On the Classification of the Group of Units for Some Symbolic m-Plithogenic Rings

Lee Xu
University of Chinese Academy of Sciences, CAS, Mathematics Department, Beijing, China
Leexu1244@yahoo.com

Abstract

This paper is dedicated to study and to find the symbolic m-plithogenic units in many symbolic plithogenic rings for some special values of m, where we present a full classification of many different symbolic n-plithogenic group of units as direct products of well-known groups by building suitable and well-defined algebraic isomorphisms.

Keywords: Algebraic unit; Group of units; Classification; Symbolic plithogenic ring

1. Introduction

The computation of invertible elements (units) in commutative algebraic rings is one of the most important problems in algebra, as well as the classification of the group generated by all units in a ring. The theory of logical rings based on neutrosophic sets and their generalizations began with many works, see [14-16, 20-22], where we can find the concept of neutrosophic ring, neutrosophic ideals, and homomorphisms. In [1], symbolic 2-plithogenic rings were defined for the first time, and then they were generalized for higher orders [23-27] with many related algebraic substructures based on them, such as spaces, matrices, and special elements [2-6, 17-19]. In [7-13], the group of units of several logical rings was studied, such as n-cyclic refined neutrosophic units, and plithogenic units. This has motivated us study and to find the symbolic m-plithogenic units in many symbolic plithogenic rings for some special values of m, where we present a full classification of many different symbolic n-plithogenic group of units as direct products of well-known groups by building suitable and well-defined algebraic isomorphisms.

2. Main Discussion

Definition:
Let R be a ring with $U(R)$ as its group of units.
Let $3-sp_R=\{x+yp_1+zp_2+tp_3\ ;\ x,y,z,t\in R\}$ be the corresponding 3-plithogenic ring, then:
$U(3-sp_R)=\{X\in 3-sp_R\ ;\ \exists\, Y\in 3-sp_R : XY=YX=1\}$ is called the group of units of $3-sp_R$.
Theorem:
$U(3-sp_R)\cong U(R)\times U(R)\times U(R)\times U(R)$
Proof:
Define $f: 3-sp_R\to R\times R\times R\times R$ such that:
$f(x+yp_1+zp_2+tp_3)=(x,x+y,x+y+z,x+y+z+t)$,
If $x_0+y_0p_1+z_0p_2+t_0p_3=x_1+y_1p_1+z_1p_2+t_1p_3$, then $x_0=x_1, y_0=y_1, z_0=z_1, t_0=t_1$, hence $f(x_0+y_0p_1+z_0p_2+t_0p_3)=f(x_1+y_1p_1+z_1p_2+t_1p_3)$.

DOI: https://doi.org/10.54216/NIF.040103
Received: December 11, 2023 Accepted: June 24, 2024

Assume that: $X = x_0 + y_0p_1 + z_0p_2 + t_0p_3, Y = x_1 + y_1p_1 + z_1p_2 + t_1p_3$, we have:$X + Y = (x_0 + x_1) + (y_0 + y_1)p_1 + (z_0 + z_1)p_2 + (t_0 + t_1)p_3$,
$f(X + Y) = (a, b, c, d)$; where:

$$\begin{cases} a = x_0 + x_1, b = x_0 + x_1 + y_0 + y_1 \\ c = x_0 + x_1 + y_0 + y_1 + z_0 + z_1 \\ d = x_0 + x_1 + y_0 + y_1 + z_0 + z_1 + t_0 + t_1 \end{cases}$$

