New Operators Using Neutrosophic Crisp Open Set

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Abstract

In this paper, we introduce new sets in neutrosophic crisp topology called neutrosophic crisp frontier, neutrosophic crisp border and neutrosophic crisp exterior with the help of neutrosophic crisp open sets in neutrosophic crisp topological space. Also, we discuss the basic and important properties of them and the relations between them. Finally, many examples are presented.

Keywords: Neutrosophic crisp open set; neutrosophic crisp closed set; neutrosophic crisp frontier; neutrosophic crisp border; Neutrosophic crisp exterior.

1. Introduction

Neutrosophy which has many applications in different fields of sciences such as topology introduced by F. Smarandache [1,2], as a new branch of Philosophy. A.S. Mashhour et al. [3] in 1983, as a generalization of topological space was introduced supra topological space. The most generalization of topological spaces, was neutrosophic topological spaces [4] defined in 2014. Also, neutrosophic crisp topological spaces [5].

Recently, P. Iswarya and K. Bageerathi,[6] studied neutrosophic frontier and semi-frontier in neutrosophic topological spaces. Separation axioms in neutrosophic crisp topological space and neutrosophic crisp point were first introduced by A. Alnafee et al. in [7].

Recently, the neutrosophic crisp set have applications in image processing [8],[9], the field of geographic information systems[10] and possible applications to database[11]. Also, neutrosophic sets [12] have applications in the medical field [13-16]. For more deatails about neutrosophic topology see [17-21].

In this paper, we use the neutrosophic crisp sets to introduce neutrosophic crisp frontier, neutrosophic crisp border and neutrosophic crisp exterior and discuss their properties in neutrosophic crisp topological space. Also, we study some basic properties of this new neutrosophic crisp concepts.

2. Preliminaries

In this part, we recall definitions and which are useful in this paper.

Definition 2.1. [5]

Let $X \neq \emptyset$ be a fixed set. A neutrosophic crisp set $(\mathrm{N_c.S})$ U is an object with the $U = < U_1, U_2, U_3 >$ shape ; U_1, U_2 and U_3 are subsets of X.

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Definition 2.4. [5]

A neutrosophic crisp topology (NCT) on a non-empty set χ is a family Ţ of neutrosophic crisp subsets in χ may be satisying the following axioms:

1. X_N and $\varnothing_N$ belong to Ţ.
2. T is closed under finite intersection.
3. T is closed under arbitrary union.

The pair (χ,Ţ)is neutrosophic crisp topological space (NCTS) in Ţ. Moreover, the elements in Ţ are said to be neutrosophic crisp open sets (NCOS). A neutrosophic crisp set F is closed (NCCS) if and only if its complement F^c is neutrosophic crisp open set.

Definition 2.5. [7]

Let χ be a non-empty set. And x,y,z∈ χ, then:

a. x_{N_1} □□□{x},∅,∅ > is called a neutrosophic crisp point (NCP_{N_1}) in χ .

b. y_{N_2} □□□∅,{y},∅ > is called a neutrosophic crisp point (NCP_{N_2}) in χ.

c. z_{N_3} □□□∅,∅,{z} > is called a neutrosophic crisp point (NCP_{N_3}) in χ .

The set of all neutrosophic crisp points (NCP_{N_1}, NCP_{N_2}, NCP_{N_3}) is denoted by NCP_N

3. Neutrosophic crisp frontier in neutrosophic crisp topological space:

In this section, we introduce neutrosophic crisp frontier and discuss their properties in neutrosophic crisp topological spaces.

Definition 3.1.

Let (X,Ψ_N) be NCTS and H, be a neutrosophic crisp set, then:

The neutrosophic crisp frontier of a neutrosophic crisp subset A (N_cFr(A))

N_cFr(H) = $N_c$cl(H) ∩ $N_c$cl(H^c).

Example 3.2.

Let $\chi = \{a, b, c\}$, $Ţ = \{\emptyset_N, X_N, A\}$, $A = \{<\{a\}, \emptyset, \emptyset>\}$.

Then (χ,Ţ) neutrosophic crisp topological space, let $H = \{<\{a\}, \{a, b\}, \{a, c\}>\}$

N_cFr(H) = $N_c$cl(H) ∩ $N_c$cl(H^c) $= X_N \cap \emptyset_N = \emptyset_N$.

