



On Neutrosophic of BE-Algebra

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Abstract

The BE-Algebra was presented by Kim in 2007. After that, several authors studied this type of logic concept in algebra. In this paper, we introduce more properties and remarks of BE-Algebra. Note that $(A, *, 1)$ is called BE-algebra if $\forall a \in A, b, c \in A$: collect $a * a = 1, a * 1 = 1, 1 * a = a$ and $a * (b * c) = b * (a * c)$. In addition, a Neutrosophic BE-filter FI subset of the Neutrosophic BE-algebra is Neutrosophic BE-algebra AI is Neutrosophic BE-subalgebra AI. Some new results and the criterion to determine some properties of BE-algebra and several relationships with another algebra namely Hibert algebras (H-algebra).

Keywords: BE-algebra; Neutrosophic set; BE-filter; BE-subalgebra; Atom element

1. Introduction

In 1965, Zadeh [1] discussed the fuzzy theorem upon realizing the inability of fragile sets to deal with fully algebraic structures. Zadeh presented in his work everything related to this theory out of his desire to develop the mathematical structures at that time. At the end of this study, Zadeh recommended that this theory could be developed by exploiting its prominent weaknesses. In 1998, Samarandaj [2] took advantage of these points and developed this theory by introducing a new, interesting theory known as the Neutrosophic Theory as an extension of the Fuzziness Theory. Through this theory, Samarandaj addressed all the failures that occurred with Zadeh and highlighted the important strengths of this theory. This theory has gained the approval of a large number of researchers around the world by linking it with other branches of mathematics such as algebra [3, 4] topology [5,6], complex analysis [7,8], statistics [9,10], and many other branches [11,12]. Among these works, Agboola [13] presented the idea of the neutrosophic algebra. Al-Quran et al [14] presented the idea of the complex neutrosophic algebra theory. Romdhini et al [15, 16] hand over the idea of the neutrosophic ring theory. Zail et al [17] established neutrosophic Ω -BCK-algebra, BCK-algebra. Jun et al. [18] discovered neutrosophic \mathcal{N} -structures used to BCK/BCI-algebras. Al-Sharqi et al. [19, 20] used the idea of neutrosophic to relate to different algebraic structures. Abed et al. [21-24] mixes the idea of the neutrosophic with many algebraic concepts. As mentioned above we conclude from what was mentioned above that this idea applies to many algebraic concepts and is applicable to solving many problems of daily life [25-26].

In other side, some types of algebras have been studied by Isieki [27]. In 2007, Kim introduced BE-algebra as a generalization of BCK-algebra [28]. In addition, filter concept by details can find it in [29]. Note that several authors studied filter as a type of algebra by [30]. In [31], the authors studied commutative with bounded property of BE-algebras but in [32] Ciungu presented commutative pseudo-BE-algebras in details. More results of filter inside BE-algebras introduced by Meng [33]. Note that commutative ideals were studied in BE-algebras by Rezaci [34] such that let A be a BE-algebra, So, A is a commutative if $(x * y) * y = (y * x) * x$, and y in A .

In this work, we will work on merging the concept of neutrosophic with a new type of algebra to generate a new idea called Neutrosophic of BE-Algebra. Based on previous algebraic ideas mentioned in the second part of this work, we will work to present a large number of results about this idea presented in the third part of this work.

2. Preliminaries with some Tools

Definition (2.1). [10] Let U be an initial universe. Fuzzy set means $S: U \rightarrow [0,1]$. Therefore, we have a degree of membership of d in FU , d in U .

Example (2.2). Let $U = \{3, 5, 7, 8, 10\}$, so the FS $H = \{(3,0), (5, 0.1), (7, 0.2), (8, 0.8), (10, 1)\}$. Therefore, the number 2 is not maximum, but 2 has the membership equal 0.

Remark (2.3): By Example (2.2), we explain the neutrosophic concept by the following $L(I)=K+2I: K, 2 \in \mathbb{R}$ or, K is a specified part on $L(I)$ and ZI defined with $Z_1I+Z_2I=(Z_1+Z_2)I$.

