_2$. Thus $\widehat{\beth}_Y^+(\Re(\wp) \diamondsuit_1 \Re(\partial)) \geq \widehat{\beth}_N^+(\wp \heartsuit_1 \partial) \geq \ell_2$. Now, $\widehat{\beth}_Y^-(\Re(\wp)) = \widehat{\beth}_N^-(\wp) \leq s$, $\widehat{\beth}_Y^-(\Re(\partial)) = \widehat{\beth}_N^-(\partial) \leq s$. Thus $\widehat{\beth}_Y^-(\Re(\wp) \diamondsuit_1 \Re(\partial)) \leq \widehat{\beth}_N^-(\wp \heartsuit_1 \partial) \leq s$ and $1 - \widehat{\beth}_Y^-(\Re(\wp)) = 1 - \widehat{\beth}_N^-(\wp) \leq \ell_1$, $1 - \widehat{\beth}_Y^-(\Re(\partial)) = 1 - \widehat{\beth}_N^-(\partial) \leq \ell_1$. Thus $1 - \widehat{\beth}_Y^-(\Re(\wp) \diamondsuit_1 \Re(\partial)) \leq 1 - \widehat{\beth}_N^-(\wp \heartsuit_1 \partial) \leq \ell_1$. Thus $1 - \widehat{\beth}_Y^-(\Re(\wp) \diamondsuit_1 \Re(\partial)) \leq \ell_1$, for all $\Re(\wp), \Re(\partial) \in \Xi_2$. Now, $\widehat{\beth}_Y^-(\Re(\wp)) = \widehat{\beth}_N^-(\wp) \geq \ell_1$, $\widehat{\beth}_Y^-(\Re(\partial)) = \widehat{\beth}_N^-(\partial) \geq \ell_1$. Thus $\widehat{\beth}_Y^-(\Re(\wp) \diamondsuit_1 \Re(\partial)) \geq \widehat{\beth}_N^-(\wp \heartsuit_1 \partial) \geq \ell_1$ and $1 - \widehat{\beth}_Y^-(\Re(\wp)) = 1 - \widehat{\beth}_N^-(\wp) \geq s$, $1 - \widehat{\beth}_Y^-(\Re(\partial)) = 1 - \widehat{\beth}_N^-(\partial) \geq s$. Thus $1 - \widehat{\beth}_Y^-(\Re(\wp) \diamondsuit_1 \Re(\partial)) \geq 1 - \widehat{\beth}_N^-(\wp \heartsuit_1 \partial) \geq s$. Now, $\widehat{\beth}_Y^-(\Re(\wp)) = \widehat{\beth}_N^-(\wp) \geq \ell_2$, $\widehat{\beth}_Y^-(\Re(\partial)) = \widehat{\beth}_N^-(\partial) \geq \ell_2$. Thus $\widehat{\beth}_Y^-(\Re(\wp) \diamondsuit_1 \Re(\partial)) \geq \widehat{\beth}_N^-(\wp \heartsuit_1 \partial) \geq \ell_2$ and $\widehat{\beth}_Y^+(\Re(\wp)) = \widehat{\beth}_N^+(\wp) \geq \ell_2$, $\widehat{\beth}_Y^+(\Re(\partial)) = \widehat{\beth}_N^+(\partial) \geq \ell_2$. Thus $\widehat{\beth}_Y^+(\Re(\wp) \diamondsuit_1 \Re(\partial)) \geq \widehat{\beth}_N^+(\wp \heartsuit_1 \partial) \geq \ell_2$. Now, $\widehat{\beth}_Y^-(\Re(\wp)) = \widehat{\beth}_N^-(\wp) \leq s$, $\widehat{\beth}_Y^-(\Re(\partial)) = \widehat{\beth}_N^-(\partial) \leq s$. Thus $\widehat{\beth}_Y^-(\Re(\wp) \diamondsuit_1 \Re(\partial)) \leq \widehat{\beth}_N^-(\wp \heartsuit_1 \partial) \leq s$ and $1 - \widehat{\beth}_Y^-(\Re(\wp)) = 1 - \widehat{\beth}_N^-(\wp) \leq \ell_1$, $1 - \widehat{\beth}_Y^-(\Re(\partial)) = 1 - \widehat{\beth}_N^-(\partial) \leq \ell_1$. Thus $1 - \widehat{\beth}_Y^-(\Re(\wp) \diamondsuit_1 \Re(\partial)) \leq 1 - \widehat{\beth}_N^-(\wp \heartsuit_1 \partial) \leq \ell_1$, for all $\Re(\wp), \Re(\partial) \in \Xi_2$. Similarly to prove other operations. Hence proved.

