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Efficient Neutrosophic Optimization for Minimum Cost Flow Problems

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Abstract

In the domain of optimization, linear programming (LP) is recognized as an exceptionally effective method for ensuring the most favorable outcomes. Within the context of LP, the minimum cost flow (MCF) problem is fundamental, with its primary objective being to reduce the transportation costs for a single item moving through a network, under the constraints related to capacity. This network is made up of supply nodes, directed arcs, and demand nodes and each arc has an associated cost and capacity constraint, these factors are certain. However, in practical scenarios, these factors are susceptible to variation due to causal uncertainty. The neutrosophic set theory has surfaced as a challenging approach to tackle the uncertainty that is often encountered in optimization processes. In this manuscript, our primary objective is to address the minimal cost flow (MCF) problem while accounting for the uncertainty inherent in the neutrosophic set. We specifically focus on the cost aspect as SVTN numbers and introduce a new approach based on a customized ranking function handmade for the MCF problem a pioneering endeavor within the field of neutrosophic sets. Additionally, we present numerical example to validate the effectiveness and robustness of our model.

Keywords: LPP; Minimal cost flow; Uncertainty; Neutrosophic set; SVTN numbers; Triangular neutrosophic MCF problem.

1 Introduction

In various industries and domains, decision-making and problem-solving are facilitated by Operations Research (OR), a pivotal discipline. Process optimization is permitted and elaborate decision scenarios are tackled by this powerful tool. Within the field, LP is recognized as a key optimization technique. The main aim of LP is to find optimal values for decision variables that satisfy all constraints while the objective functions are max or min.LP has a wide range of applications, including network flow problems, Job scheduling, transportation problems, resource allocation, and so on. The various researchers working in the field of optimization problems as Klein and Hannan[1] presented a technique for solving the MOILP problem that identifies some or all efficient solutions and Cavory et al.[2] propose an innovative method that combines a genetic algorithm with a scheduler to address job shop scheduling problems under linear constraints. Furthermore, Fisher et al.[3]

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introduce a new approach that leverages the primal-dual ascent algorithm to address both the traveling salesman problem and the network scheduling problem. Ren and Gao[4] established a MILP model for the integrated plan and assessment of DER systems. For solving possibilistic LP models Lai and Hwang[5] introduced a new method to tackle an auxiliary MOLP model to address LP problems that had uncertain objectives and constraint coefficients. Nazemi and Omidi[6] have proposed an optimization approach to address the maximum flow problem arising from extensive applications for neural networks in various environments. Chen[7] proposed a new version of the MCF problem. This version added a fixed cost to the model and aimed to reduce the average cost.

Network problems primarily encompass three distinct types of network problems: the shortest path, maximal cost flow, and minimal cost flow problems. The Minimal Cost Flow (MCF) is a significant and practical network flow model that includes demand nodes, supply nodes, and linear flow costs on the edges of a network graph. The objective of the MCF problem is to transmit flow from a collection of supply nodes, through the arcs of a network, to a set of demand nodes, all at a total minimal cost. To solve the classical MCF problem many researchers in this field such as Minoux[8] address the challenge of solving integer MCF problems involving convex separable cost functions on the arcs, while also considering truthfulness constraints on the flows. Goldberg's[9] scaling push-relabel method represents a significant theoretical advancement in MCF algorithms. In contrast, Goldfarb and Jin[10] have proposed a novel algorithm to solve the MCF problem. During each phase, the algorithm ensures at least one flow augmentation by directly utilizing the given data, the original network, and a scaling factor. Additionally, Gopalakrishnan et al.[11] explore how the unique least-squares properties of node and arc incidence matrices in network flow problems can be effectively utilized in a quick primal-dual least-squares algorithm to solve the MCF problem. Furthermore, Brand et al.[12] introduce new randomized algorithms that improve runtime for solving MCF problems with polynomially bounded capacities and cost, treating them as LP problems with two-sided constraints.

Building upon the above literature, the classical MCF problem encompasses factors such as cost, supply, demand, and capacity. While the standard MCF formulation assumes these factors are certain and definite, but real-world scenarios introduce uncertainty, leading us to explore novel approaches. To handle these situations there are some theories like probability theory, vagueness theory, and fuzzy theory have emerged to address uncertainty in real situations. Notably, Zadeh[13] introduced the concept of fuzzy sets in 1965 as a means to handle uncertain real-world conditions. Using fuzzy logic numerous researchers have made significant contributions to the field of fuzzy MCF problems.

