T-Spherical Fuzzy Valued Neutrosophic Aggregation Operators: Applications in Multi-Attribute Decision Making

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Abstract

This paper aims to introduce and explore the innovative concept of T-spherical fuzzy valued neutrosophic sets (T-SFVNSs) in the context of multi-attribute decision making (MADM). The T-SFVNSs are utilized to develop two key aggregation operators: the T-spherical fuzzy valued neutrosophic weighted average operator (T-SFVNWAO) and the T-spherical fuzzy valued neutrosophic weighted geometric operator (T-SFVNWGO). These operators are defined based on the operational rules of T-SFVNNs. The properties of these operators, including idempotency, boundedness, and monotonicity, are rigorously examined and established. To demonstrate the practicality and relevance of the T-SFVN operators, an algorithm and a numerical application are presented. The algorithm illustrates the step-by-step implementation of these operators, while the numerical application showcases their effectiveness in real-world scenarios. Additionally, a comparative analysis is conducted to evaluate the accuracy and performance of the proposed operators in relation to existing ones.

Keywords: Aggregation Operators; Decision Making; Neutrosophic Set; Optimization; T-Spherical Fuzzy Valued Neutrosophic Sets.

1 Introduction

Decision theory has emerged as a pivotal field of study across various scientific disciplines, particularly in addressing the challenges of decision-making under uncertainty. The evaluation of alternatives and the selection of the most preferable option are central to the decision-making process.MADM, originally proposed by Churchman et al.,[1] involves ranking alternatives or determining the best choice based on the evaluation of multiple attributes associated with each alternative. In conventional MADM problems, decision-makers typically provide deterministic measurements to express their preferences. However, due to several factors such as time constraints, limited decision-making capabilities, and the growing complexity and uncertainty

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of problems, decision-makers often encounter difficulties in providing deterministic measurements for solving MADM problems. To address this challenge, Zadeh[2] introduced the concept of fuzzy sets (FS) to handle fuzzy information, enabling a more flexible representation of attribute values in uncertain MADM problems. Nevertheless, there are cases where fuzzy sets alone fail to adequately capture the nuances of vague, ambiguous, incomplete, and indeterminate information in MADM problems. To effectively handle such cases, several generalizations and variations of FSs have been developed. Notably, picture fuzzy sets (PFS),[3] spherical fuzzy sets(SFS),[4] T-spherical fuzzy sets(T-SFSs)[5] and neutrosophic sets(NSs)[6] have emerged as notable extensions in this field.

PFS has emerged as an advanced extension, encompassing the features of both FS and intuitionistic fuzzy set (IFS).[7] PFS introduces distinct characteristics through its truth membership function (T), indeterminacy membership function (I), and falsity membership function (F), all residing within the standard interval of [0, 1]. It is essential to note that the sum of these functions must satisfy the constraint of being greater than or equal to 0 and less than or equal to 1, specifically $0 \leq T + I + F \leq 1$. The primary objective of PFS is to effectively capture and express the essence of indeterminacy, surpassing the limitations imposed by traditional FS and IFS. Building upon the foundations of PFS, further advancements have led to the development of SFS and T-SFS. SFS introduces membership grades that adhere to the condition $0 \leq T^2 + I^2 + F^2 \leq 1$, deviating from the previous constraint of $0 \leq T + I + F \leq 1$ observed in PFS. The concept of the SFS was significantly enhanced by Mahmood et al.[5] They introduced a groundbreaking framework known as T-SFS, which incorporates a novel constraint: $0 \leq T^q + I^q + F^q \leq 1$, where $q \in \mathbb{Z}$ and $q \geq 1$. This constraint enables decision makers to operate within a more flexible environment, effectively avoiding information loss during the decision-making process. T-SFSs possess the remarkable ability to represent a broader range of fuzzy information compared to SFSs. The applications of both SFS and T-SFS span across various domains, particularly in the realm of decision making.[8–11] These innovative frameworks have proven invaluable in tackling complex decision-making problems, offering enhanced versatility and precision to decision makers.

In a similar vein, Smarandache[6] made remarkable contributions to the field by introducing the notion of NSs as an intricate generalization of IFS and FS. NSs were meticulously crafted to address the intricacies and multifaceted nature of challenges encountered in the realm of MADM. NSs exhibit three fundamental components, namely the membership function (MF), non-membership function (NMF), and indeterminacy term (IMF). The noteworthy characteristic of NSs lies in the constraint that the sum of these components must not exceed three, thereby facilitating a more comprehensive and precise representation of real-world data. Consequently, NSs have garnered substantial attention and interest from researchers globally, resulting in extensive scholarly investigations and analyses. Expanding on this foundational work, Ali and Smarandache[12] ventured to extend SNSs from the real space to the complex space, thereby introducing the innovative concept of complex neutrosophic sets (CNS). This progressive development was further advanced with the introduction of Q-complex neutrosophic sets[13] and fuzzy parameterized complex neutrosophic soft expert sets,[14] all within the same overarching framework. Continuing this trajectory of breakthroughs, Al-Sharqi et al.[15] successfully amalgamated NSs and soft sets under the interval complex value, culminating in a unified and comprehensive approach.

In recent times, there has been a significant focus on integrating the characteristics of NSs, IFSs, and Pythagorean fuzzy sets (PyFS) to improve accuracy and enhance aggregation operators (AOs) for dealing with data inaccuracies. Bhowmik and Pal[16] introduced the concept of intuitionistic fuzzy valued neutrosophic set(IFVNS) and its operators, with a condition that the sum of its membership functions (MFs) should not exceed two. Building upon this, Unver et al.[17] expanded the definition of IFVNS by introducing IF neutrosophic multisets (IFNMSs) and explored algebraic operations between IFVNSs to develop various AOs. Palanikumar et al.[18] discussed a novel generalization called Pythagorean neutrosophic normal interval-valued weighted geometric (PNNIVWG) and devised an algorithm to handle MADM problems using these operators. Chellamani and Ajay[19] proposed graphical concepts using the Dombi operator within Pythagorean neutrosophic fuzzy graphs (PyNFG), while Ajay and Chellamani[20] utilized soft parameters for MCDM scenarios in the PyFVNS environment. Palanikumar and Arulmozhi[21] introduced a fresh approach to AOs by incorporating parameterized factors in the PyFVNS framework, and they proposed a score function that combined TOPSIS and VIKOR techniques. Rajan and Krishnaswamy[22] developed clustering methods based on similarity measures between PyFVNSs. Recently, Bozyigit et al.[23] redefined PyFVNS, where each component of the NS consists of a PyFVS satisfying the condition: $T^2 + F^2 \leq 1$. However, the limitations of IFVNSs and PyFVNSs arise from their narrow applicability to decision-making problems where evaluation values are represented using IF and PyF values, which may not fully capture the complete decision-related information. This article aims to expand the scope of IFVNSs and PyFVNSs by incorporating T- spherical fuzzy values into the construction of

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the SNS to introduce an innovative concept known as T-spherical fuzzy-valued neutrosophic sets (T-SFVNS) and its aggregation operators.

2 Literature Review

In this section, we will present a brief summary of the key concepts of NS, T-SFS, IFVNS, and PyFVNS to establish a foundational understanding. Additionally, we will introduce the notion of T-SFVNS along with its corresponding score functions.

Definition 2.1. [6] Let $\mathcal{U}$ be a universal set. A NS $\mathcal{M}$ in $\mathcal{U}$ is a structure of the form

$$\mathcal{M} = \{< u; \mathcal{T}_{\mathcal{M}}(u), \mathcal{I}_{\mathcal{M}}(u), \mathcal{F}_{\mathcal{M}}(u) >: u \in \mathcal{U}\},$$

where the mappings $\mathcal{T}_{\mathcal{M}}; \mathcal{I}_{\mathcal{M}}; \mathcal{F}_{\mathcal{M}} : \mathcal{U} \rightarrow]^{-}0; 1^{+}[$ represent the TM, IM and FM functions, respectively with $^{-}0 \leq \mathcal{T}_{\mathcal{M}} + \mathcal{I}_{\mathcal{M}} + \mathcal{F}_{\mathcal{M}} \leq 3^{+}$.

The concept of T-SFS was introduced and defined by Mahmood et al.[5] in the following manner.

Definition 2.2. [5] A the T-SFS $\mathfrak{L}$ on the finite set $\mathfrak{Q}$ is portrayed as follows.

$$\mathfrak{L} = \{(\mathfrak{h}, \mathfrak{T}_{\mathfrak{L}}(\mathfrak{h}), \mathfrak{I}_{\mathfrak{L}}(\mathfrak{h}), \mathfrak{F}_{\mathfrak{L}}(\mathfrak{h})) : \mathfrak{h} \in \mathfrak{Q}\},$$

where $\mathfrak{T}_{\mathfrak{L}}(\mathfrak{h}), \mathfrak{I}_{\mathfrak{L}}(\mathfrak{h})$ and $\mathfrak{F}_{\mathfrak{L}}(\mathfrak{h}) \in [0,1]$ and $0 \leq (\mathfrak{T}_{\mathfrak{L}}(\mathfrak{h}))^q + (\mathfrak{I}_{\mathfrak{L}}(\mathfrak{h}))^q + (\mathfrak{F}_{\mathfrak{L}}(\mathfrak{h}))^q \leq 1$ $(q \geq 1)$, for all $\mathfrak{h} \in \mathfrak{Q}$. The refusal degree of $\mathfrak{h}$ to $\mathfrak{Q}$ is determined by

$$\mathfrak{B}_{\mathfrak{L}}(\mathfrak{h}) = \Big(1 - \big[(\mathfrak{T}_{\mathfrak{L}}(\mathfrak{h}))^q + (\mathfrak{I}_{\mathfrak{L}}(\mathfrak{h}))^q + (\mathfrak{F}_{\mathfrak{L}}(\mathfrak{h}))^q\big]\Big)^{1/q}.$$

Unver et al.[17] redefined the definition of IFVNS, which stands as follows.

Definition 2.3. [17] An IFVNS $\mathcal{Q}$ in a universe $\hat{\Delta}$ with a generic element u in $\hat{\Delta}$ is characterized as:

$$\mathcal{Q} = \{< \hat{\delta}; \mathbb{T}_{\mathcal{Q}}(\hat{\delta}), \mathbb{I}_{\mathcal{Q}}(\hat{\delta}), \mathbb{F}_{\mathcal{Q}}(\hat{\delta}) >: \hat{\delta} \in \hat{\Delta}\},$$

where $\mathbb{T}_{\mathcal{Q}}, \mathbb{I}_{\mathcal{Q}}$ and $\mathbb{F}_{\mathcal{Q}}$ represent the membership, indeterminacy membership and non-membership neutrosophic values, each of them is an IF value, where $\forall \hat{\delta} \in \hat{\Delta}$, $\mathbb{T}_{\mathcal{Q}} = \big(\hat{\zeta}_{\mathcal{Q},\mathbb{T}}(\hat{\delta}), \hat{\varpi}_{\mathcal{Q},\mathbb{T}}(\hat{\delta})\big)$ such that $\hat{\zeta}_{\mathcal{Q},\mathbb{T}}(\hat{\delta}), \hat{\varpi}_{\mathcal{Q},\mathbb{T}}(\hat{\delta}) \in [0,1]$ with the condition $\hat{\zeta}_{\mathcal{Q},\mathbb{T}}(\hat{\delta}) + \hat{\varpi}_{\mathcal{Q},\mathbb{T}}(\hat{\delta}) \leq 1$, $\mathbb{I}_{\mathcal{Q}} = \big(\hat{\zeta}_{\mathcal{Q},\mathbb{I}}(\hat{\delta}), \hat{\varpi}_{\mathcal{Q},\mathbb{I}}(\hat{\delta})\big)$ such that $\hat{\zeta}_{\mathcal{Q},\mathbb{I}}(\hat{\delta}), \hat{\varpi}_{\mathcal{Q},\mathbb{I}}(\hat{\delta}) \in [0,1]$ with the condition $\hat{\zeta}_{\mathcal{Q},\mathbb{I}}(\hat{\delta}) + \hat{\varpi}_{\mathcal{Q},\mathbb{I}}(\hat{\delta}) \leq 1$, $\mathbb{F}_{\mathcal{Q}} = \big(\hat{\zeta}_{\mathcal{Q},\mathbb{F}}(\hat{\delta}), \hat{\varpi}_{\mathcal{Q},\mathbb{F}}(\hat{\delta})\big)$ such that $\hat{\zeta}_{\mathcal{Q},\mathbb{F}}(\hat{\delta}), \hat{\varpi}_{\mathcal{Q},\mathbb{F}}(\hat{\delta}) \in [0,1]$ with the condition $\hat{\zeta}_{\mathcal{Q},\mathbb{F}}(\hat{\delta}) + \hat{\varpi}_{\mathcal{Q},\mathbb{F}}(\hat{\delta}) \leq 1$.