Thus $f(X + Y) = (x_0, x_0 + y_0, x_0 + y_0 + z_0, x_0 + y_0 + z_0 + t_0) + (x_1, x_1 + y_1, x_1 + y_1 + z_1, x_1 + y_1 + z_1 + t_1) = f(X) + f(Y)$.
Also, $X \cdot Y = x_0x_1 + p_1(x_0y_1 + y_0x_1 + y_0y_1) + p_2(x_0z_1 + y_0z_1 + z_0z_1 + z_0x_1 + z_0y_1) + p_3(x_0t_1 + y_0t_1 + z_0t_1 + t_0x_1 + t_0y_1 + t_0z_1 + t_0t_1)$,
$f(XY) = (x_0, x_0 + y_0, x_0 + y_0 + z_0, x_0 + y_0 + z_0 + t_0)(x_1, x_1 + y_1, x_1 + y_1 + z_1, x_1 + y_1 + z_1 + t_1) = f(X)f(Y)$.
If $f(X) = 0$, then $X = 0$.
For every $(x, y, z, t) \in (R)^4$, there exists:
$X = x + (y - x)p_1 + (z - y)p_2 + (t - z)p_3 \in 3 - sp_R$ such that:
$f(X) = (x, y, z, t)$, hence (f) is a ring isomorphism.
This means that $3 - sp_R \cong R \times R \times R \times R$, and:
$U(3 - sp_R) \cong U(R) \times U(R) \times U(R) \times U(R)$.

Example:

For $R = (Z_3, +,\cdot)$, then $3 - sp_{Z_3} = \{x + yp_1 + zp_2 + tp_3 \ ; \ x, y, z, t \in Z_3\}$.
$U(3 - sp_{Z_3}) \cong Z_2 \times Z_2 \times Z_2 \times Z_2$.

To find all units in $3 - sp_{Z_3}$, we write the units of $(Z_3)^4$:

$e_1 = (1,1,1,1), e_2 = (1,1,1,2)$,
$e_3 = (1,1,2,1), e_4 = (1,2,1,1)$,
$e_5 = (2,1,1,1), e_6 = (2,2,2,2)$,
$e_7 = (2,2,2,1), e_8 = (2,2,1,2)$,
$e_9 = (2,1,2,2), e_{10} = (1,2,2,2)$,
$e_{11} = (1,1,2,2), e_{12} = (1,2,1,2)$,
$e_{13} = (2,2,1,1), e_{14} = (2,1,2,1)$,
$e_{15} = (2,1,1,2), e_{16} = (1,2,2,1)$.
Thus: $U(3 - sp_R) = \{f^{-1}(e_1) = 1, f^{-1}(e_2) = 1 + p_3$,
$f^{-1}(e_3) = 1 + p_2 + 2p_3, f^{-1}(e_4) = 1 + p_1 + 2p_2$,
$f^{-1}(e_5) = 2 + 2p_1, f^{-1}(e_6) = 2$,
$f^{-1}(e_7) = 2 + 2p_3, f^{-1}(e_8) = 2 + 2p_2 + p_3$,
$f^{-1}(e_9) = 2 + 2p_1 + p_2, f^{-1}(e_{10}) = 1 + p_1$,
$f^{-1}(e_{11}) = 1 + p_2, f^{-1}(e_{12}) = 1 + p_1 + 2p_2 + p_3$,
$f^{-1}(e_{13}) = 2 + 2p_2, f^{-1}(e_{14}) = 2 + 2p_1 + p_2 + 2p_3$,
$f^{-1}(e_{15}) = 2 + 2p_1 + p_3, f^{-1}(e_{16}) = 1 + p_1 + 2p_3\}$.

Example:

For $R = (Z_4, +,\cdot)$, $U(3 - sp_{Z_4}) \cong Z_2 \times Z_2 \times Z_2 \times Z_2$.
The units of $R \times R \times R \times R$ are:
$e_1 = (1,1,1,1), e_2 = (1,1,1,3)$,
$e_3 = (1,1,3,1), e_4 = (1,3,1,1)$,
$e_5 = (3,1,1,1), e_6 = (3,3,3,3)$,
$e_7 = (3,3,3,1), e_8 = (3,3,1,3)$,
$e_9 = (3,1,3,3), e_{10} = (1,3,3,3)$,
$e_{11} = (1,1,3,3), e_{12} = (3,3,1,1)$,
$e_{13} = (1,3,3,1), e_{14} = (3,1,1,3)$,
$e_{15} = (3,1,3,1), e_{16} = (1,3,1,3)$.

