Optimizing Weibull Distribution Parameters for Improved Earthquake Modeling in Japan: A Comparative Approach

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Abstract

The Weibull distribution is considered one of the important distributions used in reliability and in the distribution of survival times and in neutrosophic prediction. This paper contained an estimate of a two-parameter Weibull distribution using three estimation methods. One of these methods for estimating parameters is the traditional method, which represents the estimation of the greatest possibility, and the other two methods are estimation using the sine algorithm. and cosine (SCA) and the Ant Colony Algorithm (ACO). The simulation method was used to compare the methods, and it was found that the best method for estimating the parameters of the Weibull distribution is the sine and cosine algorithm (SCA) method. Then, real data was used, represented by the intensity of the earthquake in Japan (Richter) for the period July 1, 2023, to July 16, 2023, to estimate the rate of earthquake intensity in Japan, since Japan is one of the countries most exposed to earthquakes, and it was shown from The results are that the average magnitude of the earthquake in Japan in the period studied is 3.328593, which can be said. Weak buildings may be greatly damaged, but strong buildings are not greatly damaged. Also, a neutrosophic simulation of the same set of data will be suggested for future applications.

Keywords: Sine Cosine Algorithm (SCA); Ant Colony Optimization (ACO); Statistical inference; Maximum likelihood estimation, neutrosophic prediction, neutrosophic simulation.

1. Introduction

One of the most popular statistical models is Weibull distribution, which is, along with other models, very often used to analyze the type of data that reflects failures and extreme events, such as earthquakes. From its versatile shaping which enables frequency variations of seismic to be portrayed clearly, its principal function is to characterize seismic variations. Nevertheless, it is uncertainty both of shape and scale parameters that limits the usefulness of the Weibull distribution.

However, while MLE has so far been the prevalent modality of parameter estimation, it is to be noted that it is only feasible to optimize the likelihood function in case of regular and rudimentary data. With instances like this, it really becomes hard to use MLE in earthquake data because it is complex. This indispensable role has therefore necessitated researchers to review and improve the existing techniques as well as development of new advanced computational optimization techniques like the sine cosine algorithm (SCA) and ant colony optimization (ACO) for Weibull parameter estimation. These algorithms do so by employing a process which imitates nature phenomenon in an iterative way to look for ideal parameters. The research up to the moment recorded also the evidence that SCA and ACO algorithm has a capability to deal with noise, outliers and variance in seismic data nodelike than legacy approaches such as MLE.

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Nevertheless, there is a shortfall of undertaking these approaches, which are primarily concerns about outperforming the MLE, SCA, and ACO models for the estimation of Weibull parameters in the earthquake frequency modeling. The absence knowledge is indeed startling, considering the fundamental role of the probability distribution in the seismic hazard prediction models, particularly in earthquake-affected nations, including Japan. The realistic modelling of earthquakes is not only the primary step in the dare risk reduction and resilience but also is the main step.

Based on this of this, the study will examine the efficiency of MLE and SCA as well and ACO for the determination of Weibull parameters of historical earthquake frequency data of Japan. Indicators such as prediction accuracy, computation time and model fit will be some of the factors that will be measured for different techniques. The results will make it possible to obtain a lot of information regarding the best estimation way for the Weibull parameter used in seismic hazard modelling. This will give us the huge possibility to fine-tune probability distributions grid for future earthquake probabilities in Japan, which could mean life-saving measures. As it is Japan which frequently encounters earthquakes, these research efforts of mine can finally help people enhance their resilience meaning that lives get to be saved. The designed framework can also help conduct a comparative study that can help refine Weibull parameter estimation technique for failure data analysis in other industries as well.

In this study, we will focus our attention on the accuracy of estimating the parameters of this distribution for the purpose of calculating the arithmetic mean (average) of the earthquake intensity for the data that was taken and for the aforementioned period. This helps specialists in determining the average strength of the earthquake during a specific period of time.

On the other hand, we suggest some of future generalizations and studies by replacing the classical Weibull's distribution with its neutrosophic version. For more information about neutrosophic distributions check [15-18].

2. Material and methods

2.1 Weibull distribution

The Weibull distribution is a continuous probability distribution frequently used to model lifetimes, reliability, and survival analysis in various fields such as engineering, medicine, and finance. It's characterized by two parameters: the shape parameter (often denoted as "k") and the scale parameter (often denoted as "λ") [1].

The probability density function (PDF) of the Weibull distribution is given by [2]:

$$f(x;\ k,\lambda) = \left(\frac{k}{\lambda}\right)\left(\frac{x}{\lambda}\right)^{(k-1)} e^{-\left(\frac{x}{\lambda}\right)^{k}} \qquad (1)$$

where x is the random variable (usually representing time or a continuous positive variable), k is the shape parameter It determines the shape of the distribution curve, λ is the scale parameter (also called the "scale factor" or "scale parameter"). It affects the spread or scale of the distribution.

1 .Shape Parameter (k):

- When okay > 1: The distribution has a decreasing risk fee, indicating that failure fees lower over time. This is often used to model systems with "put on-out" screw ups.

- When k = 1: The distribution turns into the exponential distribution, which has a consistent hazard charge. This is often used for systems with regular failure quotes.

- When 0 < k < 1: The distribution has an growing hazard rate, indicating that failure costs boom over time. This is once in a while used to model structures with "infant mortality" failures.

2 .Scale Parameter (λ):

- λ determines the scale or common lifetime of the distribution.

- Larger values of λ cause distributions which can be extra spread out.

- Smaller values of λ result in distributions which are more focused around the foundation [3].

The Weibull distribution can take various forms depending on the values of the shape and scale parameters. It's a flexible distribution that may version a wide range of failure patterns and is frequently chosen based totally at the traits of the statistics being analyzed.

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Keep in mind that while the Weibull distribution is widely used, there are also other distributions like the exponential, normal, and log-normal distributions that might be suitable for different types of data and scenarios.

2.2 Mean and Variance of Distribution

If $X \sim Weibull(k, \lambda)$,then the following hold [4].

The mean of X is $E[X] = \lambda\, \Gamma\left(1 + \frac{1}{k}\right)$ (2)

The variance of X is $Var(X) = \lambda^2 \left[\Gamma\left(1 + \frac{2}{k}\right) - \left[\Gamma\left(1 + \frac{1}{k}\right)\right]^2\right]$ (3)

2.3 Statistical inference

In this paper, two methods for estimating the parameters of distributions will be addressed, namely, the Maximum Likelihood method and The Sine Cosine Algorithm, Ant Colony Algorithm which will be presented in this section as follows:

2.3.1 Maximum Likelihood Estimation (MLE)

To estimate the parameters of a Weibull distribution using Maximum Likelihood Estimation (MLE), you would follow these steps [2], [5].

Let's assume you have a sample of data points x_1, x_2, ..., x_n.

1. Likelihood Function

The likelihood function for the Weibull distribution is given by:

$$L(k, \lambda \mid x_1, x_2, \dots, x_n) = \Pi \left[\left(\frac{k}{\lambda}\right) \left(\frac{x_i}{\lambda}\right)^{(k-1)} e^{-\left(\frac{x_i}{\lambda}\right)^k} \right] \quad (4)$$

2. Log-Likelihood Function

Take the natural logarithm:

$$\ln\left(L(k, \lambda \mid x_1, x_2, \dots, x_n)\right) = \sum_{i=1}^{n} [\, ln(k) - ln(\lambda) + (k - 1) * ln\left(\frac{x_i}{\lambda}\right) - (\frac{x_i}{\lambda})^k \,]$$

3. Partial Derivatives

Differentiate the log-likelihood function concerning k and λ:

$$\frac{\partial ln(L)}{\partial k} = \sum_{i=1}^{n} \left[\frac{1}{k} - ln\left(\frac{x_i}{\lambda}\right) + ln\left(\frac{x_i}{\lambda}\right) (\frac{x_i}{\lambda})^k \right]$$

$$\frac{\partial ln(L)}{\partial \lambda} = \sum_{i=1}^{n} \left[\frac{-1}{\lambda} + \left(\frac{k}{\lambda}\right)\left(\frac{x_i}{\lambda}\right)^{(k-1)}\right]$$