Bagherian[14] approaches the MCF problem by considering parameters such as arc capacities, costs, and supply or demand as interval-valued FN. Muruganandam and Srinivasan[15] present an innovative algorithm for solving the uncertain transportation problem, where supply, demand, capacity, and cost are represented as trapezoidal FN. They aim to minimize transportation costs in a capacitated network. Meanwhile, Bozhenyuk et al.[16] propose a novel approach that addresses both maximum flow and MCF problems in network scenarios. They take into account parameters such as capacities and transport costs of one flow unit, represented as triangular FN. Additionally, Akram et al.[17] introduce an innovative approach that incorporates trapezoidal pentagonal fuzzy (TrPF) numbers for capacities and flow parameters, along with a ranking method for defuzzification. Konstantakopoulos et al.[18] developed a methodology for classifying multiple vehicle routing problem variants related to goods transportation—a common challenge faced by logistics and distribution companies. Researchers have widely applied theories and their extensions, such as fuzzy theory and its extensions, to address problems involving uncertainty. Effectively addressing uncertainty in a fuzzy environment through vague reasoning becomes challenging when challenged with indeterminate and inconsistent information. Addressing indeterminacy is a crucial responsibility for fostering unambiguous data, leading to the emergence of the neutrosophic concept for explicit indeterminacy analysis, due to which F. Smarandache[19] introduce the concept of the Neutrosophic Set (NS) in 1999.

Neutrosophic sets, an advanced mathematical concept, extend fuzzy logic to effectively manage data that is unclear, inconsistent, or incomplete. These sets focus on three critical measures: falsity (the degree to which information is not true), indeterminacy (when information cannot be determined), and truth. unlike fuzzy sets, which have membership levels between 0 and 1, neutrosophic sets operate within a unique range just above 0

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and just below 1. They find practical applications in modeling real-world scenarios with incomplete, inconsistent, or uncertain information. To handle these situations some researchers who work in neutrosophic environments such as Das and Edalatpanah[20] developed a new framework for solving ILP problems with TrNNS using an aggregate ranking function. In their research, Das[21] investigates a transportation problem involving pentagonal Neutrosophic numbers, where parameters are uncertain. Meanwhile, Chakraborty et al.[22] propose innovative methods to de-neutrosoficate trapezoidal neutrosophic numbers, aiming to convert them into crisp numbers. Additionally, Biswas and Dey[23] address MOLP problems under a neutrosophic environment, leveraging neutrosophic fuzzy approaches. Their technique considers three distinct membership degrees to provide decision-makers with potential values for optimization problems. Sinika and Ramesh[24] contribute to this field by proposing a novel de-neutrophication strategy using interval numbers instead of crisp numbers. Their article provides an overview of this approach, introduces a new ranking technique based on interval numbers, and explores extended neutrosophic LP. Furthermore, Das and Chakraborty[25] introduce a novel pentagonal neutrosophic (PN) approach for solving LP problems. This method relies on a ranking function and transforms the problem into its related to crisp LP form and Karak et al.[26] propose a method to order SVTN numbers based on their values and ambiguities. By applying a ratio ranking function, they transform neutrosophic LP problems into crisp LP problems, which are then solved using computational methods.

1.1 Motivation and Novelties

The Neutrosophic set theory is a well-established approach for addressing uncertainty in optimization problems. The notion of the MCF problem within the context of the neutrosophic environment has been investigated by a select handful of scholars. The basic advantage of the neutrosophic set lies in its ability to aid decision-makers by taking into account the degrees of truth, falsity, and indeterminacy. Notably, the degree of indeterminacy is often regarded as an independent factor, contributing significantly to the decision-making process. Our approach aims to overcome limitations, which tend to inflate the number of constraints larger than the original problem. To address this drawback, we propose a novel solution strategy that leverages a ranking function. This approach offers improved efficiency and effectiveness in solving MCF problems with complex cost considerations. The novelties of this manuscript are as follows:

- We propose a new method that utilizes a ranking function to address the MCF problem, accounting for the uncertainty inherent in neutrosophic sets a pioneering contribution in the field of neutrosophic set literature.
- It provides a more efficient and realistic representation by taking into account all facets including falsity, indeterminacy, and truthiness degree of the decision-making process.

1.2 Objective:

The manuscript details numerous challenges with current methods for tackling minimal cost flow (MCF) problems in real-life environments, which have led to the creation of a new approach that incorporates neutrosophic logic into MCF problems. In this article, we present an innovative solution methodology for addressing the MCF problem with neutrosophic cost parameters. The main objective of this manuscript is as follows:

- To tackle the MCF problems considering the uncertainty of the neutrosophic set, focusing especially on the cost.
- To develop a new approach for solving NMFP with neutrosophic cost within the context of neutrosophic uncertainty, where the cost factor is a pivotal element.

1.3 List of abbreviations throughout the article

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List of abbreviations

NS:	"neutrosophic set."
TrNNs:	"Triangular neutrosophic numbers."
MCF:	"Minimum cost flow."
T:	"Initial node."
TrpNNs:	"Trapezoidal neutrosophic numbers."
DER:	"Distributed energy resources.".
SVTN:	"Single-valued triangular neutrosophic."
MOLP:	"multi-objective linear programming."
NMFP:	"Neutrosophic minimal flow problem."
C:	"Maximum capacity of the arc."
TrNMCF:	"Triangular neutrosophic minimum cost flow."
SOLP:	"Single-objective linear programming."
ILP:	"Integer linear programming."
H:	"Terminal node."
FN:	"Fuzzy numbers."
MF:	"Membership function."

1.4 Structure of the Manuscript:

The structure of this article unfolds as follows: We commence with the introduction of basic definitions in the preliminary section2. We articulate both the classical and Neutrosophic minimal cost flow problems and our proposed methodology is then presented in Section 3. Following this, in Section 4, To demonstrate its efficacy, we furnish an example. Finally, we conclude our article in the conclusion section.