The units of $3 - sp_{Z_4}$ are:

$U(3 - sp_{Z_4}) = \{f^{-1}(e_1) = 1, f^{-1}(e_2) = 1 + 2p_3$,
$f^{-1}(e_3) = 1 + 2p_2 + 2p_4, f^{-1}(e_4) = 1 + 2p_1 + 2p_2$,
$f^{-1}(e_5) = 3 + 2p_1, f^{-1}(e_{11}) = 1 + 2p_2, f^{-1}(e_7) = 3 + 2p_4$,

DOI: https://doi.org/10.54216/NIF.040103
Received: December 11, 2023 Accepted: June 24, 2024

$f^{-1}(e_8)=3+2p_2+2p_4, f^{-1}(e_9)=3+2p_1+2p_2,$
$f^{-1}(e_{10})=1+2p_1, f^{-1}(e_6)=3, f^{-1}(e_{12})=3+2p_2,$
$f^{-1}(e_{13})=1+2p_1+2p_4, f^{-1}(e_{14})=3+2p_1+2p_4,$
$f^{-1}(e_{15})=3+2p_1+2p_2+2p_4, f^{-1}(e_{16})=1+2p_1+2p_2+2p_4\}.$

Example:

For $R=(Z_1,+,\cdot)$, $U(3-sp_Z)\cong Z_2\times Z_2\times Z_2\times Z_2$.

The units of Z^4 are:

$e_1=(1,1,1,1), e_2=(1,1,1,-1),$
$e_3=(1,1,-1,1), e_4=(1,-1,1,1), e_5=(-1,1,1,1),$
$e_6=(-1,-1,-1,-1), e_7=(-1,-1,-1,1),$
$e_8=(-1,-1,1,-1), e_9=(-1,1,-1,-1),$
$e_{10}=(1,-1,-1,-1), e_{11}=(1,1,-1,-1),$
$e_{12}=(-1,-1,1,1), e_{13}=(1,-1,-1,1), e_{14}=(-1,1,1,-1),$
$e_{15}=(-1,1,-1,1), e_{16}=(1,-1,1,-1).$

The units of $3-sp_Z$ are:

$U(3-sp_Z)=\{f^{-1}(e_1)=1, f^{-1}(e_2)=1-2p_3,$
$f^{-1}(e_3)=1-2p_2+2p_3, f^{-1}(e_4)=1-2p_1+2p_2,$
$f^{-1}(e_5)=-1+2p_1, f^{-1}(e_6)=-1,$
$f^{-1}(e_7)=-1+2p_3, f^{-1}(e_8)=-1+2p_2-2p_3,$
$f^{-1}(e_9)=-1+2p_1-2p_2, f^{-1}(e_{10})=1-2p_1,$
$f^{-1}(e_{11})=1-2p_2, f^{-1}(e_{12})=-1+2p_2, f^{-1}(e_{13})=1-2p_1+2p_3,$
$f^{-1}(e_{14})=-1+2p_1-2p_3, f^{-1}(e_{15})=-1+2p_1-2p_2+2p_3, f^{-1}(e_{16})=1-2p_1+2p_2-2p_3\}.$

Example:

For $R=(Z_5,+,\cdot)$, then $U(R)\cong Z_4$and $U(3-sp_{Z_5})\cong Z_4\times Z_4\times Z_4\times Z_4$.

We will find 101 units of 256 possible units:

