Linguistic Neutrosophic Semi-Connectedness and Semi-Compactness

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Abstract

The notions of semi-connectedness and semi-compactness in linguistic neutrosophic topological space are presented and some of their properties are discussed in this study. Further, the idea of linguistic neutrosophic semi-compact space is instigated and investigated.

Keywords: Linguistic neutrosophic semi-connectedness; Linguistic neutrosophic extremely disconnected; Linguistic neutrosophic super semi-connected,Linguistic neutrosophic semi-compactness; Linguistic neutrosophic semi-compact space.

1. Introduction

The compactness property extends to topological spaces the property of closed and bounded subsets of the real line. It is useful and fundamental to think of compactness and connectedness in the context of not only general topology, but also other advanced branches of mathematics. Various stronger and weaker forms of compactness and connectedness have been explored by many researchers. In 1963, Norman Levine[10] initially talked about semi-open sets and semi-continuity concepts. Dorsett[4] introduced and researched the concept of semi-compact spaces. Chang[2] introduced fuzzy topological spaces. Coker[3] defined a hybrid topological space by utilizing intuitionistic fuzzy sets[1].

Neutrosophic set is developed by Smarandache[13] by combining indeterminacy membership functions with truth and falsity memberships. Further neutrosophic topological space has been found by Salama and Alblowi[12]. Meanwhile, Gayathri and Helen[7] instigated the notion linguistic neutrosophic topological spaces. In this article, the idea of compactness and connectedness are studied and some characterization of the above are interpreted in linguistic neutrosophic topological space.

The article comprises of four sections of which the first section deals with the introduction part and the second section contains the basic ideas and definitions. Linguistic neutrosophic semi-connectedness and some of its properties are studied in chapter three. Final section concern with the concepts linguistic neutrosophic semi-compactness and linguistic neutrosophic semi-compact space in linguistic neutrosophic topological space.

2. Preliminaries

Definition 2.1[13]: Let S be a space of points (objects), with a generic element in x denoted by S. A neutrosophic set A in S is characterized by a truth-membership function T_A, an indeterminacy

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membership function I_A and a falsity-membership function F_A. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$. That is $T_A: S \to]0^-, 1^+[$, $I_A: S \to]0^-, 1^+[$, $F_A: S \to]0^-, 1^+[$

There is no restriction on the sum of $T_A(x), I_A(x)$ and $F_A(x)$, so $0^- \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+$.

Definition 2.2[13]: Let S be a space of points (objects), with a generic element in x denoted by S. A single valued neutrosophic set (SVNS) A in S is characterized by truth-membership function T_A, indeterminacy-membership function I_A and falsity-membership function F_A. For each point s in S, $T_A(x), I_A(x), F_A(x) \in [0,1]$.

When S is continuous, a SVNS A can be written as $A = \int \langle T(x), I(x), F(x)\rangle / x \in S$.

When S is discrete, a SVNS A can be written as $A = \sum \langle T(x_i), I(x_i), F(x_i)\rangle / x_i \in S$.

Definition 2.3[8]: Let $Q = \{s_0, s_1, s_2, \ldots, s_t\}$ be a linguistic term set (LTS) with odd cardinality $t + 1$ and $\underline{Q} = \{s_h / s_0 \le s_h \le s_t, h \in [0, t]\}$. Then, a linguistic single valued neutrosophic set A is defined by, $A = \{\langle x, s_\theta(x), s_\psi(x), s_\sigma(x)\rangle | x \in S\}$, where $s_\theta(x), s_\psi(x), s_\sigma(x) \in \underline{Q}$ represent the linguistic truth, linguistic indeterminacy and linguistic falsity degrees of S to A, respectively, with condition $0 \le \theta + \psi + \sigma \le 3t$. This triplet $(s_\theta, s_\psi, s_\sigma)$ is called a linguistic single valued neutrosophic number.