Example 3.3.

Let (X,Ψ_N) be NCTS, then:

N_cFr(X_N) = $N_c$cl(X_N) ∩ $N_c$cl($X_N{}^c$)= $N_c$cl(X_N) ∩ $N_c$cl($\emptyset_N$) $= X_N \cap \emptyset_N = \emptyset_N$.

N_cFr($\emptyset_N$) = $N_c$cl($\emptyset_N$) ∩ $N_c$cl($\emptyset_N{}^c$)= $N_c$cl($\emptyset_N$) ∩ $N_c$cl(X_N) $= \emptyset_N \cap X_N = \emptyset_N$.

Remark 3.4.

Let (X,Ψ_N) be NCTS and H be a neutrosophic crisp subset, then:

N_cFr(H) is NCCS.

Proof:

Since N_cFr(H) = $N_c$cl(H) ∩ $N_c$cl(H^c), then N_cFr(H) is NCCS.

Theorem 3.5.

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Let (X,Ψ_N) be NCTS and H be a neutrosophic crisp subset, then:

$N_cFr(H) = N_cFr(H^c)$.

Proof.

Let H be a neutrosophic crisp subset in NCTS (X,Ψ_N). Then by Definition 3.1 $N_cFr(H) = N_ccl(H) \cap N_ccl(H^c) = N_ccl(H^c) \cap N_ccl(H) = N_ccl(H^c) \cap (N_ccl(H^c)^c)$. But, by Definition 3.1 this is equal to $N_cFr(H^c)$.

Hence $N_cFr(H) = N_cFr(H^c)$.

Theorem 3.6.

Let (X,Ψ_N) be NCTS and H be a neutrosophic crisp subset, then:

$N_cFr(H) = N_ccl(H) - N_cint(H)$.

Proof.

Let H be a neutrosophic crisp subset in NCTS. Since $(N_ccl(H))^c = N_cint(H^c)$,then $(N_ccl(H^c))^c = N_cint(H)$

We have that, $N_cFr(H) = N_ccl(H) \cap (N_ccl(H^c)) = N_ccl(H) \cap (N_cint(H))^c$. By using

$N - M = N \cap M^c$, $N_cFr(H) = N_ccl(H) - N_cint(H)$. Hence $N_cFr(H) = N_ccl(H) - N_cint(H)$.

Theorem 3.7.

Let (X,Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

A is NCCS set in X if and only if $N_cFr(A) \subseteq A$.

Proof.

$\Rightarrow$:Let A be a NCCS set in X. Then by Definition 3.1, $N_cFr(A) = N_ccl(A) \cap N_ccl(A^c) \subseteq N_ccl(A)=A$. Therefore $N_cFr(A) \subseteq A$.

$\Leftarrow$: Let $N_cFr(A) \subseteq A$. Then $N_ccl(A) - N_cint(A) \subseteq A$. Since $N_cint(A) \subseteq A$,

we conclude that $N_ccl(A) = A$ and hence A is NCCS.

Theorem 3.8.

Let (X,Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

If A is a NCOS set in X , then $N_cFr(A) \subseteq A^c$.

Proof.

Let A be a NCOS set in X. then A^c is NCCS set in X . By Theorem 3.6,

$N_cFr(A^c) \subseteq A^c$ and by Theorem 3.5, we get $N_cFr(A) \subseteq A^c$.

Theorem 3.9.

Let $A \subseteq B$ and B be any NCCS set in X . Then $N_cFr(A) \subseteq B$.

Proof.

Since $A \subseteq B$, then $N_ccl(A) \subseteq N_ccl(B)$. By Definition 3.1, $N_cFr(A) = N_ccl(A) \cap N_ccl(A^c) \subseteq N_ccl(B) \cap N_ccl(A^c) \subseteq N_ccl(B)$. Since B be any NCCS set, then $N_ccl(B) = B$. Hence $N_cFr(A) \subseteq B$.

Theorem 3.10.

Let (X,Ψ_N) be NCTS and A be a neutrosophic crisp subset in X, then $N_ccl(N_cFr(A)) \subseteq N_cFr(A)$.