$e_1=(1,1,1,1), e_2=(1,1,1,2), e_3=(1,1,2,1),$
$e_4=(1,2,1,1), e_5=(2,1,1,1), e_6=(1,1,1,3),$
$e_7=(1,1,3,1), e_8=(1,3,1,1), e_9=(3,1,1,1),$
$e_{10}=(3,3,3,3), e_{11}=(1,1,1,4), e_{12}=(1,1,4,1),$
$e_{13}=(1,4,1,1), e_{14}=(4,1,1,1), e_{15}=(4,4,4,4),$
$e_{16}=(2,2,2,1), e_{17}=(2,2,1,2), e_{18}=(2,1,2,2),$
$e_{19}=(1,2,2,2), e_{20}=(3,3,3,1), e_{21}=(3,3,1,3),$
$e_{22}=(3,1,3,3), e_{23}=(1,3,3,3)e_{24}=(4,4,4,1),$
$e_{25}=(4,4,1,4), e_{26}=(4,1,4,4), e_{27}=(1,4,4,4),$
$e_{28}=(2,2,2,3), e_{29}=(2,2,3,2), e_{30}=(2,3,2,2),$
$e_{31}=(3,2,2,2), e_{32}=(2,2,2,4), e_{33}=(2,2,4,2),$
$e_{34}=(2,4,2,2), e_{35}=(4,2,2,2), e_{36}=(3,3,3,4),$
$e_{37}=(3,3,4,3), e_{38}=(3,4,3,3), e_{39}=(4,3,3,3),$
$e_{40}=(1,1,2,2), e_{41}=(2,2,1,1), e_{42}=(1,2,2,1),$
$e_{43}=(2,1,1,2), e_{44}=(1,2,1,2), e_{45}=(2,1,2,1),$
$e_{46}=(3,3,1,1), e_{47}=(1,1,3,3), e_{48}=(3,1,1,3),$
$e_{49}=(1,3,3,1), e_{50}=(3,1,3,1), e_{51}=(1,3,1,3),$
$e_{52}=(4,4,1,1), e_{53}=(1,1,4,4), e_{54}=(1,4,4,1),$
$e_{55}=(4,1,1,4), e_{56}=(1,4,1,4), e_{57}=(4,1,4,1),$
$e_{58}=(2,2,3,3), e_{59}=(3,3,2,2), e_{60}=(2,3,3,2),$
$e_{61}=(3,2,2,3), e_{62}=(3,2,3,2), e_{63}=(2,3,2,3),$
$e_{64}=(2,2,4,4), e_{65}=(4,4,2,2), e_{66}=(2,4,4,2),$
$e_{67}=(4,2,2,4), e_{68}=(2,4,2,4), e_{69}=(4,2,4,2),$
$e_{70}=(3,3,4,4), e_{71}=(4,4,3,3), e_{72}=(4,3,3,4),$
$e_{73}=(3,4,4,3), e_{74}=(3,4,3,4), e_{75}=(4,3,4,3),$
$e_{76}=(2,2,2,2), e_{77}=(4,4,4,4), e_{78}=(1,2,3,4),$
$e_{79}=(2,1,3,4), e_{80}=(1,2,4,3), e_{81}=(2,1,4,3),$
$e_{82}=(1,3,2,4), e_{83}=(1,3,4,2)e_{84}=(3,1,2,4),$

DOI: https://doi.org/10.54216/NIF.040103
Received: December 11, 2023 Accepted: June 24, 2024

$e_{85}=(3,1,4,2), e_{86}=(1,4,2,3), e_{87}=(1,4,3,2),$
$e_{88}=(4,1,2,3), e_{89}=(4,1,3,2), e_{90}=(2,3,1,4),$
$e_{91}=(2,3,4,1), e_{92}=(3,2,1,4), e_{93}=(3,2,4,1)$
$e_{94}=(2,4,1,3), e_{95}=(2,4,3,1), e_{96}=(4,2,1,3),$
$e_{97}=(4,2,3,1), e_{98}=(3,4,1,2), e_{99}=(3,4,2,1),$
$e_{100}=(4,3,1,2), e_{101}=(4,3,2,1).$

The corresponding units of $U(3-sp_{Z_5})$ are:

