# Neutrosophic Near Algebra Over Neutrosophic Field

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**Abstract**

This piece of paper aims to learn neutrosophic near algebra and neutrosophic sub near algebra. This paper is summarized with the suitable definitions and theorems of neutrosophic near algebra and neutrosophic sub near algebra. It has also been demonstrated that the direct product of neutrosophic near algebra is a neutrosophic near algebra and the intersection of neutrosophic sub near algebra is a neutrosophic near algebra on a neutrosophic field. It also examined the union of couple of neutrosophic near algebras is a neutrosophic near algebra.

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**Keywords:** Neutrosophic field; Weak neutrosophic near algebra; Strong neutrosophic near algebra

## 1. Introduction

The idea of near ring was introduced by G. Pilz [6]. The system of algebra with a couple of binary operations that which satisfies the ring's axioms, with the appropriate anomaly of one distributive law, is also known as a near ring. A near algebra is a near ring that admits field as a right operator domain. Near algebra was brought under the fledlight of the society by H. Brown [5]. The operators only precisely form a near algebra in P. Jordan's quantum mechanical theory. In this research of near algebra, it is so interesting as an axiomatic question and also for cocrete reasons. The study of neutralities' scope, nature, and interactions with the extra ideational spectra is worked through the new area of philosophy is known as neutrosophy. F.Smarandache [7] created a route for the entire globe in the year 1995 by presenting the full notion of neutrosophic set (NS) and neutrosophic logic. All sets, including classical sets, normal fuzzy sets, intuitionist fuzzy sets, and interval fuzzy sets, were all epitomised by F. Smarandache. Indeterminacy is incorporated into the fuzzy logic's extension known as neutrosophic logic. The proportion of reality in a set of which all the elements are contained in another set that is subset (T), the level of uncertainty in a subset (I), and the rate of falsity in a subset (F) are all anticipated to be consumed by every aim or objective in neutrosophic logic, where T, I, and F are authenticate or non-authenticate subsets of the non-standard unit interlude]- 0,1+ [. Operations research, engineering, science, game theory, information technology, law, and politics are just a few of the industries that employ neutrosophic logic. Neutosophic algebraic structures were acquaint with by Kandaswamy and F. Smarandache in 2006. A prearranged algebraic syntax $(Y,*)$ and a innovative algebraic syntax $(Y(I),*) =< Y, I >$ were formed by bringing the indeterminate component $I$ with the components of the pre-arranged syntax.

Smarandache and Kandaswamy together built some of the neutrosophic algebraic syntaxes because of the the uncertain quality $I$, which is that $I.I = I^2 = I.$ The idea of neutrosophic groups (NG) and neutrosophic semi groups, neutrosophic ring (NR), neutrosophic field (NF), neutrosophic linear space (NLS), neutrosophic near ring was being studied by A.A.A. Agboola [1,2,3,4]. T. Nagaiah et al. [10] were studied neutrosophic algebra. Neutrosophic sets are very important and usefull in artificial intelligence, neural networks, developmental programming, neutrosophic changing systems, and quantum mechanics. Neutrosophic logic has wide applications in science, engineering, IT, law, politics, economics etc. In the present research, we introduce the idea of



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neutrosophic near algebra and neutrosophic sub near algebra over a neutrosophic area. Entire paper, $Q$ means a (right) near algebra (NA) over a field $X$.

## 2. Related Work

In this, we recollect some of the basic perceptions of NS from the literature.

**Definition: 2.1. [2]** Let $(G,*)$ be any group and $G(I) =< G \cup I >$. Then $(G(I),*)$ is termed a NG engendered by $G$ and $I$ of the binary operation $*$. The indeterminacy factor $I$ is that $I * I = I$. If $*$ is standard multiplication, then $I * I * \dots .* I = I^n = I$ and if $*$ is ordinary addition, then $I * I * \dots .* I = nI = I$ for $n \in N$.
If $a * b = b * a$ for all $a, b \in G(I)$, we elivate that $G(I)$ can be commutated. Diversely, $G(I)$ is coined a non-commutative NG.

**Definition: 2.2.[2]** $(X, +, .)$ Le be any field and $X(I) = \langle X \cup I \rangle$ is a NS engendered by $X$ and $I$. Then $(X(I), +, .)$ is termed a NF.
The zero componant $0 \in X$ is being interpreted by $0 + 0I$ in $X(I)$ & $1 \in X$ is explained by $1 + 0I$ in $X(I)$.

**Definition: 2.3.[1]** Let $(N, +, .)$ be a near ring. The triple $(N(I), +, .)$ is termed a NNR if $(i)\ \tau + y = (b\ , gI) + (r, dI) = (b + r, (g + d)I)$,
$(ii) - \tau = -(b\ , gI) = (-b, -gI)$,
$(iii)\ \tau . y = (b\ , gI).(r, pI) = (br\ , (bp + gr + gp)I)\ \forall\ \tau, y \in N(I), b, g, r, p \in N$.
The zero component in $(N, +)$ is designated by $(0,0)$ in $(N(I), +)$. Any component $\tau \in N$ is designated by $(b, 0)$ in $N(I)$. $I$ in $N(I)$ is few times designated by $(0, I)$ in $N(I)$.

**Definition: 2.4.[2]** Let $(V, +, .)$ be any linear space over a field $X$ and $V(I) = \langle V \cup I \rangle$ be a NS engendered by $V$ & $I$. The triple $(V(I), +, \ .)$ is termed a feeble NLS on a field $X$. If $V(I)$ is a NLS over a NF $X(I)$, then $V(I)$ is termed a firm NLS.

The components of $V(I)$ are said neutrosophic vectors (NV) and components of $X(I)$ are said neutrosophic scalars. If $\wp = b + gI, \beth = r + pI$ where $b, g, r, p$ are vectors in $V$ and $\lambda = \lambda_1 + \lambda_2 I$ where $\lambda_1, \lambda_2$ are scalars in $X$, we define
$\wp + \beth = (\ b + gI) + (r + pI) = (\ b + r) + (g + p)I$ and
$\lambda\wp = (\lambda_1 + \lambda_2 I)(\ b + gI) = \lambda_1 b + (\lambda_1 j + \lambda_2 b + \lambda_2 g)I$.

**Definition: 2.5.** A linear space of $Q$ on area $X$ is a NA on which that multiplication is demarcated such as
$(i)\ (Q,\ .)$ is a semi group,
$(ii)(g + r).p = g.p + r.p\ \ \forall\ g, r, p \in Q$ and
$(iii)\ \lambda(gr) = (\lambda g)r, \forall\ \lambda \in X, g, r \in Q$.

**Definition: 2.6. [10]** Let $Y$ be algebra over a field $X$. The set engendered by $Y$ and $I$ is designated by $\langle Y \cup I \rangle = Y(I) = \{g + rI / g, r \in Y\}$ is entitled a weak NA over a field $X$. If $Y(I)$ is a NA over a NF $X(I)$ then $Y(I)$ is entitled a strong NA. The essentials of $Y(I)$ are entitled NV and the essentials of $X(I)$ are entitled neutrosophic scalars.

## 3. Neutrosophic Near algebra

The idea of Neutrosophic near algebra (NNA) and Neutrosophic sub near algebra (NSNA) is introduced in this section.

**Definition: 3.1.** A NNA $Q(I)$ on a NF $X(I)$ is a linear space $Q(I)$ on a NF $X(I)$ on what multiplication is demarcated as
$(i)\ (Q(I)\ , .)$ is a semi group,
$(ii)(g + r).p = g.p + r.p\ \forall\ g, r, p \in Q(I)$,
$(iii)\ \lambda(jr) = (\lambda j)r, \forall\ \lambda \in X, \forall\ g, r \in Q(I)$.

The essentials of $Q(I)$ are being entitled NV and the essentials of $X(I)$ are entitled neutrosophic scalars.



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**Definition: 3.2.** Let $Q$ be a NA over a field $X$ and $Q(I) = \{g + rI/g, r \in Q\} = \langle Q \cup I \rangle$ is a NS initiated by $Q$ and $I$. If $(Q(I), +, \ .)$ is a NNA over $X$, and then $Q(I)$ is names a "weak NNA". If $Q(I)$ is a NNA over a NF $X(I)$, and then $Q(I)$ is entitled a "strong NNA".

**Example: 3.3.** Let $V(I)$ be a NLS over a NF $X(I)$. Consider that set of all operators $T(V(I)) = \{\varrho/\varrho : V(I) \to V(I)\}$, forms a NNA which is not a NA involves addition and multiplication of scalars at the point level

$(\varrho + \zeta)(g + rI) = \varrho(g + rI) + \zeta(g + rI)\ \forall \varrho, \zeta \in T(V(I)), g + rI \in V(I)$,

$(\lambda.\varrho)(g + rI) = \lambda(\varrho(g + rI)), \forall\ \varrho \in T(V(I)),\ g + rI \in\ V(I), \lambda = \lambda_1 + \lambda_2 I \in X(I)$ and the function composition as the multiplication by

$(\varrho\zeta)(g + rI) = (\varrho o \zeta)(g + rI) = \varrho\left(\zeta((g + rI))\right), \forall\ \varrho, \zeta \in T(V(I)), g + rI \in V(I).$

I) Let $\varrho, \zeta \in T(V(I))$. Then

$(\varrho.\zeta)(g + rI) = \varrho\left(\zeta((g + rI))\right) = \varrho(g_1 + r_1 I) = g_2 + r_2 I \in V(I)$

Hence $\varrho.\zeta \in\ T(V(I)), \forall\ g_1 + r_1 I, g_2 + r_2 I \in\ V(I)$.

Now $\forall\ g + rI \in\ V(I)\ and\ g, r \in V$,

$$\begin{aligned}[(\varrho.\zeta).\omega](g + rI) &= (\varrho.\zeta)(\omega(g + rI))\\ &= \varrho[\zeta(\omega(g + rI))]\\ &= \varrho[(\zeta.\omega)(g + rI)]\\ &= [\varrho.(\zeta.\omega)](g + rI).\end{aligned}$$

Therefore $(T(V(I)), .)$ is a semi group.

II) Let $\varrho, \zeta, \omega \in T(V(I))$. For every $g + rI \in\ V(I)$,

$$\begin{aligned}[(\varrho + \zeta).\omega](g + rI) &= (\varrho + \zeta)(\omega(g + rI)) = (\varrho + \zeta)(g_1 + r_1 I)\\ &= \varrho(g_1 + rI) + \zeta(g_1 + r_1 I) = \varrho(\omega(g + rI)) + \zeta(\omega(g + rI))\\ &= (\varrho.\omega)(g + rI) + (\zeta.\omega)(g + rI)\\ &= (\varrho.\omega + \zeta.\omega)(g + rI).\end{aligned}$$

Thus $T(V(I))$ is satisfying the right distributive law. But $[\varrho.(\zeta + \omega)](g + rI) = \varrho((\zeta + \omega)(g + rI))$

$$\begin{aligned}&= \varrho(\zeta(g + rI) + \omega(g + rI))\\ &= \varrho((g_1 + r_1 I) + (g_2 + r_2 I))\\ &= \varrho((g_1 + j_2) + (r_1 + r_2)I)\\ &\neq [(\varrho.\zeta) + (\varrho.\omega)](g + rI).\end{aligned}$$

Thus $T(V(I))$ is not satisfying the left distributive law.

*III)* $\forall\ \varrho, \zeta \in T(V(I)),\ g + rI \in\ V(I),\ \lambda \in X(I)$.

$$\begin{aligned}(\lambda(\varrho.\zeta))(g + rI) &= \left(\lambda\left(\varrho(\zeta(g + rI))\right)\right)\\ &= \lambda(\varrho(g_1 + r_1 I))\\ &= \lambda(g_2 + r_2 I\ ),\\ ((\lambda\varrho).\zeta)(g + rI) &= (\lambda\varrho)(\zeta(g + rI))\\ &= (\lambda\varrho)((g_1 + r_1 I))\\ &= \lambda(\varrho(g_1 + r_1 I))\\ &= \lambda(g_2 + r_2 I\ ),\\ (\varrho(\lambda\zeta))(g + rI) &= \varrho((\lambda\zeta))(g + rI) = \varrho\left(\lambda(\zeta(g + rI))\right)\\ &= \varrho(\lambda(g_1 + r_1 I)) \quad \neq ((\lambda\varrho).\zeta)(g + rI).\end{aligned}$$

**Definition: 3.4.** