Secondary K-Range Symmetric Neutrosophic Fuzzy Matrices

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Abstract

This paper introduces and explores the concept of secondary k-Range Symmetric (RS) Neutrosophic Fuzzy Matrices (NFM) and establishes its properties and relationships with other symmetric and secondary symmetric NFMs. The study defines secondary k-RS NFMs and provides insightful numerical examples to illustrate their characteristics. The paper investigates the interconnections among s-k-RS, s-RS, k-RS, and RS NFMs, discuss on their mutual relations. Additionally, the necessary and sufficient conditions for a given NFM to qualify as a s-k-RS NFM are identified. The research demonstrates that k-symmetry implies k-RS, and vice versa, contributing to a comprehensive understanding between different types of symmetries in NFMs. Graphical representations of RS, column symmetric, and kernel symmetric adjacency and incidence NFMs are presented, unveiling intriguing patterns and relationships. While every adjacency NFM is symmetric, range symmetric, column symmetric, and kernel symmetric, the incidence matrix satisfies only kernel symmetric conditions. The study further establishes that every range symmetric adjacency NFM is a kernel symmetric adjacency NFM, though the converse does not hold in general. The existence of multiple generalized inverses of NFMs in Fn is explored, with additional equivalent conditions for certain g-inverses of s-κ-RS NFMs to retain the s-κ-RS property. We conclude by characterizing the generalized inverses belonging to specific sets λ {1, 2}, λ {1, 2, 3}, and λ {1, 2, 4} of s-k-RS NFMs, providing a comprehensive framework for understanding the structure and properties of secondary k-Range Symmetric Neutrosophic Fuzzy Matrices. This research contributes to the mathematical literature by introducing a novel class of NFMs and establishing their fundamental properties and relationships, presenting new perspectives on matrix theory in the context of neutrosophic fuzzy logic.

Keywords: Neutrosophic fuzzy matrices; s- Range symmetric; Adjacency Neutrosophic fuzzy matrices; Incidence Neutrosophic fuzzy matrices; Moore penrose inverse.

1.Introduction

In the realm of mathematical foundations, Zadeh [1] introduced the concept of fuzzy sets (FS), which has since become a fundamental tool for handling imprecise and uncertain information. The properties and applications of fuzzy matrices have been extensively studied, with Meenakshi [2] delving into their intricacies. Jaya Shree [3] contributed to the field by investigating Secondary κ-KS Fuzzy Matrices, while Shyamal and Pal [4] explored the realm of interval-valued fuzzy matrices. The notation λ {1} represents a regular fuzzy matrix, and its set of all g-inverses is denoted by P'$\mathcal{G}$. A fuzzy matrix is considered Range Symmetric (RS) and Kernel Symmetric (KS) when R(λ^T) = R(λ) and N(λ^T) = N(λ), respectively. It is noteworthy that, unlike for complex matrices, the equivalence of range and KS concepts does not hold universally for Neutrosophic Fuzzy Matrices (NFM).

The study of secondary symmetric matrices, characterized by symmetric entries relative to the secondary diagonal, was pioneered by Ann Lee [5]. Antoni, Cantoni, and Butler Paul [6] emphasized the significance of per-symmetric matrices

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concerning both diagonals in communication theory. Kim and Roush [7] delved into the exploration of generalized fuzzy matrices, contributing to the broader understanding of matrix theory. Water and Hill [8] extended the theory by introducing s-real and s-Hermitian matrices as a generalization of k-real and k-Hermitian matrices. Meenakshi and Krishanmoorthy [9] furthered this exploration by studying secondary k-Hermitian matrices, while Meenakshi and Krishnamoorthy, along with G. Ramesh [10], discussed s-k-EP matrices. Jaya Shree [12] characterized Secondary κ-KS Fuzzy Matrices, and Meenakshi and Jaya Shree [13] explored k-range symmetric matrices. Shyamal and Pal [14] contributed to the field with a study on IV Fuzzy matrices, and Meenakshi and Kalliraja [15] focused on Regular IV Fuzzy matrices.
The introduction of Atanassov's intuitionistic fuzzy sets [16] added another layer to handle partial information, focusing on truth membership and falsity-membership values. However, intuitionistic fuzzy sets fall short in handling ambiguous and contradictory data. In response, Smarandache [17] proposed the concept of neutrosophic sets, providing a mathematical framework for addressing inconsistent, imprecise, and indeterminate data. Anandhkumar et al. [18] delved into the realm of generalized symmetric neutrosophic fuzzy matrices, and their work was extended in [19] with a discussion on partial orderings, characterizations, and generalizations of k-idempotent neutrosophic fuzzy matrices. Anandan and Uthra [44] have studied a modified Fuzzy Topisis method using cosine similarities and ochiai coefficients. Anandan,Manimaran and Uthra [45] have discussed response optimization of machining parameters using vikor method under fuzzy environment.

This paper introduces and extends the concept of Range Symmetric Neutrosophic Fuzzy Matrices (RS NFM). Section 2 elaborates on the definition of a range symmetric neutrosophic fuzzy matrix, while Section 3 provides a graphical representation of Range Symmetric, Column Symmetric, and Kernel Symmetric adjacency and incidence NFMs. Section 4 delves into various generalized inverses of matrices in NFM, establishing comparable standards for g-inverses of a s-k RS fuzzy matrix to exhibit s-k RS properties. The paper concludes by characterizing the generalized inverses of a s-κ RS corresponding to specific sets $\lambda\{\{1, 2\}$, $\lambda\{\{1, 2, 3\}$, and $\lambda\{\{1, 2, 4\}$, contributing to the advancement of understanding and applications in the domain of Neutrosophic Fuzzy Matrices.

1.1 Research gap

While the concepts of range symmetric fuzzy matrices (RS FM) and k-KS matrices have been introduced and explored by Meenakshi and Jayashri, there exists a notable research gap in extending these ideas to Secondary k-RS Neutrosophic Fuzzy Matrices (NFM). The current literature lacks a comprehensive investigation into the properties, characterizations, and applications of Secondary k-RS NFMs, particularly within the context of hybrid real matrices. The existing studies have primarily focused on range symmetry and k-KS properties separately in the realm of fuzzy matrices.