$f^{-1}(e_1)=1, f^{-1}(e_2)=1+p_3, f^{-1}(e_3)=1+p_2+4p_3,$
$f^{-1}(e_4)=1+p_1+4p_2, f^{-1}(e_5)=2+4p_1, f^{-1}(e_6)=1+2p_3,$
$f^{-1}(e_7)=1+2p_2+3p_3, f^{-1}(e_8)=1+2p_1+3p_2, f^{-1}(e_9)=3+3p_1, f^{-1}(e_{10})=3, f^{-1}(e_{11})=1+$ $+3p_3, f^{-1}(e_{12})=1+3p_2+2p_3, f^{-1}(e_{13})=1+3p_1+2p_2, f^{-1}(e_{14})=4+2p_1, f^{-1}(e_{15})=4, f^{-1}(e_{16})=2+$ $4p_3, f^{-1}(e_{17})=2+4p_2+p_3, f^{-1}(e_{18})=2+4p_1+p_2, f^{-1}(e_{19})=1+p_1, f^{-1}(e_{20})=3+3p_3, f^{-1}(e_{21})=3+$ $3p_2+2p_3, f^{-1}(e_{22})=3+3p_1+2p_2, f^{-1}(e_{23})=1+2p_1, f^{-1}(e_{24})=4+2p_3, f^{-1}(e_{25})=4+2p_2+$ $3p_3, f^{-1}(e_{26})=4+2p_1+3p_2, f^{-1}(e_{27})=1+3p_1, f^{-1}(e_{28})=2+p_3, f^{-1}(e_{29})=2+p_2+4p_3, f^{-1}(e_{30})=$ $2+p_1+4p_2, f^{-1}(e_{31})=3++4p_1, f^{-1}(e_{32})=2+2p_3, f^{-1}(e_{33})=2+2p_2+3p_3, f^{-1}(e_{34})=2+2p_1+$ $3p_2, f^{-1}(e_{35})=4+3p_1, f^{-1}(e_{36})=3+p_3, f^{-1}(e_{37})=3+p_2+4p_3, f^{-1}(e_{38})=3+p_1+4p_2, f^{-1}(e_{39})=$ $4+4p_1, f^{-1}(e_{40})=1+p_2, f^{-1}(e_{41})=2+4p_2, f^{-1}(e_{42})=1+p_1+4p_3, f^{-1}(e_{43})=2+4p_1+p_3, f^{-1}(e_{44})=$ $1+p_1+4p_2+p_3, f^{-1}(e_{45})=2+4p_1+p_2+4p_3, f^{-1}(e_{46})=3+3p_2, f^{-1}(e_{47})=1+2p_2, f^{-1}(e_{48})=3+$ $3p_1+2p_3, f^{-1}(e_{49})=1+2p_1+3p_3, f^{-1}(e_{50})=3+3p_1+2p_2+3p_3, f^{-1}(e_{51})=1+2p_1+3p_2+$ $2p_3, f^{-1}(e_{52})=4+2p_2, f^{-1}(e_{53})=1+3p_2, f^{-1}(e_{54})=1+3p_1+2p_3, f^{-1}(e_{55})=4+2p_1+3p_3, f^{-1}(e_{56})=$ $1+3p_1+2p_2+3p_3, f^{-1}(e_{57})=4+2p_1+3p_2+2p_3, f^{-1}(e_{58})=2+p_2, f^{-1}(e_{59})=3+4p_2, f^{-1}(e_{60})=2+$ $p_1+4p_3, f^{-1}(e_{61})=3+4p_1+p_3, f^{-1}(e_{62})=3+p_2+4p_3, f^{-1}(e_{63})=2+p_1+4p_2+p_3, f^{-1}(e_{64})=2+$ $2p_2, f^{-1}(e_{65})=4+3p_2, f^{-1}(e_{66})=2+2p_1+3p_3, f^{-1}(e_{67})=4+3p_1+2p_3, f^{-1}(e_{68})=2+2p_1+3p_2+$ $2p_1+2p_3, f^{-1}(e_{69})=4+3p_1+2p_2+3p_3, f^{-1}(e_{70})=3+p_2, f^{-1}(e_{71})=4+4p_2, f^{-1}(e_{72})=4+4p_1+$ $p_3, f^{-1}(e_{73})=3+p_1+4p_3, f^{-1}(e_{74})=3+p_1+4p_2+p_3, f^{-1}(e_{75})=4+4p_1+p_2+4p_3, f^{-1}(e_{76})=$ $2, f^{-1}(e_{77})=4, f^{-1}(e_{78})=1+p_1+p_2+p_3, f^{-1}(e_{79})=2+4p_1+2p_2+p_3, f^{-1}(e_{80})=1+p_1+2p_2+$ $4p_3, f^{-1}(e_{81})=2+4p_1+3p_2+4p_3, f^{-1}(e_{82})=1+2p_1+4p_2+2p_3, f^{-1}(e_{83})=1+2p_1+p_2+$ $3p_3, f^{-1}(e_{84})=3+3p_1+p_2+2p_3, f^{-1}(e_{85})=3+3p_1+3p_2+3p_3, f^{-1}(e_{86})=1+3p_1+3p_2+$ $p_3, f^{-1}(e_{87})=1+3p_1+4p_2+4p_3, f^{-1}(e_{88})=4+2p_1+p_2+p_3, f^{-1}(e_{89})=4+2p_1+2p_2+$ $4p_3, f^{-1}(e_{90})=2+p_1+3p_2+3p_3, f^{-1}(e_{91})=2+p_1+p_2+2p_3, f^{-1}(e_{92})=3+4p_1+4p_2+$ $3p_3, f^{-1}(e_{93})=3+4p_1+2p_2+2p_3, f^{-1}(e_{94})=2+2p_1+2p_2+2p_3, f^{-1}(e_{95})=2+2p_1+4p_2+$ $3p_3, f^{-1}(e_{96})=4+3p_1+4p_2+2p_3, f^{-1}(e_{97})=4+3p_1+p_2+3p_3, f^{-1}(e_{98})=3+p_1+2p_2+$ $p_3, f^{-1}(e_{99})=3+p_1+3p_2+4p_3, f^{-1}(e_{100})=4+4p_1+3p_2+p_3, f^{-1}(e_{101})=4+4p_1+4p_2+4p_3.$
By continuoing the same argument, we get the rest of the units.

