Several arithmetic operations on neutrosophics type-2 fuzzy sets

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Abstract

In this paper, we explore the theoretical foundations of neutrosophics type-2 fuzzy sets by investigating its algebraic properties, demonstrating how neutrosophics type-2 fuzzy sets can generalize and extend existing operations in Type-1 and traditional Type-2 fuzzy sets. We also provide illustrative examples to clarify the practical applications of these operations, showcasing the potential of neutrosophics type-2 fuzzy sets in areas requiring sophisticated decision-making tools and uncertainty management.

Keywords: Type-2 fuzzy set; Intuitionistic type-2 fuzzy set; Neutrosophics type-2 fuzzy set

1 Introduction

Enormous progress has been made in solving fuzzy and uncertain issues since the advent of fuzzy set theory see for example.[1,3,18] The neutrosophic set is one of the expansions of the conventional fuzzy sets that scholars have developed throughout the years and in (1998), Smarandache [10] defined the concept of a neutrosophic set as a generalization of Atanassov's intuitionistic fuzzy set. Also, he introduced neutrosophic logic, neutrosophic set and its applications in.[7,11] In particular, Wang et al. [13] introduced the notion of a single valued neutrosophic set.

Concurrently, type-2 fuzzy sets—first introduced by Zadeh (1975) [16] and subsequently defined by Mendel (2001) [8] expand the concept of the classical fuzzy set by permitting the membership functions to be fuzzy. This expansion is especially helpful in situations when there is uncertainty stemming from both the data and the imprecision in assigning membership ratings.

Neutronosophic type-2 fuzzy sets (NT2FS), which have garnered much in this paper, are created by combining neutrosophic and type-2 fuzzy sets together. As a result of their increased capacity to capture imprecision and uncertainty, these sets are very helpful in domains like artificial intelligence, optimization, and decision-making.

Building on the work of Smarandache and Mendel, this study examines some basic arithmetic operations on neutrosophic type-2 fuzzy sets and investigates their applicability in several domains. Our goal in formalizing these procedures is to show how important they are for improving the precision and adaptability of models that handle uncertainty in the actual world.

Moreover, in this paper, we introduce the concept of the neutrosophics type 2 fuzzy sets (NT2FS) are an extension of traditional type 2 fuzzy sets that allow for modeling uncertainty in a more sophisticated way. And introduce the notions of basic set operations and focus on the algebraic properties of these sets with several illustrative examples.

The rest of the paper is organized as follows. The preliminary concepts of our study are presented in Section2. In Section3, we propose the neutrosophics type-2 fuzzy set (NT2FS) and give examples, the geometrical interpretation of neutrosophics type -2 fuzzy set, some set-theoretic operations of neutrosophics type -2 fuzzy set including union, intersection, and complement are defined.

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2 Preliminaries

Consider a bounded lattice $(L, \leq, 1_L, 0_L)$ and a universe X. Goguen [6] introduced L-fuzzy sets as objects A characterized by some membership function $\mu_A : X \to L$. His approach has generalized the crisp sets (in which case $L = \{0, 1\}$ and μ_A is just the characteristic function defined as follows,

$$\chi_A(x) = \begin{cases} 1, & \text{if } x \in A \\ 0, & \text{if } x \notin A \end{cases}$$

of the crisp subset A of X), Zadeh [15] proposed the concept of the type-1 fuzzy set or fuzzy sets (here $L = [0, 1]$)., i.e. $\mu_A : X \to [0, 1]$. The following example, we use a fuzzy set to represent the concept **1 or so**. We can use different functions to model this concept. Following figure represents the concept **1 or so** using three different fuzzy set A, B and C.

We can represent the fuzzy set using tuple notation as,

$$A = \{\langle x, \mu_A(x)\rangle \mid x \in X\} = \{\langle -2, 0.3\rangle, \langle 0, 0.5\rangle, \langle 1, 1\rangle, \langle 2, 0.5\rangle, \langle 3, 0.2\rangle \ldots\}$$

In the same way we create the fuzzy set B and C.

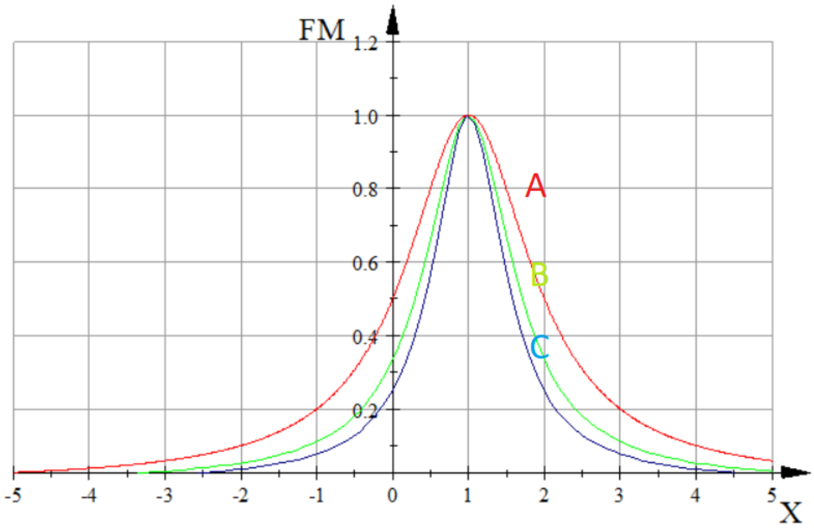

Figure 1: Fuzzy representation of concept **1 or so**

The concept of a type-2 fuzzy set

The concept of type-2 fuzzy set was introduced by Zadeh it is an extension of the ordinary fuzzy set of type-1 fuzzy set type-1 fuzzy set .[16,17] The overviews of type-2 fuzzy sets were given in Mendel .[8] Since ordinary fuzzy sets and interval-valued fuzzy sets are special cases of type-2 fuzzy sets, Takac proposed that type-2 fuzzy sets are very practical in the circumstances where there are more uncertainties .[12]

Definition 2.1. [2,9] A type-2 fuzzy set A, denoted $\widetilde{A}$, is a set membership function $\mu_{\widetilde{A}}$ on universe $X \times [0, 1]$ into $[0, 1]$, i.e.

$$\widetilde{A} = \{((x, u), \mu_{\widetilde{A}}(x, u)) \mid \forall x \in X, \forall u \in J_x \subset [0, 1]\} \; (1)$$

Where, $0 \leq \mu_{\widetilde{A}}(x, u) \leq 1$ and $J_x = \{(x, u) : u \in [0, 1], \mu_{\widetilde{A}}(x, u) > 0\}$.