Definition (2.3). We say N is a neutrosophic submodule of neutrosophic module M . Also, N is a neutrosophic small submodule of M .

Definition (2.4). [7] Let $\phi \neq D$ be a set. A fuzzy set $H = \{ \langle d, \mu_H(d) \rangle \mid d \text{ belong to } D \}$ is named membership function and denoted by the following:

$$\mu_H: D \rightarrow [0, 1], \mu_H(d) \text{ is the degree of membership of } d \text{ in the fuzzy subset } H \text{ with } d \text{ in } D.$$

Definition (2.5). [6, 7] Let H be a universal set. The neutrosophic H , in short Neu (H) is

$$B = \{ \langle \emptyset, T_K(\emptyset), I_K(\emptyset), F_K(\xi) \rangle : \emptyset \text{ in } H \} \ni T_K, I_K, F_K : H \rightarrow [0, 1].$$

such that T refer to True, I refer to Indeterminacy and F refer to False.

Let $(A, *, \circ)$ be an algebra. So, it is called neutrosophic BCK-algebra if $\{ [T((x*y) * (x*z))] * T(z*y) = 0, [I(x*y) * (x*z)] * I(z*y) = 0, [F(x*y) * (x*z)] * F(z*y) = 0 \}, \{ T[x * (x*y)] * T(y) = 0, I[x * (x*y)] * I(y) = 0, F[x * (x*y)] * F(y) = 0 \}, \{ T[(x*x) = 0], I[(x*x) = 0], F[(x*x) = 0] \}, \{ T[(0*x) = 0], I[(0*x) = 0], F[(0*x) = 0] \}$ and $\{ T[((x*y)=(y*x)) = 0, \text{ so } x=y], I[((x*y)=(y*x)) = 0, \text{ so } x=y], F[((x*y)=(y*x)) = 0, \text{ so } x=y] \}$ for all x, y, z in A .

3. Neutrosophic BE-algebra

In this section we focus of Neutrosophic BE-algebra when any element x belongs to BE-algebra is called neutrosophic Hilbert algebra and neutrosophic Quasi-algebra. However, before that we must present several concepts related to BE-algebra.

Definition 3.1. Note that $(A, *, 1)$ is called BE-algebra if $\forall a \in A, b, c \in A$: collect $1 - a * a = 1$

$$2 - a * 1 = 1$$

$$3 - 1 * a = a$$

$$4 - a * (b * c) = b * (a * c).$$

Remark 3.2. $(A, *, 1)$ of type (2,0) and a relation (\leq) on A defined by $a \leq b \leftrightarrow a * b = 1 \leftrightarrow a \dot{-} b = 1$

Example 3.3. Let $AI = \{ 1I, aI, cI, dI, eI \}$ define with binary operation by :

$$1I * 1I = 1I \quad aI * 1I = 1I \quad bI * 1I = 1I \quad cI * 1I = 1I \quad dI * 1I = 1I \quad eI * 1I = 1I$$

$$1I * aI = aI \quad aI * aI = 1I \quad bI * aI = 1I \quad cI * aI = 1I \quad dI * aI = 1I \quad eI * aI = 1I$$

$$1I * bI = bI \quad aI * bI = aI \quad bI * bI = 1I \quad cI * bI = bI \quad dI * bI = aI \quad eI * bI = 1I$$

$$1I * dI = bI \quad aI * dI = cI \quad bI * dI = cI \quad cI * dI = aI \quad dI * dI = 1I \quad eI * dI = 1I$$

$$1I * eI = eI \quad aI * eI = dI \quad bI * eI = cI \quad cI * eI = bI \quad dI * eI = aI \quad eI * eI = 1I$$

Therefore we say $(AI, *, 1)$ is Neutrosophic BE-algebra.