However, the hybridization of these concepts in the form of Secondary k-RS NFMs introduces a novel and unexplored dimension to the understanding of matrix theory. The research gap lies in the absence of a thorough exploration of the implications of Secondary k-RS NFMs on the structure of hybrid real matrices and their applications in the domain of fuzzy matrices. Furthermore, the characterization of Secondary k-RS NFMs remains underdeveloped in the current literature. There is a need for alternative and comprehensive characterizations of Secondary k-RS NFMs to enhance the theoretical foundation and practical applications of this matrix class. The lack of such characterizations hinders a deeper understanding of the structural properties of Secondary k-RS NFMs and their potential implications in various mathematical contexts.

Additionally, while g-inverses associated with regular matrices have been extensively studied, their relationship with Secondary k-RS NFMs has not been adequately addressed. Establishing a characterization of the set of all inverses using Secondary k-RS NFMs would contribute significantly to bridging this gap and extending the applicability of these matrices in the broader field of mathematical research. In summary, the research gap identified in this study pertains to the exploration of Secondary k-RS NFMs, including their properties, characterizations, and applications in hybrid real matrices and fuzzy matrices. Addressing this gap would not only contribute to the theoretical understanding of matrix theory but also open avenues for innovative applications in diverse mathematical contexts.

1.2 Literature Review

The study of various symmetric and secondary symmetric matrices in the context of fuzzy and intuitionistic fuzzy matrices has been a rich field of research. Meenakshi and Jaya Shree [20] delved into the properties of k-kernel symmetric matrices, while Meenakshi and Krishanmoorthy [21] characterized secondary k-Hermitian matrices, contributing to the understanding of Hermitian properties in fuzzy matrices. In a related vein, Meenakshi and Jaya Shree

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[22] extended their exploration to k-range symmetric matrices, further expanding the scope of symmetry concepts in fuzzy matrices.

The work of Anandhkumar, Said Broumi have studied [23] Characterization of Fuzzy, Intuitionistic Fuzzy and Neutrosophic Fuzzy Matrices introduced the concept of secondary κ-Kernel Symmetric Fuzzy Matrices, adding a layer of complexity to the existing understanding of kernel symmetry in fuzzy matrices. Shyamal and Pal [24] contributed to the field with an exploration of Interval Valued Fuzzy matrices, providing insights into handling imprecise data. Meenakshi and Kalliraja [25] focused on Regular Interval Valued Fuzzy matrices, establishing a foundation for understanding regularity in the interval-valued context. Anandhkumar [26] extended the discussion to Kernel and k-kernel Intuitionistic Fuzzy matrices, providing insights into the intersection of kernel properties and intuitionistic fuzzy logic. Jaya Shree [27] discussed Secondary κ-Range Symmetric Fuzzy Matrices, contributing to the understanding of range symmetry in the context of secondary fuzzy matrices. Anandhkumar et al. [28] studied Generalized Symmetric Neutrosophic Fuzzy Matrices, broadening the scope of symmetry concepts to the neutrosophic fuzzy domain.

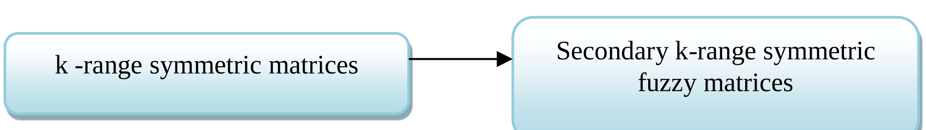
Kaliraja and Bhavani [29] explored Interval Valued Secondary *κ*-Range Symmetric Fuzzy Matrices, extending the understanding of range symmetry to the interval-valued setting. Baskett and Katz [30] Theorems on products of EPr matrices. The notion of kernel symmetric matrices for fuzzy matrices was addressed by Meenakshi and Krishnamoorthy [31]. Meenakshi, Krishnamoorthy, and Ramesh [32] delved into the study of s-k-EP matrices, contributing to the exploration of EP matrices as a generalization of k-hermitian matrices. Meenakshi and Krishnamoorthy [33] introduced s-k hermitian matrices as a generalization of secondary hermitian and hermitian matrices. The extension of the concept into s-k kernel symmetric Intuitionistic Fuzzy matrices was explored, establishing equivalent conditions for various g-inverses. Meenakshi and Krishnamoorthy [34] have discussed on κ-EP matrices. Shyamal and Pal [35] have studied Interval valued Fuzzy matrices.

The study of k-symmetric matrices was initiated by Ann Lec [36], with Elumalai and Rajesh kannan [37] expanding the understanding to k-Symmetric Circulant Matrices. Elumalai and Arthi [38] contributed to the exploration of properties of k-CentroSymmetric and k-Skew CentroSymmetric Matrices. Gunasekaran and Mohana [39] studied k-symmetric Double stochastic, s-symmetric Double stochastic, s-k-symmetric Double stochastic Matrices, broadening the applicability of symmetry concepts to double stochastic matrices. Anandhkumar et.al [40] has studied On various Inverse of Neutrosophic Fuzzy Matrices. Anandhkumar et.al [41] has discussed Reverse Sharp and Left-T Right-T Partial Ordering on Neutrosophic Fuzzy Matrices. Anandhkumar et.al [42] has focused on Reverse Tilde (T) and Minus Partial Ordering on Intuitionistic Fuzzy Matrices.Anandhkumar et.al [43] Interval Valued Secondary k-Range Symmetric Neutrosophic Fuzzy Matrices. Anandhkumar et.al [46] have discussed, Pseudo Similarity of Neutrosophic Fuzzy matrices. Anandhkumar et.al [47] has applied secondary k-column symmetric Neutrosophic Fuzzy Matrices. Bobin et.al [48] has studied Decision Making using cubic Hypersoft Topsis Method.

In summary, the literature reviewed presents a comprehensive exploration of various symmetric and secondary symmetric matrices in the context of fuzzy and intuitionistic fuzzy matrices. The studies cover a wide range of properties, characterizations, and applications, contributing to the advancement of matrix theory in the domain of imprecise and uncertain information. The research landscape is rich and varied, providing a solid foundation for further explorations and advancements in this interdisciplinary field.