4. Set Derivatives to Zero

Set the derivatives equal to zero and solve for k and λ:

$$\frac{1}{k} \sum_{i=1}^{n} [1 - (\frac{x_i}{\lambda})^k] = 0$$

$$\frac{\sum_{i=1}^{n} [(\frac{x_i}{\lambda})^k] - n}{\lambda} = 0$$

5. Solve for Parameters

Solve the above equations for k and λ simultaneously.

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$$\hat{\lambda} = \left[n^{-1} \sum_{i=1}^{n} x_i^k \right]^{1/k} \quad (5)$$

$$\hat{k} = \left[\left(\sum_{i=1}^{n} x_i^k \ln x_i \right) \left(\sum_{i=1}^{n} x_i \right)^{-1} - n^{-1} \sum_{i=1}^{n} \ln x_i \right]^{-1} \quad (6)$$

The value of k has to be obtained from (6) by the use of standard iterative procedures (i.e. Newton Raphson method) and then used in (5) to obtain λ [6].

2.3.2 The sine-cosine algorithm (SCA)

The Sine-cosine algorithm, taking inspiration from sine and cosine functions, is a population-based optimization algorithm designed to tackle optimization concerns such as determining parameters linked to distributions like the Weibull distribution. Interested in knowing how to use SCA in order to estimate Weibull distribution parameters [7], [8].

1.Initialization:Sets of Weibull parameters can be represented by a population of sine wave solutions that serve as candidates. For each parameter (shape and scale parameters), determine the appropriate bounds for the search space [9], [10].

2. Encoding Parameters Map k to frequency (ωk) and λ to amplitude (Aλ):

$$k = g(\omega k)$$

$$\lambda = h(A\lambda)$$

Where g() and h() are encoding functions.

3. Fitness Function

$$f(x;\ k, \lambda) = \text{Weibull PDF}$$

$$F(x;\ k, \lambda) = \text{Weibull CDF}$$

$$\text{Error} = \sum_{i=1}^{n} [F(x_i;\ k, \lambda) - y_i]^2 \quad (7)$$

Where $y_i = \sin(x_i)$ or $\cos(x_i)$

4. Update Sine Waves

$$\omega k \leftarrow \omega k + \alpha \cos(t) \sin(\omega k t + \varphi k)$$

$$A\lambda \leftarrow A\lambda + \alpha \cos(t) \sin(A\lambda t + \varphi \lambda)$$

α is learning rate. t is iteration counter.

9. Decode Parameters

$$k = g - 1(\omega k)$$

$$\lambda = h - 1(A\lambda)$$

Where g-1() and h-1() are decoding functions, reverse of encoding functions.

$$g(\omega k) = 1 + \omega k\, h(A\lambda) = 1 + 0.5A \quad (8)$$

Then:

$$g - 1(\omega k) = \omega k - 1$$
$$h - 1(A\lambda) = 2A\lambda - 1$$

10. Validation

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Calculate errors: AIC and BIC [11].

It's vital to note that applying the SCA to parameter estimation for the Weibull distribution requires cautious parameter tuning, dealing with of boundary constraints, and validation of the acquired effects. Additionally, the success of the SCA relies upon on elements along with the selection of algorithm parameters, the encoding scheme, and the complexity of the optimization problem [8].

As with any optimization method, it is advocated to very well take a look at and validate the technique on each synthetic and actual-world data earlier than applying it to critical packages.

2.3.3 Ant Colony Optimization (ACO)

Ant Colony Optimization (ACO) is a metaheuristic algorithm inspired by the foraging behavior of ants. It can be used to solve a whole lot of optimization issues, which include the estimation of Weibull parameter distribution [12].

The simple idea of ACO is to use a colony of artificial ants to search for the best technique to a trouble. Each ant starts at a random node inside the problem space and then moves to a neighboring node primarily based on a chance this is decided by using the pheromones which have been deposited on the nodes. The pheromones are deposited by using ants as they flow via the trouble area, and that they indicate the likelihood that a specific node is a great answer.