Bozyigit et al.[23] have extended the concept of IFVNS by introducing a novel extension called PyFVNS, as explained in the following.

Definition 2.4. [23] A PyFVNS $\mathcal{W}$ in a universe $\hat{\Delta}$ with a generic element u in $\hat{\Delta}$ is characterized as:

$$\mathcal{W} = \{< \hat{\delta}; \mathbb{T}_{\mathcal{W}}(\hat{\delta}), \mathbb{I}_{\mathcal{W}}(\hat{\delta}), \mathbb{F}_{\mathcal{W}}(\hat{\delta}) >: \hat{\delta} \in \hat{\Delta}\},$$

where $\mathbb{T}_{\mathcal{W}}, \mathbb{I}_{\mathcal{W}}$ and $\mathbb{F}_{\mathcal{W}}$ represent the membership, indeterminacy membership and non-membership neutrosophic values, each of them is a PyF value, where $\forall \hat{\delta} \in \hat{\Delta}$, $\mathbb{T}_{\mathcal{W}} = \big(\hat{\zeta}_{\mathcal{W},\mathbb{T}}(\hat{\delta}), \hat{\varpi}_{\mathcal{W},\mathbb{T}}(\hat{\delta})\big)$ such that $\hat{\zeta}_{\mathcal{W},\mathbb{T}}(\hat{\delta}), \hat{\varpi}_{\mathcal{W},\mathbb{T}}(\hat{\delta}) \in [0,1]$ with the condition $(\hat{\zeta}_{\mathcal{W},\mathbb{T}}(\hat{\delta}))^2 + (\hat{\varpi}_{\mathcal{W},\mathbb{T}}(\hat{\delta}))^2 \leq 1$, $\mathbb{I}_{\mathcal{W}} = \big(\hat{\zeta}_{\mathcal{W},\mathbb{I}}(\hat{\delta}), \hat{\varpi}_{\mathcal{W},\mathbb{I}}(\hat{\delta})\big)$ such that $\hat{\zeta}_{\mathcal{W},\mathbb{I}}(\hat{\delta}), \hat{\varpi}_{\mathcal{W},\mathbb{I}}(\hat{\delta}) \in [0,1]$ with the condition $(\hat{\zeta}_{\mathcal{W},\mathbb{I}}(\hat{\delta}))^2 + (\hat{\varpi}_{\mathcal{W},\mathbb{I}}(\hat{\delta}))^2 \leq 1$, $\mathbb{F}_{\mathcal{W}} = \big(\hat{\zeta}_{\mathcal{W},\mathbb{F}}(\hat{\delta}), \hat{\varpi}_{\mathcal{W},\mathbb{F}}(\hat{\delta})\big)$ such that $\hat{\zeta}_{\mathcal{W},\mathbb{F}}(\hat{\delta}), \hat{\varpi}_{\mathcal{W},\mathbb{F}}(\hat{\delta}) \in [0,1]$ with the condition $(\hat{\zeta}_{\mathcal{W},\mathbb{F}}(\hat{\delta}))^2 + (\hat{\varpi}_{\mathcal{W},\mathbb{F}}(\hat{\delta}))^2 \leq 1$.

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Here, we present the formal definitions of T-SFVNS and T-SFVNN, along with the corresponding score functions (SFs) of T-SFVNN.

Definition 2.5. Let $\hat{\mathcal{U}}$ be a universe. A T-SFVNS $\mathcal{S}$ over $\hat{\mathcal{U}}$ is signified by $\mathcal{S} = \{\langle u, \mathcal{T}_\mathcal{S}, \mathcal{I}_\mathcal{S}, \mathcal{F}_\mathcal{S}\rangle : u \in \hat{\mathcal{U}}\}$, where $\mathcal{T}_\mathcal{S}, \mathcal{I}_\mathcal{S}$ and $\mathcal{F}_\mathcal{S}$ represent the membership, indeterminacy membership and non-membership neutrosophic values, each of them is a T- spherical fuzzy value, where $\forall u \in \hat{\mathcal{U}}, q \geq 1, \mathcal{T}_\mathcal{S} = \big(\mu_{\mathcal{S},\mathcal{T}}(u), \omega_{\mathcal{S},\mathcal{T}}(u), \nu_{\mathcal{S},\mathcal{T}}(u)\big)$ such that $\mu_{\mathcal{S},\mathcal{T}}(u), \omega_{\mathcal{S},\mathcal{T}}(u), \nu_{\mathcal{S},\mathcal{T}}(u) \in [0,1]$, subject to the condition $(\mu_{\mathcal{S},\mathcal{T}}(u))^q + (\omega_{\mathcal{S},\mathcal{T}}(u))^q + (\nu_{\mathcal{S},\mathcal{T}}(u))^q \leq 1$, $\mathcal{I}_\mathcal{S} = \big(\mu_{\mathcal{S},\mathcal{I}}(u), \omega_{\mathcal{S},\mathcal{I}}(u), \nu_{\mathcal{S},\mathcal{I}}(u)\big)$ such that $\mu_{\mathcal{S},\mathcal{I}}(u), \omega_{\mathcal{S},\mathcal{I}}(u), \nu_{\mathcal{S},\mathcal{I}}(u) \in [0,1]$, subject to the condition $(\mu_{\mathcal{S},\mathcal{I}}(u))^q + (\omega_{\mathcal{S},\mathcal{I}}(u))^q + (\nu_{\mathcal{S},\mathcal{I}}(u))^q \leq 1, \mathcal{F}_\mathcal{S} = \big(\mu_{\mathcal{S},\mathcal{F}}(u), \omega_{\mathcal{S},\mathcal{F}}(u), \nu_{\mathcal{S},\mathcal{F}}(u)\big)$ such that $\mu_{\mathcal{S},\mathcal{F}}(u), \omega_{\mathcal{S},\mathcal{F}}(u), \nu_{\mathcal{S},\mathcal{F}}(u) \in [0,1]$, subject to the condition $(\mu_{\mathcal{S},\mathcal{F}}(u))^q + (\omega_{\mathcal{S},\mathcal{F}}(u))^q + (\nu_{\mathcal{S},\mathcal{F}}(u))^q \leq 1$. By definition, $0 \leq \mathcal{T}_\mathcal{S} + \mathcal{I}_\mathcal{S} + \mathcal{F}_\mathcal{S} \leq 3$. A T-SFVNS $\mathcal{S}$ over $\hat{\mathcal{U}}$ can be written as:

$$\mathcal{S} = \{\langle u, \big(\mu_{\mathcal{S},\mathcal{T}}(u), \omega_{\mathcal{S},\mathcal{T}}(u), \nu_{\mathcal{S},\mathcal{T}}(u)\big), \big(\mu_{\mathcal{S},\mathcal{I}}(u), \omega_{\mathcal{S},\mathcal{I}}(u), \nu_{\mathcal{S},\mathcal{I}}(u)\big), \big(\mu_{\mathcal{S},\mathcal{F}}(u), \omega_{\mathcal{S},\mathcal{F}}(u), \nu_{\mathcal{S},\mathcal{F}}(u)\big)\rangle : u \in \hat{\mathcal{U}}\}.$$

Definition 2.6. A collection of $\Gamma = \big\langle \big(\mu_\mathcal{T}, \omega_\mathcal{T}, \nu_\mathcal{T}\big), \big(\mu_\mathcal{I}, \omega_\mathcal{I}, \nu_\mathcal{I}\big), \big(\mu_\mathcal{F}, \omega_\mathcal{F}, \nu_\mathcal{F}\big)\big\rangle$ is called T-SFVNN with $(\mu_\mathcal{T})^q + (\omega_\mathcal{T})^q + (\nu_\mathcal{T})^q \leq 1,\ (\mu_\mathcal{I})^q + \omega_\mathcal{I})^q + (\nu_\mathcal{I})^q \leq 1$ and $(\mu_\mathcal{F})^q + (\omega_\mathcal{F})^q + (\nu_\mathcal{F})^q \leq 1$, $(q \geq 1)$.

In this part, we provide the definitions of several functions, including the score function (SF), accuracy function (AF), quadratic SF (QSF), and quadratic AF (QAF).

Definition 2.7. Let $\Gamma = \big\langle \big(\mu_\mathcal{T}, \omega_\mathcal{T}, \nu_\mathcal{T}\big), \big(\mu_\mathcal{I}, \omega_\mathcal{I}, \nu_\mathcal{I}\big), \big(\mu_\mathcal{F}, \omega_\mathcal{F}, \nu_\mathcal{F}\big)\big\rangle$ be T-SFVNN. Then the SF on Γ is defined as $\Pi_\Gamma = \Pi(\Gamma) = \frac{1}{3}\Big[\big[(\mu_\mathcal{T})^q - (\nu_\mathcal{T})^q\big] + \Big(1 - \big[(\mu_\mathcal{I})^q - (\nu_\mathcal{I})^q\big]\Big) + \Big(1 - \big[(\mu_\mathcal{F})^q - (\nu_\mathcal{F})^q\big]\Big)\Big], q \geq 1$.

Definition 2.8. Let $\Gamma = \big\langle \big(\mu_\mathcal{T}, \omega_\mathcal{T}, \nu_\mathcal{T}\big), \big(\mu_\mathcal{I}, \omega_\mathcal{I}, \nu_\mathcal{I}\big), \big(\mu_\mathcal{F}, \omega_\mathcal{F}, \nu_\mathcal{F}\big)\big\rangle$ be T-SFVNN. Then the AF $\urcorner$ on Γ is defined as $\urcorner_\Gamma = \urcorner(\Gamma) = \Big[\big[(\mu_\mathcal{T})^q + (\omega_\mathcal{T})^q + (\nu_\mathcal{T})^q\big] - \big[(\mu_\mathcal{F})^q + (\omega_\mathcal{F})^q + (\nu_\mathcal{F})^q\big]\Big], q \geq 1$.

Definition 2.9. Let Γ_1 and Γ_2 be two T-SFVNNs.

1. If $\Pi_{\Gamma_1} < \Pi_{\Gamma_2}$, then $\Gamma_1 < \Gamma_2$,
2. If $\Pi_{\Gamma_1} > \Pi_{\Gamma_2}$, then $\Gamma_1 > \Gamma_2$,
3. If $\Pi_{\Gamma_1} = \Pi_{\Gamma_2}$ and $\urcorner_{\Gamma_1} < \urcorner_{\Gamma_2}$, then $\Gamma_1 < \Gamma_2$,
4. If $\Pi_{\Gamma_1} = \Pi_{\Gamma_2}$ and $\urcorner_{\Gamma_1} > \urcorner_{\Gamma_2}$, then $\Gamma_1 > \Gamma_2$.

Definition 2.10. Let $\Gamma = \big\langle \big(\mu_\mathcal{T}, \omega_\mathcal{T}, \nu_\mathcal{T}\big), \big(\mu_\mathcal{I}, \omega_\mathcal{I}, \nu_\mathcal{I}\big), \big(\mu_\mathcal{F}, \omega_\mathcal{F}, \nu_\mathcal{F}\big)\big\rangle$ be a T-SFVNN. Then the QSF on Γ is defined as $\Omega_\Gamma = \Omega(\Gamma) = \frac{1}{3}\Big[\big[(\mu_\mathcal{T})^{2q} - (\nu_\mathcal{T})^{2q}\big] + \Big(1 - \big[(\mu_\mathcal{I})^{2q} - (\nu_\mathcal{I})^{2q}\big]\Big) + \Big(1 - \big[(\mu_\mathcal{F})^{2q} - (\nu_\mathcal{F})^{2q}\big]\Big)\Big]$, $q \geq 1$.