Remark 2.2. A type-2 fuzzy set A, can also be expressed as

$$\widetilde{A} = \int_{x \in X} \int_{u \in J_x} \mu_{\widetilde{A}}(x, u) \mid (x, u)$$

$\int\int$ denotes union over all admissible x and u. For discrete universes of discourse $\int$ is replaced by $\sum$.

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Example 2.3. Let $X = \{10, 16, 22, 28, 34, 40, 46\}$ the set of temperature. then, the concept of " hot " temperature be represented by a type-2 fuzzy set $\widetilde{A}$, the primary membership of the points of X is $J_{10} = \{0.00, 0.20\} = J_{46}$, $J_{16} = \{0.20, 0.40, 0.60\} = J_{40}$, $J_{22} = \{0.60, 0.80, 1.00\} = J_{28}$ and $J_{34} = \{0.30, 0.50, 0.70, 1.00\}$, respectively.
The discrete type-2 fuzzy set $\widetilde{A}$ is given by,

$$\widetilde{A} = \left\{ \left((x, u), \mu_{\widetilde{A}}(x, u) \right) : x \in X, u \in J_x \subseteq [0, 1] \right\}$$

Accordingly, discrete type-2 fuzzy set $\widetilde{A}$ can be represented as:

$$\widetilde{A} = \left\{ \begin{array}{l} \left((10, u), \mu_{\widetilde{A}}(10, u)\right), u \in J_{10}+ \\ \left((16, u), \mu_{\widetilde{A}}(16, u)\right), u \in J_{16}+ \\ \left((22, u), \mu_{\widetilde{A}}(22, u)\right), u \in J_{22}+ \\ \left((28, u), \mu_{\widetilde{A}}(28, u)\right), u \in J_{28}+ \\ \left((34, u), \mu_{\widetilde{A}}(34, u)\right), u \in J_{34}+ \\ \left((40, u), \mu_{\widetilde{A}}(40, u)\right), u \in J_{40}+ \\ \left((46, u), \mu_{\widetilde{A}}(46, u)\right), u \in J_{46} \end{array} \right\}$$

$$\widetilde{A} = \left\{ \begin{array}{l} ((10, 0.00), 1.00), ((10, 0.20), 0.30), ((16, 0.20), 0.60), ((16, 0.40), 1.00), \\ ((16, 0.60), 0.70), ((22, 0.60), 0.40), ((22, 0.80), 0.70), ((22, 1.00), 1.00), \\ ((28, 0.60), 0.40), ((28, 0.80), 0.70), ((28, 1.00), 1.00), ((34, 0.30), 0.20), \\ ((34, 0.50), 0.60), ((34, 0.7), 1.00), ((34, 1.00), 0.40), ((40, 0.20), 0.30), \\ ((40, 0.40), 1.00), ((40, 0.60), 0.40), ((46, 0.00), 1.00), ((46, 0.20), 0.60) \end{array} \right\}$$

A type-2 fuzzy set for defining the concept of âhotâ temperature that we will denote by $\widetilde{A}$, can be as depicted below:

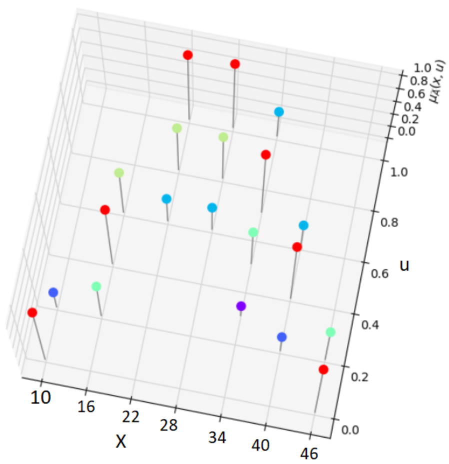

Figure 2: Representation of a type-2 fuzzy set for defining the concept of âhotâ temperature.

2.1 Neutrosophic fuzzy set

This section contains the basic definitions and properties of Neutrosophic fuzzy set and some related notions that will be needed throughout this paper.

Definition 2.4. [7] Let X be a nonempty set. A neutrosophic set (NS, for short) A on X is an object of the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle \mid x \in X\}$ characterized by a membership function $\mu_A : X \to]^{-}0, 1^{+}[$ and an indeterminacy function $\sigma_A : X \to]^{-}0, 1^{+}[$ and a non-membership function $\nu_A : X \to]^{-}0, 1^{+}[$ which satisfy the condition:

$$^{-}0 \leq \mu_A(x) + \sigma_A(x) + \nu_A(x) \leq 3^{+}, \text{ for any } x \in X.$$

Certainly, intuitionistic fuzzy sets are neutrosophic sets by setting $\sigma_A(x) = 1 - \mu_A(x) - \nu_A(x)$.

Next, we show the notion of single valued neutrosophic set as an instance of neutrosophic set which can be used in real scientific and engineering applications.

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Example 2.5. Let us consider picture fuzzy sets A, B, C in $X = \{a_1, a_2, a_3\}$, The full description of picture fuzzy set A, i.e.

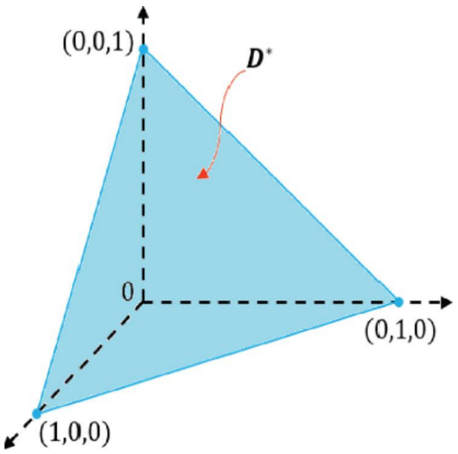
$$A = \{(\mu_A(a_1), \eta_A(a_1), \nu_A(a_1))/a_1, (\mu_A(a_2), \eta_A(a_2), \nu_A(a_2))/a_2, (\mu_A(a_3), \eta_A(a_3), \nu_A(a_3))/a_3\}$$

For example,

$$\begin{aligned} A &= \{(0.8, 0.1, 0)/a_1, (0.4, 0.2, 0.3)/a_2, (0.5, 0.3, 0)/a_3\} \\ B &= \{(0.3, 0.3, 0.2)/a_1, (0.7, 0.1, 0.1)/a_2, (0.4, 0.3, 0.2)/a_3\} \\ C &= \{(0.3, 0.4, 0.1)/a_1, (0.6, 0.2, 0.1)/a_2, (0.4, 0.3, 0.1)/a_3\} \end{aligned}$$