Table:1 Extension of Neutrosophic Fuzzy Matrices based on previous works

References	**Extension of Neutrosophic Fuzzy Matrices from Fuzzy Matrices**	**Year**
[20]	On k-kernel symmetric matrices	2009
[22]	On k -range symmetric matrices	2009
[12]	Secondary k-Kernel Symmetric Fuzzy Matrices	2014
[27]	Secondary k-range symmetric fuzzy matrices	2018
[29]	Interval Valued Secondary κ-Range Symmetric Fuzzy Matrices	2022
Proposed	Secondary k-Range Symmetric Neutrosophic Fuzzy Matrices	



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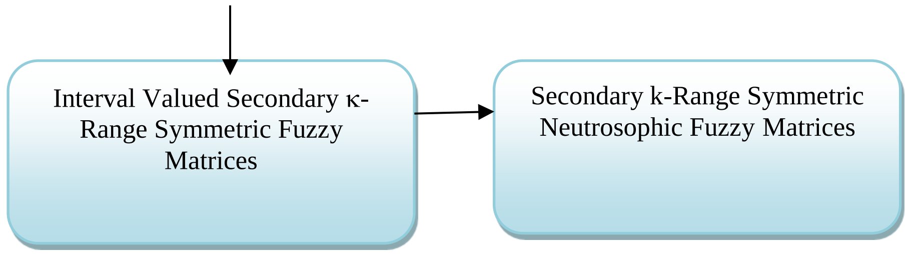

Figure 1: Flowchart of research Gap

From Table 1 and the process flow, it is evident that previous studies have focused on k-Kernel, k-range, Secondary k-Kernel, and Secondary k-range using fuzzy matrices. However, there is a noticeable research gap in these studies within a Neutrosophic environment. Based on this observation, we have addressed this gap by establishing results for K-range and Secondary k-range in neutrosophic fuzzy matrices.

1.3 Novelties

- We have introduced the concept of Secondary k-RS in Neutrosophic Fuzzy Matrices, playing a pivotal role in the hybrid fuzzy structure. Its application in NFM has been thoroughly explored, and detailed results have been studied.
- We present equivalent characterizations of Secondary k-RS Neutrosophic Fuzzy Matrices, shedding light on their structural properties. Additionally, various examples of Secondary k-RS Neutrosophic Fuzzy Matrices are provided to illustrate the concept.
- The study delves into various g-inverses associated with regular matrices, establishing a characterization of the set of all inverses through the utilization of Secondary s-k-RS Neutrosophic Matrices.

1.4 Notations

λ^{T} = Transpose of the matrix λ.

λ^{+} = Moore-Penrose inverse of λ.

R(λ) = Row space of λ.

N(λ) = Null space of λ.

RS = Range symmetric.

KS = Kernel symmetric.

Fn = Square Neutrosophic Fuzzy Matrices.

2. Range Symmetric Neutrosophic Fuzzy Matrices

Definition: 2.1 Let λ be a NFM, if R[λ] = R[λ^T] then λ is called as RS.

Example:2.1 Consider a NFM, $\lambda = \begin{bmatrix} <0.3,0.5,0.4> & <0,0,1> & <0.7,0.2,0.5> \\ <0,0,1> & <0,0,1> & <0,0,1> \\ <0.7,0.2,0.5> & <0,0,1> & <0.3,0.2,0.4> \end{bmatrix}$,

The following matrices are not RS

$$\lambda = \begin{bmatrix} <1,1,0> & <1,1,0> & <0,0,1> \\ <0,0,1> & <1,1,0> & <1,1,0> \\ <0,0,1> & <0,0,1> & <1,1,0> \end{bmatrix},\ \lambda^T = \begin{bmatrix} <1,1,0> & <0,0,1> & <0,0,1> \\ <1,1,0> & <1,1,0> & <0,0,1> \\ <0,0,1> & <1,1,0> & <1,1,0> \end{bmatrix},$$

$$\left(<1,1,0> \quad <1,1,0> \quad <0,0,1> \right) \in R(\lambda), \qquad \left(<1,1,0> \quad <1,1,0> \quad <0,0,1> \right) \in R(\lambda^T)$$

$$\left(<0,0,1> \quad <1,1,0> \quad <1,1,0> \right) \in R(\lambda), \qquad \left(<0,0,1> \quad <1,1,0> \quad <1,1,0> \right)^T \in R(\lambda^T)$$

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$(<0,0,1> \quad <0,0,1> \quad <1,1,0>) \in R(\lambda)$, $\quad (<0,0,1> \quad <0,0,1> \quad <1,1,0>) \notin R(\lambda^T)$

$R(\lambda) \notin R(\lambda^T)$

Definition 2.2: A NFM $\lambda \in F_n$ is s-symmetric NFM $\Leftrightarrow$ $\lambda = V\lambda^T V$.

Example:2.2 Consider a NFM $\lambda = \begin{bmatrix} <0.4,0.3,0.2> & <0,0,1> & <0.5,0.4,0.3> \\ <0,0,1> & <0,0,1> & <0,0,1> \\ <0.5,0.4,0.3> & <0,0,1> & <0.3,0.2,0.4> \end{bmatrix}$,

$$V = \begin{bmatrix} <0,0,0> & <0,0,0> & <1,1,0> \\ <0,0,0> & <1,1,0> & <0,0,0> \\ <1,1,0> & <0,0,0> & <0,0,0> \end{bmatrix}$$

Definition 2.3: A NFM $\lambda \in F_n$ is s-RS NFM $\Leftrightarrow$ $R(\lambda) = R(V\lambda^T V)$.

Example:2.3 Consider a NFM $\lambda = \begin{bmatrix} <0.7,0.4,0.5> & <0,0,1> & <0.8,0.2,0.1> \\ <0,0,1> & <0,0,1> & <0,0,1> \\ <0.8,0.2,0.1> & <0,0,1> & <0.5,0.7,0.3> \end{bmatrix}$,

$$V = \begin{bmatrix} <0,0,0> & <0,0,0> & <1,1,0> \\ <0,0,0> & <1,1,0> & <0,0,0> \\ <1,1,0> & <0,0,0> & <0,0,0> \end{bmatrix}$$

Definition 2.4: A NFM $\lambda \in F_n$ is s-k-RS NFM $\Leftrightarrow$ $R(\lambda) = R(KV\lambda^T VK)$.

