MBJ-Neutrosophic WI Ideals in Lattice Wajsberg Algebra

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Abstract

In this study, we introduce the concepts of MBJ-Neutrosophic WI-ideal and MBJ-Neutrosophic lattice ideal of lattice Wajsberg algebras. We demonstrate that every MBJ-Neutrosophic WI-ideal of lattice Wajsberg algebra is an MBJ-Neutrosophic lattice ideal of lattice Wajsberg algebra. Additionally, we talk about its opposite. Furthermore, we discover that in lattice H-Wajsberg algebra, every MBJ-Neutrosophic lattice ideal is an MBJ-Neutrosophic WI-ideal.

Keywords: Wajsberg algebra (WA); Lattice Wajsberg algebra(LWA); WI-ideal; MBJ-Neutrosophic WI-ideal; MBJ-Neutrosophic lattice ideal.

1. Introduction

Different kinds of uncertainties are faced in a wide variety of real-world circumstances and in many complex systems, including biological, behavioral, and chemical ones. The fuzzy set was first presented by L.A. Zadeh [4] in 1965 to handle uncertainties in various practical applications, and K. Atanassov introduced the intuitionistic fuzzy set on a universe X in 1983 as a generalization of the fuzzy set. By extending the concepts of classic set, (intuitionistic) fuzzy set, and interval valued (intuitionistic) fuzzy set, Smarandache ([14], [15], and [16]) created the concept of neutrosophic set. The membership functions for truth, falsehood, and indeterminacy in the neutrosophic set are fuzzy sets. We use the interval valued fuzzy set as a basis for thinking about a generalization of neutrosophic set.

The idea of Wajsberg algebra was first put out by Mordchaj Wajsberg [1] in 1935.Lattice valued logic is developing as a study area that has a significant impact on the advancement of algebraic logic, computer science, and artificial intelligence technologies. In 1984, Font etal.[3] proposed Wajsberg algebra's lattice structure, examined its features, and expanded Wajsberg algebra as a substitute model for the infinite-valued Lukasiewicz logic.

Lattice Wajsberg algebras are an algebraic structure that are created by combining a lattice with a Wajsberg algebra. Font, Rodriguez, and Torrens [3] presented the idea of lattice Wajsberg algebras in 1984 and analyzed some of its features. Filter theory is crucial for the overall advancement of lattice Wajsberg algebras. In a lattice Wajsberg algebra, they presented the idea of implicative filters and looked into their characteristics. The concepts of fuzzy implicative and anti-fuzzy implicative filters of lattice Wajsberg algebras were proposed by Basheer Ahamed and Ibrahim [7,18], who also established several properties using examples. Another significant advancement in lattice Wajsberg algebras is the theory of ideals. The Wajsberg implicative ideal (WI-ideal) of lattice Wajsberg algebra was presented by the authors [8], who also deduced various features.

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In this paper, we introduce the notions of MBJ-Neutrosophic WI-ideal and MBJ-Neutrosophic lattice ideal of lattice Wajsberg algebras. We show that every MBJ-Neutrosophic WI-ideal of lattice Wajsberg algebra is a MBJ-Neutrosophic latttice ideal of lattice Wajsberg algebra. Also, we discuss its converse part. Further, we obtain every MBJ-Neutrosophic lattice ideal is an MBJ-Neutrosophic WI-ideal in lattice H-Wajsberg algebra.

2. Preliminaries

2.1. Definition [3]: Let (L, →,*, 1) be an algebra with an unary operation *, and a binary operation → is called a Wajsberg algebra (W-algebra) if and only if it satisfies the following axioms for all x, y, z ∈ L