Definition 2.4[7]: For a linguistic neutrosophic topology τ_{LN}, the collection of linguistic neutrosophic sets should obey,

1. $0_{LN}, 1_{LN} \in \tau_{LN}$
2. $K_1 \cap K_2 \in \tau_{LN}$ for any $K_1, K_2 \in \tau_{LN}$
3. $\cup K_i \in \tau_{LN}, \forall \{K_i : i \in J\} \subseteq \tau_{LN}$

We call, the pair (S_{LN}, τ_{LN}), a linguistic neutrosophic topological space.

3. Linguistic Neutrosophic Semi-Connectedness

Definition 3.1: Let A_{LN} be a linguistic neutrosophic set of (S_{LN}, τ_{LN}). Then A_{LN} is said to be

1. linguistic neutrosophic semi-open set if there exists a linguistic neutrosophic open set $B_{LN} \in \tau_{LN}$ such that $B_{LN} \subseteq A_{LN} \subseteq LNCl(B_{LN})$.

2. linguistic neutrosophic semi-closed set if there exists a linguistic neutrosophic closed set $B_{LN} \in \tau_{LN}$ such that $LNInt(B_{LN}) \subseteq A_{LN} \subseteq B_{LN}$.

Definition 3.2: Let (S_{LN}, τ_{LN}) be a linguistic neutrosophic topological space. A linguistic neutrosophic semi separation on S_{LN} is a pair E_{LN} and F_{LN} of non void linguistic neutrosophic semi open sets such that $S_{LN} = E_{LN} \cup F_{LN}$, where $E_{LN} \cap F_{LN} = \phi$.

Definition 3.3: A linguistic neutrosophic topological space (S_{LN}, τ_{LN}) is said to be a linguistic neutrosophic semi connected space if there exists no linguistic neutrosophic semi separations in S_{LN}. Suppose if S_{LN} has such linguistic neutrosophic semi separation, then (S_{LN}, τ_{LN}) is a linguistic neutrosophic semi disconnected space.

Theorem 3.4: Let A_{LN} and B_{LN} be linguistic neutrosophic semi separations in a linguistic neutrosophic topological space (S_{LN}, τ_{LN}). If K_{LN} is a linguistic neutrosophic semi connected subspace of S_{LN}, then one has either $K_{LN} \subseteq A_{LN}$ or $K_{LN} \subseteq B_{LN}$.

Proof:

As A_{LN} and B_{LN} are linguistic neutrosophic semi open sets, we have $K_{LN} \cap A_{LN}$ and $K_{LN} \cap B_{LN}$ are also linguistic neutrosophic semi open sets. Thus, $K_{LN} \cap A_{LN}$ and $K_{LN} \cap B_{LN}$ are linguistic neutrosophic semi separations of K_{LN}, which is a contradiction. Therefore, either $K_{LN} \cap A_{LN}$ or $K_{LN} \cap B_{LN}$ is an empty set and hence we have either $K_{LN} \subseteq A_{LN}$ or $K_{LN} \subseteq B_{LN}$.

Theorem 3.5: Let K_{LN} is a linguistic neutrosophic semi connected subspace of a linguistic neutrosophic topological space (S_{LN}, τ_{LN}). If $K_{LN} \subseteq M_{LN} \subseteq LNSCl(K_{LN})$, then M_{LN} is linguistic neutrosophic semi connected.

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Proof:

Suppose M_{LN} is not linguistic neutrosophic semi connected, then there exists non void linguistic neutrosophic semi open sets E_{LN} and F_{LN} such that these sets form a linguistic neutrosophic semi separation of M_{LN}. Then, we have either $K_{LN} \subseteq E_{LN}$ or $K_{LN} \subseteq F_{LN}$. Suppose $K_{LN} \subseteq E_{LN}$, then $M_{LN} \subseteq LNSCl(K_{LN}) \subseteq LNSCl(E_{LN})$. So, $M_{LN} \cap B_{LN} \subseteq LNSCl(E_{LN}) \cap B_{LN} = E_{LN} \cap F_{LN} = \phi$. Hence, $E_{LN} \subseteq M_{LN} = \phi$, a contradiction. Thus, M_{LN} is linguistic neutrosophic semi connected.