3. Results

We will write all the computed units in table called the units table of the symbolic plithogenic ring.

Table 1: Units table of $3-sp_{Z_3}$

Unit in $(Z_3)^4$	Corresponding Unit in $3-sp_{Z_3}$	
(1,1,1,1)	1	1
(1,1,1,2)	$1+p_3$	2
(1,1,2,1)	$1+p_2+2p_3$	3
(1,2,1,1)	$1+p_1+2p_2$	4
(2,1,1,1)	$2+2p_1$	5
(2,2,2,2)	2	6
(2,2,2,1)	$2+2p_3$	7
(2,2,1,2)	$2+2p_2+p_3$	8
(2,1,2,2)	$2+2p_1+p_2$	9
(1,2,2,2)	$1+p_1$	10
(1,1,2,2)	$1+p_2$	11

DOI: https://doi.org/10.54216/NIF.040103
Received: December 11, 2023 Accepted: June 24, 2024

(1,2,1,2)	$1 + p_1 + 2p_2 + p_3$	12
(2,2,1,1)	$2 + 2p_2$	13
(2,1,2,1)	$2 + 2p_1 + p_2 + 2p_3$	14
(2,1,1,2)	$2 + 2p_1 + p_3$	15
(1,2,2,1)	$1 + p_1 + 2p_3$	16

Table 2: Units table of $3 - sp_{Z_4}$

Unit in $(Z_4)^4$	**Corresponding Unit in $3 - sp_{Z_4}$**	
(1,1,1,1)	1	1
(1,1,1,3)	$1 + 2p_3$	2
(1,1,3,1)	$1 + 2p_2 + 2p_4$	3
(1,3,1,1)	$1 + 2p_1 + 2p_2$	4
(3,1,1,1)	$3 + 2p_1$	5
(3,3,3,3)	3	6
(3,3,3,1)	$3 + 2p_4$	7
(3,3,1,3)	$3 + 2p_2 + 2p_4$	8
(3,1,3,3)	$3 + 2p_1 + 2p_2$	9
(1,3,3,3)	$1 + 2p_1$	10
(1,1,3,3)	$1 + 2p_2$	11
(3,3,1,1)	$3 + 2p_2$	12
(1,3,3,1)	$1 + 2p_1 + 2p_4$	13
(3,1,1,3)	$3 + 2p_1 + 2p_4$	14
(3,1,3,1)	$3 + 2p_1 + 2p_2 + 2p_4$	15
(1,3,1,3)	$1 + 2p_1 + 2p_2 + 2p_4$	16