(W1) $1 \to x = x$

(W2) $x \to y \to y \to z \to x \to z = 1$

(W3) $x \to y \to y = (y \to x) \to x$

(W4) $(x^* \to y^*) \to y \to x = 1$

2.2. Proposition [3]: The W-algebra (L, →, *, 1) satisfies the following equations and implications for all x, y, z ∈ L

(1) $x \to x = 1$

(2) If $x \to y = y \to x = 1$, then $x = y$

(3) $x \to 1 = 1$

(4) $x \to y \to x = 1$

(5) If $x \to y = y \to z = 1$, then $x \to z = 1$

(6) $x \to y \to z \to x \to z \to y = 1$

(7) $x \to y \to z = y \to (x \to z)$

(8) $x \to 0 = x \to 1^* = x^*$

(9) $(x^*)^* = x$

(10) $x^* \to y^* = y \to x$

(11) If $x \leq y$ then $y \to z \leq x \to z$

(12) $(x \vee y)^* = (x^* \wedge y^*)$

(13) $(x \wedge y)^* = (x^* \vee y^*)$

(14) $(x \vee y) \to z = (x \to z) \wedge (y \to z)$

(15) $x \to (y \wedge z) = (x \to y) \wedge (x \to z)$

(16) $(x \to y) \vee (y \to x) = 1$

(17) $x \to (y \vee z) = (x \to y) \vee (x \to z)$

(18) $(x \wedge y) \to z = (x \to z) \vee (y \to z)$

(19) $(x \wedge y) \vee z = (x \vee z) \wedge (y \vee z)$

(20) $(x \wedge y) \to z = (x \to y) \vee (x \to z)$

(21) $x \leq y \to z$ if and only if $y \leq x \to z$

(22) If $x \leq y$ then $z \to x \leq z \to y$

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2.3. Definition [3]: The W-algebra L is called a Lattice W-algebra if it satisfies the following conditions for all x, y ∈ L,

(1) A partial ordering "≤" on L such that x ≤ y if and only if x → y = 1

(2) x∨y = (x → y) → y

(3) x ∧ y = ((x* → y*) → y*) *

Thus (L, ∨,∧ , ∗,→,0,1) is a Lattice W-algebra with lower bound 0 and an upper bound 1.

2.4. Definition [17]: Let X be a non-empty set. A MBJ-neutrosophic set of the form $A = \{\langle\varsigma; M_A(\varsigma), \widetilde{B_A}(\varsigma), J_A(\varsigma)/\varsigma \in X\}\rangle\}$ where M_A and J_A are fuzzy sets in X, which are called a truth membership function and a false membership function, respectively and $\widetilde{B_A}$ is an IVF set in X which is called an indeterminate interval valued membership function. For the sake of simplicity, we shall use the symbol $A = (M_A, \widetilde{B_A}, J_A)$ for the MBJ-Neutrosophic set $A = \{\langle\varsigma; M_A(\varsigma), \widetilde{B_A}(\varsigma), J_A(\varsigma)/\varsigma \in X\}\rangle\}$.

In an MBJ-Neutrosophic set $A = (M_A, \widetilde{B_A}, J_A)$ in X we take $\widetilde{B_A}: X \longrightarrow [I], \varsigma \longrightarrow [B_A^-(\varsigma), B_A^+(\varsigma)]$ with $B_A^-(\varsigma) = B_A^+(\varsigma)$ then $A = (M_A, \widetilde{B_A}, J_A)$ is a neutrosophic set in X.

2.5. Definition [3]. Let L be a lattice. An ideal I of L is a nonempty subset of L is called a lattice ideal, if it satisfies the following axioms for all x, y∈ I

(i) x∈I , y∈ L and y≤x imply y∈ I
(ii) x, y∈I implies x∨y∈ I

2.6. Definition [7]. Let A be a lattice Wajsberg algebra. Let I be a nonempty subset of A, then I is called WI ideal of lattice Wajsberg algebra A satisfies,

(i) 0 ∈ I
(ii) (x →y) *∈ I and y∈I imply x∈I for all x, y∈ A.

3. MBJ-Neutrosophic WI-ideals in Lattice Wajsberg algebra:

3.1. Definition: A MBJ-N set A = (M_A, $\widetilde{\mathcal{B}_A}$, J_A) in a LWA Lis called a MBJ-N WI-ideal of'L if the following attributes are true.