Now, we consider the set D^* defined by

$$D^* = \left\{x = (x_1, x_2, x_3) : x \in [0,1]^3 \text{ and } 0 \le x_1 + x_2 + x_3 \le 3\right\}$$



Consider the order relation $\le_1$ on D^*, defined by:

$$x \le_1 y \Leftrightarrow \begin{cases} ((x_1 < y_1) \wedge (x_3 \ge y_3)) \\ \vee \\ ((x_1 = y_1) \wedge (x_3 > y_3)) \\ \vee \\ ((x_1 = y_1) \wedge (x_3 = y_3) \wedge (x_2 \le y_2)) \end{cases}$$

Lemma 2.6. *For each $x, y \in D^*$, we define:*

$$\inf(x,y) = \begin{cases} \min(x,y), & \text{if } x \le_1 y \text{ or } y \le_1 x, \\ ((x_1 \wedge y_1), 1 - (x_1 \wedge y_1) - (x_3 \vee y_3), (x_3 \vee y_3)), & \text{else.} \end{cases}$$
$$\sup(x,y) = \begin{cases} \max(x,y), & \text{if } x \le_1 y \text{ or } y \le_1 x, \\ ((x_1 \vee y_1), 0, (x_3 \wedge y_3)), & \text{else.} \end{cases}$$

Then, $(D^*, \le_1)$ is a complete lattice .

Using this lattice, we easily see that with every picture fuzzy set $A = \{(x, \mu_A(x), \eta_A(x), \nu_A(x)) : x \in X\}$ corresponds an D^*-fuzzy set, i.e., a mapping $A : X \to D^*,\ x \mapsto A(x) = (\mu_A(x), \eta_A(x), \nu_A(x))$. We denote the units of D^* by $1_{D^*} = (1, 0, 0)$ and $0_{D^*} = (0, 0, 1)$, respectively.

A single valued neutrosophic set

Definition 2.7. [13] Let X be a nonempty set. A single valued neutrosophic set (SVNS, for short) A on X is an object of the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle \mid x \in X\}$ characterized by a truth-membership function $\mu_A : X \to [0,1]$, an indeterminacy-membership function $\sigma_A : X \to [0,1]$ and a falsity-membership function $\nu_A : X \to [0,1]$.

The class of single valued neutrosophic sets on X is denoted by $SVN(X)$.

For any two SVNSs A and B on a set X, several operations are defined (see, e.g., [13,14]). Here we will present only those which are related to the present paper.

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(i) $A \subseteq B$ if $\mu_A(x) \leq \mu_B(x)$ and $\sigma_A(x) \leq \sigma_B(x)$ and $\nu_A(x) \geq \nu_B(x),$ for all $x \in X,$

(ii) $A = B$ if $\mu_A(x) = \mu_B(x)$ and $\sigma_A(x) = \sigma_B(x)$ and $\nu_A(x) = \nu_B(x),$ for all $x \in X,$

(iii) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \nu_A(x) \vee \nu_B(x)\rangle \mid x \in X\},$

(iv) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \nu_A(x) \wedge \nu_B(x)\rangle \mid x \in X\},$

(v) $\overline{A} = \{\langle x, 1 - \nu_A(x), 1 - \sigma_A(x), 1 - \mu_A(x)\rangle \mid x \in X\}.$

2.2 Neutrosophics fuzzy t-norms and neutrosophics fuzzy t-conorms

Now we define neutrosophics fuzzy t-norms and neutrosophics fuzzy t-conorms and will give some classes of conjuction operators and some classes of disjunction operators for neutrosophics fuzzy sets. Neutrosophics fuzzy t-norms are direct extension of fuzzy t-norms and of intuitionistic fuzzy t-norms .[5]

Definition 2.8. A neutrosophics fuzzy t-norm is an $(D^*)^2 \to D^*$ mapping T satisfying the following conditions:

1. $T(x, y) = T(y, x)$, $\forall x, y \in D^*$ (Commutativity),
2. $T(x, T(y, z)) = T(T(x, y), z)$, $\forall x, y, z \in D^*$ (Associativity),
3. $T(x, y) \leq_1 T(x, z)$, $\forall x, y, z \in D^*$, $y \leq_1 z$ (Monotonicity) and
4. $T(1_{D^*}, x) \in I(x)$, $\forall x \in D^*$ (Boundary condition).

Definition 2.9. A neutrosophics fuzzy t-norm is an $(D^*)^2 \to D^*$ mapping S satisfying the following conditions:

1. $S(x, y) = S(y, x)$, $\forall x, y \in D^*$ (Commutativity),
2. $S(x, S(y, z)) = S(S(x, y), z)$, $\forall x, y, z \in D^*$ (Associativity),
3. $S(x, y) \leq_1 S(x, z)$, $\forall x, y, z \in D^*$, $y \leq_1 z$ (Monotonicity) and
4. $S(0_{D^*}, x) \in I(x)$, $\forall x \in D^*$ (Boundary condition).

From now on, we denote $x \wedge y = \min(x, y)$, $x \vee y = \max(x, y)$ for all $x, y \in [0, 1]$.
Now, we will give some examples about the neutrosophics fuzzy t-norms

Example 2.10. Some neutrosophics fuzzy t-norms, for all $x, y \in D^*$

1. $T_{\inf}(x, y) = \inf\{x, y\} = \begin{cases} (x_1 \wedge y_1, 1 - (x_1 \wedge y_1) - (x_3 \vee y_3), x_3 \vee y_3) & \text{if } x \parallel_{\leq_1} y \\ x \wedge y & \text{otherwise} \end{cases}.$
2. $T_{\min}(x, y) = (x_1 \wedge y_1, x_2 \wedge y_2, x_3 \vee y_3) = x \wedge_{\min} y.$

Example 2.11. Some neutrosophics fuzzy t-conorms, for all $x, y \in D^*$.

1. $S_{\sup}(x, y) = \sup\{x, y\} = \begin{cases} (x_1 \vee y_1, 0, x_3 \wedge y_3) & \text{if } x \parallel_{\leq_1} y \\ x \vee y & \text{otherwise} \end{cases}.$
2. $S_{\max}(x, y) = (x_1 \vee y_1, x_2 \wedge y_2, x_3 \wedge y_3) = x \vee_{\max} y.$

Definition 2.12. [4] A neutrosophics fuzzy t-norm T is called representable iff there exist two t-norms t_1, t_2 and a t-conorm s_3 on $[0, 1]$ satisfying, for all $x, y \in D^*$,

$$T(x, y) = (t_1(x_1, y_1), t_2(x_2, y_2), s_3(x_3, y_3)).$$

Definition 2.13. [4] A neutrosophics fuzzy t-norm S is called representable iff there exist two t-norms t_1, t_2 and a t-conorm s_3 on $[0, 1]$ satisfying, for all $x, y \in D^*$,

$$S(x, y) = (s_3(x_1, y_1), t_2(x_2, y_2), t_1(x_3, y_3)).$$

In this paper we consider that T and S is representable.

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3 Neutrosophics Type-2 Fuzzy Set

In this section, we introduce the concept of the neutrosophics type-2 fuzzy set,

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Definition 3.1. An neutrosophics type-2 fuzzy set $\widetilde{A}$ note by NT2FS on X is defined as an object of the following form:

$$\widetilde{A} = \left\{ \left((x,u), \mu_{\widetilde{A}}(x,u), \eta_{\widetilde{A}}(x,u), \nu_{\widetilde{A}}(x,u) \right) : x \in X, u \in J_x \subseteq [0,1] \right\} \tag{2}$$

Where $\mu_{\widetilde{A}}(x,u) \in [0,1]$ is called the degree of positive membership of (x,u) in $X \times J_x$, $\eta_{\widetilde{A}}(x,u) \in [0,1]$ is called the degree of neutral membership of (x,u) in $X \times J_x$ and $\nu_{\widetilde{A}}(x,u) \in [0,1]$ is called the degree of negative membership of (x,u) in $X \times J_x$, and where $\mu_{\widetilde{A}}, \eta_{\widetilde{A}}$ and $\nu_{\widetilde{A}}$ satisfy the following condition:

$$\mu_{\widetilde{A}}(x,u) + \eta_{\widetilde{A}}(x,u) + \nu_{\widetilde{A}}(x,u) \leq 1, \text{ for any } (x,u) \in X \times J_x$$

For a discrete universe of discourse, an NT2FS can be represented as:

$$\widetilde{A} = \int_{x \in X} \left(\int_{u \in J_x} \left(\mu_{\widetilde{A}}(x,u), \eta_{\widetilde{A}}(x,u), \nu_{\widetilde{A}}(x,u) \right) \mid u \right) \mid x, \quad J_x \subset [0,1]$$

In the continuous case, however, $\int$ is substituted by $\sum$, resulting in the following representation for the continuous universe:

$$\widetilde{A} = \sum_{x \in X} \left(\sum_{u \in J_x} \left(\mu_{\widetilde{A}}(x,u), \eta_{\widetilde{A}}(x,u), \nu_{\widetilde{A}}(x,u) \right) \mid u \right) \mid x, \quad J_x \subset [0,1]$$

Example 3.2. Let a NT2FS $\widetilde{A}$ represent the set young. The degree of positive membership function $\mu_{\widetilde{A}}(x,u)$ of $\widetilde{A}$ is **youth**, the degree of neuter membership $\eta_{\widetilde{A}}(x,u)$ and The degree of negative membership function $\nu_{\widetilde{A}}(x,u)$ of $\widetilde{A}$ are the degree of **youthness** and the degree of **adultness**., respectively. Let $X = \{7, 14, 16\}$ be the set, and the primary membership of the points of X is $J_7 = \{0.8, 0.9, 1.0\}$, $J_{14} = \{0.6, 0.7, 0.8\}$, and $J_{16} = \{0.4, 0.5, 0.6\}$, respectively. Then, the discrete NT2FS $\widetilde{A}$ is given by,

$$\widetilde{A} = \left\{ \left((x,u), \mu_{\widetilde{A}}(x,u), \eta_{\widetilde{A}}(x,u), \nu_{\widetilde{A}}(x,u) \right) : x \in X, u \in J_x \subseteq [0,1] \right\}$$

$$\widetilde{A} = \left\{ \begin{array}{l} ((7,u), \mu_{\widetilde{A}}(7,u), \eta_{\widetilde{A}}(7,u), \nu_{\widetilde{A}}(7,u)), u \in J_7 = \{0.8, 0.9, 1.0\} + \\ ((14,u), \mu_{\widetilde{A}}(14,u), \eta_{\widetilde{A}}(14,u), \nu_{\widetilde{A}}(14,u)), u \in J_7 = \{0.8, 0.9, 1.0\} + \\ ((16,u), \mu_{\widetilde{A}}(16,u), \eta_{\widetilde{A}}(16,u), \nu_{\widetilde{A}}(16,u)), u \in J_7 = \{0.8, 0.9, 1.0\} + \\ ((7,u), \mu_{\widetilde{A}}(7,u), \eta_{\widetilde{A}}(7,u), \nu_{\widetilde{A}}(7,u)), u \in J_{14} = \{0.6, 0.7, 0.8\} + \\ ((14,u), \mu_{\widetilde{A}}(14,u), \eta_{\widetilde{A}}(14,u), \nu_{\widetilde{A}}(14,u)), u \in J_{14} = \{0.6, 0.7, 0.8\} + \\ ((16,u), \mu_{\widetilde{A}}(16,u), \eta_{\widetilde{A}}(16,u), \nu_{\widetilde{A}}(16,u)), u \in J_{14} = \{0.6, 0.7, 0.8\} + \\ ((7,u), \mu_{\widetilde{A}}(7,u), \eta_{\widetilde{A}}(7,u), \nu_{\widetilde{A}}(7,u)), u \in J_{16} = \{0.4, 0.5, 0.6\} + \\ ((14,u), \mu_{\widetilde{A}}(14,u), \eta_{\widetilde{A}}(14,u), \nu_{\widetilde{A}}(14,u)), u \in J_{16} = \{0.4, 0.5, 0.6\} + \\ ((16,u), \mu_{\widetilde{A}}(16,u), \eta_{\widetilde{A}}(16,u), \nu_{\widetilde{A}}(16,u)), u \in J_{16} = \{0.4, 0.5, 0.6\} \end{array} \right\}$$

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$$\widetilde{A}=\left\{\begin{array}{l}((7,0.8),\mu_{\widetilde{A}}(7,0.8),\eta_{\widetilde{A}}(7,0.8),\nu_{\widetilde{A}}(7,0.8))+\\((14,0.8),\mu_{\widetilde{A}}(14,0.8),\eta_{\widetilde{A}}(14,0.8),\nu_{\widetilde{A}}(14,0.8))+\\((16,0.8),\mu_{\widetilde{A}}(16,0.8),\eta_{\widetilde{A}}(16,0.8),\nu_{\widetilde{A}}(16,0.8))+\\((7,0.9),\mu_{\widetilde{A}}(7,0.9),\eta_{\widetilde{A}}(7,0.9),\nu_{\widetilde{A}}(7,0.9))+\\((14,0.9),\mu_{\widetilde{A}}(14,0.9),\eta_{\widetilde{A}}(14,0.9),\nu_{\widetilde{A}}(14,0.9))+\\((16,0.9),\mu_{\widetilde{A}}(16,0.9),\eta_{\widetilde{A}}(16,0.9),\nu_{\widetilde{A}}(16,0.9))+\\((7,1.0),\mu_{\widetilde{A}}(7,1.0),\eta_{\widetilde{A}}(7,1.0),\nu_{\widetilde{A}}(7,1.0))+\\((14,1.0),\mu_{\widetilde{A}}(14,1.0),\eta_{\widetilde{A}}(14,1.0),\nu_{\widetilde{A}}(14,1.0))+\\((16,1.0),\mu_{\widetilde{A}}(16,1.0),\eta_{\widetilde{A}}(16,1.0),\nu_{\widetilde{A}}(16,1.0))+\\((7,0.6),\mu_{\widetilde{A}}(7,0.6),\eta_{\widetilde{A}}(7,0.6),\nu_{\widetilde{A}}(7,0.6))+\\((14,0.6),\mu_{\widetilde{A}}(14,0.6),\eta_{\widetilde{A}}(14,0.6),\nu_{\widetilde{A}}(14,0.6))+\\((16,0.6),\mu_{\widetilde{A}}(16,0.6),\eta_{\widetilde{A}}(16,0.6),\nu_{\widetilde{A}}(16,0.6))+\\((7,0.7),\mu_{\widetilde{A}}(7,0.7),\eta_{\widetilde{A}}(7,0.7),\nu_{\widetilde{A}}(7,0.7))+\\((14,0.7),\mu_{\widetilde{A}}(14,0.7),\eta_{\widetilde{A}}(14,0.7),\nu_{\widetilde{A}}(14,0.7))+\\((16,0.7),\mu_{\widetilde{A}}(16,0.7),\eta_{\widetilde{A}}(16,0.7),\nu_{\widetilde{A}}(16,0.7))+\\((7,0.8),\mu_{\widetilde{A}}(7,0.8),\eta_{\widetilde{A}}(7,0.8),\nu_{\widetilde{A}}(7,0.8))+\\((14,0.8),\mu_{\widetilde{A}}(14,0.8),\eta_{\widetilde{A}}(14,0.8),\nu_{\widetilde{A}}(14,0.8))+\\((16,0.8),\mu_{\widetilde{A}}(16,0.8),\eta_{\widetilde{A}}(16,0.8),\nu_{\widetilde{A}}(16,0.8))+\\((7,0.4),\mu_{\widetilde{A}}(7,0.4),\eta_{\widetilde{A}}(7,0.4),\nu_{\widetilde{A}}(7,0.4))+\\((14,0.4),\mu_{\widetilde{A}}(14,0.4),\eta_{\widetilde{A}}(14,0.4),\nu_{\widetilde{A}}(14,0.4))+\\((16,0.4),\mu_{\widetilde{A}}(16,0.4),\eta_{\widetilde{A}}(16,0.4),\nu_{\widetilde{A}}(16,0.4))+\\((7,0.5),\mu_{\widetilde{A}}(7,0.5),\eta_{\widetilde{A}}(7,0.5),\nu_{\widetilde{A}}(7,0.5))+\\((14,0.5),\mu_{\widetilde{A}}(14,0.5),\eta_{\widetilde{A}}(14,0.5),\nu_{\widetilde{A}}(14,0.5))+\\((16,0.5),\mu_{\widetilde{A}}(16,0.5),\eta_{\widetilde{A}}(16,0.5),\nu_{\widetilde{A}}(16,0.5))+\\((7,0.6),\mu_{\widetilde{A}}(7,0.6),\eta_{\widetilde{A}}(7,0.6),\nu_{\widetilde{A}}(7,0.6))+\\((14,0.6),\mu_{\widetilde{A}}(14,0.6),\eta_{\widetilde{A}}(14,0.6),\nu_{\widetilde{A}}(14,0.6))+\\((16,0.6),\mu_{\widetilde{A}}(16,0.6),\eta_{\widetilde{A}}(16,0.6),\nu_{\widetilde{A}}(16,0.6))\end{array}\right\}$$

$$\widetilde{A}=\left\{\begin{array}{l}((7,0.8),0.1,0.1,0.3)+((14,0.8),0.2,0.00,0.5)+((16,0.8),0.4,0.2,0.3)+\\((7,0.9),0.4,0.1,0.2)+((14,0.9),0.35,0.45,0.10)+((16,0.9),0.28,0.10,0.4)+\\((7,1.0),0.4,0.5,0.01)+((14,1.0),0.14,0.27,0.30)+((16,1.0),0.20,0.29,0.4)+\\((7,0.6),0.2,0.4,0.3)+((14,0.6),0.13,0.20,0.4)+((16,0.6),0.36,0.18,0.40)+\\((7,0.7),0.41,0.00,0.45)+((14,0.7),0.37,0.12,0.48)+((16,0.7),0.31,0.14,0.49)+\\((7,0.8),0.28,0.10,0.34)+((14,0.8),0.45,0.00,0.50)+((16,0.8),0.27,0.4,0.29)+\\((7,0.4),0.27,0.20,0.30)+((14,0.4),0.21,0.50,0.24)+((16,0.4),0.28,0.40,0.38)+\\((7,0.5),0.20,0.41,0.32)+((14,0.5),0.28,0.45,0.30)+((16,0.5),0.27,0.47,0.38)+\\((7,0.6),0.24,0.47,0.34)+((14,0.6),0.23,0.49,0.31)+((16,0.6),0.24,0.49,0.30))\end{array}\right\}$$

4 Operations on neutrosophics type-2 fuzzy set

In this section, we also present fundamental operations, such as union, intersection, and complement on the proposed NT2FS, which are similar to several existing set-theoretic operations on fuzzy sets. Let's consider the two NT2FS, denoted below as $\widetilde{A}$ and $\widetilde{B}$ on X, in this context.

$$\widetilde{A}=\int\limits_{x\in X}\left(\int\limits_{u\in J_x}\left(\mu_{\widetilde{A}}(x,u),\eta_{\widetilde{A}}(x,u),\nu_{\widetilde{A}}(x,u)\right)\mid u\right)\mid x,\quad J_x\subset[0,1]$$

And

$$\widetilde{B}=\int\limits_{x\in X}\left(\int\limits_{\nu\in J'_x}\left(\mu_{\widetilde{A}}(x,\nu),\eta_{\widetilde{A}}(x,\nu),\nu_{\widetilde{A}}(x,\nu)\right)\mid \nu\right)\mid x,\quad J'_x\subset[0,1]$$

Then, the union of $\widetilde{A}$ and $\widetilde{B}$ is defined as:

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$$\widetilde{A} \cup \widetilde{B} = \int\limits_{x \in X} \left(\int\limits_{w \in J_x \cup J_x'} \left(\mu_{\widetilde{A} \cup \widetilde{B}}(x,w), \eta_{\widetilde{A} \cup \widetilde{B}}(x,w), \nu_{\widetilde{A} \cup \widetilde{B}}(x,w) \right) \mid w \right) \mid x, \quad J_x \cup J_x^{'} \subset [0,1]$$

where:

$$\mu_{\widetilde{A} \cup \widetilde{B}}(x) = S \left(\int\limits_{u \in J_x} \mu_{\widetilde{A}}(x,u) \mid u, \int\limits_{u \in J_x} \mu_{\widetilde{B}}(x,v) \mid v \right)$$

By using the extension principle, we obtain,

$$\mu_{\widetilde{A} \cup \widetilde{B}}(x,w) = \int\limits_{u \in J_x} \left(\int\limits_{\nu \in J_x'} \left(\mu_{\widetilde{A}}(x,u) \wedge \mu_{\widetilde{B}}(x,\nu) \right) \right) \mid S(u,v)$$

Where $S(u,v)$ is the t-conorm of u and v see definition above i.e.,

$$\mu_{\widetilde{A} \cup \widetilde{B}}(x,w) = \int\limits_{u \in J_x} \left(\int\limits_{\nu \in J_x'} \left(\mu_{\widetilde{A}}(x,u) \wedge \mu_{\widetilde{B}}(x,\nu) \right) \right) \mid S(u,v)$$

Similarly,

$$\eta_{\widetilde{A} \cup \widetilde{B}}(x,w) = \int\limits_{u \in J_x} \left(\int\limits_{\nu \in J_x'} \left(\eta_{\widetilde{A}}(x,u) \wedge \eta_{\widetilde{B}}(x,\nu) \right) \right) \mid S(u,v)$$

And

$$\nu_{\widetilde{A} \cup \widetilde{B}}(x,w) = \int\limits_{u \in J_x} \left(\int\limits_{\nu \in J_x'} \left(\nu_{\widetilde{A}}(x,u) \vee \nu_{\widetilde{B}}(x,\nu) \right) \right) \mid S(u,v)$$

The intersection of of $\widetilde{A}$ and $\widetilde{B}$ is is defined as:

$$\widetilde{A} \cap \widetilde{B} = \int\limits_{x \in X} \left(\int\limits_{w \in J_x \cup J_x'} \left(\mu_{\widetilde{A} \cap \widetilde{B}}(x,w), \eta_{\widetilde{A} \cap \widetilde{B}}(x,w), \nu_{\widetilde{A} \cap \widetilde{B}}(x,w) \right) \mid w \right) \mid x, \quad J_x \cup J_x^{'} \subset [0,1]$$

With,

$$\mu_{\widetilde{A} \cap \widetilde{B}}(x,w) = \int\limits_{u \in J_x} \left(\int\limits_{\nu \in J_x'} \left(\mu_{\widetilde{A}}(x,u) \wedge \mu_{\widetilde{B}}(x,\nu) \right) \right) \mid T(u,v)$$

,

$$\eta_{\widetilde{A} \cap \widetilde{B}}(x,w) = \int\limits_{u \in J_x} \left(\int\limits_{\nu \in J_x'} \left(\eta_{\widetilde{A}}(x,u) \wedge \eta_{\widetilde{B}}(x,\nu) \right) \right) \mid T(u,v)$$

And

$$\nu_{\widetilde{A} \cap \widetilde{B}}(x,w) = \int\limits_{u \in J_x} \left(\int\limits_{\nu \in J_x'} \left(\nu_{\widetilde{A}}(x,u) \vee \nu_{\widetilde{B}}(x,\nu) \right) \right) \mid T(u,v)$$

Where $T(u,v)$ is the t-norm of u and v.

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The complement of $\widetilde{A}$ is $\widetilde{A}^c$ defined as:

$$\widetilde{A}^c = \int_{x\in X}\left(\int_{u\in J_x}\left(\nu_{\widetilde{A}}(x,u),\eta_{\widetilde{A}}(x,u),\mu_{\widetilde{A}}(x,u)\right)\mid u\right)\mid x,\quad J_x\subset[0,1]$$

In addition, there are some more operations on NT2FSs that are defined below.
$\widetilde{A}\subset\widetilde{B}$ iff $\mu_{\widetilde{A}}(x,u)\le\mu_{\widetilde{A}}(x,\nu)$, $\eta_{\widetilde{A}}(x,u)\le\eta_{\widetilde{A}}(x,\nu)$ and $\nu_{\widetilde{A}}(x,u)\ge\nu_{\widetilde{A}}(x,\nu)$ for all $x\in X$.
And
$\widetilde{A}=\widetilde{B}$ iff $\mu_{\widetilde{A}}(x,u)=\mu_{\widetilde{A}}(x,\nu)$, $\eta_{\widetilde{A}}(x,u)=\eta_{\widetilde{A}}(x,\nu)$ and $\nu_{\widetilde{A}}(x,u)=\nu_{\widetilde{A}}(x,\nu)$ for all $x\in X$.
We present the following example to illustrate the properties of NT2FS as mentioned below

Example 4.1. From the example 3.2, let $\widetilde{A}$ and $\widetilde{B}$ be two NT2FSs representing the set young.

Let $X=\{7,14,16\}$ be the set, and the primary membership of the points of X is $J_7=\{0.8,0.9,1.0\}$, $J_{14}=\{0.6,0.7,0.8\}$, and $J_{16}=\{0.4,0.5,0.6\}$, respectively. Then, the discrete NT2FS $\widetilde{A}$ is given by,

$$\widetilde{A}=\left\{\begin{array}{l}((7,0.8),0.1,0.1,0.3)+((14,0.8),0.2,0.00,0.5)+((16,0.8),0.4,0.2,0.3)+\\((7,0.9),0.4,0.1,0.2)+((14,0.9),0.35,0.45,0.10)+((16,0.9),0.28,0.10,0.4)+\\((7,1.0),0.4,0.5,0.01)+((14,1.0),0.14,0.27,0.30)+((16,1.0),0.20,0.29,0.4)+\\((7,0.6),0.2,0.4,0.3)+((14,0.6),0.13,0.20,0.4)+((16,0.6),0.36,0.18,0.40)+\\((7,0.7),0.41,0.00,0.45)+((14,0.7),0.37,0.12,0.48)+((16,0.7),0.31,0.14,0.49)+\\((7,0.8),0.28,0.10,0.34)+((14,0.8),0.45,0.00,0.50)+((16,0.8),0.27,0.4,0.29)+\\((7,0.4),0.27,0.20,0.30)+((14,0.4),0.21,0.50,0.24)+((16,0.4),0.28,0.40,0.38)+\\((7,0.5),0.20,0.41,0.32)+((14,0.5),0.28,0.45,0.30)+((16,0.5),0.27,0.47,0.38)+\\((7,0.6),0.24,0.47,0.34)+((14,0.6),0.23,0.49,0.31)+((16,0.6),0.24,0.49,0.30))\end{array}\right\}$$

And the discrete NT2FS $\widetilde{B}$ is given by,

$$\widetilde{B}=\left\{\begin{array}{l}((7,0.8),0.17,0.18,0.31)+((14,0.8),0.21,0.00,0.52)+((16,0.8),0.45,0.24,0.33)+\\((7,0.9),0.41,0.12,0.23)+((14,0.9),0.37,0.48,0.19)+((16,0.9),0.29,0.14,0.42)+\\((7,1.0),0.41,0.52,0.03)+((14,1.0),0.15,0.28,0.34)+((16,1.0),0.27,0.28,0.42)+\\((7,0.6),0.24,0.41,0.35)+((14,0.6),0.14,0.27,0.42)+((16,0.6),0.37,0.19,0.47)+\\((7,0.7),0.44,0.12,0.47)+((14,0.7),0.38,0.18,0.49)+((16,0.7),0.33,0.19,0.50)+\\((7,0.8),0.30,0.20,0.44)+((14,0.8),0.27,0.30,0.35)+((16,0.8),0.27,0.41,0.30)+\\((7,0.4),0.26,0.19,0.27)+((14,0.4),0.19,0.33,0.22)+((16,0.4),0.26,0.39,0.38)+\\((7,0.5),0.20,0.41,0.32)+((14,0.5),0.21,0.42,0.30)+((16,0.5),0.25,0.45,0.35)+\\((7,0.6),0.21,0.44,0.33)+((14,0.6),0.21,0.41,0.31)+((16,0.6),0.22,0.43,0.29))\end{array}\right\}$$

1. Then for $x=7$, the union operation of $\widetilde{A}$ and $\widetilde{B}$ is

$$\left(\mu_{\widetilde{A}\cup\widetilde{B}}(7,w),\eta_{\widetilde{A}\cup\widetilde{B}}(7,w),\nu_{\widetilde{A}\cup\widetilde{B}}(7,w)\right)=\left(\mu_{\widetilde{A}}(7,u),\eta_{\widetilde{A}}(7,u),\nu_{\widetilde{A}}(7,u)\right)\cup\left(\mu_{\widetilde{B}}(7,v),\eta_{\widetilde{B}}(7,v),\nu_{\widetilde{B}}(7,v)\right)$$

So,

$$\left\{\begin{array}{l}\mu_{\widetilde{A}\cup\widetilde{B}}(7,w)=\mu_{\widetilde{A}}(7,u)\vee_{\max}\mu_{\widetilde{B}}(7,v)\\\eta_{\widetilde{A}\cup\widetilde{B}}(7,w)=\eta_{\widetilde{A}}(7,u)\vee_{\max}\eta_{\widetilde{B}}(7,v)\\\nu_{\widetilde{A}\cup\widetilde{B}}(7,w)=\nu_{\widetilde{A}}(7,u)\vee_{\max}\nu_{\widetilde{B}}(7,v)\end{array}\right.$$

$$(\widetilde{A}\cup\widetilde{B})_{x=7}=\left\{\begin{array}{l}((7,0.8),0.1,0.1,0.3)\vee_{\max}((7,0.8),0.17,0.18,0.31)\\((7,0.9),0.4,0.1,0.2)\vee_{\max}((7,0.9),0.41,0.12,0.23)\\((7,1.0),0.4,0.5,0.01)\vee_{\max}((7,1.0),0.41,0.52,0.03)\\((7,0.6),0.2,0.4,0.3)\vee_{\max}((7,0.6),0.24,0.41,0.35)\\((7,0.7),0.41,0.00,0.45)\vee_{\max}((7,0.7),0.44,0.12,0.47)\\((7,0.8),0.28,0.10,0.34)\vee_{\max}((7,0.8),0.30,0.20,0.44)\\((7,0.4),0.27,0.20,0.30)\vee_{\max}((7,0.4),0.26,0.19,0.27)\\((7,0.5),0.20,0.41,0.32)\vee_{\max}((7,0.5),0.20,0.41,0.32)\\((7,0.6),0.24,0.47,0.34)\vee_{\max}((7,0.6),0.21,0.44,0.33)\end{array}\right\}$$

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$$(\widetilde{A}\cup\widetilde{B})_{x=7}=\left\{\begin{array}{l}((7,0.8),0.1,0.1,0.3)\vee_{\max}((7,0.8),0.17,0.18,0.31)=((7,0.8),0.17,0.10,0.3)\\((7,0.9),0.4,0.1,0.2)\vee_{\max}((7,0.9),0.41,0.12,0.23)=((7,0.9),0.41,0.1,0.2)\\((7,1.0),0.4,0.5,0.01))\vee_{\max}((7,1.0),0.41,0.52,0.03)=((7,1.0),0.41,0.5,0.01)\\((7,0.6),0.2,0.4,0.3)\vee_{\max}((7,0.6),0.24,0.41,0.35)=((7,0.6),0.24,0.4,0.3)\\((7,0.7),0.41,0.00,0.45)\vee_{\max}((7,0.7),0.44,0.12,0.47)=((7,0.7),0.44,0.00,0.45)\\((7,0.8),0.28,0.10,0.34)\vee_{\max}((7,0.8),0.30,0.20,0.44)=((7,0.8),0.28,0.10,0.34)\\((7,0.4),0.27,0.20,0.30)\vee_{\max}((7,0.4),0.26,0.19,0.27)=((7,0.4),0.27,0.19,0.27)\\((7,0.5),0.20,0.41,0.32)\vee_{\max}((7,0.5),0.20,0.41,0.32)=((7,0.5),0.20,0.41,0.32)\\((7,0.6),0.24,0.47,0.34)\vee_{\max}((7,0.6),0.21,0.44,0.33)=((7,0.6),0.24,0.44,0.33)\end{array}\right\}$$