Definition 2.11. Let $\Gamma = \big\langle \big(\mu_\mathcal{T}, \omega_\mathcal{T}, \nu_\mathcal{T}\big), \big(\mu_\mathcal{I}, \omega_\mathcal{I}, \nu_\mathcal{I}\big), \big(\mu_\mathcal{F}, \omega_\mathcal{F}, \nu_\mathcal{F}\big)\big\rangle$ be T-SFVNN. Then the QAF $\sqsupset$ on Γ is defined as $\sqsupset_\Gamma = \sqsupset(\Gamma) = \Big[\big[(\mu_\mathcal{T})^{2q} + (\omega_\mathcal{T})^{2q} + (\nu_\mathcal{T})^{2q}\big] - \big[(\mu_\mathcal{F})^{2q} + (\omega_\mathcal{F})^{2q} + (\nu_\mathcal{F})^{2q}\big]\Big], q \geq 1$.

QSF and QAF can be used to compare two T-SFVNNs as follows.

Definition 2.12. Let Γ_1 and Γ_2 be two q-ROFVNNs.

1. If $\Omega_{\Gamma_1} < \Omega_{\Gamma_2}$, then $\Gamma_1 < \Gamma_2$,
2. If $\Omega_{\Gamma_1} > \Omega_{\Gamma_2}$, then $\Gamma_1 > \Gamma_2$,
3. If $\Omega_{\Gamma_1} = \Omega_{\Gamma_2}$ and $\sqsupset_{\Gamma_1} < \sqsupset_{\Gamma_2}$, then $\Gamma_1 < \Gamma_2$,
4. If $\Omega_{\Gamma_1} = \Omega_{\Gamma_2}$ and $\sqsupset_{\Gamma_1} > \sqsupset_{\Gamma_2}$, then $\Gamma_1 > \Gamma_2$.

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3 T-SFVN Aggregation Operators

Here, we define the T-SFVNWA operator and discuss its properties.

Definition 3.1. Let $\Gamma_\varepsilon = \{\langle ({}^\varepsilon\mu_{\mathcal{T}}, {}^\varepsilon\omega_{\mathcal{T}}, {}^\varepsilon\nu_{\mathcal{T}}), ({}^\varepsilon\mu_{\mathcal{I}}, {}^\varepsilon\omega_{\mathcal{I}}, {}^\varepsilon\nu_{\mathcal{I}}), ({}^\varepsilon\mu_{\mathcal{F}}, {}^\varepsilon\omega_{\mathcal{F}}, {}^\varepsilon\nu_{\mathcal{F}})\rangle : \varepsilon = 1, ..., n\}$ be a collection of T-SFVNNs. The T-SFVNWA operator is characterized by the transformation $T - SFVNWA : T - SFVNN(\hat{\mathcal{U}}) \longrightarrow T - SFVNN(\hat{\mathcal{U}})$ and defined as :

$$T - SFVNWA(\Gamma_1, \Gamma_2, ..., \Gamma_n) = \eta_1\Gamma_1 \oplus \eta_2\Gamma_2 \oplus ...\eta_n\Gamma_n,$$

where $\eta_\varepsilon \in [0, 1]$ is the weight of Γ_ε, $\forall\varepsilon = 1, ..., n$ and $\sum_{\varepsilon=1}^{n} \eta_\varepsilon = 1$.

Definition 3.2. Let $\Gamma_\varepsilon = \{\langle ({}^\varepsilon\mu_{\mathcal{T}}, {}^\varepsilon\omega_{\mathcal{T}}, {}^\varepsilon\nu_{\mathcal{T}}), ({}^\varepsilon\mu_{\mathcal{I}}, {}^\varepsilon\omega_{\mathcal{I}}, {}^\varepsilon\nu_{\mathcal{I}}), ({}^\varepsilon\mu_{\mathcal{F}}, {}^\varepsilon\omega_{\mathcal{F}}, {}^\varepsilon\nu_{\mathcal{F}})\rangle : \varepsilon = 1, ..., n\}$ be a collection of T-SFVNNs and $\eta = (\eta_1, \eta_2, ...\eta_n)$ be the weight vector of Γ_ε. Then,

$$T - SFVNWA(\Gamma_1, \Gamma_2, ..., \Gamma_n) = \Big\langle\Big([1 - \prod_{\varepsilon=1}^{n}(1 - ({}^\varepsilon\mu_{\mathcal{T}})^q)^{\eta_\varepsilon}]^{\frac{1}{q}}, \prod_{\varepsilon=1}^{n}({}^\varepsilon\omega_{\mathcal{T}})^{\eta_\varepsilon}, \prod_{\varepsilon=1}^{n}({}^\varepsilon\nu_{\mathcal{T}})^{\eta_\varepsilon}\Big), \Big(\prod_{\varepsilon=1}^{n}({}^\varepsilon\mu_{\mathcal{I}})^{\eta_\varepsilon}, [1 - \prod_{\varepsilon=1}^{n}(1 - ({}^\varepsilon\omega_{\mathcal{I}})^q)^{\eta_\varepsilon}]^{\frac{1}{q}}, [1 - \prod_{\varepsilon=1}^{n}(1 - ({}^\varepsilon\nu_{\mathcal{I}})^q)^{\eta_\varepsilon}]^{\frac{1}{q}}\Big), \Big(\prod_{\varepsilon=1}^{n}({}^\varepsilon\mu_{\mathcal{F}})^{\eta_\varepsilon}, [1 - \prod_{\varepsilon=1}^{n}(1 - ({}^\varepsilon\omega_{\mathcal{F}})^q)^{\eta_\varepsilon}]^{\frac{1}{q}}, [1 - \prod_{\varepsilon=1}^{n}(1 - ({}^\varepsilon\nu_{\mathcal{F}})^q)^{\eta_\varepsilon}]^{\frac{1}{q}}\Big)\Big\rangle, q \geq 1. \quad (1)$$

The T-SFVNWA operator has the following properties.

Proposition 3.3. *Idempotency Property: Let* $\Gamma_\varepsilon = \{\langle ({}^\varepsilon\mu_{\mathcal{T}}, {}^\varepsilon\omega_{\mathcal{T}}, {}^\varepsilon\nu_{\mathcal{T}}), ({}^\varepsilon\mu_{\mathcal{I}}, {}^\varepsilon\omega_{\mathcal{I}}, {}^\varepsilon\nu_{\mathcal{I}}), ({}^\varepsilon\mu_{\mathcal{F}}, {}^\varepsilon\omega_{\mathcal{F}}, {}^\varepsilon\nu_{\mathcal{F}})\rangle : \varepsilon = 1, ..., n\}$ *be a collection of T-SFVNNs. If* $\Gamma_\varepsilon = \Gamma = \langle (\mu_{\mathcal{T}}, \omega_{\mathcal{T}}, \nu_{\mathcal{T}}), (\mu_{\mathcal{I}}, \omega_{\mathcal{I}}, \nu_{\mathcal{I}}), (\mu_{\mathcal{F}}, \omega_{\mathcal{F}}, \nu_{\mathcal{F}})\rangle, \forall\varepsilon = 1, ..., n$. *Then,* $T - SFVNWA(\Gamma_1, \Gamma_2, ..., \Gamma_n) = \Gamma = \langle (\mu_{\mathcal{T}}, \omega_{\mathcal{T}}, \nu_{\mathcal{T}}), (\mu_{\mathcal{I}}, \omega_{\mathcal{I}}, \nu_{\mathcal{I}}), (\mu_{\mathcal{F}}, \omega_{\mathcal{F}}, \nu_{\mathcal{F}})\rangle$.

Proof. Since $\Gamma_\varepsilon = \Gamma = \langle (\mu_{\mathcal{T}}, \omega_{\mathcal{T}}, \nu_{\mathcal{T}}), (\mu_{\mathcal{I}}, \omega_{\mathcal{I}}, \nu_{\mathcal{I}}), (\mu_{\mathcal{F}}, \omega_{\mathcal{F}}, \nu_{\mathcal{F}})\rangle, \forall\varepsilon = 1, ..., n$. Then, based on Definition 3.2,

$$T - SFVNWA(\Gamma_1, \Gamma_2, ..., \Gamma_n) = \Big\langle\Big([1 - \prod_{\varepsilon=1}^{n}(1 - ({}^\varepsilon\mu_{\mathcal{T}})^q)^{\eta_\varepsilon}]^{\frac{1}{q}}, \prod_{\varepsilon=1}^{n}({}^\varepsilon\omega_{\mathcal{T}})^{\eta_\varepsilon}, \prod_{\varepsilon=1}^{n}({}^\varepsilon\nu_{\mathcal{T}})^{\eta_\varepsilon}\Big), \Big(\prod_{\varepsilon=1}^{n}({}^\varepsilon\mu_{\mathcal{I}})^{\eta_\varepsilon}, [1 - \prod_{\varepsilon=1}^{n}(1 - ({}^\varepsilon\omega_{\mathcal{I}})^q)^{\eta_\varepsilon}]^{\frac{1}{q}}, [1 - \prod_{\varepsilon=1}^{n}(1 - ({}^\varepsilon\nu_{\mathcal{I}})^q)^{\eta_\varepsilon}]^{\frac{1}{q}}\Big), \Big(\prod_{\varepsilon=1}^{n}({}^\varepsilon\mu_{\mathcal{F}})^{\eta_\varepsilon}, [1 - \prod_{\varepsilon=1}^{n}(1 - ({}^\varepsilon\omega_{\mathcal{F}})^q)^{\eta_\varepsilon}]^{\frac{1}{q}}, [1 - \prod_{\varepsilon=1}^{n}(1 - ({}^\varepsilon\nu_{\mathcal{F}})^q)^{\eta_\varepsilon}]^{\frac{1}{q}}\Big)\Big\rangle,$$

$$= \Big\langle\Big(\big[1-(1-(\mu_{\mathcal{T}})^q)^{\sum_{\varepsilon=1}^{n}\eta_\varepsilon}\big]^{\frac{1}{q}}, (\omega_{\mathcal{T}})^{\sum_{\varepsilon=1}^{n}\eta_\varepsilon}, (\nu_{\mathcal{T}})^{\sum_{\varepsilon=1}^{n}\eta_\varepsilon}\Big), \Big((\mu_{\mathcal{I}})^{\sum_{\varepsilon=1}^{n}\eta_\varepsilon}, \big[1-(1-(\omega_{\mathcal{I}})^q)^{\sum_{\varepsilon=1}^{n}\eta_\varepsilon}\big]^{\frac{1}{q}}, \big[1-(1 - (\nu_{\mathcal{I}})^q)^{\sum_{\varepsilon=1}^{n}\eta_\varepsilon}\big]^{\frac{1}{q}}\Big), \Big((\mu_{\mathcal{F}})^{\sum_{\varepsilon=1}^{n}\eta_\varepsilon}, \big[1 - (1 - (\omega_{\mathcal{F}})^q)^{\sum_{\varepsilon=1}^{n}\eta_\varepsilon}\big]^{\frac{1}{q}}, \big[1 - (1 - (\nu_{\mathcal{F}})^q)^{\sum_{\varepsilon=1}^{n}\eta_\varepsilon}\big]^{\frac{1}{q}}\Big)\Big\rangle,$$

$$= \Big\langle\Big(\big[1 - (1 - (\mu_{\mathcal{T}})^q)\big]^{\frac{1}{q}}, (\omega_{\mathcal{T}}), (\nu_{\mathcal{T}})\Big), \Big((\mu_{\mathcal{I}}), \big[1 - (1 - (\omega_{\mathcal{I}})^q)\big]^{\frac{1}{q}}, \big[1 - (1 - (\nu_{\mathcal{I}})^q)\big]^{\frac{1}{q}}\Big), \Big((\mu_{\mathcal{F}}), \big[1 - (1 - (\omega_{\mathcal{F}})^q)\big]^{\frac{1}{q}}, \big[1 - (1 - (\nu_{\mathcal{F}})^q)\big]^{\frac{1}{q}}\Big)\Big\rangle,$$

$$= \langle (\mu_{\mathcal{T}}, \omega_{\mathcal{T}}, \nu_{\mathcal{T}}), (\mu_{\mathcal{I}}, \omega_{\mathcal{I}}, \nu_{\mathcal{I}}), (\mu_{\mathcal{F}}, \omega_{\mathcal{F}}, \nu_{\mathcal{F}})\rangle = \Gamma.$$