$$(\forall \varsigma_1 \in L)\ (M_A(0) \geq M_A(\varsigma_1), \widetilde{\mathcal{B}_A}(0) \geq \widetilde{\mathcal{B}_A}(\varsigma_1), J_A(0) \leq J_A(\varsigma_1)) \qquad (1)$$

$$\text{And } (\forall \varsigma_1, \varsigma_2 \in L)\begin{pmatrix} M_A(\varsigma_1) \geq \min\{M_A((\varsigma_1 \to \varsigma_2)'), M_A(\varsigma_2)\} \\ \widetilde{\mathcal{B}_A}(\varsigma_1) \geq \text{rmin}\{\widetilde{\mathcal{B}_A}((\varsigma_1 \to \varsigma_2)'), \widetilde{\mathcal{B}_A}(\varsigma_2)\} \\ J_A(\varsigma_1) \leq \max\{J_A((\varsigma_1 \to \varsigma_2)'), J_A(\varsigma_2)\} \end{pmatrix} \qquad (2)$$

The set of all MBJ-NWI-ideals of L is denoted by MBJ-NWI(L)

3.2. Example:

Let A = {0, a, b, c, d, r, s, t, 1} be a set with Figure (1) as a partial ordering. Define a quasi-complement " ∗ " and a binary operation " → " on A as in Table (1) and Table(2).

Figure (1)

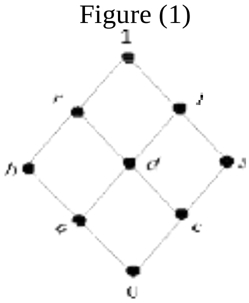

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Table 1:

ς_1	x* $\varsigma_1{}^*$
0	1
a	t
b	b
c	r
d	d
r	c
s	b
t	a
1	0

Table 2:

x	**00 0**	**A a**	**B**	**C c**	d	r	s	t	1
0	1	1	1	1	1	1	1	1	1
a	t	1	1	t	1	1	t	1	1
b	b	t	1	s	t	1	s	t	1
c	r	r	r	1	1	1	1	1	1
d	d	r	r	t	1	1	t	1	1
r	c	d	r	s	t	1	s	t	1
s	b	b	b	r	r	r	1	1	1
t	a	b	b	d	r	r	t	1	1
1	0	a	b	c	d	r	s	t	1

Define ∨ and ∧ operations on A as follows,

$(\varsigma_1 \vee \varsigma_2) = (\varsigma_1 \to \varsigma_2) \to \varsigma_2$,

$(\varsigma_1 \wedge \varsigma_2) = ((\varsigma_1{}^* \to \varsigma_2{}^*) \to \varsigma_2{}^*)^*$ for all $\varsigma_1, \varsigma_2 \in A$.

Then (A, ∨ , ∧ , *, 0, 1) is a lattice Wajsberg algebra.

Consider an MBJ-neutrosophic set $S = (M_A, \widetilde{B_A}, J_A)$ on A as

$$M_A(\varsigma_1) = \begin{cases} 1 \ if \ \varsigma_1 \in \{0, b\} for \ every \ \varsigma_1 \in A \\ 0.6 \ otherwise \ for \ all \ \varsigma_1 \in A \end{cases}$$

$$\tilde{B}_A(\varsigma_1) = \begin{cases} [0.5, 0.6] \ if \ \varsigma_1 \in \{0, b\} for \ every \ \varsigma_1 \in A \\ [0.3, 0.4] \ otherwise \ for \ all \ \varsigma_1 \in A \end{cases}$$

$$J_A(\varsigma_1) = \begin{cases} 0 \ if \ \varsigma_1 \in \{0, b\} for \ every \ \varsigma_1 \in A \\ 0.4 \ otherwise \ for \ all \ \varsigma_1 \in A \end{cases}$$

Then S is an MBJ-neutrosophic WI-ideal.

In the same Example 3.2, let us consider an MBJ-neutrosophic set $S = (M_A, \widetilde{B_A}, J_A)$ on A as

$$M_A(\varsigma_1) = \begin{cases} 1 \ if \ \varsigma_1 \in \{a, b\} for \ every \ \varsigma_1 \in A \\ 0.42 \ otherwise \ for \ all \ \varsigma_1 \in A \end{cases}$$

$$\tilde{B}_A(\varsigma_1) = \begin{cases} [0.8, 0.9] \ if \ \varsigma_1 \in \{a, b\} for \ every \ \varsigma_1 \in A \\ [0.3, 0.4] \ otherwise \ for \ all \ \varsigma_1 \in A \end{cases}$$