In a similar way we find $(\widetilde{A}\cup\widetilde{B})_{x=14}$ and $(\widetilde{A}\cup\widetilde{B})_{x=16}$. Then, the union of $\widetilde{A}$ and $\widetilde{B}$ is defined as:

$$\widetilde{A}\cup\widetilde{B}=\left\{\ (\widetilde{A}\cup\widetilde{B})_{x=7},(\widetilde{A}\cup\widetilde{B})_{x=14},(\widetilde{A}\cup\widetilde{B})_{x=16}\ \right\}$$

2. Now, for $x=7$, the intersection operation of $\widetilde{A}$ and $\widetilde{B}$ is

$$(\mu_{\widetilde{A}\cap\widetilde{B}}(7,w),\eta_{\widetilde{A}\cap\widetilde{B}}(7,w),\nu_{\widetilde{A}\cap\widetilde{B}}(7,w))=(\mu_{\widetilde{A}}(7,u),\eta_{\widetilde{A}}(7,u),\nu_{\widetilde{A}}(7,u))\cap(\mu_{\widetilde{B}}(7,v),\eta_{\widetilde{B}}(7,v),\nu_{\widetilde{B}}(7,v))$$

So,

$$\left\{\begin{array}{c}\mu_{\widetilde{A}\cap\widetilde{B}}(7,w)=\mu_{\widetilde{A}}(7,u)\wedge_{\min}\mu_{\widetilde{B}}(7,v)\\\eta_{\widetilde{A}\cap\widetilde{B}}(7,w)=\eta_{\widetilde{A}}(7,u)\wedge_{\min}\eta_{\widetilde{B}}(7,v)\\\nu_{\widetilde{A}\cap\widetilde{B}}(7,w)=\nu_{\widetilde{A}}(7,u)\wedge_{\min}\nu_{\widetilde{B}}(7,v)\end{array}\right.$$

$$(\widetilde{A}\cap\widetilde{B})_{x=7}=\left\{\begin{array}{l}((7,0.8),0.1,0.1,0.3)\wedge_{\min}((7,0.8),0.17,0.18,0.31)\\((7,0.9),0.4,0.1,0.2)\wedge_{\min}((7,0.9),0.41,0.12,0.23)\\((7,1.0),0.4,0.5,0.01)\wedge_{\min}((7,1.0),0.41,0.52,0.03)\\((7,0.6),0.2,0.4,0.3)\wedge_{\min}((7,0.6),0.24,0.41,0.35)\\((7,0.7),0.41,0.00,0.45)\wedge_{\min}((7,0.7),0.44,0.12,0.47)\\((7,0.8),0.28,0.10,0.34)\wedge_{\min}((7,0.8),0.30,0.20,0.44)\\((7,0.4),0.27,0.20,0.30)\wedge_{\min}((7,0.4),0.26,0.19,0.27)\\((7,0.5),0.20,0.41,0.32)\wedge_{\min}((7,0.5),0.20,0.41,0.32)\\((7,0.6),0.24,0.47,0.34)\wedge_{\min}((7,0.6),0.21,0.44,0.33)\end{array}\right\}$$

$$(\widetilde{A}\cap\widetilde{B})_{x=7}=\left\{\begin{array}{l}((7,0.8),0.1,0.1,0.3)\wedge_{\min}((7,0.8),0.17,0.18,0.31)=((7,0.8),0.1,0.10,0.31)\\((7,0.9),0.4,0.1,0.2)\wedge_{\min}((7,0.9),0.41,0.12,0.23)=((7,0.9),0.4,0.1,0.23)\\((7,1.0),0.4,0.5,0.01))\wedge_{\min}((7,1.0),0.41,0.52,0.03)=((7,1.0),0.4,0.5,0.03)\\((7,0.6),0.2,0.4,0.3)\wedge_{\min}((7,0.6),0.24,0.41,0.35)=((7,0.6),0.2,0.4,0.35)\\((7,0.7),0.41,0.00,0.45)\wedge_{\min}((7,0.7),0.44,0.12,0.47)=((7,0.7),0.41,0.00,0.47)\\((7,0.8),0.28,0.10,0.34)\wedge_{\min}((7,0.8),0.30,0.20,0.44)=((7,0.8),0.28,0.10,0.44)\\((7,0.4),0.27,0.20,0.30)\wedge_{\min}((7,0.4),0.26,0.19,0.27)=((7,0.4),0.26,0.19,0.30)\\((7,0.5),0.20,0.41,0.32)\wedge_{\min}((7,0.5),0.20,0.41,0.32)=((7,0.5),0.20,0.41,0.32)\\((7,0.6),0.24,0.47,0.34)\wedge_{\min}((7,0.6),0.21,0.44,0.33)=((7,0.6),0.21,0.44,0.34)\end{array}\right\}$$

In a similar way we find $(\widetilde{A}\cap\widetilde{B})_{x=14}$ and $(\widetilde{A}\cap\widetilde{B})_{x=16}$. Then, the interaction of $\widetilde{A}$ and $\widetilde{B}$ is defined as:

$$\widetilde{A}\cap\widetilde{B}=\left\{\ (\widetilde{A}\cap\widetilde{B})_{x=7},(\widetilde{A}\cap\widetilde{B})_{x=14},(\widetilde{A}\cap\widetilde{B})_{x=16}\ \right\}$$

3. Similar to the above in a direct way, we find $\widetilde{A}^c$.

5 Conclusions

Neutrosophics Type-2 Fuzzy Sets are a useful addition to conventional fuzzy systems as they offer a more complex and nuanced way to manage uncertainty. In domains where a high tolerance for imprecision and incomplete information is required, their capacity to encompass uncertainty at several levels provides researchers and practitioners with a strong method for managing ambiguity. This presents a potential option for future study and applications.

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