Theorem 3.6: A linguistic neutrosophic topological space (S_{LN}, τ_{LN}) is linguistic neutrosophic semi connected if and only if both linguistic neutrosophic semi open and linguistic neutrosophic semi closed sets are only ϕ and S_{LN}.

Proof:

Necessity Part: Let the linguistic neutrosophic topological space (S_{LN}, τ_{LN}) be linguistic neutrosophic semi connected. Suppose if K_{LN} is both linguistic neutrosophic semi open and linguistic neutrosophic semi closed set in (S_{LN}, τ_{LN}) that is different from ϕ and S_{LN}. Then, $(K_{LN})^c$ is also a linguistic neutrosophic semi open set. Thus,K_{LN} and $(K_{LN})^c$ forms a linguistic neutrosophic semi separation of S_{LN}, which is a contradiction. Therefore, both linguistic neutrosophic semi open and linguistic neutrosophic semi closed sets are only ϕ and S_{LN}.

Sufficiency Part: Let K_{LN} and J_{LN} be a linguistic neutrosophic semi separation of S_{LN} and $K_{LN} \neq S_{LN}$. Since $S_{LN} = K_{LN} \cup J_{LN}, K_{LN} = (J_{LN})^c$. This shows that K_{LN} is both linguistic neutrosophic semi open and linguistic neutrosophic semi closed set in S_{LN} that is different from ϕ and S_{LN}, which is a contradiction. So, (S_{LN}, τ_{LN}) is linguistic neutrosophic semi connected.

Theorem 3.7: Let (S_{LN}, τ_{LN}) and (T_{LN}, η_{LN}) be two linguistic neutrosophic topological spaces. And $f_{LN}: (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ be a linguistic neutrosophic semi continuous function. Then if (S_{LN}, τ_{LN}) is linguistic neutrosophic semi connected then, (T_{LN}, η_{LN}) is linguistic neutrosophic semi connected.

Proof:

Suppose (T_{LN}, η_{LN}) is not linguistic neutrosophic semi connected, then there exists a linguistic neutrosophic semi separation K_{LN} and J_{LN} of (T_{LN}, η_{LN}) . So, $S_{LN} = (f_{LN})^{-1}(K_{LN} \cap J_{LN}) = (f_{LN})^{-1}(K_{LN}) \cap (f_{LN})^{-1}(J_{LN})$. And $(f_{LN})^{-1}(K_{LN}) \cap (f_{LN})^{-1}(J_{LN}) = \phi$.

Clearly, K_{LN} and J_{LN} are different from ϕ, and so $(f_{LN})^{-1}(K_{LN})$ and $(f_{LN})^{-1}(J_{LN})$ forms a linguistic neutrosophic semi separation of S_{LN}, which is a contradiction. Therefore, (T_{LN}, η_{LN}) is linguistic neutrosophic semi-connected.

Theorem 3.8: Let (S_{LN}, τ_{LN}) and (T_{LN}, η_{LN}) be two linguistic neutrosophic topological spaces. And $f_{LN}: (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ be a linguistic neutrosophic semi-irresolute and onto function and if (S_{LN}, τ_{LN}) is linguistic neutrosophic semi-connected space then, (T_{LN}, η_{LN}) is linguistic neutrosophic semi-connected.

Proof:

Suppose that (T_{LN}, η_{LN}) is not linguistic neutrosophic semi-connected, then there lies a proper linguistic neutrosophic clopen set B_{LN} in (T_{LN}, η_{LN}). As the map f_{LN} is linguistic neutrosophic semi-irresolute, $(f_{LN})^{-1}(B_{LN}) \in LNSO(S_{LN}, \tau_{LN})$ and $(f_{LN})^{-1}(B_{LN}) \in LNSC(S_{LN}, \tau_{LN})$, which is contrariety.