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$$J_A(\varsigma_1) = \begin{cases} 0 \; if \; \varsigma_1 \in \{a, b\} for \; every \; \varsigma_1 \in A \\ 0.46 \; otherwise \; for \; all \; \varsigma_1 \in A \end{cases}$$

Then S is not a MBJ-neutrosophic WI-ideal of A for

$M_A(t) < min\{M_A((t \to b)^*), M_A(b)\}$

$\widetilde{\mathcal{B}_A}(t) < rmin\{\widetilde{\mathcal{B}_A}((t \to b)^*), \widetilde{\mathcal{B}_A}(b)\}$

$J_A(t) > max\{J_A((t \to b)^*), J_A(b)\}$

3.3. Proposition:

Every MBJ-NWI-ideal $A=(M_A,\widetilde{\mathcal{B}_A},J_A)$ of L accomplish the following assertions.

$$(\forall \varsigma_1,\varsigma_2 \in L)\left(\varsigma_1 \leq \varsigma_2 \Rightarrow \begin{cases} M_A(\varsigma_1) \geq M_A(\varsigma_2) \\ \widetilde{\mathcal{B}_A}(\varsigma_1) \geq \widetilde{\mathcal{B}_A}(\varsigma_2) \\ J_A(\varsigma_1) \leq J_A(\varsigma_2) \end{cases} \right) \quad (3)$$

Proof: Let $A \in NWI(L)$ and $\varsigma_1,\varsigma_2 \in L$ such that $\varsigma_1 \leq \varsigma_2$. Since $(\varsigma_1 \to \varsigma_2)' = 0$,

We have, $M_A(\varsigma_1) \geq \min\{M_A((\varsigma_1 \to \varsigma_2)'), M_A(\varsigma_2)\} = \min\{M_A(0), M_A(\varsigma_2)\} = M_A(\varsigma_2)$,

$\widetilde{\mathcal{B}_A}(\varsigma_1) \geq \text{rmin}\{\widetilde{\mathcal{B}_A}((\varsigma_1 \to \varsigma_2)'), \widetilde{\mathcal{B}_A}(\varsigma_2)\} = \text{rmin}\{\widetilde{\mathcal{B}_A}(0), \widetilde{\mathcal{B}_A}(\varsigma_2)\} = \widetilde{\mathcal{B}_A}(\varsigma_2)$,

$J_A(\varsigma_1) \leq \max\{J_A((\varsigma_1 \to \varsigma_2)'), J_A(\varsigma_2)\} = \max\{J_A(0), J_A(\varsigma_2)\} = J_A(\varsigma_2)$.

3.4. Proposition: Every MBJ-N WI-ideal $A = (M_A,\widetilde{\mathcal{B}_A},J_A)$ of L accomplish the following assertions.

$$(\forall \varsigma_1,\varsigma_2,\varsigma_3 \in L)\left(\varsigma_1 \leq \varsigma_2' \to \varsigma_3 \Rightarrow \begin{cases} M_A(\varsigma_1) \geq \min\{M_A(\varsigma_2), \; M_A(\varsigma_3)\} \\ \widetilde{\mathcal{B}_A}(\varsigma_1) \geq \text{r min}\{\widetilde{\mathcal{B}_A}(\varsigma_2), \widetilde{\mathcal{B}_A}(\varsigma_3)\} \\ J_A(\varsigma_1) \leq \max\{J_A(\varsigma_2), \; J_A(\varsigma_3)\}) \end{cases} \right) \quad (4)$$

Proof: Let $A \in NWI(L)$ such that for all $\varsigma_1,\varsigma_2,\varsigma_3 \in L, \varsigma_1 \leq \varsigma_2' \to \varsigma_3$.

Then $1 = \varsigma_1 \to (\varsigma_2' \to \varsigma_3) = \varsigma_3' \to (\varsigma_1 \to \varsigma_2) = (\varsigma_1 \to \varsigma_2)' \to \varsigma_3$,

And so $(\varsigma_1 \to \varsigma_2)' \to \varsigma_3)' = 0$.