□

Proposition 3.4. *Boundedness Property: Let* $\Gamma_\varepsilon = \{\langle ({}^\varepsilon\mu_{\mathcal{T}}, {}^\varepsilon\omega_{\mathcal{T}}, {}^\varepsilon\nu_{\mathcal{T}}), ({}^\varepsilon\mu_{\mathcal{I}}, {}^\varepsilon\omega_{\mathcal{I}}, {}^\varepsilon\nu_{\mathcal{I}}), ({}^\varepsilon\mu_{\mathcal{F}}, {}^\varepsilon\omega_{\mathcal{F}}, {}^\varepsilon\nu_{\mathcal{F}})\rangle : \varepsilon = 1, ..., n\}$ *be a collection of T-SFVNNs. If* $\Gamma^- = \langle (\mu^-_{\mathcal{T}}, \omega^+_{\mathcal{T}}, \nu^+_{\mathcal{T}}), (\mu^+_{\mathcal{I}}, \omega^-_{\mathcal{I}}, \nu^-_{\mathcal{I}}), (\mu^+_{\mathcal{F}}, \omega^-_{\mathcal{F}}, \nu^-_{\mathcal{F}})\rangle$ *and* $\Gamma^+ = \langle (\mu^+_{\mathcal{T}}, \omega^-_{\mathcal{T}}, \nu^-_{\mathcal{T}}), (\mu^-_{\mathcal{I}}, \omega^+_{\mathcal{I}}, \nu^+_{\mathcal{I}}), (\mu^-_{\mathcal{F}}, \omega^+_{\mathcal{F}}, \nu^+_{\mathcal{F}})\rangle$, *where,* $\mu^-_{\mathcal{T}} = \min_\varepsilon\{{}^\varepsilon\mu_{\mathcal{T}}\}, \mu^+_{\mathcal{T}} = \max_\varepsilon\{{}^\varepsilon\mu_{\mathcal{T}}\}, \mu^-_{\mathcal{I}} = \min_\varepsilon\{{}^\varepsilon\mu_{\mathcal{I}}\}, \mu^+_{\mathcal{I}} = \max_\varepsilon\{{}^\varepsilon\mu_{\mathcal{I}}\}, \mu^-_{\mathcal{F}} = \min_\varepsilon\{{}^\varepsilon\mu_{\mathcal{F}}\}, \mu^+_{\mathcal{F}} = \max_\varepsilon\{{}^\varepsilon\mu_{\mathcal{F}}\}, \omega^-_{\mathcal{T}} = \min_\varepsilon\{{}^\varepsilon\omega_{\mathcal{T}}\}, \omega^+_{\mathcal{T}} = \max_\varepsilon\{{}^\varepsilon\omega_{\mathcal{T}}\}, \omega^-_{\mathcal{I}} = \min_\varepsilon\{{}^\varepsilon\omega_{\mathcal{I}}\}, \omega^+_{\mathcal{I}} = \max_\varepsilon\{{}^\varepsilon\omega_{\mathcal{I}}\}, \omega^-_{\mathcal{F}} = \min_\varepsilon\{{}^\varepsilon\omega_{\mathcal{F}}\}, \omega^+_{\mathcal{F}} = \max_\varepsilon\{{}^\varepsilon\omega_{\mathcal{F}}\}, \nu^-_{\mathcal{T}} = \min_\varepsilon\{{}^\varepsilon\nu_{\mathcal{T}}\}, \nu^+_{\mathcal{T}} = \max_\varepsilon\{{}^\varepsilon\nu_{\mathcal{T}}\}, \nu^-_{\mathcal{I}} = \min_\varepsilon\{{}^\varepsilon\nu_{\mathcal{I}}\}, \nu^+_{\mathcal{I}} = \max_\varepsilon\{{}^\varepsilon\nu_{\mathcal{I}}\}, \nu^-_{\mathcal{F}} = \min_\varepsilon\{{}^\varepsilon\nu_{\mathcal{F}}\}, \nu^+_{\mathcal{F}} = \max_\varepsilon\{{}^\varepsilon\nu_{\mathcal{F}}\}$. *Then,*
$\Gamma^- \leq T - SFVNWA(\Gamma_1, \Gamma_2, ..., \Gamma_n) \leq \Gamma^+$

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Proof. Since $\mu_{\mathcal{T}}^{-} \leq {}^{\varepsilon}\mu_{\mathcal{T}} \leq \mu_{\mathcal{T}}^{+}$, then , for $q \geq 1$, we obtain

$(\mu_{\mathcal{T}}^{-})^q \leq ({}^{\varepsilon}\mu_{\mathcal{T}})^q \leq (\mu_{\mathcal{T}}^{+})^q \Rightarrow 1-(\mu_{\mathcal{T}}^{-})^q \geq 1-({}^{\varepsilon}\mu_{\mathcal{T}})^q \geq 1-(\mu_{\mathcal{T}}^{+})^q \Rightarrow (1-(\mu_{\mathcal{T}}^{-})^q)^{\eta_\varepsilon} \geq (1-({}^{\varepsilon}\mu_{\mathcal{T}})^q)^{\eta_\varepsilon} \geq (1-(\mu_{\mathcal{T}}^{+})^q)^{\eta_\varepsilon} \Rightarrow \prod_{\varepsilon=1}^{n}(1-(\mu_{\mathcal{T}}^{-})^q)^{\eta_\varepsilon} \geq \prod_{\varepsilon=1}^{n}(1-({}^{\varepsilon}\mu_{\mathcal{T}})^q)^{\eta_\varepsilon} \geq \prod_{\varepsilon=1}^{n}(1-(\mu_{\mathcal{T}}^{+})^q)^{\eta_\varepsilon} \Rightarrow 1-\prod_{\varepsilon=1}^{n}(1-(\mu_{\mathcal{T}}^{-})^q)^{\eta_\varepsilon} \leq 1-\prod_{\varepsilon=1}^{n}(1-({}^{\varepsilon}\mu_{\mathcal{T}})^q)^{\eta_\varepsilon} \leq 1-\prod_{\varepsilon=1}^{n}(1-(\mu_{\mathcal{T}}^{+})^q)^{\eta_\varepsilon} \Rightarrow \big[1-\prod_{\varepsilon=1}^{n}(1-(\mu_{\mathcal{T}}^{-})^q)^{\eta_\varepsilon}\big]^{\frac{1}{q}} \leq \big[1-\prod_{\varepsilon=1}^{n}(1-({}^{\varepsilon}\mu_{\mathcal{T}})^q)^{\eta_\varepsilon}\big]^{\frac{1}{q}} \leq \big[1-\prod_{\varepsilon=1}^{n}(1-(\mu_{\mathcal{T}}^{+})^q)^{\eta_\varepsilon}\big]^{\frac{1}{q}}$, since, $\big[1-\prod_{\varepsilon=1}^{n}(1-(\mu_{\mathcal{T}}^{-})^q)^{\eta_\varepsilon}\big]^{\frac{1}{q}} = \mu_{\mathcal{T}}^{-}$ and $\big[1-\prod_{\varepsilon=1}^{n}(1-(\mu_{\mathcal{T}}^{+})^q)^{\eta_\varepsilon}\big]^{\frac{1}{q}} = \mu_{\mathcal{T}}^{+}$. Then, $\mu_{\mathcal{T}}^{-} \leq \big[1-\prod_{\varepsilon=1}^{n}(1-({}^{\varepsilon}\mu_{\mathcal{T}})^q)^{\eta_\varepsilon}\big]^{\frac{1}{q}} \leq \mu_{\mathcal{T}}^{+}$.

Similarly, since $\omega_{\mathcal{I}}^{-} \leq {}^{\varepsilon}\omega_{\mathcal{I}} \leq \omega_{\mathcal{I}}^{+}$, and $\omega_{\mathcal{F}}^{-} \leq {}^{\varepsilon}\omega_{\mathcal{F}} \leq \omega_{\mathcal{F}}^{+}$, we obtain, $\omega_{\mathcal{I}}^{-} \leq \big[1-\prod_{\varepsilon=1}^{n}(1-({}^{\varepsilon}\omega_{\mathcal{I}})^q)^{\eta_\varepsilon}\big]^{\frac{1}{q}} \leq \omega_{\mathcal{I}}^{+}$ and $\omega_{\mathcal{F}}^{-} \leq \big[1-\prod_{\varepsilon=1}^{n}(1-({}^{\varepsilon}\omega_{\mathcal{F}})^q)^{\eta_\varepsilon}\big]^{\frac{1}{q}} \leq \omega_{\mathcal{F}}^{+}$ and since $\nu_{\mathcal{I}}^{-} \leq {}^{\varepsilon}\nu_{\mathcal{I}} \leq \nu_{\mathcal{I}}^{+}$, and $\nu_{\mathcal{F}}^{-} \leq {}^{\varepsilon}\nu_{\mathcal{F}} \leq \nu_{\mathcal{F}}^{+}$, we obtain, $\nu_{\mathcal{I}}^{-} \leq \big[1-\prod_{\varepsilon=1}^{n}(1-({}^{\varepsilon}\nu_{\mathcal{I}})^q)^{\eta_\varepsilon}\big]^{\frac{1}{q}} \leq \nu_{\mathcal{I}}^{+}$ and $\nu_{\mathcal{F}}^{-} \leq \big[1-\prod_{\varepsilon=1}^{n}(1-({}^{\varepsilon}\nu_{\mathcal{F}})^q)^{\eta_\varepsilon}\big]^{\frac{1}{q}} \leq \nu_{\mathcal{F}}^{+}$.

Now, since $\omega_{\mathcal{T}}^{-} \leq {}^{\varepsilon}\omega_{\mathcal{T}} \leq \omega_{\mathcal{T}}^{+} \Rightarrow (\omega_{\mathcal{T}}^{-})^{\eta_\varepsilon} \leq ({}^{\varepsilon}\omega_{\mathcal{T}})^{\eta_\varepsilon} \leq (\omega_{\mathcal{T}}^{+})^{\eta_\varepsilon} \Rightarrow \prod_{\varepsilon=1}^{n}(\omega_{\mathcal{T}}^{-})^{\eta_\varepsilon} \leq \prod_{\varepsilon=1}^{n}({}^{\varepsilon}\omega_{\mathcal{T}})^{\eta_\varepsilon} \leq \prod_{\varepsilon=1}^{n}(\omega_{\mathcal{T}}^{+})^{\eta_\varepsilon}$, since, $\prod_{\varepsilon=1}^{n}(\omega_{\mathcal{T}}^{-})^{\eta_\varepsilon} = \omega_{\mathcal{T}}^{-}$ and $\prod_{\varepsilon=1}^{n}(\omega_{\mathcal{T}}^{+})^{\eta_\varepsilon} = \omega_{\mathcal{T}}^{+}$. Then, $\omega_{\mathcal{T}}^{-} \leq \prod_{\varepsilon=1}^{n}({}^{\varepsilon}\omega_{\mathcal{T}})^{\eta_\varepsilon} \leq \omega_{\mathcal{T}}^{+}$.

In the same manner, we obtain $\nu_{\mathcal{T}}^{-} \leq \prod_{\varepsilon=1}^{n}({}^{\varepsilon}\nu_{\mathcal{T}})^{\eta_\varepsilon} \leq \nu_{\mathcal{T}}^{+}$, $\mu_{\mathcal{I}}^{-} \leq \prod_{\varepsilon=1}^{n}({}^{\varepsilon}\mu_{\mathcal{I}})^{\eta_\varepsilon} \leq \mu_{\mathcal{I}}^{+}$, and $\mu_{\mathcal{F}}^{-} \leq \prod_{\varepsilon=1}^{n}({}^{\varepsilon}\mu_{\mathcal{F}})^{\eta_\varepsilon} \leq \mu_{\mathcal{F}}^{+}$.