2 Preliminaries

Within this portion, we introduce some basic operations and definitions that are used in the manuscript.

Definition 2.1. Neutrosophic Set:[27] A set $\widetilde{P}_{neu}$ in the discourse universal set V, is said to be NS if $\widetilde{P}_{neu} = \left\{\left(v, \left[T_{\widetilde{P}_{neu}}(v), I_{\widetilde{P}_{neu}}(v), F_{\widetilde{P}_{neu}}(v)\right]\right) : v \in V\right\}$ Where $F_{\widetilde{P}_{neu}}(v) : V \to [0,1]$, $I_{\widetilde{P}_{neu}}(v) : V \to [0,1]$, and $T_{\widetilde{P}_{neu}}(v) : V \to [0,1]$ are defined as the falsity-MF $F_{\widetilde{P}_{neu}}(v)$, indeterminacy-MF $I_{\widetilde{P}_{neu}}(v)$ and truth MF $T_{\widetilde{P}_{neu}}(v)$, of an element v in $\widetilde{P}_{neu}$ and respectively. $T_{\widetilde{P}_{neu}}(v), I_{\widetilde{P}_{neu}}(v), F_{\widetilde{P}_{neu}}(v)$ satisfy the condition

$$0 \leq T_{\widetilde{P}_{neu}}(v) + I_{\widetilde{P}_{neu}}(v) + F_{\widetilde{P}_{neu}}(v) \leq 3$$

A neutrosophic set is said to be an SVTN set,[28] if v is a single-valued independent variable.

Definition 2.2. Single-valued triangular neutrosophic (SVTN) number:[29]
A set $\widetilde{PN}_{neu} = \left\langle \left(p_n^1, p_n^2, p_n^3\right), \left(\alpha_n^1, \alpha_n^2, \alpha_n^3\right), \left(d_n^1, d_n^2, d_n^3\right)\right\rangle$ is called SVTN number if it defines a truth MF $T_{\widetilde{PN}_{neu}}(v)$, indeterminacy MF, $I_{\widetilde{PN}_{neu}}(v)$ and the falsity-MF $F_{\widetilde{PN}_{neu}}(v)$ of an element v in $\widetilde{PN}_{neu}$ is defined as,

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$$T_{\widetilde{PN}_{neu}}(v)=\left\{\begin{array}{cc}\left(\frac{v-p_n^1}{p_n^2-p_n^1}\right) & p_n^1\leq v<p_n^2\\ 1 & p_n^2=v\\ \left(\frac{p_n^3-v}{p_n^3-p_n^2}\right) & p_n^2<v\leq p_n^3\\ 0 & \text{otherwise}\end{array}\right\},$$

$$I_{\widetilde{PN}_{neu}}(v)=\left\{\begin{array}{cc}\left(\frac{\alpha_n^2-v}{\alpha_n^2-\alpha_n^1}\right) & \alpha_n^1\leq v<\alpha_n^2\\ 0 & \alpha_n^2=v\\ \left(\frac{v-\alpha_n^2}{\alpha_n^3-\alpha_n^2}\right) & \alpha_n^2<v\leq \alpha_n^3\\ 1 & \text{otherwise}\end{array}\right\}\text{ and}$$

$$F_{\widetilde{PN}_{neu}}(v)=\left\{\begin{array}{cc}\left(\frac{\mathrm{d}_n^2-v}{\mathrm{d}_n^2-\mathrm{d}_n^1}\right) & \mathrm{d}_n^1\leq v<\mathrm{d}_n^2\\ 0 & \mathrm{d}_n^2=v\\ \left(\frac{v-\mathrm{d}_n^2}{\mathrm{d}_n^3-\mathrm{d}_n^2}\right) & \mathrm{d}_n^2<v\leq \mathrm{d}_n^3\\ 1 & \text{otherwise}\end{array}\right\}$$

with the condition $0\leq T_{\widetilde{PN}_{neu}}(v)+I_{\widetilde{PN}_{neu}}(v)+F_{\widetilde{PN}_{neu}}(v)\leq 3$

Definition 2.3. Arithmetic Operations for SVTN number:[30]

Let $\widetilde{PN}_{neu}=\left\langle\left(p_n^1,p_n^2,p_n^3\right),\left(\alpha_n^1,\alpha_n^2,\alpha_n^3\right),\left(d_n^1,d_n^2,d_n^3\right)\right\rangle$ and $\widetilde{QN}_{neu}=\left\langle\left(q_n^1,q_n^2,q_n^3\right),\left(\beta_n^1,\beta_n^2,\beta_n^3\right),\left(f_n^1,f_n^2,f_n^3\right)\right\rangle$ be two SVTN numbers and $\Gamma>0$ then-

(i) $\widetilde{PN}_{neu}\oplus\widetilde{QN}_{neu}=\left\langle\begin{array}{c}\left(p_n^1+q_n^1,p_n^2+q_n^2,p_n^3+q_n^3\right),\left(\alpha_n^1+\beta_n^1,\alpha_n^2+\beta_n^2,\alpha_n^3+\beta_n^3\right),\\ \left(d_n^1+f_n^1,d_n^2+f_n^2,d_n^3+f_n^3\right)\end{array}\right\rangle$