By(2),we get that

$M_A(\varsigma_1) \geq \min\{M_A((\varsigma_1 \to \varsigma_2)'), M_A(\varsigma_2)\}$

$\geq \min\{\min\{M_A(((\varsigma_1 \to \varsigma_2)' \to \varsigma_3)'), M_A(\varsigma_3)\}, M_A(\varsigma_2)\}$

$= \min\{\min\{M_A(0), M_A(\varsigma_3)\}, M_A(\varsigma_2)\}$

$= \min\{M_A(\varsigma_3), M_A(\varsigma_2)\}$

$\widetilde{\mathcal{B}_A}(\varsigma_1) \geq \text{r min}\{\widetilde{\mathcal{B}_A}((\varsigma_1 \to \varsigma_2)'), \widetilde{\mathcal{B}_A}(\varsigma_2)\}$

$\geq \text{rmin}\{\text{rmin}\{\widetilde{\mathcal{B}_A}(((\varsigma_1 \to \varsigma_2)' \to \varsigma_3)'), \widetilde{\mathcal{B}_A}(\varsigma_3)\}, \widetilde{\mathcal{B}_A}(\varsigma_2)\}$

$= \text{rmin}\{\text{rmin}\{\widetilde{\mathcal{B}_A}(0), \widetilde{\mathcal{B}_A}(\varsigma_3)\}, \widetilde{\mathcal{B}_A}(\varsigma_2)\}$

$= \text{rmin}\{\widetilde{\mathcal{B}_A}(\varsigma_3), \widetilde{\mathcal{B}_A}(\varsigma_2)\}$, and

$J_A(\varsigma_1) \leq \max\{J_A((\varsigma_1 \to \varsigma_2)'), J_A(\varsigma_2)\}$

$\leq \max\{\max\{J_A(((\varsigma_1 \to \varsigma_2)' \to \varsigma_3)'), J_A(\varsigma_3)\}, J_A(\varsigma_2)\}$

$= \max\{\max\{J_A(0), J_A(\varsigma_3)\}, J_A(\varsigma_2)\}$

$= \max\{J_A(\varsigma_3), J_A(\varsigma_2)\}$.

Hence the proof.

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3.5. Definition: A MBJ-Nset A=(M_A,$\widetilde{\mathcal{B}_A}$,$J_A$) in L is called a MBJ-Lattice ideal of L if it satisfies(3) and ($\forall\varsigma_1,\varsigma_2 \in L$)

$$\begin{pmatrix} M_A(\varsigma_1 \vee \varsigma_2) \geq \min\{M_A(\varsigma_1), M_A(\varsigma_2)\} \\ \widetilde{\mathcal{B}_A}(\varsigma_1 \vee \varsigma_2) \geq \mathrm{rmin}\{\widetilde{\mathcal{B}_A}(\varsigma_1), \widetilde{\mathcal{B}_A}(\varsigma_2)\} \\ J_A(\varsigma_1 \vee \varsigma_2) \leq \max\{J_A(\varsigma_1), J_A(\varsigma_2)\} \end{pmatrix} \quad (5)$$

3.6. Example: Let L be the Lattice implication algebra as in Example 3.2 and A=(M_A,$\tilde{B}_A$,J_A) be a MBJ-N set in L which is defined by

$$M_A(\varsigma_1)=\begin{cases} 1 \; if \; \varsigma_1 \in \{0,d\} for \; every \; \varsigma_1 \in A \\ 0.6 \;\; otherwise \; for \; all \; \varsigma_1 \in A \end{cases}$$

$$\widetilde{\mathcal{B}_A}(\varsigma_1)=\begin{cases} [0.5,0.6] \; if \; \varsigma_1 \in \{0,d\} for \; every \; \varsigma_1 \in A \\ [0.3,0.4] \;\; otherwise \; for \; all \; \varsigma_1 \in A \end{cases}$$

$$J_A(\varsigma_1)=\begin{cases} 0 \; if \; \varsigma_1 \in \{0,d\} for \; every \; \varsigma_1 \in A \\ 0.4 \;\; otherwise \; for \; all \; \varsigma_1 \in A \end{cases}$$

Then A= (M_A,$\widetilde{\mathcal{B}_A}$,$J_A$) is an MBJ-neutrosophic lattice ideal of A.