In the case of Weibull parameter estimation, the problem space is the set of all possible values for the Weibull distribution parameters ok and c. The ants begin at random values for ok and c and then circulate to neighboring values based totally at the pheromones that have been deposited on the nodes. The pheromones are deposited with the aid of ants as they circulate via the problem area, and that they imply the chance that a specific cost for okay and c is a good answer.

The ants preserve to move through the hassle space till they reach a termination circumstance, such as a most range of iterations or a certain level of fitness. The pleasant answer determined using any ant is then again as the envisioned value of the Weibull distribution parameters [13].

ACO is powerful in estimating Weibull parameter distribution in a variety of packages, together with wind electricity, reliability engineering, and production. It is a noticeably easy algorithm to put in force, and it isn't always as computationally costly as a few other strategies for estimating Weibull parameter distribution [7].

Here is a more targeted explanation of how ACO may be used to estimate Weibull parameter distribution:

1. Initialize the pheromones on all nodes inside the trouble area to a small cost.

2. Create a colony of ants.

3. For each ant:

a. Start at a random node within the trouble space.

b. Move to a neighboring node based totally on the pheromones that have been deposited at the nodes.

c. Deposit pheromones on the node that it simply visited.

d. Repeat steps three-four until the ant reaches a termination condition.

4.The nice solution located using any ant is then back because the estimated fee of the Weibull distribution parameters.

The pheromones in ACO play an vital position in guiding the ants in the direction of right answers. The ants are much more likely to transport to nodes that have a high awareness of pheromones, which suggests that those nodes are suitable answers. As the ants flow thru the problem space, they deposit pheromones on the nodes that they go to. This causes the pheromone attention to boom on those nodes, which makes them even more attractive to the other ants. This technique continues till the ants attain a termination circumstance, together with a most wide variety of iterations or a positive level of fitness [14].

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ACO is an effective algorithm that may be used to remedy lots of optimization issues, such as the estimation of Weibull parameter distribution. It is a notably easy set of rules to enforce, and it is not as computationally high-priced as a few different strategies for estimating Weibull parameter distribution.

3. Results

3.1 Simulation study

The furnished simulation includes evaluating three optimization algorithms for estimating parameters of a Weibull distribution, primarily based on exclusive synthetic records samples. The algorithms are Maximum Likelihood Estimation (MLE), Ant Colony Optimization (ACO), and Sine-Cosine Algorithm (SCA).

In this simulation, information with regarded Weibull parameters (form and scale) is generated. For numerous sample sizes (10, 20, 50, and 100), the experiment is repeated one thousand times. For every experiment, the subsequent steps arise:

1. Synthetic data is generated from a Weibull distribution using the known true parameters.
2. MLE is applied to estimate shape and scale parameters from the generated data.
3. The ACO algorithm is employed to optimize the Weibull parameters.
4. The SCA algorithm is used to optimize the Weibull parameters.

The Mean Squared Error (MSE) between the true parameters and the estimated parameters calculated for each method (MLE, ACO, SCA) in every experiment. The average MSE values are tn computed across elements for each method at each sample size

The simulation aims to compare the performance of the three optimization algorithms in terms of parameter estimation accuracy for different sample sizes. It demonstrates how the choice of optimization method affects the accuracy of parameter estimation in Weibull distribution fitting.

Table 1: Mean Squared Error (MSE) of Different Methods for Estimating

k	λ	Sample Size	MALE	ACO	SCA
3	2	10	2.201	0.687	0.512
		20	2.178	0.596	0.467
		50	1.158	0.523	0.427
		100	1.143	0.477	0.396
4	3	10	3.275	0.836	0.665
		20	2.248	0.768	0.615
		50	2.225	0.697	0.571
		100	1.210	0.647	0.535
5	4	10	3.372	0.102	0.888
		20	3.341	0.006	0.826
		50	2.315	0.945	0.773
		100	1.294	0.890	0.727

Table (1) showed that for all sample sizes and true parameters, SCA performed better than ACO and MLE. Therefore, it can be deduced that SCA provides better parameter estimation for Weibull than ACO and MLE.