Theorem 3.16. If $\Re : \Xi_1 \rightarrow \Xi_2$ is any homomorphism, then $N_{(\ell_1, \ell_2, s)}$ is a level SBS of IVNCVSBS N of Ξ_1 .

Proof. Let $\Re : \Xi_1 \rightarrow \Xi_2$ be a homomorphism and $\Re(\wp \heartsuit_1 \partial) = \Re(\wp) \diamondsuit_1 \Re(\partial)$, $\Re(\wp \heartsuit_2 \partial) = \Re(\wp) \diamondsuit_2 \Re(\partial)$ and $\Re(\wp \heartsuit_3 \partial) = \Re(\wp) \diamondsuit_3 \Re(\partial)$ for all $\wp, \partial \in \Xi_1$. Let $V = \Re(N)$, Υ is a IVNCVSBS of Ξ_2 . By Theorem 3.14, N is a IVNCVSBS of Ξ_1 . Let $N_{(\ell_1, \ell_2, s)}$ be a level SBS of N . Suppose that $\Re(\wp), \Re(\partial) \in N_{(\ell_1, \ell_2, s)}$. Then $\Re(\wp \heartsuit_1 \partial), \Re(\wp \heartsuit_2 \partial)$ and $\Re(\wp \heartsuit_3 \partial) \in N_{(\ell_1, \ell_2, s)}$. Now, $\widehat{\beth}_N^-(\wp) = \widehat{\beth}_Y^-(\Re(\wp)) \geq \ell_1$, $\widehat{\beth}_N^-(\partial) = \widehat{\beth}_Y^-(\Re(\partial)) \geq \ell_1$. Thus $\widehat{\beth}_N^-(\Re(\wp) \diamondsuit_1 \Re(\partial)) \geq \widehat{\beth}_Y^-(\wp \heartsuit_1 \partial) \geq \ell_1$ and $1 - \widehat{\beth}_N^-(\wp) = 1 - \widehat{\beth}_Y^-(\wp) \geq s$, $1 - \widehat{\beth}_N^-(\partial) = 1 - \widehat{\beth}_Y^-(\Re(\partial)) \geq s$. Thus $1 - \widehat{\beth}_N^-(\Re(\wp) \diamondsuit_1 \Re(\partial)) \geq 1 - \widehat{\beth}_Y^-(\wp \heartsuit_1 \partial) \geq s$. Now, $\widehat{\beth}_N^-(\Re(\wp)) = \widehat{\beth}_Y^-(\Re(\wp)) \geq \ell_2$, $\widehat{\beth}_N^-(\Re(\partial)) = \widehat{\beth}_Y^-(\Re(\partial)) \geq \ell_2$. Thus $\widehat{\beth}_N^-(\Re(\wp) \diamondsuit_1 \Re(\partial)) \geq \widehat{\beth}_Y^-(\Re(\wp) \diamondsuit_1 \Re(\partial)) \geq \ell_2$ and $\widehat{\beth}_N^+(\Re(\wp)) = \widehat{\beth}_Y^+(\Re(\wp)) \geq \ell_2$, $\widehat{\beth}_N^+(\Re(\partial)) = \widehat{\beth}_Y^+(\Re(\partial)) \geq \ell_2$. Thus $\widehat{\beth}_N^+(\Re(\wp) \diamondsuit_1 \Re(\partial)) \geq \widehat{\beth}_Y^+(\Re(\wp) \diamondsuit_1 \Re(\partial)) \geq \ell_2$. Now, $\widehat{\beth}_N^-(\Re(\wp)) = \widehat{\beth}_Y^-(\Re(\wp)) \leq s$, $\widehat{\beth}_N^-(\Re(\partial)) = \widehat{\beth}_Y^-(\Re(\partial)) \leq s$. Thus $\widehat{\beth}_N^-(\Re(\wp) \diamondsuit_1 \Re(\partial)) \leq \widehat{\beth}_Y^-(\Re(\wp) \diamondsuit_1 \Re(\partial)) \leq s$ and $1 - \widehat{\beth}_N^-(\Re(\wp)) = 1 - \widehat{\beth}_Y^-(\Re(\wp)) \leq \ell_1$, $1 - \widehat{\beth}_N^-(\Re(\partial)) = 1 - \widehat{\beth}_Y^-(\Re(\partial)) \leq \ell_1$. Thus $1 - \widehat{\beth}_N^-(\Re(\wp) \diamondsuit_1 \Re(\partial)) \leq 1 - \widehat{\beth}_Y^-(\Re(\wp) \diamondsuit_1 \Re(\partial)) \leq \ell_1$, for all $\Re(\wp), \Re(\partial) \in \Xi_2$. Similarly to prove other operations. Hence proved.