We discuss the relationship between a MBJ-NLI-ideal and a MBJ-N Lattice ideal.

3.7. Theorem: Every MBJ-NLI-ideal is a MBJ-N Lattice ideal.

Proof: Let A = (M_A,$\widetilde{\mathcal{B}_A}$,$J_A$)∈NLI(L). The condition (3) is valid.

Since$((\varsigma_1 \vee \varsigma_2)\to\varsigma_2)'$

$= (((\varsigma_1\to\varsigma_2)\to\varsigma_2)\to\varsigma_2)'$

$=(\varsigma_1\to\varsigma_2)'$

$\leq (\varsigma_1')'$ for allς_1, $\varsigma_2 \in L$,

by(3) and(2),we have

$M_A(\varsigma_1 \vee \varsigma_2) \geq \min\{M_A(((\varsigma_1 \vee \varsigma_2)\to\varsigma_2)'), MA(\varsigma_2)\} \geq \min\{M_A(\varsigma_1), M_A(\varsigma_2)\}$,

$\widetilde{\mathcal{B}_A}(\varsigma_1 \vee \varsigma_2) \geq \mathrm{rmin}\{\widetilde{\mathcal{B}_A}((\varsigma_1 \vee \varsigma_2)\to\varsigma_2)'), \widetilde{\mathcal{B}_A}(\varsigma_2)\} \geq \mathrm{rmin}\{\widetilde{\mathcal{B}_A}(\varsigma_1), \widetilde{\mathcal{B}_A}(\varsigma_2)\}$,

$J_A(\varsigma_1 \vee \varsigma_2) \leq \max\{J_A(((\varsigma_1 \vee \varsigma_2)\to\varsigma_2)'), JA(\varsigma_2)\} \leq \max\{J_A(\varsigma_1), J_A(\varsigma_2)\}$

Hence, A=(M_A,$\widetilde{\mathcal{B}_A}$,$J_A$) is a MBJ-N Latticeideal.

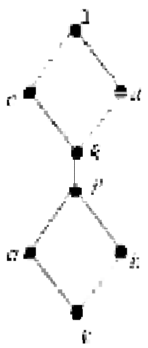
By the example given below, we observed the falsity of the converse of the Theorem 3.7.

3.8. Example:

Let A={0, a, b, p, q,c,d,1} be a set with Figure(2) as a partial ordering.Define a quasi complement " $*$ " and a binary operation " $\to$ " on A as in Table (3) and Table (4).

Figure (2)

Define ∨ and ∧ operations on A as follows



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$(\varsigma_1 \vee \varsigma_2) = (\varsigma_1 \rightarrow \varsigma_2) \rightarrow \varsigma_2$,

$(\varsigma_1 \wedge \varsigma_2) = ((\varsigma_1{}^{*} \rightarrow \varsigma_2{}^{*}) \rightarrow \varsigma_2{}^{*})^{*}$ for all $\varsigma_1, \varsigma_2 \in A$

Table 1:

ς_1	**x*** $\varsigma_1{}^{*}$
0	1
a	b
b	a
p	0
q	0
c	0
d	0
1	0

Table 2:

ς_1	**00 0**	**A a**	**b**	**C p**	q	C	d	1
0	1	1	1	1	1	1	1	1
a	b	1	b	1	1	1	1	1
b	a	a	1	1	1	1	1	1
p	0	a	b	1	1	1	1	1
q	0	a	b	p	1	1	1	1
c	0	a	b	p	d	1	d	1
d	0	a	b	p	c	C	1	1
1	0	a	b	p	q	C	d	1

Let $A=(M_A, \tilde{B}_A, J_A)$ be a MBJ-N set in L which is defined by

$$M_A(\varsigma_1)=\begin{cases} 1 \ if \ \varsigma_1 \in \{0,b,d\} for \ every \ \varsigma_1 \in A \\ \quad 0.7 \ \ otherwise \ for \ all \ \varsigma_1 \in A \end{cases}$$

$$\widetilde{\mathcal{B}_A}(\varsigma_1)=\begin{cases} [0.5,0.6] \ if \ \varsigma_1 \in \{0,b,d\} for \ every \ \varsigma_1 \in A \\ \quad [0.3,0.4] \ \ otherwise \ for \ all \ \varsigma_1 \in A \end{cases}$$

$$J_A(\varsigma_1)=\begin{cases} 0 \ if \ \varsigma_1 \in \{0,b,d\} for \ every \ \varsigma_1 \in A \\ \quad 0.3 \ \ otherwise \ for \ all \ \varsigma_1 \in A \end{cases}$$

Thus we have $A=(M_A, \tilde{B}_A, J_A)$ is a MBJ-neutrosophic lattice ideal of A, but not a MBJ-neutrosophic WI-ideal of A for

$$M_A(p) < \min\{M_A((p \rightarrow d)^*), M_A(d)\}$$

$$\widetilde{\mathcal{B}_A}(p) < \mathrm{rmin}\{\widetilde{\mathcal{B}_A}((p \rightarrow d)^*), \widetilde{\mathcal{B}_A}(d)\}$$

$$J_A(p) > \max\{J_A((p \rightarrow d)^*), J_A(d)\}$$

Now we investigate that under which condition, a MBJ-N Lattice ideal can be aMBJ-NLI-ideal.

3.9. Theorem:

In a LatticeH-implication algebra L, every MBJ-N Lattice ideal is a MBJ-NLI-ideal.

Proof: Let $A= (M_A, \widetilde{\mathcal{B}_A}, J_A)$ be a MBJ-N Lattice ideal of a Lattice H-implication algebra L.

Moreover, since $0 \leq \varsigma_1$ for all $\varsigma_1 \in L$, it follows from(1) that

$M_A(0) \geq M_A(\varsigma_1), \widetilde{\mathcal{B}_A}(0) \geq \widetilde{\mathcal{B}_A}(\varsigma_1)$ and $J_A(0) \leq J_A(\varsigma_1)$.

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Also, from $\varsigma_1 \leq \varsigma_1 \vee \varsigma_2$ for all $\varsigma_1, \varsigma_2 \in L$, by(3)and(5)we get that,

$M_A(\varsigma_1) \geq M_A(\varsigma_1 \vee \varsigma_2) = M_A(\varsigma_2 \vee (\varsigma_1' \vee \varsigma_2)')$

$= M_A(\varsigma_2 \vee (\varsigma_1 \rightarrow \varsigma_2)')$

$\geq \min\{M_A(\varsigma_2), M_A((\varsigma_1 \rightarrow \varsigma_2)')\}$,

$\widetilde{\mathcal{B}_A}(\varsigma_1) \geq \widetilde{\mathcal{B}_A}(\varsigma_1 \vee \varsigma_2) = \widetilde{\mathcal{B}_A}(\varsigma_2 \vee (\varsigma_1' \vee \varsigma_2)')$

$= \widetilde{\mathcal{B}_A}(\varsigma_2 \vee (\varsigma_1 \rightarrow \varsigma_2)')$

$\geq r\min\{\widetilde{\mathcal{B}_A}(\varsigma_2), \widetilde{\mathcal{B}_A}((\varsigma_1 \rightarrow \varsigma_2)')\}$,

And $J_A(\varsigma_1) \leq J_A(\varsigma_1 \vee \varsigma_2) = J_A(\varsigma_2 \vee (\varsigma_1' \vee \varsigma_2)'$

$= J_A(\varsigma_2 \vee (\varsigma_1 \rightarrow \varsigma_2)')$

$\leq \max\{J_A(\varsigma_2), J_A((\varsigma_1 \rightarrow \varsigma_2)')\}$.

Therefore, $A = (M_A, \widetilde{\mathcal{B}_A}, J_A) \in$ MBJ-NLI(L).

4. Conclusion:

In this paper, we have introduced the definitions of MBJ-neutrosophic WI-ideal and MBJ-neutrosophic lattice ideal of lattice Wajsberg algebra. We have discussed some of their properties with illustrations. Also, we have shown that every MBJ-neutrosophic WI-ideal of lattice Wajsberg algebra is an MBJ-neutrosophic lattice ideal of lattice Wajsberg algebra. But the converse part is true only in the lattice H-Wajsberg algebras. We hope that more links of logics emerge by the stipulating of this work.

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