Now, let $T-SFVNWA(\Gamma_1,\Gamma_2,...,\Gamma_n) = \Gamma = \big\langle(\mu_{\mathcal{T}},\ \omega_{\mathcal{T}},\ \nu_{\mathcal{T}}),(\mu_{\mathcal{I}},\ \omega_{\mathcal{I}},\ \nu_{\mathcal{I}}),(\mu_{\mathcal{F}},\ \omega_{\mathcal{F}},\ \nu_{\mathcal{F}})\big\rangle$. Then,

$\Pi_\Gamma = \Pi(\Gamma) = \frac{1}{3}\Big[\big[(\mu_{\mathcal{T}})^q-(\nu_{\mathcal{T}})^q\big]+\Big(1-\big[(\mu_{\mathcal{I}})^q-(\nu_{\mathcal{I}})^q\big]\Big)+\Big(1-\big[(\mu_{\mathcal{F}})^q-(\nu_{\mathcal{F}})^q\big]\Big)\Big] \geq \Pi_\Gamma = \Pi(\Gamma) = \frac{1}{3}\Big[\big[(\mu_{\mathcal{T}}^{-})^q-(\nu_{\mathcal{T}}^{+})^q\big]+\Big(1-\big[(\mu_{\mathcal{I}}^{+})^q-(\nu_{\mathcal{I}}^{-})^q\big]\Big)+\Big(1-\big[(\mu_{\mathcal{F}}^{+})^q-(\nu_{\mathcal{F}}^{-})^q\big]\Big)\Big] = \Pi(\Gamma^-)$, and

$\Pi(\Gamma) = \frac{1}{3}\Big[\big[(\mu_{\mathcal{T}})^q-(\nu_{\mathcal{T}})^q\big]+\Big(1-\big[(\mu_{\mathcal{I}})^q-(\nu_{\mathcal{I}})^q\big]\Big)+\Big(1-\big[(\mu_{\mathcal{F}})^q-(\nu_{\mathcal{F}})^q\big]\Big)\Big] \leq \Pi_\Gamma = \Pi(\Gamma) = \frac{1}{3}\Big[\big[(\mu_{\mathcal{T}}^{+})^q-(\nu_{\mathcal{T}}^{-})^q\big]+\Big(1-\big[(\mu_{\mathcal{I}}^{-})^q-(\nu_{\mathcal{I}}^{+})^q\big]\Big)+\Big(1-\big[(\mu_{\mathcal{F}}^{-})^q-(\nu_{\mathcal{F}}^{+})^q\big]\Big)\Big] = \Pi(\Gamma^+)$. This implies $\Gamma^- \leq T-SFVNWA(\Gamma_1,\Gamma_2,...,\Gamma_n) \leq \Gamma^+$. □

Proposition 3.5. *Monotonicity Property: Let* $\Gamma_\varepsilon = \big\{\big\langle({}^{\varepsilon}\mu_{\mathcal{T}},\ {}^{\varepsilon}\omega_{\mathcal{T}},\ {}^{\varepsilon}\nu_{\mathcal{T}}),({}^{\varepsilon}\mu_{\mathcal{I}},\ {}^{\varepsilon}\omega_{\mathcal{I}},\ {}^{\varepsilon}\nu_{\mathcal{I}}),({}^{\varepsilon}\mu_{\mathcal{F}},\ {}^{\varepsilon}\omega_{\mathcal{F}},\ {}^{\varepsilon}\nu_{\mathcal{F}})\big\rangle : \varepsilon = 1,...,n\big\}$ *and* $\Gamma_\varepsilon^* = \big\{\big\langle({}^{\varepsilon}\mu_{\mathcal{T}}^*,\ {}^{\varepsilon}\omega_{\mathcal{T}}^*,\ {}^{\varepsilon}\nu_{\mathcal{T}}^*),({}^{\varepsilon}\mu_{\mathcal{I}}^*,\ {}^{\varepsilon}\omega_{\mathcal{I}}^*,\ {}^{\varepsilon}\nu_{\mathcal{I}}^*),({}^{\varepsilon}\mu_{\mathcal{F}}^*,\ {}^{\varepsilon}\omega_{\mathcal{F}}^*,\ {}^{\varepsilon}\nu_{\mathcal{F}}^*)\big\rangle : \varepsilon = 1,...,n\big\}$ *be two collections of T-SFVNNs. If* ${}^{\varepsilon}\mu_{\mathcal{T}} \leq^{\varepsilon} \mu_{\mathcal{T}}^*$, ${}^{\varepsilon}\omega_{\mathcal{T}} \geq^{\varepsilon} \omega_{\mathcal{T}}^*$, ${}^{\varepsilon}\nu_{\mathcal{T}} \geq^{\varepsilon} \nu_{\mathcal{T}}^*$, ${}^{\varepsilon}\mu_{\mathcal{I}} \geq^{\varepsilon} \mu_{\mathcal{I}}^*$, ${}^{\varepsilon}\omega_{\mathcal{I}} \leq^{\varepsilon} \omega_{\mathcal{I}}^*$, ${}^{\varepsilon}\nu_{\mathcal{I}} \leq^{\varepsilon} \nu_{\mathcal{I}}^*$, ${}^{\varepsilon}\mu_{\mathcal{F}} \geq^{\varepsilon} \mu_{\mathcal{F}}^*$, ${}^{\varepsilon}\omega_{\mathcal{F}} \leq^{\varepsilon} \omega_{\mathcal{F}}^*$ *and* ${}^{\varepsilon}\nu_{\mathcal{F}} \leq^{\varepsilon} \nu_{\mathcal{F}}^*$, $\forall \varepsilon = 1,2,...,n$. *Then,* $T-SFVNWA(\Gamma_1,\Gamma_2,...,\Gamma_n) \leq T-SFVNWA(\Gamma_1^*,\Gamma_2^*,...,\Gamma_n^*)$.

Proof. Since ${}^{\varepsilon}\mu_{\mathcal{T}} \leq^{\varepsilon} \mu_{\mathcal{T}}^*$, then , for $q \geq 1$, we obtain

$({}^{\varepsilon}\mu_{\mathcal{T}})^q \leq ({}^{\varepsilon}\mu_{\mathcal{T}}^*)^q \Rightarrow 1-({}^{\varepsilon}\mu_{\mathcal{T}})^q \geq 1-({}^{\varepsilon}\mu_{\mathcal{T}}^*)^q \Rightarrow (1-({}^{\varepsilon}\mu_{\mathcal{T}})^q)^{\eta_\varepsilon} \geq (1-({}^{\varepsilon}\mu_{\mathcal{T}}^*)^q)^{\eta_\varepsilon} \Rightarrow \prod_{\varepsilon=1}^{n}(1-({}^{\varepsilon}\mu_{\mathcal{T}})^q)^{\eta_\varepsilon} \geq \prod_{\varepsilon=1}^{n}(1-({}^{\varepsilon}\mu_{\mathcal{T}}^*)^q)^{\eta_\varepsilon} \Rightarrow 1-\prod_{\varepsilon=1}^{n}(1-({}^{\varepsilon}\mu_{\mathcal{T}})^q)^{\eta_\varepsilon} \leq 1-\prod_{\varepsilon=1}^{n}(1-({}^{\varepsilon}\mu_{\mathcal{T}}^*)^q)^{\eta_\varepsilon} \Rightarrow \big[1-\prod_{\varepsilon=1}^{n}(1-({}^{\varepsilon}\mu_{\mathcal{T}})^q)^{\eta_\varepsilon}\big]^{\frac{1}{q}} \leq$

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$\big[1-\prod_{\varepsilon=1}^{n}(1-({}^{\varepsilon}\mu^{*}_{\mathcal{T}})^{q})^{\eta_\varepsilon}\big]^{\frac{1}{q}}$. Similarly, since ${}^{\varepsilon}\omega_{\mathcal{I}} \leq^{\varepsilon} \omega^{*}_{\mathcal{I}}$, ${}^{\varepsilon}\omega_{\mathcal{F}} \leq^{\varepsilon} \omega^{*}_{\mathcal{F}}$, ${}^{\varepsilon}\nu_{\mathcal{I}} \leq^{\varepsilon} \nu^{*}_{\mathcal{I}}$, and ${}^{\varepsilon}\nu_{\mathcal{F}} \leq^{\varepsilon} \nu^{*}_{\mathcal{F}}$, we obtain,

$\big[1-\prod_{\varepsilon=1}^{n}(1-({}^{\varepsilon}\omega_{\mathcal{I}})^{q})^{\eta_\varepsilon}\big]^{\frac{1}{q}} \leq \big[1-\prod_{\varepsilon=1}^{n}(1-({}^{\varepsilon}\omega^{*}_{\mathcal{I}})^{q})^{\eta_\varepsilon}\big]^{\frac{1}{q}}$, $\big[1-\prod_{\varepsilon=1}^{n}(1-({}^{\varepsilon}\omega_{\mathcal{F}})^{q})^{\eta_\varepsilon}\big]^{\frac{1}{q}} \leq \big[1-\prod_{\varepsilon=1}^{n}(1-({}^{\varepsilon}\omega^{*}_{\mathcal{F}})^{q})^{\eta_\varepsilon}\big]^{\frac{1}{q}}$, $\big[1-\prod_{\varepsilon=1}^{n}(1-({}^{\varepsilon}\nu_{\mathcal{I}})^{q})^{\eta_\varepsilon}\big]^{\frac{1}{q}} \leq \big[1-\prod_{\varepsilon=1}^{n}(1-({}^{\varepsilon}\nu^{*}_{\mathcal{I}})^{q})^{\eta_\varepsilon}\big]^{\frac{1}{q}}$, and $\big[1-\prod_{\varepsilon=1}^{n}(1-({}^{\varepsilon}\nu_{\mathcal{F}})^{q})^{\eta_\varepsilon}\big]^{\frac{1}{q}} \leq \big[1-\prod_{\varepsilon=1}^{n}(1-({}^{\varepsilon}\nu^{*}_{\mathcal{F}})^{q})^{\eta_\varepsilon}\big]^{\frac{1}{q}}$,

Now, since ${}^{\varepsilon}\omega_{\mathcal{T}} \geq {}^{\varepsilon}\omega^{*}_{\mathcal{T}} \Rightarrow ({}^{\varepsilon}\omega_{\mathcal{T}})^{\eta_\varepsilon} \geq ({}^{\varepsilon}\omega^{*}_{\mathcal{T}})^{\eta_\varepsilon} \Rightarrow \prod_{\varepsilon=1}^{n}({}^{\varepsilon}\omega_{\mathcal{T}})^{\eta_\varepsilon} \geq \prod_{\varepsilon=1}^{n}({}^{\varepsilon}\omega^{*}_{\mathcal{T}})^{\eta_\varepsilon}$. Also as ${}^{\varepsilon}\nu_{\mathcal{T}} \geq {}^{\varepsilon}\nu^{*}_{\mathcal{T}} \Rightarrow ({}^{\varepsilon}\nu_{\mathcal{T}})^{\eta_\varepsilon} \geq ({}^{\varepsilon}\nu^{*}_{\mathcal{T}})^{\eta_\varepsilon} \Rightarrow \prod_{\varepsilon=1}^{n}({}^{\varepsilon}\nu_{\mathcal{T}})^{\eta_\varepsilon} \geq \prod_{\varepsilon=1}^{n}({}^{\varepsilon}\nu^{*}_{\mathcal{T}})^{\eta_\varepsilon}$.

In the same manner, as ${}^{\varepsilon}\mu_{\mathcal{I}} \geq {}^{\varepsilon}\mu^{*}_{\mathcal{I}}$ and ${}^{\varepsilon}\mu_{\mathcal{F}} \geq {}^{\varepsilon}\mu^{*}_{\mathcal{F}}$ we obtain $\prod_{\varepsilon=1}^{n}({}^{\varepsilon}\mu_{\mathcal{I}})^{\eta_\varepsilon} \geq \prod_{\varepsilon=1}^{n}({}^{\varepsilon}\mu^{*}_{\mathcal{I}})^{\eta_\varepsilon}$ and $\prod_{\varepsilon=1}^{n}({}^{\varepsilon}\mu_{\mathcal{F}})^{\eta_\varepsilon} \geq \prod_{\varepsilon=1}^{n}({}^{\varepsilon}\mu^{*}_{\mathcal{F}})^{\eta_\varepsilon}$.