Definition 3.9: A linguistic neutrosophic semi-open set in a linguistic neutrosophic topological space (S_{LN}, τ_{LN}) is linguistic neutrosophic regular semi-open set if $K_{LN} = LNSInt(LNSCl(K_{LN}))$. The complement of a linguistic neutrosophic regular semi-open set is a linguistic neutrosophic regular semi-closed set.

Definition 3.10: A linguistic neutrosophic topological space (S_{LN}, τ_{LN}) is linguistic neutrosophic super semi-connected space if there lies no proper linguistic neutrosophic regular semi-open set in (S_{LN}, τ_{LN}).

Theorem 3.11: Let (S_{LN}, τ_{LN}) be a linguistic neutrosophic topological space, then the results are equivalent.

1. (S_{LN}, τ_{LN}) is a linguistic neutrosophic super semi-connected space.

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2. for every non-zero linguistic neutrosophic regular semi-open set $B_{LN}, LNSCl(B_{LN}) = 1_{LN}$.
3. for every non-zero linguistic neutrosophic regular semi-closed set $A_{LN} \neq 1_{LN}, \quad LNSInt(A_{LN}) = 0_{LN}$.
4. there lies no linguistic neutrosophic regular semi-open sets A_{LN} and B_{LN} in $A_{LN} \neq 0_{LN} \neq B_{LN}, A_{LN} \subseteq (B_{LN})^c$.
5. there lies no linguistic neutrosophic regular semi-open sets A_{LN} and B_{LN} in $A_{LN} \neq 0_{LN} \neq B_{LN}, B_{LN} = (LNSCl(A_{LN}))^c, A_{LN} = (LNSCl(B_{LN}))^c$.
6. there lies no linguistic neutrosophic regular semi-closed sets A_{LN} and B_{LN} in $A_{LN} \neq 1_{LN} \neq B_{LN}, B_{LN} = (LNSInt(A_{LN}))^c, A_{LN} = (LNSInt(B_{LN}))^c$.

Proof:

(1) ⇒ (2): Assume that there lies a $A_{LN} \neq 0_{LN}$ with $A_{LN} \in LNRSO(S_{LN}, \tau_{LN})$ and $LNSCl(A_{LN}) \neq 1_{LN}$. Let $B_{LN} = LNSInt(LNSCl(A_{LN}))^c$, then B_{LN} is a proper linguistic neutrosophic regular semi-open set, which is a contrariety. Thus, $LNSCl(A_{LN}) = 1_{LN}$.

(2) ⇒ (3): Let $A_{LN} \neq 0_{LN}$ be a linguistic neutrosophic regular semi-closed set in (S_{LN}, τ_{LN}). If $(0_{LN} \neq B_{LN}) = (A_{LN})^c$, then B_{LN} is linguistic neutrosophic regular semi-open set. From the hypothesis, $LNSCl(B_{LN}) = 1_{LN}$ which shows that $(LNSCl(B_{LN}))^c = 0_{LN}$. Now, $LNSInt((B_{LN})^c) = LNSInt(A_{LN}) = 0_{LN}$.

(3) ⇒ (4): Let the sets A_{LN} and B_{LN} be linguistic neutrosophic regular semi-open in (S_{LN}, τ_{LN}), with $A_{LN} \neq 0_{LN} \neq B_{LN}, A_{LN} \subseteq (B_{LN})^c$. As $(B_{LN})^c$ is linguistic neutrosophic regular semi-closed set, we get $(B_{LN})^c = LNSCl(LNSInt((B_{LN})^c))$ and $LNSInt((B_{LN})^c) = 0_{LN}$.

Now,
$0_{LN} \neq A_{LN} = LNSInt(LNSCl(A_{LN})) \subseteq LNSInt(LNSCl((B_{LN})^c)) = LNSInt(LNSCl(LNSCl(LNSInt((B_{LN})^c)))) = LNSInt(LNSCl(LNSInt((B_{LN})^c))) = LNSInt((B_{LN})^c) = 0_{LN}$, which results in a contradiction.

(4) ⇒ (1): Proof is as above.