Example:2.4 Consider a NFM $\lambda = \begin{bmatrix} <0.7,0.3,0.4> & <0.5,0.3,0.4> \\ <0.5,0.3,0.4> & <0.7,0.3,0.5> \end{bmatrix}$,

$$K = \begin{bmatrix} <1,1,0> & <0,0,0> \\ <0,0,0> & <1,1,0> \end{bmatrix}, \; V = \begin{bmatrix} <0,0,0> & <1,1,0> \\ <1,1,0> & <0,0,0> \end{bmatrix},$$

Definition 2.5: A NFM λ is KS NFM $\Leftrightarrow$ $N(A) = N(A^T)$.

Definition 2.6: A NFM λ is column symmetric NFM $\Leftrightarrow$ $C(A) = C(A^T)$.

Preliminary:2.1 Let the function is defined by $v(y) = (y_{k[1]}, y_{k[2]}, y_{k[3]}, \ldots, y_{k[n]}) \in F_{n\times 1}$ for $y = (y_1, y_2, \ldots, y_n) \in F_{[1\times n]}$, where V is permutation matrix its satisfied the following conditions, $VV^T = V^T V = I_n$ then $V^T = V$ and $R(\lambda) = R(V\lambda)$, $R(\lambda) = R(K\lambda)$.

Remark 2.1: Every s-k-symmetric NFM is s-k-RS NFM since $\lambda = KV\lambda^T VK$ if λ is s-k-symmetric NFM. Thus, $R(\lambda) = R(KV\lambda^T VK)$, signifying that λ is a NFM with s-k-RS.

Example 2.5. Consider a NFM, $V = \begin{bmatrix} <0,0,0> & <1,1,0> \\ <1,1,0> & <0,0,0> \end{bmatrix}$,

$$\lambda = \begin{bmatrix} <0.7,0.3,0.4> & <0.5,0.3,0.4> \\ <0.5,0.3,0.4> & <0.7,0.3,0.5> \end{bmatrix}, \; K = \begin{bmatrix} <1,1,0> & <0,0,0> \\ <0,0,0> & <1,1,0> \end{bmatrix},$$

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$$KV\lambda^T VK = \begin{bmatrix} <1,1,0> & <0,0,0> \\ <0,0,0> & <1,1,0> \end{bmatrix} \begin{bmatrix} <0,0,0> & <1,1,0> \\ <1,1,0> & <0,0,0> \end{bmatrix} \begin{bmatrix} <0.7,0.3,0.4> & <0.5,0.3,0.4> \\ <0.5,0.3,0.4> & <0.7,0.3,0.5> \end{bmatrix}$$

$$\begin{bmatrix} <0,0,0> & <1,1,0> \\ <1,1,0> & <0,0,0> \end{bmatrix} \begin{bmatrix} <1,1,0> & <0,0,0> \\ <0,0,0> & <1,1,0> \end{bmatrix}$$

$$KV\lambda^T VK = \begin{bmatrix} <0.7,0.3,0.4> & <0.5,0.3,0.4> \\ <0.5,0.3,0.4> & <0.7,0.3,0.5> \end{bmatrix} = \lambda$$

λ is symmetric, s-κ-symmetric which implies s- k-RS NFM.

Example 2.6. Consider a NFM

$$K = \begin{bmatrix} <0,0,0> & <1,1,0> & <0,0,0> \\ <1,1,0> & <0,0,0> & <0,0,0> \\ <0,0,0> & <0,0,0> & <1,1,0> \end{bmatrix}, V = \begin{bmatrix} <0,0,0> & <0,0,0> & <1,1,0> \\ <0,0,0> & <1,1,0> & <0,0,0> \\ <1,1,0> & <0,0,0> & <0,0,0> \end{bmatrix}$$

$$\lambda = \begin{bmatrix} <0,0,0> & <0,0,0> & <1,1,0> \\ <0.5,0.3,0.4> & <1,1,0> & <0,0,0> \\ <0.4,0.2,0.6> & <0.5,0.3,0.4> & <0,0,0> \end{bmatrix}$$

$$KV = \begin{bmatrix} <0,0,0> & <1,1,0> & <0,0,0> \\ <1,1,0> & <0,0,0> & <0,0,0> \\ <0,0,0> & <0,0,0> & <1,1,0> \end{bmatrix} \begin{bmatrix} <0,0,0> & <0,0,0> & <1,1,0> \\ <0,0,0> & <1,1,0> & <0,0,0> \\ <1,1,0> & <0,0,0> & <0,0,0> \end{bmatrix}$$

$$KV = \begin{bmatrix} <0,1,0> & <0,1,0> & <0,1,0> \\ <0,1,0> & <0,1,0> & <1,1,0> \\ <1,1,0> & <0,1,0> & <0,1,0> \end{bmatrix}$$

$$VK = \begin{bmatrix} <0,0,0> & <0,0,0> & <1,1,0> \\ <0,0,0> & <1,1,0> & <0,0,0> \\ <1,1,0> & <0,0,0> & <0,0,0> \end{bmatrix} \begin{bmatrix} <0,0,0> & <1,1,0> & <0,0,0> \\ <1,1,0> & <0,0,0> & <0,0,0> \\ <0,0,0> & <0,0,0> & <1,1,0> \end{bmatrix}$$

$$VK = \begin{bmatrix} <0,1,0> & <0,1,0> & <1,1,0> \\ <1,1,0> & <0,1,0> & <0,1,0> \\ <0,1,0> & <1,1,0> & <0,1,0> \end{bmatrix}$$

$$\lambda^T VK = \begin{bmatrix} <0.5,0.8,0.4> & <0.4,0.8,0.6> & <0,0,0.4> \\ <0,0.7,0> & <0.5,0.7,0> & <0,0.7,0> \\ <0,0,0> & <0,0,0> & <1,0,0> \end{bmatrix}$$

$$KV\lambda^T VK = \begin{bmatrix} <0,1,0> & <0,1,0> & <0,1,0> \\ <0,1,0> & <0,1,0> & <1,1,0> \\ <1,1,0> & <0,1,0> & <0,1,0> \end{bmatrix} \begin{bmatrix} <0.5,0.8,0.4> & <0.4,0.8,0.6> & <0,0,0.4> \\ <0,0.7,0> & <0.5,0.7,0> & <0,0.7,0> \\ <0,0,0> & <0,0,0> & <1,0,0> \end{bmatrix}$$

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$$KV\lambda^T VK = \begin{bmatrix} <0,0,0> & <0,0.2,0> & <0,0,0> \\ <0,0,0> & <0,0,0> & <1,0,0> \\ <0.5,0,0> & <0.4,0,0> & <0,0,0> \end{bmatrix} \neq \lambda$$

$\lambda \neq KV\lambda^T VK$ is not s- κ –symmetric iff not s- κ-RS.