Now, let $T-SFVNWA(\Gamma_1,\Gamma_2,...,\Gamma_n)=\Gamma=\big\langle\big(\mu_{\mathcal{T}},\,\omega_{\mathcal{T}},\,\nu_{\mathcal{T}}\big),\big(\mu_{\mathcal{I}},\,\omega_{\mathcal{I}},\,\nu_{\mathcal{I}}\big),\big(\mu_{\mathcal{F}},\,\omega_{\mathcal{F}},\,\nu_{\mathcal{F}}\big)\big\rangle$ and

$T-SFVNWA(\Gamma^{*}_1,\Gamma^{*}_2,...,\Gamma^{*}_n)=\Gamma^{*}=\big\langle\big(\mu^{*}_{\mathcal{T}},\,\omega^{*}_{\mathcal{T}},\,\nu^{*}_{\mathcal{T}}\big),\big(\mu^{*}_{\mathcal{I}},\,\omega^{*}_{\mathcal{I}},\,\nu^{*}_{\mathcal{I}}\big),\big(\mu^{*}_{\mathcal{F}},\,\omega^{*}_{\mathcal{F}},\,\nu^{*}_{\mathcal{F}}\big)\big\rangle$. Then,

$\Pi(\Gamma)=\frac{1}{3}\Big[\big[(\mu_{\mathcal{T}})^{q}-(\nu_{\mathcal{T}})^{q}\big]+\Big(1-\big[(\mu_{\mathcal{I}})^{q}-(\nu_{\mathcal{I}})^{q}\big]\Big)+\Big(1-\big[(\mu_{\mathcal{F}})^{q}-(\nu_{\mathcal{F}})^{q}\big]\Big)\Big] \leq \frac{1}{3}\Big[\big[(\mu^{*}_{\mathcal{T}})^{q}-(\nu^{*}_{\mathcal{T}})^{q}\big]+\Big(1-\big[(\mu^{*}_{\mathcal{I}})^{q}-(\nu^{*}_{\mathcal{I}})^{q}\big]\Big)+\Big(1-\big[(\mu^{*}_{\mathcal{F}})^{q}-(\nu^{*}_{\mathcal{F}})^{q}\big]\Big)\Big]=\Pi(\Gamma^{*})$.

This implies $T-SFVNWA(\Gamma_1,\Gamma_2,...,\Gamma_n) \leq T-SFVNWA(\Gamma^{*}_1,\Gamma^{*}_2,...,\Gamma^{*}_n)$.

□

In this part, we present the T-SFVNWG operator along with its associated properties.

Definition 3.6. Let $\Gamma_\varepsilon=\big\{\big\langle\big({}^{\varepsilon}\mu_{\mathcal{T}},\,{}^{\varepsilon}\omega_{\mathcal{T}},\,{}^{\varepsilon}\nu_{\mathcal{T}}\big),\big({}^{\varepsilon}\mu_{\mathcal{I}},\,{}^{\varepsilon}\omega_{\mathcal{I}},\,{}^{\varepsilon}\nu_{\mathcal{I}}\big),\big({}^{\varepsilon}\mu_{\mathcal{F}},\,{}^{\varepsilon}\omega_{\mathcal{F}},\,{}^{\varepsilon}\nu_{\mathcal{F}}\big)\big\rangle:\varepsilon=1,...,n\big\}$ be a collection of T-SFVNNs. The T-SFVNWG operator is characterized by the transformation $T-SFVNWG: T-SFVNN(\hat{\mathcal{U}}) \longrightarrow T-SFVNN(\hat{\mathcal{U}})$ and defined as :

$$T-SFVNWG(\Gamma_1,\Gamma_2,...,\Gamma_n)=\Gamma_1^{\eta_1}\otimes\Gamma_2^{\eta_2}\otimes...\Gamma_n^{\eta_n},$$

where $\eta_\varepsilon\in[0,1]$ is the weight of Γ_ε, $\forall\varepsilon=1,...,n$ and $\sum_{\varepsilon=1}^{n}\eta_\varepsilon=1$.

Definition 3.7. Let $\Gamma_\varepsilon=\big\{\big\langle\big({}^{\varepsilon}\mu_{\mathcal{T}},\,{}^{\varepsilon}\omega_{\mathcal{T}},\,{}^{\varepsilon}\nu_{\mathcal{T}}\big),\big({}^{\varepsilon}\mu_{\mathcal{I}},\,{}^{\varepsilon}\omega_{\mathcal{I}},\,{}^{\varepsilon}\nu_{\mathcal{I}}\big),\big({}^{\varepsilon}\mu_{\mathcal{F}},\,{}^{\varepsilon}\omega_{\mathcal{F}},\,{}^{\varepsilon}\nu_{\mathcal{F}}\big)\big\rangle:\varepsilon=1,...,n\big\}$ be a collection of T-SFVNNs and $\eta=(\eta_1,\eta_2,...\eta_n)$ be the weight vector of Γ_ε. Then,

$$T-SFVNWG(\Gamma_1,\Gamma_2,...,\Gamma_n)=\Big\langle\Big(\prod_{\varepsilon=1}^{n}({}^{\varepsilon}\mu_{\mathcal{T}})^{\eta_\varepsilon},\big[1-\prod_{\varepsilon=1}^{n}\big(1-({}^{\varepsilon}\omega_{\mathcal{T}})^{q}\big)^{\eta_\varepsilon}\big]^{\frac{1}{q}},\big[1-\prod_{\varepsilon=1}^{n}\big(1-({}^{\varepsilon}\nu_{\mathcal{T}})^{q}\big)^{\eta_\varepsilon}\big]^{\frac{1}{q}}\Big),\Big(\big[1-\prod_{\varepsilon=1}^{n}\big(1-({}^{\varepsilon}\mu_{\mathcal{I}})^{q}\big)^{\eta_\varepsilon}\big]^{\frac{1}{q}},\prod_{\varepsilon=1}^{n}({}^{\varepsilon}\omega_{\mathcal{I}})^{\eta_\varepsilon},\prod_{\varepsilon=1}^{n}({}^{\varepsilon}\nu_{\mathcal{I}})^{\eta_\varepsilon}\Big),\Big(\big[1-\prod_{\varepsilon=1}^{n}\big(1-({}^{\varepsilon}\mu_{\mathcal{F}})^{q}\big)^{\eta_\varepsilon}\big]^{\frac{1}{q}},\prod_{\varepsilon=1}^{n}({}^{\varepsilon}\omega_{\mathcal{F}})^{\eta_\varepsilon},\prod_{\varepsilon=1}^{n}({}^{\varepsilon}\nu_{\mathcal{F}})^{\eta_\varepsilon}\Big)\Big\rangle. \quad (2)$$

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The T-SFVNWG operator possesses the following properties, which are stated here without proof, as the proof is analogous to that of the T-SFVNWA operator.

Proposition 3.8. *Idempotency Property: Let* $\Gamma_\varepsilon = \{\langle({}^\varepsilon\mu_\mathcal{T}, {}^\varepsilon\omega_\mathcal{T}, {}^\varepsilon\nu_\mathcal{T}), ({}^\varepsilon\mu_\mathcal{I}, {}^\varepsilon\omega_\mathcal{I}, {}^\varepsilon\nu_\mathcal{I}), ({}^\varepsilon\mu_\mathcal{F}, {}^\varepsilon\omega_\mathcal{F}, {}^\varepsilon\nu_\mathcal{F})\rangle : \varepsilon = 1, ..., n\}$ *be a collection of T-SFVNNs. If* $\Gamma_\varepsilon = \Gamma = \langle(\mu_\mathcal{T}, \omega_\mathcal{T}, \nu_\mathcal{T}), (\mu_\mathcal{I}, \omega_\mathcal{I}, \nu_\mathcal{I}), (\mu_\mathcal{F}, \omega_\mathcal{F}, \nu_\mathcal{F})\rangle, \forall\varepsilon = 1, ..., n$. *Then,* $T - SFVNWG(\Gamma_1, \Gamma_2, ..., \Gamma_n) = \Gamma = \langle(\mu_\mathcal{T}, \omega_\mathcal{T}, \nu_\mathcal{T}), (\mu_\mathcal{I}, \omega_\mathcal{I}, \nu_\mathcal{I}), (\mu_\mathcal{F}, \omega_\mathcal{F}, \nu_\mathcal{F})\rangle$.

Proposition 3.9. *Boundedness Property: Let* $\Gamma_\varepsilon = \{\langle({}^\varepsilon\mu_\mathcal{T}, {}^\varepsilon\omega_\mathcal{T}, {}^\varepsilon\nu_\mathcal{T}), ({}^\varepsilon\mu_\mathcal{I}, {}^\varepsilon\omega_\mathcal{I}, {}^\varepsilon\nu_\mathcal{I}), ({}^\varepsilon\mu_\mathcal{F}, {}^\varepsilon\omega_\mathcal{F}, {}^\varepsilon\nu_\mathcal{F})\rangle : \varepsilon = 1, ..., n\}$ *be a collection of T-SFVNNs. If* $\Gamma^- = \langle(\mu_\mathcal{T}^-, \omega_\mathcal{T}^+, \nu_\mathcal{T}^+), (\mu_\mathcal{I}^+, \omega_\mathcal{I}^-, \nu_\mathcal{I}^-), (\mu_\mathcal{F}^+, \omega_\mathcal{F}^-, \nu_\mathcal{F}^-)\rangle$ *and* $\Gamma^+ = \langle(\mu_\mathcal{T}^+, \omega_\mathcal{T}^-, \nu_\mathcal{T}^-), (\mu_\mathcal{I}^-, \omega_\mathcal{I}^+, \nu_\mathcal{I}^+), (\mu_\mathcal{F}^-, \omega_\mathcal{F}^+, \nu_\mathcal{F}^+)\rangle$, *where,* $\mu_\mathcal{T}^- = \min_\varepsilon\{{}^\varepsilon\mu_\mathcal{T}\}, \mu_\mathcal{T}^+ = \max_\varepsilon\{{}^\varepsilon\mu_\mathcal{T}\}, \mu_\mathcal{I}^- = \min_\varepsilon\{{}^\varepsilon\mu_\mathcal{I}\}, \mu_\mathcal{I}^+ = \max_\varepsilon\{{}^\varepsilon\mu_\mathcal{I}\}, \mu_\mathcal{F}^- = \min_\varepsilon\{{}^\varepsilon\mu_\mathcal{F}\}, \mu_\mathcal{F}^+ = \max_\varepsilon\{{}^\varepsilon\mu_\mathcal{F}\}, \omega_\mathcal{T}^- = \min_\varepsilon\{{}^\varepsilon\omega_\mathcal{T}\}, \omega_\mathcal{T}^+ = \max_\varepsilon\{{}^\varepsilon\omega_\mathcal{T}\}, \omega_\mathcal{I}^- = \min_\varepsilon\{{}^\varepsilon\omega_\mathcal{I}\}, \omega_\mathcal{I}^+ = \max_\varepsilon\{{}^\varepsilon\omega_\mathcal{I}\}, \omega_\mathcal{F}^- = \min_\varepsilon\{{}^\varepsilon\omega_\mathcal{F}\}, \omega_\mathcal{F}^+ = \max_\varepsilon\{{}^\varepsilon\omega_\mathcal{F}\}, \nu_\mathcal{T}^- = \min_\varepsilon\{{}^\varepsilon\nu_\mathcal{T}\}, \nu_\mathcal{T}^+ = \max_\varepsilon\{{}^\varepsilon\nu_\mathcal{T}\}, \nu_\mathcal{I}^- = \min_\varepsilon\{{}^\varepsilon\nu_\mathcal{I}\}, \nu_\mathcal{I}^+ = \max_\varepsilon\{{}^\varepsilon\nu_\mathcal{I}\}, \nu_\mathcal{F}^- = \min_\varepsilon\{{}^\varepsilon\nu_\mathcal{F}\}, \nu_\mathcal{F}^+ = \max_\varepsilon\{{}^\varepsilon\nu_\mathcal{F}\}$. *Then,*
$\Gamma^- \leq T - SFVNWG(\Gamma_1, \Gamma_2, ..., \Gamma_n) \leq \Gamma^+$

Proposition 3.10. *Monotonicity Property: Let* $\Gamma_\varepsilon = \{\langle({}^\varepsilon\mu_\mathcal{T}, {}^\varepsilon\omega_\mathcal{T}, {}^\varepsilon\nu_\mathcal{T}), ({}^\varepsilon\mu_\mathcal{I}, {}^\varepsilon\omega_\mathcal{I}, {}^\varepsilon\nu_\mathcal{I}), ({}^\varepsilon\mu_\mathcal{F}, {}^\varepsilon\omega_\mathcal{F}, {}^\varepsilon\nu_\mathcal{F})\rangle : \varepsilon = 1, ..., n\}$ *and* $\Gamma_\varepsilon^* = \{\langle({}^\varepsilon\mu_\mathcal{T}^*, {}^\varepsilon\omega_\mathcal{T}^*, {}^\varepsilon\nu_\mathcal{T}^*), ({}^\varepsilon\mu_\mathcal{I}^*, {}^\varepsilon\omega_\mathcal{I}^*, {}^\varepsilon\nu_\mathcal{I}^*), ({}^\varepsilon\mu_\mathcal{F}^*, {}^\varepsilon\omega_\mathcal{F}^*, {}^\varepsilon\nu_\mathcal{F}^*)\rangle : \varepsilon = 1, ..., n\}$ *be two collections of T-SFVNNs. If* ${}^\varepsilon\mu_\mathcal{T} \leq^\varepsilon \mu_\mathcal{T}^*$, ${}^\varepsilon\omega_\mathcal{T} \geq^\varepsilon \omega_\mathcal{T}^*$, ${}^\varepsilon\nu_\mathcal{T} \geq^\varepsilon \nu_\mathcal{T}^*$, ${}^\varepsilon\mu_\mathcal{I} \geq^\varepsilon \mu_\mathcal{I}^*$, ${}^\varepsilon\omega_\mathcal{I} \leq^\varepsilon \omega_\mathcal{I}^*$, ${}^\varepsilon\nu_\mathcal{I} \leq^\varepsilon \nu_\mathcal{I}^*$, ${}^\varepsilon\mu_\mathcal{F} \geq^\varepsilon \mu_\mathcal{F}^*$, ${}^\varepsilon\omega_\mathcal{F} \leq^\varepsilon \omega_\mathcal{F}^*$ *and* ${}^\varepsilon\nu_\mathcal{F} \leq^\varepsilon \nu_\mathcal{F}^*$, $\forall\varepsilon = 1, 2, ..., n$. *Then,* $T - SFVNWG(\Gamma_1, \Gamma_2, ..., \Gamma_n) \leq T - SFVNWG(\Gamma_1^*, \Gamma_2^*, ..., \Gamma_n^*)$.

4 The Versatile Applicability of T-SFVN Operators in MADM

This section underscores the practicality and significance of employing the T-SFVN operators in decision-making processes. To establish their efficacy, we present a MADM problem wherein the evaluation results are expressed using T-SFVNNs. We employ the T-SFVNWA and T-SFVNWG operators to address this MADM problem. To proceed, we hypothesize that the alternatives, denoted as $\mathfrak{B}i = 1, 2, ..., n$, can be derived from decision makers (DMs) who assess attributes $\mathfrak{C}j = 1, 2, ..., m$ with corresponding weights $\eta_{j=1,2,...,m}$ adhering to the condition $\eta_j \in [0, 1]$ and $\sum_{j=1}^{m} \eta_j = 1$ for all $j = 1, 2, ..., m$. Expert participation is solicited to evaluate the T-SFVN data for each attribute, aiding in the selection of the optimal candidate. To facilitate this process and determine the optimal candidate, we propose the subsequent algorithm.

Algorithm 1:

Step 1: The attributes evaluated for each alternative are presented in the form of T-SFVNNs, constituting a decision matrix.

Step 2: To maintain consistency among the attributes, the resulting decision matrix, which comprises two types of attributes, is normalized using the following equation.

$$\Gamma_\varepsilon = \begin{cases} \langle({}^\varepsilon\mu_\mathcal{T}, {}^\varepsilon\omega_\mathcal{T}, {}^\varepsilon\nu_\mathcal{T}), ({}^\varepsilon\mu_\mathcal{I}, {}^\varepsilon\omega_\mathcal{I}, {}^\varepsilon\nu_\mathcal{I}), ({}^\varepsilon\mu_\mathcal{F}, {}^\varepsilon\omega_\mathcal{F}, {}^\varepsilon\nu_\mathcal{F})\rangle & \text{for benefit attributes} \\ \langle({}^\varepsilon\mu_\mathcal{F}, {}^\varepsilon\omega_\mathcal{F}, {}^\varepsilon\nu_\mathcal{F}), ({}^\varepsilon\nu_\mathcal{I}, 1 - {}^\varepsilon\omega_\mathcal{I}, {}^\varepsilon\mu_\mathcal{I}), ({}^\varepsilon\mu_\mathcal{T}, {}^\varepsilon\omega_\mathcal{T}, {}^\varepsilon\nu_\mathcal{T})\rangle & \text{for cost attributes} \end{cases} \tag{3}$$

Step 3: By employing either the T-SFVNWA or T-SFVNWG operators, the multiple attribute values for each candidate are combined into a single value denoted as $\mathfrak{K}_{i=1,2,...,n}$.

Step 4: The computation of the Score Function (SF) for each candidate is carried out according to Definition 2.7.

Step 5: The candidate with the highest score value is deemed the optimal candidate.

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Figure 1 depicts the intricate process of the groundbreaking and innovative method introduced in this research.

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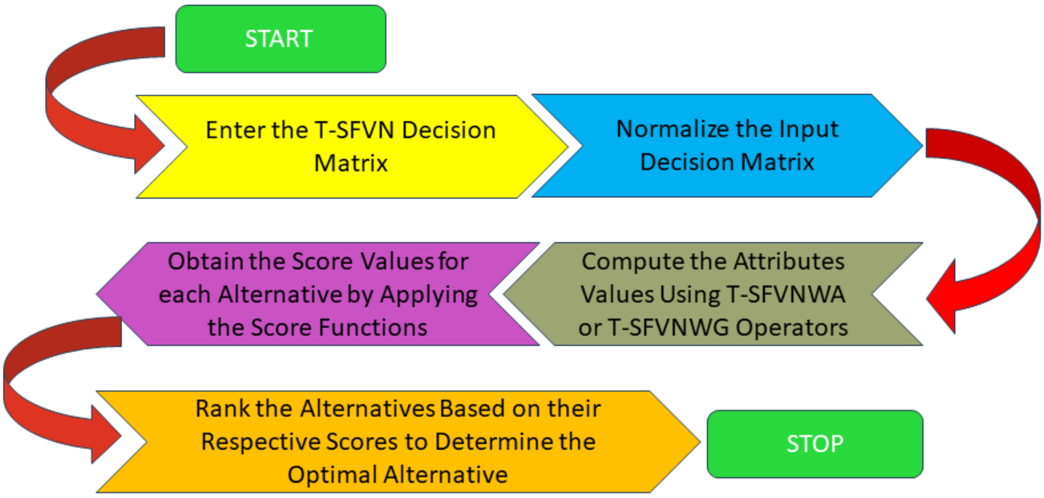

Figure 1: Procedural Workflow for the Proposed Method

4.1 Practical Illustration

In this part, we apply the aforementioned algorithm to address the following decision-making problem:

A company is evaluating four chatbot platforms to implement on their website. Let $\mathfrak{B}_{i=1,2,3,4}$ be a set of four chatbot platforms. They are interested in optimizing for a combination of five attributes $\mathfrak{C}_1$: user satisfaction, $\mathfrak{C}_2$: technical features, $\mathfrak{C}_3$: ease of implementation, $\mathfrak{C}_4$: cost and $\mathfrak{C}_5$: chatbot's security and compliance features. Each chatbot platform has different scores across these attributes, and the company needs to consider the relative importance of each attribute when making their decision. Suppose that the weights of attributes respectively are $0.2, 0.1, 0.1, 0.3, 0.3$. This is a MADM problem where multiple attributes need to be considered to choose the best chatbot platform among the options available, accounting for both the technical and security needs of the company. In this case study the quality of the alternatives $\mathfrak{B}_i$ with respect to attributes $\mathfrak{C}_j$ are expressed by T-SFVNNs with $q = 4$. Subsequently, we will use the proposed algorithm to choose the best chatbot platform as discussed below.

Step 1: The decision makers conducted evaluations of each alternative based on their attributes using T-SFVN values, resulting in the construction of the decision matrix presented in Table 1.

Table 1: The Initial Decision Matrix

	$\mathfrak{B}_1$	$\mathfrak{B}_2$
$\mathfrak{C}_1$	⟨(0.9 , 0.2 , 0.3) , (0.6 , 0.3, 0.9) ,(0.5 , 0.4, 0.7) ⟩	⟨(0.9 , 0.2, 0.6),(0.5 , 0.4, 0.7),(0.6 , 0.5, 0.8) ⟩
$\mathfrak{C}_2$	⟨(0.6 ,0.2, 0.1) , (0.5 ,0.5, 0.9) ,(0.8 , 0.2, 0.7) ⟩	⟨(0.8 , 0.2, 0.5),(0.4 , 0.5, 0.8),(0.5 , 0.3, 0.7) ⟩
$\mathfrak{C}_3$	⟨(0.7 , 0.6, 0.3) , (0.3 , 0.4, 0.7) ,(0.1 , 0.5, 0.4) ⟩	⟨(0.6 , 0.1, 0.8),(0.3 , 0.4,0.9),(0.7 , 0.1, 0.8) ⟩
$\mathfrak{C}_4$	⟨(0.5 , 0.4, 0.3) , (0.2 , 0.6, 0.5) ,(0.8 , 0.4, 0.7) ⟩	⟨(0.6 , 0.2, 0.7),(0.8 , 0.5, 0.2),(0.7 , 0.5, 0.4) ⟩
$\mathfrak{C}_5$	⟨(0.5 , 0.5, 0.3) , (0.8 , 0.1, 0.7) ,(0.6 , 0.2, 0.7) ⟩	⟨(0.8 , 0.2, 0.7), (0.6 , 0.3, 0,8),(0.6, 0.2, 0.9) ⟩
	$\mathfrak{B}_3$	$\mathfrak{B}_4$
$\mathfrak{C}_1$	⟨(0.5 , 0.3, 0.1), (0.8 , 0.2, 0.5),(0.5 , 0.4, 0.4) ⟩	⟨(0.6 , 0.3, 0.2),(0.9 , 0.4, 0.6), (0.5 , 0.2, 0.5) ⟩
$\mathfrak{C}_2$	⟨(0.8 , 0.2, 0.6), (0.6 , 0.4, 0.8), (0.4 , 0.4, 0.7) ⟩	⟨(0.8 , 0.1,0.7),(0.5 , 0.4,0.3), (0.2 , 0.3, 0.7) ⟩
$\mathfrak{C}_3$	⟨(0.7 , 0.5, 0.2), (0.3 , 0.4, 0.5),(0.7 , 0.5, 0.1) ⟩	⟨(0.5 , 0.3, 0.6),(0.7 , 0.1, 0.9),(0.1 , 0.5, 0.2) ⟩
$\mathfrak{C}_4$	⟨(0.6 , 0.3, 0.5), (0.8 , 0.5, 0.7),(0.2 , 0.4, 0.4) ⟩	⟨(0.7 , 0.3, 0.8),(0.2 , 0.4, 0.9), (0.5, 0.4, 0.7) ⟩
$\mathfrak{C}_5$	⟨(0.5 , 0.3, 0.1), (0.7 , 0.2,0.8) ,(0.5 , 0.2 ,0.6) ⟩	⟨(0.8 , 0.3, 0.2), (0.7 , 0.3, 0.8), (0.9, 0.5, 0.1) ⟩

Step 2: To normalize the initial decision matrix, we apply a complement operation to the cost attribute ($\mathfrak{C}_4$) in our case study. The normalized decision matrix is presented in Table 2.

Step 3: In this step, we utilize Equation (1) to calculate the T-SFVNWA operator for each alternative. The resulting values are provided below.

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Table 2: The Normalized Decision Matrix

	$\mathfrak{B}_1$	$\mathfrak{B}_2$
$\mathfrak{C}_1$	⟨(0.9 , 0.2 , 0.3) , (0.6 , 0.3, 0.9) ,(0.5 , 0.4, 0.7) ⟩	⟨(0.9 , 0.2, 0.6),(0.5 , 0.4, 0.7),(0.6 , 0.5, 0.8) ⟩
$\mathfrak{C}_2$	⟨(0.6 ,0.2, 0.1) , (0.5 ,0.5, 0.9) ,(0.8 , 0.2, 0.7) ⟩	⟨(0.8 , 0.2, 0.5),(0.4 , 0.5, 0.8),(0.5 , 0.3, 0.7) ⟩
$\mathfrak{C}_3$	⟨(0.7 , 0.6, 0.3) , (0.3 , 0.4, 0.7) ,(0.1 , 0.5, 0.4) ⟩	⟨(0.6 , 0.1, 0.8),(0.3 , 0.4,0.9),(0.7 , 0.1, 0.8) ⟩
$\mathfrak{C}_4$	⟨(0.8 , 0.4, 0.7) , (0.5 , 0.4, 0.2) ,(0.5 , 0.4, 0.3) ⟩	⟨(0.7 , 0.5, 0.4),(0.2 , 0.5, 0.8),(0.6 , 0.2, 0.7) ⟩
$\mathfrak{C}_5$	⟨(0.5 , 0.5, 0.3) , (0.8 , 0.1, 0.7) ,(0.6 , 0.2, 0.7) ⟩	⟨(0.8 , 0.2, 0.7), (0.6 , 0.3, 0,8),(0.6, 0.2, 0.9) ⟩
	$\mathfrak{B}_3$	$\mathfrak{B}_4$
$\mathfrak{C}_1$	⟨(0.5 , 0.3, 0.1), (0.8 , 0.2, 0.5),(0.5 , 0.4, 0.4) ⟩	⟨(0.6 , 0.3, 0.2),(0.9 , 0.4, 0.6), (0.5 , 0.2, 0.5) ⟩
$\mathfrak{C}_2$	⟨(0.8 , 0.2, 0.6), (0.6 , 0.4, 0.8), (0.4 , 0.4, 0.7) ⟩	⟨(0.8 , 0.1,0.7),(0.5 , 0.4,0.3), (0.2 , 0.3, 0.7) ⟩
$\mathfrak{C}_3$	⟨(0.7 , 0.5, 0.2), (0.3 , 0.4, 0.5),(0.7 , 0.5, 0.1) ⟩	⟨(0.5 , 0.3, 0.6),(0.7 , 0.1, 0.9),(0.1 , 0.5, 0.2) ⟩
$\mathfrak{C}_4$	⟨(0.2 , 0.4, 0.4), (0.7 , 0.5, 0.8),(0.6 , 0.3, 0.5) ⟩	⟨(0.5 , 0.4, 0.7),(0.9 , 0.6, 0.2), (0.7, 0.3, 0.8) ⟩
$\mathfrak{C}_5$	⟨(0.5 , 0.3, 0.1), (0.7 , 0.2,0.8) ,(0.5 , 0.2 ,0.6) ⟩	⟨(0.8 , 0.3, 0.2), (0.7 , 0.3, 0.8), (0.9, 0.5, 0.1) ⟩

$\mathfrak{K}_1 = \langle (0.7697, 0.3618, 0.3466), (0.5674, 0.3679, 0.769), (0.4712, 0.3754, 0.6286) \rangle,$
$\mathfrak{K}_2 = \langle (0.7981, 0.2456, 0.5623), (0.3728, 0.4337, 0.801), (0.5983, 0.3475, 0.8166) \rangle,$
$\mathfrak{K}_3 = \langle (0.5716, 0.3305, 0.1943), (0.6504, 0.3978, 0.7531), (0.5341, 0.3611, 0.5468) \rangle,$
$\mathfrak{K}_4 = \langle (0.6942, 0.293, 0.3684), (0.7675, 0.4742, 0.7104), (0.5125, 0.4125, 0.6515) \rangle.$

Step 4: The score value of each alternative is computed, resulting in $\Pi(\mathfrak{K}_1) = 0.9274, \Pi(\mathfrak{K}_2) = 0.9608, \Pi(\mathfrak{K}_3) = 0.8309$ and $\Pi(\mathfrak{K}_4) = 0.8026$

Step 5: Based on the results obtained in Step 4, the ranking is as follows: $\mathfrak{B}2 \geq \mathfrak{B}1 \geq \mathfrak{B}3 \geq \mathfrak{B}4$.

When utilizing the T-SFVNWG operator to aggregate attribute values in step 3, we obtain:

$\mathfrak{K}_1 = \langle (0.682, 0.449, 0.5377), (0.6615, 0.2548, 0.5183), (0.5872, 0.3, 0.5133) \rangle,$
$\mathfrak{K}_2 = \langle (0.7646, 0.3767, 0.6347), (0.4881, 0.4012, 0.7881), (0.6061, 0.2334, 0.7857) \rangle,$
$\mathfrak{K}_3 = \langle (0.4117, 0.3678, 0.3835), (0.7056, 0.3024, 0.6948), (0.5606, 0.3048, 0.4447) \rangle,$
$\mathfrak{K}_4 = \langle (0.6258, 0.335, 0.5849), (0.8315, 0.3607, 0.4571), (0.7637, 0.3393, 0.3352) \rangle.$

The score value of each alternative is computed, resulting in $\Pi(\mathfrak{K}_1) = 0.6424, \Pi(\mathfrak{K}_2) = 0.8698, \Pi(\mathfrak{K}_3) = 0.6338$ and $\Pi(\mathfrak{K}_4) = 0.4127$

Based on the computed score values, the ranking is as follows: $\mathfrak{B}2 \geq \mathfrak{B}1 \geq \mathfrak{B}3 \geq \mathfrak{B}4$.

Obviously, these two approaches have the same ranking result.

4.2 Comparative Analysis: Assessing the Proposed Methodology against Existing Approaches

In this section, we conduct a comprehensive comparison between the proposed method and other commonly utilized approaches. We aim to evaluate their respective strengths and weaknesses to determine the effectiveness of the presented method.

In addition to the T-SFVN model, several other models have been proposed in the literature to address MADM problems. Among these models, we specifically focus on IFS,[7] NSs,[6] TSFS,[5] IFVNS,[16] and PyFVNS[23] due to their relevance in this comparative analysis. In the following discussion, we provide a comparative analysis of these alternative models. To facilitate the comparison, we apply the aforementioned models to the same dataset presented in Section 4.1. Throughout this comparison, the terms T, I, F denote the degrees of membership function (MF), indeterminate membership function (IMF), and non-membership function (NMF), respectively.

Initially, the IFS model is characterized by two parameters, namely T and F, satisfying the condition $T + F \leq 1$. The AOs within this model are proposed based on this condition. However, when faced with scenarios

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where $T + F > 1$, these AOs are unable to produce the desired outcomes. Additionally, the IFS model lacks the capability to handle indeterminate situations, rendering it unsuitable for solving the DM problem presented in Section 4.1.

Secondly, the NS model comprises three distinct components, namely T, I, and F, representing the degrees of MF, IMF, and NMF which are independent of each other. Each component is represented by a single value, subject to the condition $T + I + F \leq 3$. Conversely, the T-SFVNS model is constructed by incorporating T-SF values instead of single values for the MF, IMF, and NMF degrees. It becomes evident that the NS model is incapable of effectively modeling the data presented in Table 2, as its memberships lack the ability to express three-dimensional data. In stark contrast, the T-SFVNS model exhibits a structure that enables the representation of such data, as its memberships are inherently three-dimensional.

Thirdly, the TSFS model consists of three distinct components: T, I, and F, representing the degrees of MF, IMF, and NMF, respectively. Each component is represented by a single value, satisfying the condition $T^q + I^q + F^q \leq 1$. On the contrary, the T-SFVNS model employs T-SF values instead of single values for the MF, IMF, and NMF degrees. It is important to note that the TSFS model represents only one component of the T-SFVNS model. For instance, if we consider the value from Table 2: $\big\langle(0.9\ , 0.2, 0.3), (0.6\ ,\ 0.3, 0.9), (0.5,\ 0.4,\ 0.7)\big\rangle$, it is evident that the TSFS model can describe only one component, such as $(0.9,\ 0.2, 0.3)$ or $(0.6,\ 0.3,\ 0.9)$ or $(0.5,\ 0.4,\ 0.7)$. In such cases, we would require three TSFS models to represent this data and perform additional operations to find the optimal solution. Conversely, by utilizing the T-SFVNS model, we only need one set without additional operations, thereby saving time, avoiding complex operations, and obtaining a more accurate solution.

Fourthly, the IFVNS model is an extension of the NS model, encompassing three membership functions: T, I, and F, which represent intuitionistic fuzzy values. However, it is evident that the IFVNS model is unable to represent the data in Table 2 effectively. This limitation arises from the fact that the IFVNS model's membership functions are two-dimensional, while the data in Table 2 consists of three-dimensional memberships.

Finally, PyFVNS emerges as an extension of IFVNS, sharing a similar structure but with the condition $(T)^2 + (F)^2 \leq 1$. This condition significantly expands the scope of data that can be accommodated, surpassing the limitations of IFVNS. However, due to the same inherent structure, PyFVNS falls short in representing the data found in Table 2. On the contrary, the proposed method overcomes these limitations by employing three membership functions represented by TSFS, each possessing three dimensions. As a result, T-SFVNS empowers the representation of a broader range of fuzzy information. Its flexibility lies in dynamically adjusting the parameter q to define the information expression range, making it well-suited for effectively describing uncertain information.

Table 3 presents a comparison of the current models based on several relevant criteria, including the presence of three membership degrees, representation of 3D information within each degree, the existence of constraints on 3D information within each degree, the level of flexibility of the constraints, and the ranking values.

Table 3: Comparative Analysis of Current Models: Evaluating Based on Relevant Criteria

Method	Presence of three membership degrees	Representation of 3D information within each degree	Existence of constraints on 3D information within each degree	Level of flexibility of the constraints	Ranking values
IFS[7]	x	x	Non-applicable	Non-applicable	Non-computable
NS[6]	✓	x	Non-applicable	Non-applicable	Non-computable
TSFS[5]	✓	x	Non-applicable	Non-applicable	Non-computable
IFVNS[16]	✓	x	Non-applicable	Non-applicable	Non-computable
PyFVNS[23]	✓	x	Non-applicable	Non-applicable	Non-computable
The proposed Method	✓	✓	✓	High	Algorithmic

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5 Conclusion

The manuscript presented a comprehensive exploration of T-SFVNSs and T-SFVNNs, providing formal definitions for these concepts. It also defined several score functions, such as SF, AF, QSF, and QAF. To effectively aggregate T-spherical fuzzy valued neutrosophic data, the manuscript introduced the T-SFVNWA and T-SFVNWG operators. The properties of these operators, including idempotency, boundedness, and monotonicity, are rigorously discussed and proven. Furthermore, a novel approach for MADM is proposed, specifically tailored for attribute values represented as T-SFVNSs. The proposed method is applied to rank various chatbot platforms based on their features. Attribute values are aggregated using the T-SFVNWA and T-SFVNWG operators, and the ranking results are obtained using the score functions. Surprisingly, it is found that the ranking outcomes obtained from the proposed operators are completely consistent and identical. This remarkable consistency serves as strong evidence of the exceptional accuracy and precision of these measures. The manuscript also conducts a comparative analysis of the proposed models, providing a detailed and insightful discussion to interpret and clarify the findings in relation to existing models. To further expand the scope of this research, future investigations should explore aggregation operators within the proposed model and their application in solving complex decision-making problems, see.[24–27]

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