(ii) $\widetilde{PN}_{neu}\otimes\widetilde{QN}_{neu}=\left\langle\begin{array}{c}\left(p_n^1.q_n^1,p_n^2.q_n^2,p_n^3.q_n^3\right),\left(\alpha_n^1.\beta_n^1,\alpha_n^2.\beta_n^2,\alpha_n^3.\beta_n^3\right),\\ \left(d_n^1.f_n^1,d_n^2.f_n^2,d_n^3.f_n^3\right)\end{array}\right\rangle$

(iii) $\Gamma\odot\widetilde{PN}_{neu}=\left\langle\left(\Gamma.p_n^1,\Gamma.p_n^2,\Gamma.p_n^3\right),\left(\Gamma.\alpha_n^1,\Gamma.\alpha_n^2,\Gamma.\alpha_n^3\right),\left(\Gamma.d_n^1,\Gamma.d_n^2,\Gamma.d_n^3\right)\right\rangle$

Definition 2.4. [29] Let $\widetilde{PN}_{neu}=\left\langle\left(p_n^1,p_n^2,p_n^3\right),\left(\alpha_n^1,\alpha_n^2,\alpha_n^3\right),\left(d_n^1,d_n^2,d_n^3\right)\right\rangle$ be an SVTN number and define a ranking function $De_{neu}:N(\mathbb{R})\rightarrow\mathbb{R}$ such as

$$De_{neu}\left(\widetilde{PN}_{neu}\right)=\frac{1}{12}\left[\left(p_n^1+2.p_n^2+p_n^3\right)+\left(\alpha_n^1+2.\alpha_n^2+\alpha_n^3\right)+\left(d_n^1+2.d_n^2+d_n^3\right)\right]$$

where $N(\mathbb{R})$ denotes a set of SVTN numbers characterized by a set of the real number $\mathbb{R}$.

3 Our Proposed model

Before presenting our proposed algorithm, we introduce a sub-section that discusses the existing crisp model in the minimum cost flow (MCF) problem and the approach involving the neutrosophic environment with the NMFP with neutrosophic cost.

3.1 Existing crisp model in MCF problem

Let a directed graph $G^{\eta}=(N_{\eta},\aleph_{\eta})$ is considered, where $N_{\eta}=\{1,2,3,...,t\}$ is the set of finite nodes and $\aleph_{\eta}$ represents the set of arcs.

τ_{qt}: Per-unit flow from the arc q to t.
Ck_{qt}:Cost-per-unit flow from arc q to t.
ℓ_{qt}: Lower capacity from the arc q to t.

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p_{qt}: Upper capacity from the arc q to t.
Υ_q: Represents the supply, demand, or transshipment node.

The node q is identified as a transshipment node if Υ_q equals 0. If Υ_q is less than 0, then the node q is identified as a demand node. If Υ_q is greater than 0, then the node q is identified as a supply node.

The general formulation of the mathematical model for the classical MCF problem is to be considered as follows:

$Mini\ \mathrm{Z} = \sum_{(q,t)\in\aleph_\eta} Ck_{qt}\, \tau_{qt}$

Subject to $\sum_{t:(q,t)\in\aleph_\eta} \tau_{qr} - \sum_{t:(t,q)\in\aleph_\eta} \tau_{tq} = \Upsilon_q, \quad \forall q \in N_\eta$ (Flow conservation equations)

$0 \le \ell_{qt} \le \tau_{qt} \le p_{qt} \quad \forall\,(q,t) \in \aleph_\eta$ (Flow capacity constraints)

3.2 Transformation of the crisp model MCF problem into NMFP with neutrosophic cost

In this section, consider the scenario where we substitute the cost Ck_{qt} convert into a neutrosophic cost per unit flow $\widetilde{Ck_{qt}^{\eta}}$ from arc q to t. Then the general formulation of the mathematical model for the NMFP with neutrosophic cost is to be considered as follows:

$$Mini\ \widetilde{O^{\eta}} \approx \sum_{(q,t)\in\aleph_\eta} \widetilde{Ck_{qt}^{\eta}}\, \tau_{qt} \tag{1}$$

Subject to constraints

$$\sum_{t:(q,t)\in\aleph_\eta} \tau_{qt} - \sum_{t:(t,q)\in\aleph_\eta} \tau_{tq} = \Upsilon_q, \qquad \forall q \in N_\eta \tag{2}$$

$$0 \le \ell_{qt} \le \tau_{qt} \le p_{qt} \qquad \forall\,(q,t) \in \aleph_\eta \tag{3}$$

3.3 Algorithm: A novel approach for finding the NMFP with neutrosophic cost considering as SVTN number for cost parameters

Let's consider a directed graph whose arcs denote the SVTN cost per unit flow $\widetilde{Ck_{qt}^{\eta}}$ from arc q to t. In this section, our proposed algorithm by using a new ranking function tends to provide a novel methodology for finding the NMFP with neutrosophic cost considering as SVTN number for cost parameters.
The steps of the algorithm are as follows table 1.