Table 3: Units table of $3 - sp_Z$

Unit in $(Z)^4$	**Corresponding Unit in $3 - sp_Z$**	
(1,1,1,1)	1	1
(1,1,1,-1)	$1 - 2p_3$	2
(1,1,-1,1)	$1 - 2p_2 + 2p_3$	3
(1,-1,1,1)	$1 - 2p_1 + 2p_2$	4
(-1,1,1,1)	$-1 + 2p_1$	5
(-1,-1,-1,-1)	-1	6
(-1,-1,-1,1)	$-1 + 2p_3$	7
(-1,-1,1,-1)	$-1 + 2p_2 - 2p_3$	8
(-1,1,-1,-1)	$-1 + 2p_1 - 2p_2$	9
(1,-1,-1,-1)	$1 - 2p_1$	10
(1,1,-1,-1)	$1 - 2p_2$	11
(-1,-1,1,1)	$-1 + 2p_2$	12
(1,-1,-1,1)	$1 - 2p_1 + 2p_3$	13
(-1,1,1,-1)	$-1 + 2p_1 - 2p_3$	14
(-1,1,-1,1)	$-1 + 2p_1 - 2p_2 + 2p_3$	15
(1,-1,1,-1)	$1 - 2p_1 + 2p_2 - 2p_3$	16

4. Conclusion

In this paper, we study and find the symbolic m-plithogenic units in many symbolic plithogenic rings for some special values of m, where we presented a full classification of many different symbolic n-plithogenic group of units as direct products of well-known groups by building suitable and well-defined algebraic isomorphisms.

DOI: https://doi.org/10.54216/NIF.040103
Received: December 11, 2023 Accepted: June 24, 2024

References

[1] Merkepci, H., and Abobala, M., " On The Symbolic 2-Plithogenic Rings", International Journal of Neutrosophic Science, 2023.
[2] Smarandache, F., " Introduction to the Symbolic Plithogenic Algebraic Structures (revisited)", Neutrosophic Sets and Systems, vol. 53, 2023.
[3] Taffach, N., and Ben Othman, K., " An Introduction to Symbolic 2-Plithogenic Modules Over Symbolic 2-Plithogenic Rings", Neutrosophic Sets and Systems, Vol 54, 2023.
[4] Albasheer, O., Hajjari., A., and Dalla., R., " On The Symbolic 3-Plithogenic Rings and TheirAlgebraic Properties", Neutrosophic Sets and Systems, Vol 54, 2023.
[5] Ben Othman, K., "On Some Algorithms For Solving Symbolic 3-Plithogenic Equations", Neoma Journal Of Mathematics and Computer Science, 2023.
[6] Rawashdeh, A., "An Introduction To The Symbolic 3-plithogenic Number Theory", Neoma Journal Of Mathematics and Computer Science, 2023.
[7] Von Shtawzen, O., " Conjectures For Invertible Diophantine Equations Of 3-Cyclic and 4-Cyclic Refined Integers", Journal Of Neutrosophic And Fuzzy Systems, Vol.3, 2022.
[8] Von Shtawzen, O., " On A Novel Group Derived From A Generalization Of Integer Exponents and Open Problems", Galoitica journal Of Mathematical Structures and Applications, Vol 1, 2022.
[9] Basheer, A., Ahmad, K., and Ali, R., " On Some Open Problems About n-Cyclic Refined Neutrosophic Rings and Number Theory", Journal Of Neutrosophic And Fuzzy Systems,, 2022.
[10] A. Alrida Basheer , Katy D. Ahmad , Rozina Ali., "Examples on Some Novel Diophantine Equations Derived from the Group of Units Problem in n-Cyclic Refined Neutrosophic Rings of Integers", Galoitica Journal Of Mathematical Structures And Applications, Vol.3, 2022.
[11] Sankari, H., and Abobala, M., " On The Group of Units Classification In 3-Cyclic and 4-cyclic Refined Rings of Integers And The Proof of Von Shtawzens' Conjectures", International Journal of Neutrosophic Science, 2023.
[12] Sankari, H., and Abobala, M., " On The Classification of The Group of Units of Rational and Real 2-Cyclic Refined Neutrosophic Rings", Neutrosophic Sets and Systems, 2023.
[13] Sankari, H., and Abobala, M., " On The Algebraic Homomorphisms Between Symbolic 2-Plithogenic Rings And 2-cyclic Refined Rings", Neutrosophic Sets and Systems, 2023.
[14] Abobala, M., Ziena, M.B., Doewes, R.I., Hussein, Z. The Representation of Refined Neutrosophic Matrices By Refined Neutrosophic Linear Functions, *International Journal of Neutrosophic Science*, 2022, 19(1), pp. 342–349
[15] Abobala, M., On Refined Neutrosophic Matrices and Their Applications In Refined Neutrosophic Algebraic Equations, Journal Of Mathematics, Hindawi, 2021
[16] Sankari, H. Abobala, M. (2024). On The Computational Properties of 3-Cyclic and 4-Cyclic Refined Matrices and the Diagonalization Algorithm. *International Journal of Advances in Applied Computational Intelligence*, 6(2), 37-45. DOI: https://doi.org/10.54216/IJAACI.060204
[17] Soueycatt, M., Charchekhandra, B., Abu Hakmeh, R., " On The Foundations Of Symbolic 5-Plithogenic Number Theory", Neutrosophic Sets and Systems, vol. 59, 2023.
[18] Merkepci, H., and Rawashdeh, A., " On The Symbolic 2-Plithogenic Number Theory and Integers ", Neutrosophic Sets and Systems, Vol 54, 2023.
[19] avier Gamboa-Cruzado, Renatto Oyague Guerra, Enrique Condor Tinoco, Guillermo Paucar-Carlos, Juan Gamarra Moreno, Warshine Barry. (2024). A Study of the 16-Plithogenic and 17-Plithogenic Square Real Matrices and Their Properties. Journal of International Journal of Neutrosophic Science, 23 (3), 63-76 (Doi : https://doi.org/10.54216/IJNS.230306).
[20] Smarandache F., and Abobala, M., "n-Refined Neutrosophic Vector Spaces", International Journal of Neutrosophic Science, Vol. 7, pp. 47-54. 2020.
[21] Abobala, M.,. "A Study of AH-Substructures in n-Refined Neutrosophic Vector Spaces", International Journal of Neutrosophic Science", Vol. 9, pp.74-85. 2020.
[22] Abobala, M., "On The Characterization of Maximal and Minimal Ideals In Several Neutrosophic Rings", Neutrosophic Sets and Systems, Vol. 45, 2021.
[23] Nader Mahmoud Taffach , Ahmed Hatip., "A Review on Symbolic 2-Plithogenic Algebraic Structures " Galoitica Journal Of Mathematical Structures and Applications, Vol.5, 2023.