3. Graphical Representation of Range symmetric, Column symmetric and kernel symmetric Adjacency NFM.

Definition 3.1. Adjacency NFM

An adjacency Neutrosophic Fuzzy Matrix is a square matrix that serves as a representation for a finite graph. The matrix's elements convey information regarding whether pairs of vertices within the graph are connected or not. In the specific scenario of a finite simple graph, the adjacency matrix can be described as a binary matrix, often denoted as a (1,1,0) and (0,0,1) -matrix, where the diagonal elements are uniformly set to (0,0,1).

G(V, E) denote a simple graph with n vertices. The adjacency matrix A = [a_{ij}] is a symmetric matrix defined by

$$A = [a_{ij}] = \begin{cases} (1,1,0) & \text{when } v_i \text{ is adjacent to } v_j \\ (0,0,1) & \text{otherwise} \end{cases}$$, denoted by A(G) or A_G

Example: 3.1 Consider an adjacency NFM and a corresponding graph

$$A = \begin{matrix} a \\ b \\ c \\ d \end{matrix} \begin{bmatrix} (0,0,1) & (1,1,0) & (0,0,1) & (1,1,0) \\ (1,1,0) & (0,0,1) & (1,1,0) & (0,0,1) \\ (0,0,1) & (1,1,0) & (0,0,1) & (1,1,0) \\ (1,1,0) & (0,0,1) & (1,1,0) & (0,0,1) \end{bmatrix}$$

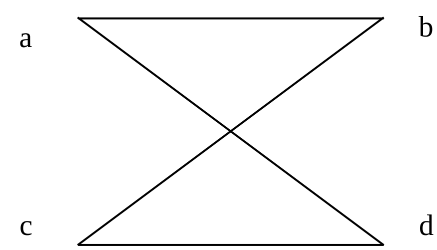

Figure 2: Graph of adjacency

Definition 3.2. Incidence NFM

G(V, E) represent a simple graph with n vertices. Let V = {V_1, V_2, …, V_n} and E = {e_1, e_2, ..., e_m}. Then, the incidence NFM I = [m_{ij}] is a $n \times m$ matrix defined by

$$I = [m_{ij}] = \begin{cases} (1,1,0) & \text{when } v_i \text{ is incident to } e_j \\ (0,0,1) & \text{otherwise} \end{cases}$$, denoted by A(G) or A_G.

Example:3.2 Consider an incidence NFM and a corresponding graph

The incidence NFM is $$I = \begin{matrix} a \\ b \\ c \\ d \end{matrix} \begin{bmatrix} (1,1,0) & (0,0,1) & (0,0,1) & (1,1,0) & (1,1,0) \\ (1,1,0) & (1,1,0) & (0,0,1) & (0,0,1) & (0,0,1) \\ (0,0,1) & (0,0,1) & (1,1,0) & (1,1,0) & (0,0,1) \\ (0,0,1) & (1,1,0) & (1,1,0) & (0,0,1) & (1,1,0) \end{bmatrix}$$

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Corresponding graph

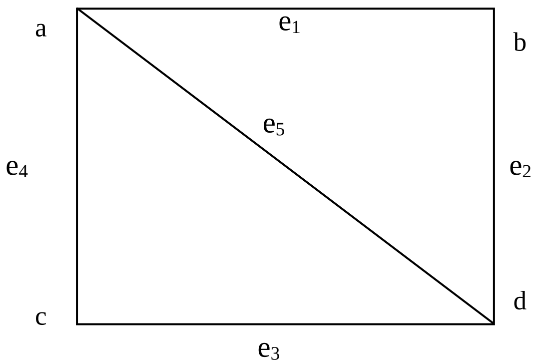

Figure 2: Graph of Incidency

3.1 Range symmetric, Column symmetric and kernel symmetric Adjacency NFM
Graph A

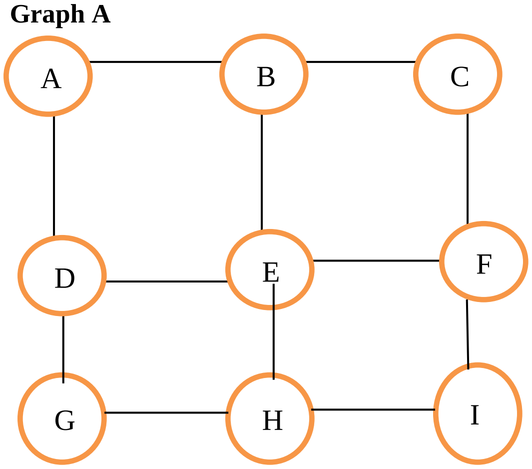

Figure 3: Graph of range symmetric Adjacency

Adjacency NFM

$$A = \begin{bmatrix} (0,0,1) & (1,1,0) & (0,0,1) & (1,1,0) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \\ (1,1,0) & (0,0,1) & (1,1,0) & (0,0,1) & (1,1,0) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \\ (0,0,1) & (1,1,0) & (0,0,1) & (0,0,1) & (0,0,1) & (1,1,0) & (0,0,1) & (0,0,1) & (0,0,1) \\ (1,1,0) & (0,0,1) & (0,0,1) & (0,0,1) & (1,1,0) & (0,0,1) & (1,1,0) & (0,0,1) & (0,0,1) \\ (0,0,1) & (1,1,0) & (0,0,1) & (1,1,0) & (0,0,1) & (1,1,0) & (0,0,1) & (1,1,0) & (0,0,1) \\ (0,0,1) & (0,0,1) & (1,1,0) & (0,0,1) & (1,1,0) & (0,0,1) & (0,0,1) & (0,0,1) & (1,1,0) \\ (0,0,1) & (0,0,1) & (0,0,1) & (1,1,0) & (0,0,1) & (0,0,1) & (0,0,1) & (1,1,0) & (0,0,1) \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (1,1,0) & (0,0,1) & (1,1,0) & (0,0,1) & (1,1,0) \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (1,1,0) & (0,0,1) & (1,1,0) & (0,0,1) \end{bmatrix}$$