Table 1: The steps of the algorithm

Steps	Algorithm
Step 01:	Formulate the mathematical model of the problem by the model of equation 1 , 2 , and 3 .
Step 02:	Transformation of the NMFP with neutrosophic cost obtained in step 01 into crisp model MCF problem by using the definition 2.4
Step 03:	Solve this MCF problem obtained in step 02 by using the existing methods or software (LINGO 18.0) and obtain the NMFP with neutrosophic cost problem.
Step 04:	Substitute all values τ_{qt} in the objective equation 1 to get the optimal value in the form of an SVTN number is $\left\langle \left(p_n^{1*}, p_n^{2*}, p_n^{3*}\right), \left(\alpha_n^{1*}, \alpha_n^{2*}, \alpha_n^{3*}\right), \left(d_n^{1*}, d_n^{2*}, d_n^{3*}\right) \right\rangle$
Step 05:	The End

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To illustrate our proposed algorithm, we consider an example, a flow network shown in Figure 1.

4 Numerical Example

Let's investigate an example of a network diagram, as presented below. This network consists of seven nodes and fourteen arcs. Each arc's cost is expressed using an SVTN number for the cost parameters. The objective is to determine the flow within the network while minimizing the overall cost. See Figure 1 for the visual representation of this flow network and using the data of table 2 .

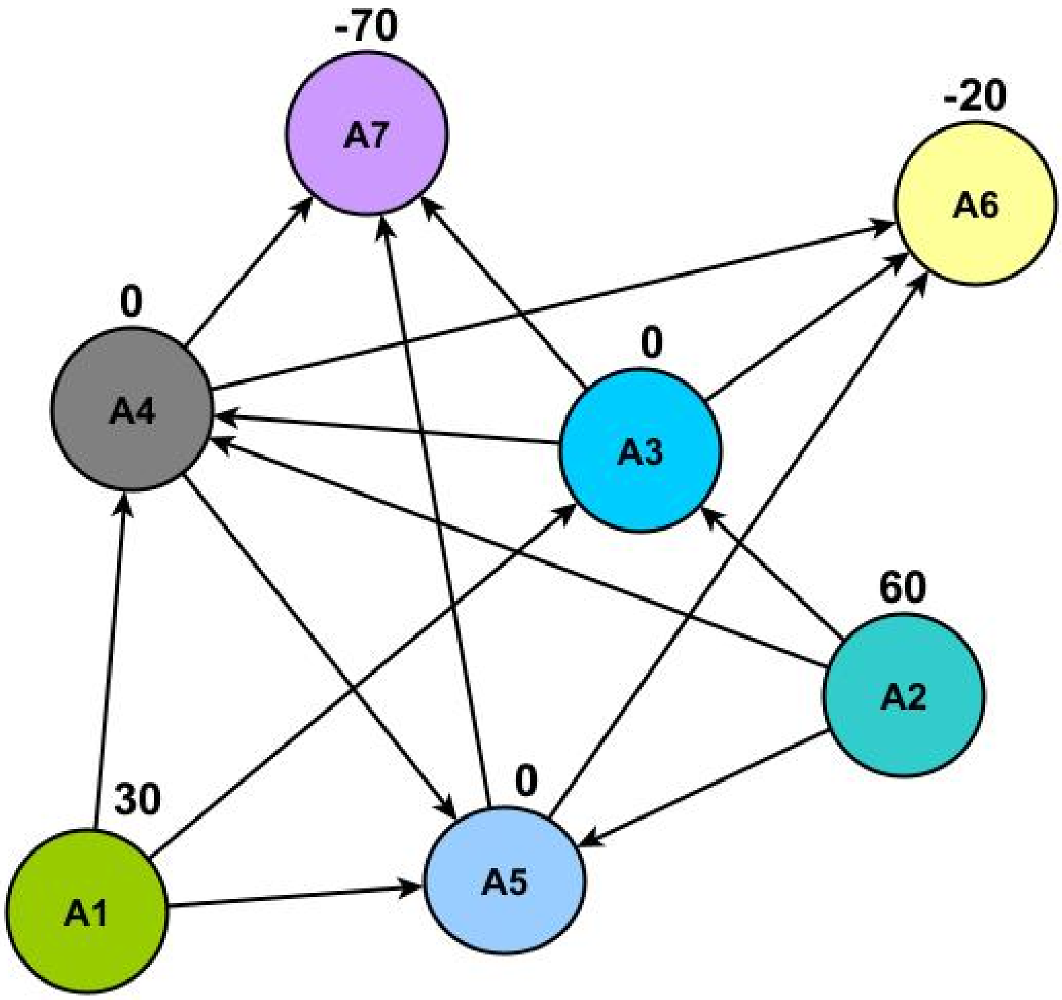

Figure 1: The network consider for Solving MCF problem under neutrosophic environment

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Table 2: Consider the below data for Solving NMFP under neiutrosophic environment