DOI: https://doi.org/10.54216/NIF.040103
Received: December 11, 2023 Accepted: June 24, 2024

[24] Nader Mahmoud Taffach , Ahmed Hatip.," A Brief Review on The Symbolic 2-Plithogenic Number Theory and Algebraic Equations ", Galoitica Journal Of Mathematical Structures and Applications, Vol.5, 2023.
[25] Abobala, M., "Neutrosophic Real Inner Product Spaces", Neutrosophic Sets and Systems, Vol. 43, 2021.
[26] P. Prabakaran, Bilal Abdallah, Turayeva Dinara Tulkunovna. (2024). On Certain Algebraic Properties of Symbolic 3-Plithogenic Real Square Matrices. Journal of International Journal of Neutrosophic Science, 23 (4), 350-357 (Doi : https://doi.org/10.54216/IJNS.230427).
[27] Necati Olgun. (2023). On The 15-Plithogenic Square Real Matrices. Galoitica: Journal of Mathematical Structures and Applications, 9 (2), 41-48 (Doi : https://doi.org/10.54216/GJMSA.090205).
[28] Noor Edin Rabeh, Othman Al-Basheer, Sara Sawalmeh, Rozina Ali. (2023). An Algebraic Approach to n-Plithogenic Square Matrices For $18 \leq n \leq 19$. Journal of Neutrosophic and Fuzzy Systems, 7 (2), 08-23 (Doi : https://doi.org/10.54216/JNFS.070201)

DOI: https://doi.org/10.54216/NIF.040103
Received: December 11, 2023 Accepted: June 24, 2024