Improved performance appears for higher sample size in both ACO and MLE. Therefore, using an average of MSE from only a sample size of 100, it appears that there is a great disparity between SCA and ACO against MLE.

MSE of SCA reduces when the shape value of the true parameters is high. Higher the value of the shape increases the fatness of the Weibull distribution and the accuracy becomes challenging to estimate since the distribution tends to get fattened further.

The MSE of the SCA can be minimized only by means of increasing the sample size. This is because there are more datasets on the true parameters of the distribution. Conducting an estimation of the distribution's true parameters is very easy when there is more data. It also gets more difficult when the sample size is smaller. Essentially, more accurate estimations are determined through a bigger sample.

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Using artificially generated data, the findings of the experiment imply that the superior method for determining the parameters of a Weibull distribution is MLE. Even though ACO and MLE can be utilized as alternate approaches, their precision turned out to be inferior compared to that of MLE.

3. Results and Discussion

The facts include the place, magnitude, and maximum seismic intensity of earthquakes that happened in Japan from July 1, 2023 to July 16, 2023.

The most sizeable earthquake became a value 7.3 earthquake that befell off the coast of Hokkaido on July 16, 2023. This earthquake had a seismic depth of two, which means that it brought about slight shaking in some areas.

There were also several other earthquakes with magnitudes of four or more at some stage in this term. These earthquakes occurred in various elements of Japan, such as Honshu, Hokkaido, and Kyushu.

The data shows that there is quite an excessive frequency of earthquakes in Japan. This is because Japan is positioned in a seismically active location. The Pacific Ring of Fire, a horseshoe-formed belt of volcanoes and earthquakes that encircles the Pacific Ocean, passes through Japan.

The facts are essential because they can help scientists apprehend the seismic pastime in Japan. This data can be used to expand early caution structures that may assist in shielding human beings from the outcomes of earthquakes.

In addition, the information may be used to examine the outcomes of earthquakes in the environment. For instance, scientists can use statistics to tune the motion of groundwater and to evaluate the harm to infrastructure.

Overall, the information presents precious insights into the seismic interest in Japan. These statistics may be used to improve our knowledge of earthquakes and to help us to mitigate their risks.

3.2.1 Parameters Estimation

Table (2) shows the results of estimations for the parameters of the Weibull distribution using three estimation methods, namely, the Maximum likelihood method (MLM) and Ant Colony Optimization (ACO) , Sine-Cosine algorithm (SCA)

Table 2: Weibull Distribution Parameters Estimators

Methods	Weibull distribution	
	$\hat{k}$	$\hat{\lambda}$
MLE	5.87494	2.67891
ACO	3.16689	3.99975
SCA	3.97162	2.00184

3.2.2 Goodness of Fit Results

The good fit (GOF) tests that were mentioned in the theoretical aspect were used to find the best match for the probability distributions on the earthquake data for each of the Japan governorates, where the results were as follows:

Table 3: shows the results of the goodness of fit

Methods	The goodness of fit criteria	
	AIC	BIC
MLE	-309.868	-304.293
ACO	-476.733	-443.158
SCA	-487.878	-496.303

Table (3) and Figure (1) show that AIC stands for the Akaike information criterion. It is a measure of how well a model fits the data, taking into account the number of parameters in the model. A lower AIC value indicates a better-fitting model.

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BIC stands for Bayesian information criterion. It is similar to AIC, but it penalizes more heavily for models with a large number of parameters. A lower BIC value also indicates a better-fitting model.

In this case, the SCA method has the lowest AIC and BIC values, so it is the best-fitting model, The MLE method has the highest AIC and BIC values, so it is the worst-fitting model.

In general, the BIC is preferred over the AIC because it is more conservative and less likely to overfit the data. However, the AIC is sometimes used when the sample size is small because it is less sensitive to the number of parameters in the model.

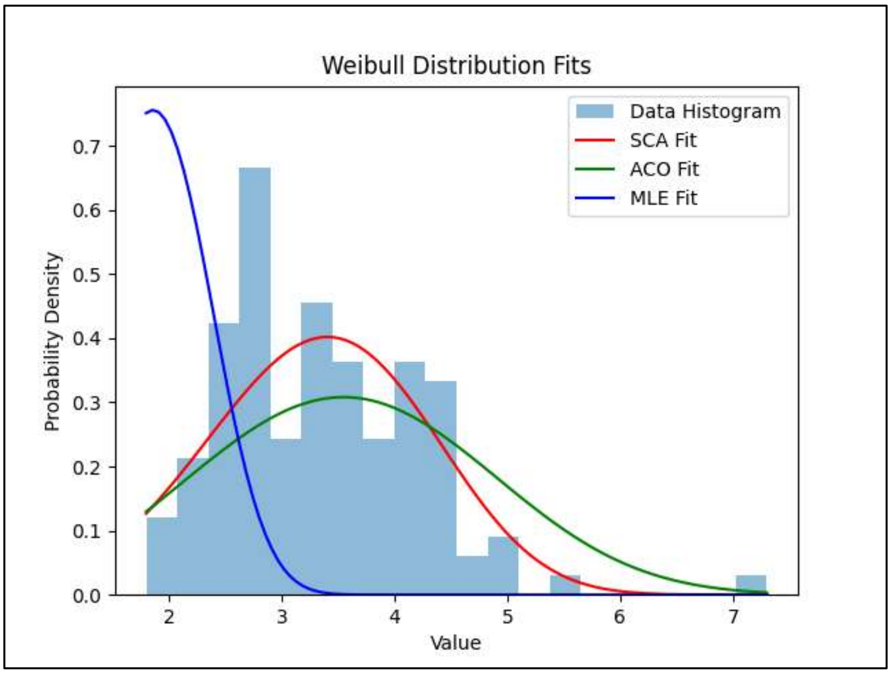

Figure 1: Weibull Distribution Fits

3.2.3 Estimate Mean of Magnitude Earthquake

You can use these parameters to get the mean and variance of this distribution after determining the most effective technique for estimating the parameters of a Weibull distribution for modeling earthquake magnitudes in actual data from Japan. The variance shows the spread or variety in earthquake magnitudes, whereas the mean indicates the average magnitude of earthquakes in the dataset. 3.328593 is the mean of The Weibull distribution, which we have determined through analyzing Japan's genuine seismic data. According to this model, it appears that Japan's typical earthquake magnitude is 3.328593.

This may be an opportunity to analyze earthquake statistics in Japan that could shed light on important aspects and facts. For example, we would investigate Weibull distribution variation when investigating the magnitudes of such natural events. In this case, the variances were measured and turned out to be Higher variance shows more amplitudes' variability while smaller variance signifies stability. These are the statistical measurements that may reveal significant details about Japan's earthquake occurrences. The first is the variance of 0.923550 which is a measure of the degree to which values are dispersed about the mean of earthquake magnitudes which is our central tendency. These statistics could help carry out a more comprehensive study and understanding of the seismicity in the area.

4. Conclusions

However, the use of ACO and the SCA optimization methods could be utilized to estimate the parameters of the Weibull function model. Widely applied in the measurement of survival rates using observed outcomes, dependable software and characterizing extreme occurrences such as earthquakes. This even outdoes using traditional approaches. In order to check on parameter estimation from synthetic data sets, SCA performed worse than MLE and ACO, but was much better in comparison to all of them. Finally, real earthquake information utility

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from Japan also supported SC as the most honest choice. In this case, estimation of Weibull mean is 3.252593, and the variance is 0.923550, which allows for acquiring information about the features of an earthquake and its variability in Japan. This enables one to comprehend the way various waves appear as well as advanced early warning and losses' quantification related to an existing geomorphological environment.

Suggestions for future applications

We suggest authors to generalize our study by using the same data and the neutrosophic version of Weibull's distribution to get the possible results in the case of indeterminacy data.