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Proof.

Let A be the neutrosophic crisp subset in X. Then by Definition 3.1, $N_c cl(N_c Fr(A)) =$

$N_c cl(N_c cl(A) \cap (N_c cl(A^c))) \subseteq (N_c cl(N_c cl(A))) \cap (N_c cl(N_c cl(A^c)))$. Since $N_c cl(A)$ and $N_c cl(A^c)$ are NCCS, then

$N_c cl(N_c Fr(A)) \subseteq N_c cl(A) \cap (N_c cl(A^c)) = N_c Fr(A)$.

Theorem 3.11.

Let (X,Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

$N_c Fr(N_c int(A)) \subseteq N_c Fr(A)$.

Proof. Let A be the neutrosophic crisp subset in X. Then

$N_c Fr(N_c int(A)) = N_c cl(N_c int(A)) \cap (N_c cl(N_c int(A))^c)$[by Definition 3.1]

$= N_c cl(N_c int(A)) \cap (N_c cl(N_c cl(A^c)))$[$(N_c int(A))^c = N_c cl(A^c)$]

$= N_c cl(N_c int(A)) \cap (N_c cl(A^c))$[$N_c cl(A^c)$ is NCCS]

$\subseteq N_c cl(A) \cap N_c cl(A^c)$[$N_c int(A) \subseteq A$]

$= N_c Fr(A)$[by Definition 3.1].

Hence $N_c Fr(N_c \delta int(A)) \subseteq (N_c Fr(A))$.

Theorem 3.12.

Let (X,Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

$N_c Fr(N_c cl(A)) \subseteq N_c Fr(A)$.

Proof.

Let A be a neutrosophic crisp subset in X. Then

$N_c Fr(N_c cl(A)) = N_c cl(N_c cl(A)) \cap (N_c cl(N_c cl(A))^c)$[by Definition 3.1]

$= N_c cl(A) \cap (N_c cl(N_c int(A^c)))$[$(N_c cl(A))^c = N_c int(A^c)$]

$\subseteq N_c cl(A) \cap N_c cl(A^c)$[$N_c int(A^c) \subseteq A^c$]

$= N_c Fr(A)$[by Definition 3.1]

Hence $N_c Fr(N_c cl(A)) \subseteq N_c Fr(A)$.

Theorem 3.13.

Let (X,Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

$N_c int(A) \subseteq A - N_c Fr(A)$

Proof.

Let A be a neutrosophic crisp subset in X. Now by Definition 3.1,

$A - N_c Fr(A) = A \cap (N_c Fr(A))^c$

$= A \cap [N_c cl(A) \cap N_c cl(A^c)]^c$

$= A \cap [N_c int(A^c) \cup N_c int(A)]$

$= [A \cap N_c int(A^c)] \cup [A \cap N_c int(A)]$

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$=[A \cap N_c int(A^c)] \cup N_c int(A)$….(1)

But $N_c int(A) \subseteq [A \cap N_c int(A^c)] \cup N_c int(A)$….(2)

Hence $N_c int(A) \subseteq A - N_c Fr(A)$[by (1) and (2)].

Theorem 3.14.

Let (X,Ψ_N) be NCTS and A, B be a neutrosophic crisp subsets, then:

$N_c Fr(A \cup B) \subseteq N_c Fr(A) \cup N_c Fr(B)$.

Proof. Let A and B be neutrosophic crisp subsets in X. Then

$N_c Fr(A \cup B) = N_c cl(A \cup B) \cap N_c cl(A \cup B)^c$[by Definition 3.1]

$= N_c cl(A \cup B) \cap N_c cl(A^c \cap B^c)$

$\subseteq [(N_c cl(A) \cup N_c cl(B)] \cap ((N_c cl(A^c))) \cap (N_c cl(B^c))$

$= [(N_c cl(A) \cup (N_c cl(B))] \cap (N_c cl(A^c))) \cap [(N_c cl(A) \cup (N_c cl(B))] \cap (N_c cl(B^c)))$

$= [(N_c cl(A) \cap N_c cl(A^c))] \cup [((N_c cl(B) \cap (N_c cl(A^c))))] \cap [(N_c cl(A) \cap (N_c cl(B^c)))$

$\cup ((N_c cl(B) \cap (N_c cl(B^c))))]$

$= [N_c Fr(A) \cup (N_c cl(B)) \cap (N_c cl(A^c))] \cap [(N_c cl(A) \cap (N_c cl(B^c))) \cup (N_c Fr(B))]$

[by Definition 3.1]

$= (N_c Fr(A) \cup N_c Fr(B)) \cap [(N_c cl(B) \cap (N_c cl(A^c))) \cup ((N_c cl(A) \cap N_c cl(B^c)))]$

$\subseteq N_c Fr(A) \cup N_c Fr(B)$.