(1) ⇒ (5): Let the sets A_{LN} and B_{LN} be linguistic neutrosophic regular semi-open in (S_{LN}, τ_{LN}), with $A_{LN} \neq 0_{LN} \neq B_{LN}, B_{LN} = (LNSCl(A_{LN}))^c$ and $A_{LN} = (LNSCl(B_{LN}))^c$. Now, $LNSInt(LNSCl(A_{LN})) = LNSInt((B_{LN})^c) = (LNSCl(B_{LN}))^c = A_{LN}, A_{LN} \neq 0_{LN}$ and $A_{LN} \neq 1_{LN}$. Suppose if $A_{LN} = 1_{LN}$, then $1_{LN} = (LNSCl(B_{LN}))^c \Rightarrow LNSCl(B_{LN}) = 0_{LN} \Rightarrow B_{LN} = 0_{LN}$. But $B_{LN} \neq 0_{LN}$. Due to the conflict, it is concluded that the statement (5) holds true.

(5) ⇒ (1): Let A_{LN} be a linguistic neutrosophic regular semi-open in (S_{LN}, τ_{LN}), with $0_{LN} \neq A_{LN} \neq 1_{LN}$. Put $B_{LN} = (LNSCl(A_{LN}))^c$. Now, $B_{LN} \neq 0_{LN}$ and B_{LN} is a linguistic neutrosophic regular semi-open in (S_{LN}, τ_{LN}). Consider $B_{LN} = (LNSCl(A_{LN}))^c \Rightarrow (LNSCl(B_{LN}))^c = (LNSCl((LNSCl(A_{LN}))^c))^c = LNSInt((LNSCl(A_{LN}))^c)^c = LNSInt(LNSCl(A_{LN})) = A_{LN}$, which results in a contradiction. Ergo, (S_{LN}, τ_{LN}) is a linguistic neutrosophic super semi-connected space.

(5) ⇒ (6): Let the sets A_{LN} and B_{LN} be linguistic neutrosophic regular semi-open in (S_{LN}, τ_{LN}), with $A_{LN} \neq 1_{LN} \neq B_{LN}$ and $B_{LN} = (LNSInt(A_{LN}))^c, A_{LN} = (LNSInt(B_{LN}))^c$. Take $K_{LN} = (A_{LN})^c$ and $H_{LN} = (B_{LN})^c$, then K_{LN} and H_{LN} are non-empty linguistic neutrosophic regular semi-open sets. Now, $H_{LN} = ((LNSInt(A_{LN}))^c)^c = LNSInt((K_{LN})^c) = (LNSCl(K_{LN}))^c = (LNSCl(H_{LN}))^c$, which is a contradiction. Thus, (S_{LN}, τ_{LN}) is a linguistic neutrosophic super semi-connected space.

(6) ⇒ (5): Proof is same as above.

4. Linguistic Neutrosophic Semi-Compactness

The concept which depends solely upon linguistic neutrosophic semi open sets is linguistic neutrosophic semi-compactness. In this section, linguistic neutrosophic compactness is discussed with some characterizations.

Definition 4.1: A linguistic neutrosophic cover is defined as the collection in which every member is a linguistic neutrosophic semi-open set.

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Definition 4.2: A collection of linguistic neutrosophic subsets of S_{LN} has the finite intersection property if for each finite collection $\{A^1{}_{LN}, A^2{}_{LN}, A^3{}_{LN}, \ldots\ldots\ldots, A^k{}_{LN}\}$ the common intersection $\bigcap_{r=1}^{n} A^r{}_{LN}$ is non-empty.

Definition 4.3: A linguistic neutrosophic topological space (S_{LN}, τ_{LN}) is linguistic neutrosophic semi-compact if each linguistic neutrosophic cover of (S_{LN}, τ_{LN}) by semi open sets has a finite linguistic neutrosophic sub-cover, or for every collection A_{LN} of S_{LN}, $\bigcap_{r=1}^{n} A^r{}_{LN} \neq \phi$.

Remark 4.4: Every linguistic neutrosophic compact space is linguistic neutrosophic semi-compact space.

Definition 4.5: A linguistic neutrosophic topological space (S_{LN}, τ_{LN}) is linguistic neutrosophic semi-Lindelof space if each linguistic neutrosophic cover of (S_{LN}, τ_{LN}) by linguistic neutrosophic semi-open sets has a countable sub-cover of S_{LN}.

Definition 4.6: A linguistic neutrosophic topological space (S_{LN}, τ_{LN}) is linguistic neutrosophic countably semi-compact space relative to (S_{LN}, τ_{LN}) if each linguistic neutrosophic countable semi-open cover has a finite linguistic neutrosophic sub-cover.

Theorem 4.7: Let H_{LN} be a linguistic neutrosophic pre-open set and $K_{LN} \subset H_{LN}$. ThenK_{LN} is linguistic neutrosophic semi-compact if and only if K_{LN} is linguistic neutrosophic semi-compact in H_{LN}.

Proof:

Necessity part: Let $K_{LN} = \{(K_{LN})_i, \in I\}$ be a linguistic neutrosophic semi-open cover of K_{LN} by the linguistic neutrosophic semi-open sets in H_{LN}. Now, $(K_{LN})_i = (M_{LN})_i \cap H_{LN}$ for each $i \in I$, where $(M_{LN})_i$ is linguistic neutrosophic semi-open. Thus, $M_{LN} = \{(M_{LN})_i, \in I\}$ is a cover of K_{LN} by linguistic neutrosophic semi-open sets in (S_{LN}, τ_{LN}). As K_{LN} is linguistic neutrosophic semi-compact, we can find $i_1, i_2, i_3, \ldots. i_n \in I$ with $K_{LN} \subset \bigcup_{j=1}^{n}((M_{LN})_i)_j$ and we have $K_{LN} \subset \bigcup_{j=1}^{n}(((M_{LN})_i)_j \cap H_{LN}) = \bigcup_{j=1}^{n}((K_{LN})_i)_j$. Therefore, K_{LN} is linguistic neutrosophic semi-compact in H_{LN}.

Sufficiency Part: Let $M_{LN} = \{(M_{LN})_i, \in I\}$ is a cover of K_{LN} by linguistic neutrosophic semi-open sets in (S_{LN}, τ_{LN}). Then $K_{LN} = \{(M_{LN})_i \cap H_{LN}, \in I\}$ is a cover of K_{LN}. Since $\{(M_{LN})_i, \in I\}$ is is linguistic neutrosophic semi-open in (S_{LN}, τ_{LN}) and H_{LN} is linguistic neutrosophic pre-open, for each $i \in I$, $(M_{LN})_i \cap H_{LN}$ is linguistic neutrosophic semi-open in H_{LN}. As K_{LN} is linguistic neutrosophic semi-compact in H_{LN}, we can find $i_1, i_2, i_3, \ldots. i_n \in I$ with $K_{LN} \subset \bigcup_{j=1}^{n}(((M_{LN})_i)_j \cap H_{LN}) \subset \bigcup_{j=1}^{n}((M_{LN})_i)_j$. Ergo, K_{LN} is linguistic neutrosophic semi-compact in S_{LN}.

Theorem 4.8: Let H_{LN} be a linguistic neutrosophic pre-open set and $K_{LN} \subset H_{LN}$. ThenK_{LN} is linguistic neutrosophic semi-Lindelof if and only if K_{LN} is linguistic neutrosophic semi-Lindelof in H_{LN}.

Proof: Proof is straight forward.

Theorem 4.9: Let $f_{LN}: (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ be a linguistic neutrosophic semi-irresolute mapping. Then the succeeding results are equivalent.

1. If K_{LN} is linguistic neutrosophic semi-Lindelof in (S_{LN}, τ_{LN}), then $f_{LN}(K_{LN})$ is linguistic neutrosophic semi-Lindelof in (T_{LN}, η_{LN}).

2. If K_{LN} is linguistic neutrosophic semi-compact in (S_{LN}, τ_{LN}), then $f_{LN}(K_{LN})$ is linguistic neutrosophic semi-compact in (T_{LN}, η_{LN}).