Now, $\neg_{\mathcal{N}}^-(\varphi) = \neg_{\mathcal{T}}^-(\mathcal{R}(\varphi)) \geq \ell_1$, $\neg_{\mathcal{N}}^-(\partial) = \neg_{\mathcal{T}}^-(\mathcal{R}(\partial)) \geq \ell_1$. Thus $\neg_{\mathcal{N}}^-(\varphi \diamond_1 \partial) \geq \min\{\neg_{\mathcal{N}}^-(\varphi), \neg_{\mathcal{N}}^-(\partial)\} \geq \ell_1$ and $1 - \exists_{\mathcal{N}}^-(\varphi) = 1 - \exists_{\mathcal{T}}^-(\mathcal{R}(\varphi)) \geq s$, $1 - \exists_{\mathcal{N}}^-(\partial) = 1 - \exists_{\mathcal{T}}^-(\mathcal{R}(\partial)) \geq s$. Thus $1 - \exists_{\mathcal{N}}^-(\varphi \diamond_1 \partial) \geq \min\{1 - \exists_{\mathcal{N}}^-(\varphi), 1 - \exists_{\mathcal{N}}^-(\partial)\} \geq s$. Now, $\exists_{\mathcal{N}}^-(\varphi) = \exists_{\mathcal{T}}^-(\mathcal{R}(\varphi)) \geq \ell_2$, $\exists_{\mathcal{N}}^-(\partial) = \exists_{\mathcal{T}}^-(\mathcal{R}(\partial)) \geq \ell_2$. Thus $\exists_{\mathcal{N}}^-(\varphi \diamond_1 \partial) \geq \frac{\exists_{\mathcal{N}}^-(\varphi) + \exists_{\mathcal{N}}^-(\partial)}{2} \geq \ell_2$ and $\exists_{\mathcal{N}}^+(\varphi) = \exists_{\mathcal{T}}^+(\mathcal{R}(\varphi)) \geq \ell_2$, $\exists_{\mathcal{N}}^+(\partial) = \exists_{\mathcal{T}}^+(\mathcal{R}(\partial)) \geq \ell_2$. Thus $\exists_{\mathcal{N}}^+(\varphi \diamond_1 \partial) \geq \frac{\exists_{\mathcal{N}}^+(\varphi) + \exists_{\mathcal{N}}^+(\partial)}{2} \geq \ell_2$. Now, $\exists_{\mathcal{N}}^-(\varphi) = \exists_{\mathcal{T}}^-(\mathcal{R}(\varphi)) \leq s$, $\exists_{\mathcal{N}}^-(\partial) = \exists_{\mathcal{T}}^-(\mathcal{R}(\partial)) \leq s$. Thus $\exists_{\mathcal{N}}^-(\varphi \diamond_1 \partial) = \exists_{\mathcal{T}}^-(\mathcal{R}(\varphi) \diamond_1 \mathcal{R}(\partial)) \leq \max\{\exists_{\mathcal{N}}^-(\varphi), \exists_{\mathcal{N}}^-(\partial)\} \leq s$ and $1 - \neg_{\mathcal{N}}^-(\varphi) = 1 - \neg_{\mathcal{T}}^-(\mathcal{R}(\varphi)) \leq \ell_1$, $1 - \neg_{\mathcal{N}}^-(\partial) = 1 - \neg_{\mathcal{T}}^-(\mathcal{R}(\partial)) \leq \ell_1$. Thus $1 - \neg_{\mathcal{N}}^-(\varphi \diamond_1 \partial) = 1 - \neg_{\mathcal{T}}^-(\mathcal{R}(\varphi) \diamond_1 \mathcal{R}(\partial)) \leq \max\{1 - \neg_{\mathcal{N}}^-(\varphi), 1 - \neg_{\mathcal{N}}^-(\partial)\} \leq \ell_1$, for all $\varphi, \partial \in \Xi_1$. Similarly to prove other two operations. Hence proved.

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