Remark 3.3. AI Neutrosophic BE-algebra is (a BE-algebra) if $\forall eI, bI \in AI$ and $(aI * bI = 1I \text{ and } bI * aI = 1I \rightarrow aI = bI)$. But Neutrosophic BE-algebra (BE-algebra) if

$$\forall a, b, c \in A, (bI * cI = 1I) \rightarrow ((aI * bI). (aI.cI) = 1I).$$

Example 3.4. Let $AI = \{ 1I, aI, cI \}$ with $*$ and defined by :

$$1I * 1I = 1I \quad aI * 1I = 1I \quad bI * 1I = 1I \quad cI * 1I = 1I$$

$$1I * aI = aI \quad aI * aI = 1I \quad bI * aI = aI \quad cI * aI = 1I$$

$$1I * bI = bI \quad aI * bI = bI \quad bI * bI = 1I \quad cI * bI = 1I$$

$$1I * cI = cI \quad aI * cI = bI \quad bI * cI = 1I \quad cI * cI = 1I$$

Then $(AI, *, 1I)$ is a Neutrosophic BE-algebra.

Note $(AI, *, 1I)$ in Example 3.5 is not (BE-algebra) and not (BE-algebra).

Definition 3.5. By $A^\wedge ICAI$ (BE-algebra) is called Neutrosophic BE-subalgebra if :

$$\forall aI, bI \in AI ; aI \in A^\wedge I, bI \in A^\wedge I \rightarrow aI, bI \in A^\wedge I$$

$1I \in A^\wedge I$, so BE-subalgebra of BE-algebra is not zero.

Definition 3.6. [6] Any Neutrosophic BE-algebra AI is called Neutrosophic self- distributive (N-S-distributive) if : $\forall aI, bI, cI \in AI * aI * (bI * cI) = (aI * bI) * (aI * cI)$.

From (1) and (2) in Definition 3.1, we get the following:

$$\forall a \in AI \rightarrow aI \leq aI$$

$$\forall a \in AI \rightarrow aI \leq 1I$$

Also, from (3), (4) in Definition 3.1, we get:

$$\forall aI \in AI \rightarrow aI \leq aI$$

$$\forall aI, dI \in AI \rightarrow aI \leq (aI * bI) * bI.$$

On the other hand, when AI is (N-S-Distributive) BE-algebra, this means $\forall aI, bI, cI \in AI \rightarrow aI \leq bI \leq cI \rightarrow aI \leq cI$

Remark 3.7. Any Neutrosophic BE-algebra AI is called Neutrosophic transitive if

$$\forall aI, bI, cI \in AI \rightarrow (bI * cI) \leq (aI * bI) * (aI * cI).$$

Therefore, if AI is (N-S-Distributive) BE-algebra, AI is a Neutrosophic transitive and hence if:

$$\forall aI, bI, cI \in AI : aI \leq bI \rightarrow (cI * aI \leq cI * bI) \wedge bI * cI \leq aI * cI.$$

Then AI is called transitive Neutrosophic BE-algebra.

Definition 3.8. Let $\emptyset \neq F \subseteq AI$ such that $AI = (AI, *, 1)$ is BE-algebra. Then FI is BE-filter in AI :

$$1) 1I \in F$$

$$2) \forall aI, bI \in F$$

$$AI : 1) 1 \in F \quad 2) \forall aI, bI \in AI ; aI \in FI \wedge aI * bI \in FI \rightarrow bI \in FI.$$

Example 3.9. $F_1 = \{1I\} \subseteq AI$ (Neutrosophic BE-algebra) is Neutrosophic BE-filter.

Remark 3.10. A Neutrosophic FI is structure in Neutrosophic BE-algebra AI is also has the following:

$$1) \forall aI, bI \in AI ; aI \in FI \wedge aI \leq bI \rightarrow bI \in FI.$$

$$2) \forall aI, bI \in AI ; aI \in FI \wedge bI \in FI \rightarrow aI * bI \in FI$$

Note that Neutrosophic BE-filter FI subset of the Neutrosophic BE-algebra is Neutrosophic BE-algebra AI is Neutrosophic BE-subalgebra AI .

Example 3.11. Let $AI = \{1I, aI, bI, cI, dI, eI\}$.

Let $F_1 = \{1I\} \subseteq AI$ Neutrosophic BE-algebra with filter property.