The given Graph is range symmetric NFM $R(A) = R(A^T)$

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Graph B

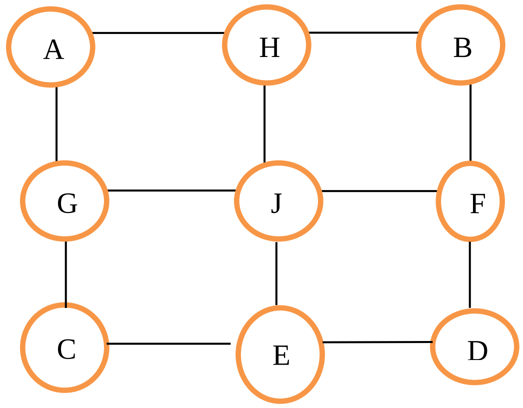

Figure 4: Graph of column symmetric Adjacency

Adjacency NFM

$$B=\begin{bmatrix}
(0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (1,1,0) & (1,1,0) & (0,0,1)\\
(0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (1,1,0) & (0,0,1) & (1,1,0) & (0,0,1)\\
(0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (1,1,0) & (0,0,1) & (1,1,0) & (0,0,1) & (0,0,1)\\
(0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (1,1,0) & (1,1,0) & (0,0,1) & (0,0,1) & (0,0,1)\\
(0,0,1) & (0,0,1) & (1,1,0) & (1,1,0) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (1,1,0)\\
(0,0,1) & (1,1,0) & (0,0,1) & (1,1,0) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (1,1,0)\\
(1,1,0) & (0,0,1) & (1,1,0) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (1,1,0)\\
(1,1,0) & (1,1,0) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (1,1,0)\\
(0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (1,1,0) & (1,1,0) & (1,1,0) & (1,1,0) & (0,0,1)
\end{bmatrix}$$

The given Graph is column symmetric NFM C(B) = C(B^T)

Graph C

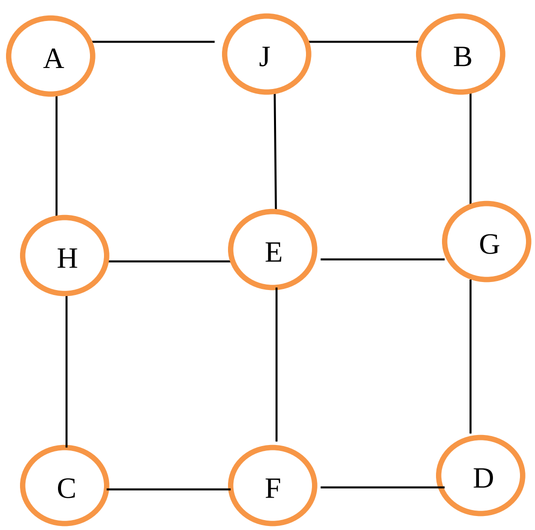

Figure 5: Graph of kernel symmetric Adjacency Adjacency Matrix

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$$C=\begin{bmatrix}(0,0,1)&(0,0,1)&(0,0,1)&(0,0,1)&(0,0,1)&(0,0,1)&(0,0,1)&(1,1,0)&(1,1,0)\\(0,0,1)&(0,0,1)&(0,0,1)&(0,0,1)&(0,0,1)&(0,0,1)&(1,1,0)&(0,0,1)&(1,1,0)\\(0,0,1)&(0,0,1)&(0,0,1)&(0,0,1)&(0,0,1)&(1,1,0)&(0,0,1)&(1,1,0)&(0,0,1)\\(0,0,1)&(0,0,1)&(0,0,1)&(0,0,1)&(0,0,1)&(1,1,0)&(1,1,0)&(0,0,1)&(0,0,1)\\(0,0,1)&(0,0,1)&(0,0,1)&(0,0,1)&(0,0,1)&(1,1,0)&(1,1,0)&(1,1,0)&(1,1,0)\\(0,0,1)&(0,0,1)&(1,1,0)&(1,1,0)&(1,1,0)&(0,0,1)&(0,0,1)&(0,0,1)&(0,0,1)\\(0,0,1)&(1,1,0)&(0,0,1)&(1,1,0)&(1,1,0)&(0,0,1)&(0,0,1)&(0,0,1)&(0,0,1)\\(1,1,0)&(0,0,1)&(1,1,0)&(0,0,1)&(1,1,0)&(0,0,1)&(0,0,1)&(0,0,1)&(0,0,1)\\(1,1,0)&(1,1,0)&(0,0,1)&(0,0,1)&(1,1,0)&(0,0,1)&(0,0,1)&(0,0,1)&(0,0,1)\end{bmatrix}$$

The given Graph is kernel symmetric Fuzzy Matrix N(C) = N(C^T)

3.2 **Incidence matrix**

$$A=\begin{matrix}a\\b\\c\\d\end{matrix}\begin{bmatrix}(1,1,0)&(0,0,1)&(0,0,1)&(0,0,1)&(0,0,1)\\(0,0,1)&(1,1,0)&(1,1,0)&(1,1,0)&(0,0,1)\\(1,1,0)&(1,1,0)&(0,0,1)&(0,0,1)&(1,1,0)\\(0,0,1)&(0,0,1)&(1,1,0)&(1,1,0)&(1,1,0)\end{bmatrix}$$

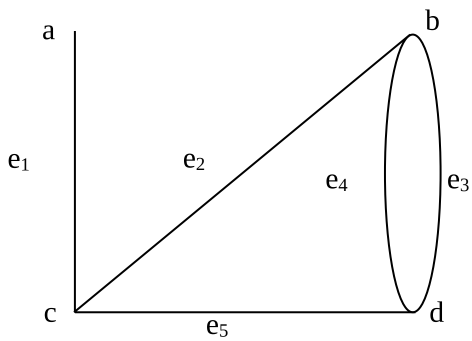

Figure 6: Graph of Incidency

The given Graph is kernel symmetric Fuzzy Matrix but not Range and Column symmetric.

Note:3.1 Every Adjacency NFM is symmetric, range symmetric, column symmetric and kernel symmetric but Incidence matrix satisfies only kernel symmetric conditions.

Note:3.2 Every range symmetric NFM is kernel symmetric NFM but kernel symmetric NFM need not be range symmetric NFM

4. Theorems and Results

Theorem 4.1: The subsequent conditions are equivalent for $\lambda \in F_n$

(i) R(λ) = R(λ^T).

(ii) $\lambda^T = \lambda$H= Kλ for several NFM H, K and $\rho(\lambda)$ = r.

Theorem 4.2 . The subsequent conditions are equivalent for $\lambda \in F_n$

(i) R(λ) = R(KVλ^TVK)

(ii) R (KVλ) = R((KVλ)^T)

(iii) R(λKV) = R((λKV)^T)

(iv) R(Vλ) = R(K(Vλ)^{T}K)

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(v) $R(\lambda K)= R(V(\lambda K)^T V)$

(vi) $R(\lambda^T)= R(KV(\lambda)VK)$

(vii) $R(\lambda) = R(\lambda^T VK)$

(viii) $R(P^T) = R(PKV)$

(ix) $\lambda = VK\lambda^T VKH_1$ for $H_1 \in F_n$

(x) $\lambda = H_1 KV\lambda^T VK$ for $H_1 \in F_n$

(xi) $\lambda^T = KV\lambda VKH$ for $H \in F_n$

(xii) $\lambda^T = HKV\lambda KV$ for $H \in F_n$

Proof: (i) ⇔ (ii) ⇔ (iv)

λ is s- κ- RS

⇔ $R(\lambda) = R(KV\lambda^T VK)$

⇔ $R(KV\lambda) = R((KV\lambda)^T$ [By Definition:2.1]

⇔ $KV\lambda$ is RS

⇔ VP is κ- RS

Hence, (i) ⇔ (ii) ⇔ (iv) hold.

(i) ⇔ (iii) ⇔ (v)

λ is s- κ - RS ⇔ $R(\lambda) = R(KV\lambda^T VK)$ [By Definition 2.4]

⇔ $R(KV\lambda) = R((KV\lambda)^T)$ [By Definition:2.5]

⇔ $R(VK(KV\lambda)(VK)^T = R((VK)\lambda^T VK(VK)^T)$

⇔ $R(\lambda KV) = R((\lambda KV)^T)$

⇔ λKV is RS

⇔ λK is s- RS

Hence, (i) ⇔ (iii) ⇔ (v) hold.

(ii) ⇔ (vii)

$KV\lambda$ is RS ⇔ $R(KV\lambda) = R((KV\lambda)^T)$

⇔ $R(\lambda) = R((KV\lambda)^T)$ [By Preliminary 2.1]

⇔ $R(\lambda) = R(\lambda^T VK)$

Hence, (ii) ⇔ (vii) hold.

(iii) iff (viii):

λVK is RS ⇔ $R(\lambda VK) = R((\lambda VK)^T)$

⇔ $R(\lambda VK) = R(\lambda^T)$ [By Preliminary 2.1]

Hence,(iii) ⇔ (viii) hold.

(i) iff (vi)

λ is s- κ- RS ⇔ $R(\lambda) = R(KV\lambda^T VK)$

⇔ $R(KV\lambda) = R((KV\lambda)^T)$ [By Preliminary 2.1]

⇔ $(KV\lambda)^T$ is RS

⇔ $\lambda^T VK$ is RS

⇔ λ^T is s- κ - RS

Hence, (i) ⇔ (vi) hold.

(i) iff (xi) iff (x)

λ is s- κ- RS ⇔ $R(\lambda) = R(KV\lambda^T VK)$

⇔ $R(\lambda^T) = R(KV\lambda VK)$

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$\Leftrightarrow$ λ^T = KVλVKH [By Theorem 2.1]

$\Leftrightarrow$ λ = H_1KVλ^TVK for $H_1 \in F_n$

Hence, (i) ⇔ (xi) ⇔ (x) hold.

(ii) ⇔ (xii) ⇔ (ix)

KVP is RS ⇔ VP is κ- RS

$\Leftrightarrow$ R(Vλ) = R(K(Vλ)^{T}K)

$\Leftrightarrow$ R(λ) = R(λ^TVK) [By Preliminary 2.1]

$\Leftrightarrow$ R (λ^T) = R(KVλ)

$\Leftrightarrow$ λ^T = HKVλ for H $\in F_n$ [By Theorem 2.1]

$\Leftrightarrow$ λ^T = HKVλKV

$\Leftrightarrow$ λ = VKλ^TVKH_1 for $H_1 \in F_n$

Hence, (ii) ⇔ (xii) ⇔ (ix) hold.

Corollary 4.1: The subsequent conditions are equivalent for $\lambda \in F_n$

(i) R(λ) = R(Vλ^TV)
(ii) R(Vλ) = R(VP)^T
(iii) R(λV) = R(λV)^T
(iv) λ is s- RS
(v) R(λ^T) = R(VλV)
(vi) R(λ) = R(λ^TV)
(vii) R(λ^T) = R(λV)
(viii) R(KVλ) = R((Vλ)^T)
(ix) λ = Vλ^TVH_1 for $H_1 \in F_n$
(x) λ = H_1Vλ^TV for $H_1 \in F_n$
(xi) λ^T = VλVH for H $\in F_n$
(xii) λ^T = HVλV for H $\in$ F

Theorem 4.3: For $\lambda \in F_n$ then any pair of the following statements indicate the other one

(i) R(λ) = R(Kλ^TK)
(ii) R(λ) = R(VKλ^TKV)
(iii) R(λ^T) = R((VKλ)^T)

Proof: (i) and (ii) iff (iii)

λ is s- κ – RS $\Rightarrow$R(λ) = R(λ^TVK) [By Theorem 2.2]

$\Rightarrow$R(KλK) = R(Kλ^TK)

Hence (i) and (ii) $\Rightarrow$ R(λ^T) = R((VλK)^T)

Hence, (iii) hold.