Proof:

(1): Let $M_{LN} = \{(M_{LN})_i, \in I\}$ is a cover of $f_{LN}(K_{LN}$ by linguistic neutrosophic semi-open sets in (T_{LN}, η_{LN}). Then, $K_{LN} = \{(f_{LN})^{-1}(K_{LN})_i, i \in I\}$ is a cover of K_{LN}. As f_{LN} is linguistic neutrosophic semi-irresolute, $(f_{LN})^{-1}((M_{LN})_i)$ is linguistic neutrosophic semi-open in (S_{LN}, τ_{LN}). As K_{LN} is linguistic neutrosophic semi-Lindelof, there lies $i_1, i_2, i_3, \ldots. i_n \in I$ with $K_{LN} \subset \bigcup_{j=1}^{\infty}(f_{LN})^{-1}((M_{LN})_i)_j$. Therefore, $f_{LN}(K_{LN}) \subset \bigcup_{j=1}^{\infty}((M_{LN})_i)_j$, which shows that $f_{LN}(K_{LN})$ is linguistic neutrosophic semi-compact in (T_{LN}, η_{LN}).

(2): Proof is same as of (1).

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Definition 4.10: A linguistic neutrosophic topological space is extremely disconnected if the linguistic neutrosophic closure of each linguistic neutrosophic open subset of (S_{LN}, τ_{LN}) is linguistic neutrosophic open.

Definition 4.11: A linguistic neutrosophic topological space is linguistic neutrosophic semi-compact space if any linguistic neutrosophic subset of (S_{LN}, τ_{LN}) which is linguistic neutrosophic semi-compact is linguistic neutrosophic semi-closed in (S_{LN}, τ_{LN}).

Theorem 4.12: A linguistic neutrosophic topological space is extremely disconnected if the intersection of any two linguistic neutrosophic semi-open subsets of (S_{LN}, τ_{LN}) is linguistic neutrosophic semi-open.

Proof: Proof is direct.

Theorem 4.13: Let (S_{LN}, τ_{LN}) be a linguistic neutrosophic semi-T_2 extremely disconnected, then the space is linguistic neutrosophic semi-compact.

Proof: Let A_{LN} be a linguistic neutrosophic semi-compact subset of (S_{LN}, τ_{LN}) and let $s \in A_{LN}$. For each $p \in A_{LN}$, there lie two different linguistic neutrosophic semi-open sets B_{LN} and D_{LN} containing s and t respectively. Then we can find $p_1, p_2, \ldots., p_n \in A_{LN}$ with $A_{LN} \subset \bigcup_{i=1}^{n} (D_{LN})_{p_i}$. Let $B_{LN} = \bigcap_{i=1}^{n} (B_{LN})_{p_i}$. Then B_{LN} is a linguistic neutrosophic semi-open set that contains s and distinct from A_{LN}. Thus, $s \in LNSCl(A_{LN})$, which results that A_{LN} is linguistic neutrosophic semi-closed.

Theorem 4.14:Let f_{LN} be a linguistic neutrosophic semi-irresolute and one to one mapping from a linguistic neutrosophic topological space (S_{LN}, τ_{LN}) into a linguistic neutrosophic semi-compact space (T_{LN}, η_{LN}), then the space (S_{LN}, τ_{LN}) is linguistic neutrosophic semi-compact.

Proof: Let A_{LN} be a linguistic neutrosophic semi-compact set in S_{LN} then $f_{LN}(A_{LN})$ is linguistic neutrosophic semi-compact set in T_{LN}. As the space (T_{LN}, η_{LN}) is linguistic neutrosophic semi-compact, $f_{LN}(A_{LN})$ is linguistic neutrosophic semi-closed in T_{LN}. Since f_{LN} is one to one and irresolute, $(f_{LN})^{-1}(f(A_{LN}))$ is linguistic neutrosophic semi-closed in S_{LN}. Hence, (S_{LN}, τ_{LN}) is linguistic neutrosophic semi-compact.

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