Let $F_2 = \{1I, aI\} \subseteq AI$ is a filter.

Let $F_3 = \{1I, cI\} \subseteq AI$ is a filter.

Let $F_4 = \{1I, aI, bI\} \subseteq AI$ is a filter.

Let $F_5 = \{1I, cI, dI\} \subseteq AI$ is a filter.

Let $F_6 = \{1I, aI, bI, cI\} \subseteq AI$ is a filter.

Let $F_7 = \{1I, cI, dI, eI\} \subseteq AI$ is a filter.

It is clear that according to Remark 3.10 $F_1, F_2, F_3, F_4, F_5, F_6, F_7$ is a Neutrosophic BE-filter because:

F_1I is not a Neutrosophic BE-filter, because:

$$aI \in F_1I \text{ and } aI * bI = aI \in F_1I \text{ but } bI \notin F_1I.$$

F_2I, F_3I . and F_4I . are Neutrosophic BE-filters in AI .

F_5I . and F_6I . are Neutrosophic BE-filters in AI .

Example 3.12. Let $AI = \{1I, aI, bI, cI, dI, eI\}$ be a set with an operation $*$ and determined by :

$$1I * 1I = 1I \quad aI * 1I = 1I \quad bI * 1I = 1I \quad cI * 1I = 1I \quad dI * 1I = 1I$$

$$1I * aI = aI \quad aI * aI = 1I \quad bI * aI = aI \quad cI * aI = aI \quad dI * aI = 1I$$

$$1I * bI = bI \quad aI * bI = bI \quad bI * bI = 1I \quad cI * bI = bI \quad dI * bI = 1I$$

$$1I * cI = cI \quad aI * cI = cI \quad bI * cI = cI \quad cI * cI = 1I \quad dI * cI = 1I$$

$$1I * dI = dI \quad aI * dI = dI \quad bI * dI = cI \quad cI * dI = bI \quad dI * dI = 1I$$

Then AI is N-S distributive BE-algebra. Note that (\leq) is given by:

$$\{(1I, 1I), (aI, 1I), (aI, aI), (bI, 1I), ((bI, bI), (cI, 1I),$$

$$(cI, aI), (cI, cI), (dI, 1I), (dI, aI), (dI, bI), (dI, cI), (dI, dI)\}.$$

and let the following Neutrosophic subset of AI :

$$F_0 = \{1I\}, F_1 = \{1I, aI\}, F_2 = \{1I, bI\}, F_3 = \{1I, aI, cI\}.$$

are Neutrosophic BE-filters. But $F_4 = \{1I, bI, dI\}$ and $F_5 = \{1I, aI, cI, dI\}$ are not BE-filters. Now we present a definition of Neutrosophic atoin Neutrosophic BE-algebra.

Definition 3.13. An element $aI \neq 1I \in AI$ is called a tom if $bI \in AI, aI \leq bI$ then $bI = aI$ or $bI = 1I$.

Note: $O(AI)$ refer to the set of all atoms in AI .

Proposition 3.14. $O(AI)$ is anti-chain.

Proof : Suppose that $aI, bI \in O(A) \ni aI \neq bI$. Assume that $bI = aI$ or $bI = 1I$. clear t is that all the above cases is not possible. So aI, bI are not comparable.

Theorem 3.15. Let AI be a Neutrosophic BE-algebra; $aI \in AI \ni 1I \neq aI$. So aI an atom if $\ni \{1I, aI\}$ is a Neutrosophic filter inside AI .

Theorem 3.16. Let AI be a neutrosophic BE-algebra and let $aI \in O(AI)$. Then

$$1) bI \in AI \rightarrow (aI * bI) * bI = aI \vee (aI * bI) * bI = 1I.$$

$$2) bI \in AI \rightarrow bI * aI = aI \vee bI * aI = 1I.$$

Theorem 3.17. Let AI be a N-S-distributive(aBE)-algebra and with $aI \in AI$. If aI satisfies (2) in Theorem 3.17, so aI is a Neutrosophic in AI .

Proof: Let $bI \in AI \in bI$ lest than and equal aI . So $1I = bI * bI \leq (bI) * (aI) \leq 1I$. Then $abI = 1I$. AI . So $aI \leq bI$. Hence $aI = bI$. Thus bI is an AI .

be any set with operation $*$ and defined by:

$$1I * 1I = 1I \quad aI * 1I = 1I \quad bI * 1I = 1I \quad cI * 1I = 1I \quad dI * 1I = 1I$$

$$1I * aI = aI \quad aI * aI = 1I \quad bI * aI = aI \quad cI * aI = aI \quad dI * aI = 1I$$

$$1I * bI = bI \quad aI * bI = bI \quad bI * bI = 1I \quad cI * bI = bI \quad dI * bI = 1I$$

$$1I * cI = cI \quad aI * cI = cI \quad bI * cI = cI \quad cI * cI = 1I \quad dI * cI = 1I$$

$$1I * dI = dI \quad aI * dI = dI \quad bI * dI = cI \quad cI * dI = bI \quad dI * dI = 1I$$

Then $(AI, *, 1I)$ is aEB-algebra and $\leq \{1I, 1I\}, (aI, 1I), (aI, aI), (bI, 1I), (cI, 1I), (cI, bI), (cI, cI)\}$. Note that $\{1I, aI\}$ and $\{1I, bI\}$ are subsets of BE-filter of AI .

Corollary 3.18. $O(AI) \cup \{1I\} \subseteq$ Neutrosophic BE_algebra AI is Neutrosophic BE-subalgebra.

Example 3.19. Let $AI = \{1I, aI, bI, cI, dI, eI\}$ be a set with an operation $*$ and:

$$\begin{aligned}
1I * 1I &= 1I & aI * 1I &= 1I & bI * 1I &= 1I & cI * 1I &= 1I & dI * 1I &= 1I \\
1I * aI &= aI & aI * aI &= 1I & bI * aI &= aI & cI * aI &= aI & dI * aI &= 1I \\
1I * bI &= bI & aI * bI &= bI & bI * bI &= 1I & cI * bI &= bI & dI * bI &= 1I \\
1I * cI &= cI & aI * cI &= cI & bI * cI &= cI & cI * cI &= 1I & dI * cI &= 1I \\
1I * dI &= dI & aI * dI &= dI & bI * dI &= cI & cI * dI &= bI & dI * dI &= 1I
\end{aligned}$$

so, $(AI, *, 1I)$ is a Neutrosophic BE-algebra. (\leq) Note that defined by:

$$\leq \{(1I, 1I), (aI, aI), (aI, 1I), (bI, bI), (bI, 1I), (cI, cI), (cI, 1I), (cI, bI), (dI, dI),$$

$(dI, 1I)\}$. Also F_1, F_2, F_3, F_4 and F_5 are Neutrosophic subset of $AI \ni F_1 I = \{1I\}$, $F_2 I = \{1I, aI\}$, $F_3 I = \{1I, bI\}$, $F_4 I = \{1I, cI\}$ and $F_5 I = \{1I, dI\}$ are Neutrosophic BE-filter inside AI . Therefore, by theorem 3.16, every element without $1I$ is Neutrosophic atoms inside Neutrosophic BE-algebra.

4. Conclusion and final Remarks

The concept of Neutrosophic BE-algebra can be considered, as a generalization and extension of Neutrosophic BCK-algebra are a BE-algebra and Neutrosophic BE-algebra. Our report presented of Neutrosophic atoms inside Neutrosophic BE-algebra. In Theorem 3.15, we satisfied an important property which say if AI is a Neutrosophic BE-algebra; $aI \in AI$ such that $1I$ not equal aI is atom if $\{1I, aI\}$ is a Neutrosophic filter inside AI . Also, we proved that if AI is a Neutrosophic BE-algebra; $aI \in AI \ni 1I \neq aI$. So aI an atom if $\exists \{1I, aI\}$ is a Neutrosophic filter inside AI . In addition, several different of extends, a Neutrosophic BE-algebra were studied. Some important new results discussed in this report.

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