(i) and (iii) iff (ii)

λ is κ- RS $\Rightarrow$R(λ) = R (Kλ^TK)

$\Rightarrow$R(KλK) = R(λ^T)

Hence (i) and (iii) $\Rightarrow$ R(KλK)= R((VλK)^T)

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$\Rightarrow$R(λ) = R(λ^TVK)

$\Rightarrow$ R(λ) = R((KVλ)^T)

$\Rightarrow$ λ is s- κ RS [By Theorem 2.2]

Therefore,(ii) hold.

(ii) and (iii) $\Rightarrow$(i)

λ is s- κ – RS $\Rightarrow$ R(λ)= R(λ^TVK)

$\Rightarrow$R(KλK) = R (Kλ^TV) [By Definition:4.5]

Hence (ii) and (iii) $\Rightarrow$ R (KλK) = R(λ^T)

$\Rightarrow$R(λ) = R(Kλ^TK)

$\Rightarrow$ λ is κ – RS

Therefore, (i) hold. Hence the Theorem

5. s- κ-Range Symmetric Regular NFM

We show the existence of several generalized inverses of NFM in F_n. and determine the conditions for different g-inverses of a s-k-RS NFM to be s-k RS NFM. Generalized inverses belonging to the sets λ {1, 2}, λ {1, 2, 3} and λ {1, 2, 4} of s-k-RS NFM are λ characterized.

Theorem 5.1: Let $\lambda \in F_n$, $Z \in F_n$ {1,2} and λZ, Zλ, are s- κ-RS NFM. Then λ is s- κ – RS NFM ⇔ Z is s- κ – RS NFM.

Proof: R(KVλ) = R(KVλZλ) ⊆ R(Zλ) [since λ = λZλ]

= R(ZVVλ) = R(ZVKKVλ) ⊆ R(KVλ)

Hence, R(KVλ) = R(Zλ)

= R(KV(Zλ)^{T}VK) [Zλ is s- κ-RS NFM]

= R(λ^T Z^T VK)

= R(Z^T VK)

= R((KVZ)^T)

R ((KVλ)^T) = R (λ^T VK)

= R(Z^T λ^T VK)

= R((KVλZ)^T)

= R(KVλZ) [Vλ is s- κ -RS]

= R(KVZ)

KVZ is RS ⇔ R(KVλ) = R((KVλ)^T)

⇔ R((KVZ)^T) = R(KVZ)

⇔ KVZ is RS

⇔ Z is s- κ- RS.

Theorem 5.2: Let $\lambda \in F_n$, Z ∈ λ {1,2,3}, R(KVλ) = R((KVZ)^T).Then λ is s-κ-RS NFM ⇔ Z is s- κ - RS NFM.

Proof: Since Z ∈ λ {1,2,3}, Hence λZλ = λ,ZλZ =Z, (λZ)^T= λZ

R ((KVλ)^T) = R(Z^T λ^T VK) [By using λZλ = λ]

= R(KV(λZ)^T)

= R((λZ)^T)

= R(λZ) [(λZ)^T= λZ]

= R(Z) [By using Z = ZλZ]

= R(KVZ)

KVλ is RS NFM ⇔ R(KVλ) = R((KVλ)^T)

⇔ R((KVZ)^T) = R(KVZ)

⇔ KVZ is RS

⇔ Z is s- κ - RS.

Theorem 5.3: Let $\lambda \in F_n$, Z ∈ λ {1,2,4}, R((KVP)^T) = R(KVZ) . Then λ is s- κ-RS NFM ⇔ Z is s- κ- RS NFM.

Proof: Since Z ∈ λ {1, 2, 4}, we have λZλ = λ, ZλZ =Z, (Zλ)^T=Zλ

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R(KV) = R(λ)
= R(Zλ) [ZλZ = Z, λZλ = λ] = N((Zλ)^T) [(Zλ)^T = Zλ]
= R($\lambda^T Z^T$)
= R (Z^T)
= R((KVZ)^T).

KVλ is RS NFM ⇔ R(KVλ) = R((KVλ)^T
⇔ R((KVZ)^T) = R(KVZ)
⇔ KVZ is RS NFM
⇔ Z is s- κ – RS NFM.

In particular for K = I, the above Theorems reduces to equivalent conditions for various g-inverses of a s-kernel symmetric NFM to be secondary kernel symmetric NFM.

Corollary 5.1: Let λ belongs to F_n, Z belongs λ(1, 2) and λZ, Zλ are s- RS NFM. Then P is s- RS NFM ⇔ Z is s- RS NFM.

Corollary 5.2: Let λ belongs to F_n , Z belongs to λ(1, 2, 3), R(KVλ) = R((VX)^T) . Then λ is s- RS NFM ⇔ Z is s- RS NFM.

Corollary 5.3: Let λ belongs to F_n, Z belongs to λ (1, 2, 4) ,R((Vλ)^T) = R(VZ) . Then λ is s- RS NFM ⇔ Z is s-RS NFM.

5. Conclusion

In conclusion, our study has successfully established significant connections and relationships among various matrix properties, such as k-symmetry, k-RS, s-RS, and RS NFM. We have provided a comprehensive characterization of the generalized inverses for specific sets, namely {1, 2}, {1, 2, 3}, and {1, 2, 4}, within the context of s-RS NFM. Our findings also present the implications of κ-symmetry, establishing its direct correlation with κ-RS while emphasizing the universality of the reverse relationship. Additionally, we present a graphical representation of Range symmetric, Column symmetric, and kernel symmetric Adjacency and Incidence NFM. Notably, every Adjacency NFM exhibits symmetry, range symmetry, column symmetry, and kernel symmetry, while the Incidence matrix adheres strictly to kernel symmetric conditions. Intriguingly, our investigation reveals that every range symmetric Adjacency NFM is a kernel symmetric Adjacency NFM, although the converse is not universally true. These results contribute valuable insights to the field, advancing our understanding of the intricate relationships within matrix properties and paving the way for further exploration in this domain. In future, we will work on related properties of secondary k-range symmetric neutrosophic fuzzy matrices.

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