T	H	Neutrosophic arc cost	C
1	4	<(0.5,2.5,4.5),(1,2,3),(1.5,3.5,5.5)>	10
3	7	<(1,5,8),(1.5,3,6.5),(4,7,9)>	90
1	3	<(1,4,7),(1,3,5),(3.5,6,7.5)>	40
3	4	<(2,4,6),(1.5,2.5,4.5),(3,5,7)>	20
1	5	<(1,3,5),(0.5,1.5,3.5),(3,4,6)>	15
4	7	<(15,20,25),(19,21,27),(17,20,24)>	20
2	5	<(1.5,2.5,3.5),(1,1.5,3),(2,3,4)>	40
4	6	<(20,25,30),(24,26,32),(22,25,29)>	30
2	4	<(1,1.5,4),(0.5,1,2.5),(2.25,3,4.25)>	80
4	5	<(10,15,20),(14,16,22),(12,15,19)>	15
2	3	<(1,2,3),(0.5,1.5,2.5),(1.5,2.5,3.5)>	50
5	7	<(15,20,25),(19,21,27),(17,20,24)>	100
3	6	<(1,5,8),(1.5,4.5,7.5),(4,6.5,9)>	100
5	6	<(13,18,23),(17,19,25),(15,18,22)>	15

Solution:
Step 01: To solve the problem, first of all, transform the above network into a mathematical model named equation 4

$$\begin{gathered} Min\left(\widetilde{O^{\eta}}\right) = \langle(0.5,2.5,4.5),(1,2,3),(1.5,3.5,5.5)\rangle\,\tau_{14} + \langle(1,4,7),(1,3,5),(3.5,6,7.5)\rangle\,\tau_{13}+ \\ \langle(1,3,5),(0.5,1.5,3.5),(3,4,6)\rangle\,\tau_{15} + \langle(1.5,2.5,3.5),(1,1.5,3),(2,3,4)\rangle\,\tau_{25}+ \\ \langle(1,1.5,4),(0.5,1,2.5),(2.25,3,4.25)\rangle\,\tau_{24} + \langle(1,2,3),(0.5,1.5,2.5),(1.5,2.5,3.5)\rangle\,\tau_{23}+ \\ \langle(1,5,8),(1.5,4.5,7.5),(4,6.5,9)\rangle\,\tau_{36} + \langle(1,5,8),(1.5,3,6.5),(4,7,9)\rangle\,\tau_{37}+ \\ \langle(2,4,6),(1.5,2.5,4.5),(3,5,7)\rangle\,\tau_{34} + \langle(15,20,25),(19,21,27),(17,20,24)\rangle\,\tau_{47}+ \\ \langle(20,25,30),(24,26,32),(22,25,29)\rangle\,\tau_{46} + \langle(10,15,20),(14,16,22),(12,15,19)\rangle\,\tau_{45}+ \\ \langle(15,20,25),(19,21,27),(17,20,24)\rangle\,\tau_{57} + \langle(13,18,23),(17,19,25),(15,18,22)\rangle\,\tau_{56}. \end{gathered} \tag{4}$$

Subject to constraints:

$$\left.\begin{gathered} \tau_{57}+\tau_{56}-\tau_{45}-\tau_{25}-\tau_{15}=0;\ 0\le\tau_{47}\le 20;\ \ 0\le\tau_{57}\le 100 \\ \tau_{15}+\tau_{14}+\tau_{13}=30;\ 0\le\tau_{14}\le 10;\ \tau_{25}+\tau_{24}+\tau_{23}=60; 0\le\tau_{15}\le 15 \\ 0\le\tau_{23}\le 50,\ \ -\tau_{36}-\tau_{46}-\tau_{56}=-20;\ 0\le\tau_{25}\le 40; 0\le\tau_{24}\le 80; \\ 0\le\tau_{37}\le 90;\ \tau_{54}+\tau_{46}+\tau_{47}-\tau_{14}-\tau_{42}-\tau_{34}=0;\ 0\le\tau_{56}\le 15; \\ 0\le\tau_{13}\le 40; 0\le\tau_{34}\le 20;\ \tau_{37}+\tau_{36}+\tau_{34}-\tau_{23}-\tau_{13}=0; 0\le\tau_{36}\le 100; \\ 0\le\tau_{45}\le 15;\ 0\le\tau_{46}\le 30;\ -\tau_{37}-\tau_{47}-\tau_{57}=-70; \end{gathered}\right\} \tag{5}$$

After solving **step: 02 and step 03**
We obtain the basic variables $\tau_{13}=30$,$\tau_{14}=0$,$\tau_{15}=0$,$\tau_{23}=50$,$\tau_{24}=0$,$\tau_{25}=10$,$\tau_{34}=0$,$\tau_{36}=10$,$\tau_{37}=70$,$\tau_{45}=0$,$\tau_{46}=0$,$\tau_{47}=0$, $\tau_{56}=10$, $\tau_{75}=0$

Step: 04 and Step 05
Substitute all basic variables τ_{qt} in the objective equation 4 to get optimal value is in the form of SVTN numbers as $\langle(305,825,1265),(355,625,1085),(670,1070,1380)\rangle$ and $\tau_{13}=30$,$\tau_{14}=0$,$\tau_{15}=0$,$\tau_{23}=50$,$\tau_{24}=0$,$\tau_{25}=10$,$\tau_{34}=0$,$\tau_{36}=10$,$\tau_{37}=70$,$\tau_{45}=0$,$\tau_{46}=0$,$\tau_{47}=0$,$\tau_{56}=10$,$\tau_{57}=0$
End