Hence, $N_c Fr(A \cup B) \subseteq N_c Fr(A) \cup N_c Fr(B)$.

Theorem 3.15.

Let (X,Ψ_N) be NCTS and A, B be a neutrosophic crisp subsets, then:

$N_c Fr(A \cap B) \subseteq (N_c Fr(A) \cap (N_c cl(B))) \cup (N_c Fr(B) \cap N_c cl(A))$.

Proof.

Let A and B be neutrosophic crisp subsets in X. Then

$N_c Fr(A \cap B) = N_c cl(A \cap B) \cap (N_c cl(A \cap B)^c)$[by Definition 3.1]

$= N_c cl(A \cap B) \cap (N_c cl(A^c \cup B^c))$

$\subseteq (N_c cl(A) \cap N_c cl(B)) \cap (N_c cl(A^c) \cup N_c cl(B^c))$

$= [(N_c cl(A) \cap N_c cl(B)) \cap N_c cl(A^c)] \cup [(N_c cl(A) \cap N_c cl(B)) \cap N_c cl(B^c)]$

$= (N_c Fr(A) \cap N_c cl(B)) \cup (N_c Fr(B) \cap N_c cl(A))$[by Definition 3.1].

Hence $N_c Fr(A \cap B) \subseteq ((N_c Fr(A) \cap (N_c cl(B))) \cup (N_c Fr(B) \cap (N_c cl(A))))$.

Theorem 3.16.

Let (X,Ψ_N) be NCTS and A, B be a neutrosophic crisp subset, then:

$N_c Fr(A \cap B) \subseteq N_c Fr(A) \cup N_c Fr(B)$.

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Proof.

Let A and B be neutrosophic subsets in the NCTS (X,Ψ_N). Then

$N_cFr(A \cap B) = N_ccl(A \cap B) \cap (N_ccl(A \cap B)^c)$[by Definition 3.1]

$= N_ccl(A \cap B) \cap (N_ccl(A^c \cup B^c)$

$\subseteq (N_ccl(A) \cap N_ccl(B)) \cap (N_ccl(A^c) \cup N_ccl(B^c))$

$= (N_ccl(A) \cap N_ccl(B)) \cap (N_ccl(A^c) \cup (N_ccl(A) \cap N_ccl(B)) \cap (N_ccl(B^c)))$

$= (N_cFr(A) \cap N_ccl(B)) \cup (N_ccl(A) \cap N_cFr(B))$[by Definition 3.1]

$\subseteq N_cFr(A) \cup (N_cFr(B)$.

Hence $N_cFr(A \cap B) \subseteq N_cFr(A) \cup N_cFr(B)$.

Theorem 3.17.

Let (X,Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

$N_cFr(N_cFr(A)) \subseteq N_cFr(A)$,

Proof.

Let A be a neutrosophic crisp subset in X. Then

$N_cFr(N_cFr(A)) = N_ccl(N_cFr(A)) \cap N_ccl(N_cFr(A)^c)$[by Definition 3.1]

$= N_ccl(N_ccl(A) \cap (N_ccl(A^c)) \cap (N_ccl(N_ccl(A)) \cap (N_ccl(A^c))^c))$[by Definition 3.1]

$\subseteq (N_ccl(N_ccl(A)) \cap (N_ccl(N_ccl(A^c))) \cap (N_ccl(N_cint(A^c))) \cup (N_cint(A)))$

$= (N_ccl(A) \cap (N_ccl(A^c)) \cap (N_ccl(N_ccl(A) \cup N_cint(A))))$

$\subseteq N_ccl(A) \cap N_ccl(A^c)$

$= N_cFr(A)$[by Definition 3.1].

Therefore $N_cFr(N_cFr(A)) \subseteq N_cFr(A)$.

(b) Again, $N_cFr(N_cFr(N_cFr(A))) \subseteq N_cFr(N_cFr(A))$.

Remark 3.18.

Let (X,Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

(a) $N_cFr(N_cFr(A)) \subseteq N_cFr(A)$,

(b) $N_cFr(N_cFr(N_cFr(A))) \subseteq N_cFr(N_c\delta Fr(A))$.

Proof.

By Theorem 3.17. $N_cFr(N_cFr(A)) \subseteq N_cFr(A)$ then, $N_cFr(N_cFr(N_cFr(A))) \subseteq N_cFr(N_cFr(A))$.

Theorem 3.19.

Let (X,Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

$(N_cFr(A))^c = N_cint(A) \cup N_cint(A^c)$.

Proof.

Let A be a neutrosophic crisp subset in X. Then by Definition 3.1, $(N_cFr(A))^c =$

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$(N_c cl(A) \cap N_c cl(A^c))^c = ((N_c cl(A))^c \cup (N_c cl(A^c))^c = N_c int(A^c) \cup N_c int(A)$. Hence $(N_c Fr(A))^c = N_c int(A) \cup N_c int(A^c)$.

4. Neutrosophic crisp border and neutrosophic crisp exterior in NCTS:

In this section, we introduce the neutrosophic crisp border, neutrosophic crisp exterior using neutrosophic crisp open sets and their properties are discussed in NCTS.

Definition 4.1.

Let (X,Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

the set $N_c Br(A) = A - N_c int(A)$ is called the neutrosophic crisp border of A.

Example 4.2.
Let $\chi = \{a, b, c\}, Ţ = \{\emptyset_N, X_N, A\}, A = \{< \{a\}, \{b\}, \{a, c\} >\}$.
Then $(\chi, Ţ)$ neutrosophic crisp topological space.

$N_c Fr(A) = A - N_c int(A) = A - A = \emptyset_N$.

Theorem 4.3.

Let (X,Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

A is a NCOS set if and only if $N_c Br(A) = \phi_N$.

Proof.

$\Rightarrow$: Suppose A is NCOS. Then $N_c int(A) = A$. Now, $N_c Br(A) =$

$A - N_c int(A) = A - A = \phi_N$.

$\Leftarrow$: Suppose $N_c Br(A) = \phi_N$. This implies, $A - N_c int(A) = \phi_N$. Therefore $A = N_c int(A)$

and hence A is NCOS.

Remark 4.4.

Let (X,Ψ_N) be NCTS, then:

$N_c Br(\phi_N) = \phi_N$ and $N_c Br(1_N) = \phi_N$.

Proof.

Since ϕ_N and 1_N are NCOS, by Theorem 4.3, $N_c Br(\phi_N) = \phi_N$ and $N_c Br(1_N) = \phi_N$.

Theorem 4.5.

Let (X,Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

$N_c Br(N_c int(A)) = \phi_N$.

Proof.

By the definition of N_c border, $N_c Br(N_c int(A)) = N_c int(A) - N_c int(N_c int(A))$. But, $N_c int(N_c int(A)) = N_c int(A)$ hence $N_c Br(N_c int(A)) = \phi_N$.

Theorem 4.6.

Let (X,Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

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the following statements are equivalent

(i) A is NCOS.

(ii) A = $N_c$int(A).

(iii) N_cBr(A) = ϕ_N.

Proof.

(i)→ (ii) Obvious.

(ii)→(iii). Suppose that A = $N_c$int(A). Then by Definition, N_cBr(A) = $N_c$int(A)− $N_c$int(A) =ϕ_N.

(iii) → (i). Let N_cBr(A) = ϕ_N. Then by Definition 4.1, A − $N_c$int(A) = ϕ_N and hence A =$N_c$int(A).

Theorem 4.7.

Let (X,Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

N_cBr(A) = A ∩ $N_c$cl(A^c).

Proof.

Since N_cBr(A) = A − $N_c$int(A)= A ∩ ($N_c$int(A))^c = A ∩ $N_c$cl (A^c).

Theorem 4.8.

Let (X,Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

N_cBr(A) ⊆ N_cFr(A).

Proof.

Since A ⊆ $N_c$cl(A), A − $N_c$int(A) ⊆ $N_c$cl(A) − $N_c$int(A). That implies, N_cBr(A) ⊆ N_cFr(A).

Definition 4.9.

Let (X,Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

The neutrosophic crisp interior of A^c is called the neutrosophic crisp exterior of A and it is denoted by N_cExt(A). That is, N_cExt(A) = $N_c$int(A^c).

Theorem 4.10.

Let (X,Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

N_cExt(A) = ($N_c$cl(A))^c.

Proof.

We know that, ($N_c$cl(A))^c = $N_c$int(A^c), then N_cExt(A) = $N_c$int(A^c) = ($N_c$cl(A))^c.

Example 4.11.

Let $\chi = \{a, b, c\}, Ţ = \{\emptyset_N , X_N , A\}, A = \{<\{a\}, \emptyset, \emptyset >\}$.

Then (χ,Ţ) neutrosophic crisp topological space, let $H = \{<\{a\}, \{a, b\}, \{a, c\}>\}$

N_cExt(A) = ($N_c$cl(A))^c= $(X_N)^c = \emptyset_N$.

Theorem 4.12.

Let (X,Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

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$N_cExt(N_cExt(A)) = N_cint(N_ccl(A)) \supseteq N_cint(A)$

Proof.

Now, $N_cExt(N_cExt(A)) = N_cExt(N_cint(A^c)) = N_cint((N_cint(A^c))^c) = N_cint(N_ccl(A)) \supseteq N_cint(A)$.

Theorem 4.13.

Let (X,Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

If $A \subseteq B$, then $N_cExt(B) \subseteq N_cExt(A)$.

Proof. Suppose $A \subseteq B$. Now, $N_cExt(B) = N_cint(B^c) \subseteq N_cint(A^c) = N_cExt(A)$.

Theorem 4.14.

Let (X,Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

$N_cExt(X_N) = \phi_N$ and $N_cExt(\phi_N) = X_N$.

Proof.

Now, $N_cExt(X_N) = N_cint((X_N)^c) = N_cint(\phi_N)$ and $N_cExt(\phi_N) = N_cint((\phi_N)^c) =$

$N_cint(X_N)$. Since ϕ_N and X_N are NCOS sets, then $N_cint(\phi_N) = \phi_N$ and $N_cint(X_N) = X_N$. Hence

$N_cExt(\phi_N) = X_N$ and $N_cExt(X_N) = \phi_N$.

Theorem 4.15.

Let (X,Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

$N_cExt(A) = N_cExt((N_cExt(A))^c)$.

Proof.

Now, $N_cExt((N_cExt(A))^c) = N_cExt((N_cint(A^c))^c) = N_cint((((N_cint(A^c))^c))^c) = N_cint(N_cint(A^c)) = N_cint(A^c) = N_cExt(A)$.

Theorem 4.16.

Let (X,Ψ_N) be NCTS and A, B be a neutrosophic crisp subsets, then:

(i) $N_cExt(A \cup B) \subseteq N_cExt(A) \cap N_cExt(B)$.

(ii) $N_cExt(A \cap B) \supseteq N_cExt(A) \cup N_cExt(B)$.

Proof.

(i) $N_cExt(A \cup B) = N_cint((A \cup B)^c) = N_cint((A^c) \cap (B^c)) \subseteq N_ccl(A^c) \cap N_ccl(B^c) =N_cExt(A) \cap N_cExt(B)$.

(ii) $N_cExt(A \cap B) = N_cint((A \cap B)^c) = N_cint((A^c) \cup (B^c)) \supseteq N_ccl(A^c) \cup N_ccl(B^c) =N_cExt(A) \cup N_cExt(B)$.

5. Conclusion

In this paper, we have defined a new concepts in neutrosophic crisp topological space by using neutrosophic crisp sets. This new concepts called neutrosophic crisp frontier**,** neutrosophic crisp border and neutrosophic crisp exterior in NCTS .Also we studied some of their basic properties and their relationship with each other. In the future, using these notions, various classes of mappings, separation axioms, and many researchers can be studied in NCTS.

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