References

[1] M. Shoaib, I. Siddiqui, Y. M. Amir, and S. U. Rehman, "Evaluation of wind power potential in Baburband (Pakistan) using Weibull distribution function," *Renewable and Sustainable Energy Reviews*, vol. 70, pp. 1343–1351, 2017.

[2] Z. Tan, "A new approach to MLE of Weibull distribution with interval data," *Reliability Engineering & System Safety*, vol. 94, no. 2, pp. 394–403, 2009.

[3] J. Mazucheli, A. F. B. Menezes, and M. E. Ghitany, "The unit-Weibull distribution and associated inference," *J. Appl. Probab. Stat*, vol. 13, no. 2, pp. 1–22, 2018.

[4] K. C. Datsiou and M. Overend, "Weibull parameter estimation and goodness-of-fit for glass strength data," *Structural Safety*, vol. 73, pp. 29–41, 2018.

[5] G. H. Lemon, "Maximum likelihood estimation for the three parameter Weibull distribution based on censored samples," *Technometrics*, vol. 17, no. 2, pp. 247–254, 1975.

[6] G. Casella and R. L. Berger, "Statistical inference (Cengage Learning)," *Belmont, CA, USA*, pp. 47–218, 2021.

[7] S. Mirjalili, "SCA: a sine cosine algorithm for solving optimization problems," *Knowledge-based systems*, vol. 96, pp. 120–133, 2016.

[8] L. Abualigah and A. Diabat, "Advances in sine cosine algorithm: a comprehensive survey," *Artificial Intelligence Review*, vol. 54, no. 4, pp. 2567–2608, 2021.

[9] K. Mohammadi, O. Alavi, A. Mostafaeipour, N. Goudarzi, and M. Jalilvand, "Assessing different parameters estimation methods of Weibull distribution to compute wind power density," *Energy Conversion and Management*, vol. 108, pp. 322–335, 2016.

[10] A. B. Gabis, Y. Meraihi, S. Mirjalili, and A. Ramdane-Cherif, "A comprehensive survey of sine cosine algorithm: variants and applications," *Artificial Intelligence Review*, vol. 54, no. 7, pp. 5469–5540, 2021.

[11] I. J. Davies, "Confidence limits for Weibull parameters estimated using linear least squares analysis," *Journal of the European Ceramic Society*, vol. 37, no. 15, pp. 5057–5064, 2017.

[12] H. Y. Xue and W. L. Ma, "Research on image restoration algorithm base on ACO-BP neural network," *Key Engineering Materials*, vol. 460, pp. 136–141, 2011.

[13] A. F. K. Purian and B. E. Sadeghian, "A Novel Method for Path Planning of Mobile Robots via Fuzzy Logic and ant Colony Algorithem in Complex Daynamic Environments," *International Journal of Fuzzy Systems and Advanced Applications*, vol. 9, pp. 1–5, 2022.

[14] S. Mazzeo and I. Loiseau, "An ant colony algorithm for the capacitated vehicle routing," *Electronic Notes in Discrete Mathematics*, vol. 18, pp. 181–186, 2004.

[15] F. Smarandache, "Neutrosophic Statistics is an extension of Interval Statistics, while Plithogenic Statistics is the most general form of statistics (second version)," International Journal of Neutrosophic Science, vol. 19, pp. 148-165, 2022.

[16] M. Y. Mustafa and Z. Y. Algamal, "Neutrosophic inverse power Lindley distribution: A modeling and application for bladder cancer patients," International Journal of Neutrosophic Science, vol. 21, pp. 216-223, 2023.

[17] Z. Khan, M. M. A. Almazah, O. Hamood Odhah, H. M. Alshanbari, and T. Mehmood, "Generalized Pareto Model: Properties and Applications in Neutrosophic Data Modeling," Mathematical Problems in Engineering, vol. 2022, pp. 1-11, 2022.

[18] M. K. H. Hassan and M. Aslam, "DUS-neutrosophic multivariate inverse Weibull distribution: properties and applications," Complex & Intelligent Systems, 2023.

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