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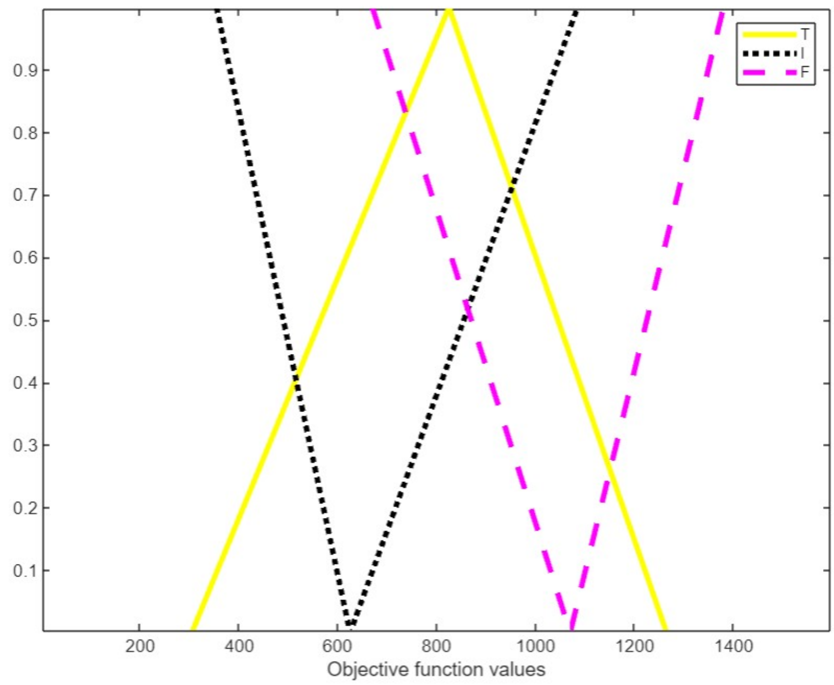

Figure 2: The graphical representation of $\langle (305, 825, 1265), (355, 625, 1085), (670, 1070, 1380) \rangle$

In Figure 2, the graphical representation of the optimal solution of a given numerical example with SVTN numbers$\langle (305, 825, 1265), (355, 625, 1085), (670, 1070, 1380) \rangle$ is shown, with the yellow line depicting truth-MF, the black dot depicting indeterminacy-MF, and the magenta dot depicting falsity-MF.

Conclusion

The minimum cost flow (MCF) problem aims to find the least cost way to move flow through a network, considering factors like cost, supply, demand, and capacity. And also discusses the neutrosophic set theory has surfaced as a challenging approach to tackle the uncertainty that is often encountered in optimization processes. In this manuscript, we introduce a new algorithm using SVTN numbers to solve the NMFP with neutrosophic cost, which also predicts crips MCF problems. By applying a ratio ranking function, they transform neutrosophic LP problems into crisp LP problems, which are then solved using computational methods. For this purpose, we used a numerical example to solve the NMFP with neutrosophic cost with SVTN numbers into crisp LP problems, by using the existing methods or software (LINGO 18.0) and obtain the NMFP with neutrosophic cost problem. The new algorithms can be used for real-world issues like assignments, job scheduling, and transportation in the future.

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References

[1] Dieter Klein and Edward Hannan. An algorithm for the multiple objective integer linear programming problem. *European Journal of Operational Research*, 9(4):378–385, 1982.

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[2] Guillaume Cavory, Remy Dupas, and Gilles Goncalves. A genetic approach to solving the problem of cyclic job shop scheduling with linear constraints. *European Journal of Operational Research*, 161(1):73–85, 2005.

[3] Marshall L Fisher, WD Northup, and JF Shapiro. Using duality to solve discrete optimization problems: Theory and computational experience. In *Nondifferentiable optimization*, pages 56–94. Springer, 2009.

[4] Hongbo Ren and Weijun Gao. A milp model for integrated plan and evaluation of distributed energy systems. *Applied Energy*, 87(3):1001–1014, 2010.

[5] Young-Jou Lai and Ching-Lai Hwang. A new approach to some possibilistic linear programming problems. *Fuzzy Sets and Systems*, 49(2):121–133, 1992.

[6] Alireza Nazemi and Farahnaz Omidi. A capable neural network model for solving the maximum flow problem. *Journal of Computational and Applied Mathematics*, 236(14):3498–3513, 2012.

[7] YL Chen. The minimal average cost flow problem. *European Journal of Operational Research*, 81(3):561–570, 1995.

[8] Michel Minoux. Solving integer minimum cost flows with separable convex cost objective polynomially. *Netflow at Pisa*, pages 237–239, 1986.

[9] Andrew V Goldberg. An efficient implementation of a scaling minimum-cost flow algorithm. *Journal of Algorithms*, 22(1):1–29, 1997.

[10] Donald Goldfarb and Zhiying Jin. A new scaling algorithm for the minimum cost network flow problem. *Operations Research Letters*, 25(5):205–211, 1999.

[11] Balaji Gopalakrishnan, Seunghyun Kong, Earl Barnes, Ellis L Johnson, and Joel S Sokol. A least-squares minimum-cost network flow algorithm. *Annals of Operations Research*, 186:119–140, 2011.

[12] Jan Van Den Brand, Yin Tat Lee, Yang P Liu, Thatchaphol Saranurak, Aaron Sidford, Zhao Song, and Di Wang. Minimum cost flows, mdps, and l1-regression in nearly linear time for dense instances. In *Proceedings of the 53rd Annual ACM SIGACT Symposium on Theory of Computing*, pages 859–869, 2021.

[13] Lofti A Zadeh. Information and control. *Fuzzy sets*, 8(3):338–353, 1965.

[14] M Bagherian. A fuzzy residual network approach to minimum cost flow problem with fuzzy parameters. *ARPN Journal of Science and Technology*, 2(10):968–975, 2012.

[15] S Muruganandam and R Srinivasan. A new algorithm for solving fuzzy transportation problems with trapezoidal fuzzy numbers. *International Journal of Recent Trends in Engineering and Research*, 2(3):428–437, 2016.

[16] Alexander Vitalievich Bozhenyuk, Evgeniya Michailovna Gerasimenko, Janusz Kacprzyk, Igor Naymovich Rozenberg, Alexander Vitalievich Bozhenyuk, Evgeniya Michailovna Gerasimenko, Janusz Kacprzyk, and Igor Naymovich Rozenberg. Maximum and minimum cost flow finding in networks in fuzzy conditions. *Flows in Networks Under Fuzzy Conditions*, pages 23–75, 2017.

[17] Muhammad Akram, Amna Habib, and Tofigh Allahviranloo. A new maximal flow algorithm for solving optimization problems with linguistic capacities and flows. *Information Sciences*, 612:201–230, 2022.

[18] Grigorios D Konstantakopoulos, Sotiris P Gayialis, and Evripidis P Kechagias. Vehicle routing problem and related algorithms for logistics distribution: A literature review and classification. *Operational research*, 22(3):2033–2062, 2022.

[19] Florentin Smarandache. A unifying field in logics. neutrosophy: Neutrosophic probability, set and logic, 1999.

[20] Sapan Kumar Das and SA Edalatpanah. A new ranking function of triangular neutrosophic number and its application in integer programming. *International Journal of Neutrosophic Science*, 4(2):82–92, 2020.

[21] Sapan Kumar Das. *Application of transportation problem under pentagonal neutrosophic environment*, volume 1. Infinite Study, 2020.

[22] Avishek Chakraborty, Sankar Prasad Mondal, Animesh Mahata, and Shariful Alam. Different linear and non-linear form of trapezoidal neutrosophic numbers, de-neutrosophication techniques and its application in time-cost optimization technique, sequencing problem. *RAIRO-Operations Research*, 55:97–118, 2021.

[23] Sanjoy Biswas and Samir Dey. A new approach to solve multi objective linear programming problem under neutrosophic environment. *International Journal of Applied and Computational Mathematics*, 9(6):135, 2023.

[24] S Sinika and G Ramesh. To enhance new interval arithmetic operations in solving linear programming problem using interval-valued trapezoidal neutrosophic numbers. *Mathematics and Statistics*, 11(4):710–725, 2023.

[25] Sapan Kumar Das and Avishek Chakraborty. A new approach to evaluate linear programming problem in pentagonal neutrosophic environment. *Complex and Intelligent Systems*, 7:101–110, 2021.

[26] Manas Karak, Pramodh Bharati, and Animesh Mahata. Ranking of neutrosophic number based on values and ambiguities and its application to linear programming problem. *Neutrosophic Sets and Systems*, 63(1):13, 2024.

[27] Nidhi Singh, Avishek Chakraborty, Soma Bose Biswas, and Malini Majumdar. Impact of social media in banking sector under triangular neutrosophic arena using mcgdm technique. *Neutrosophic Sets and Systems*, 35:153–176, 2020.

[28] Haibin Wang, Florentin Smarandache, Yanqing Zhang, and Rajshekhar Sunderraman. Single valued neutrosophic sets. *Infinite study*, 12:20110, 2010.

[29] Avishek Chakraborty, Sankar Prasad Mondal, Ali Ahmadian, Norazak Senu, Shariful Alam, and Soheil Salahshour. Different forms of triangular neutrosophic numbers, de-neutrosophication techniques, and their applications. *Symmetry*, 10(8):1–27, 2018.

[30] Mohamed Abdel-Basset, M Gunasekaran, Mai Mohamed, and Florentin Smarandache. A novel method for solving the fully neutrosophic linear programming problems. *Neural Computing and Applications*, 31:1595–1605, 2019.

[31] Mehdi Ghatee, S Mehdi Hashemi, Masoud Zarepisheh, and Esmaile Khorram. Preemptive priority-based algorithms for fuzzy minimal cost flow problem: An application in hazardous materials transportation. *Computers and Industrial Engineering*, 57(1):341–354, 2009.

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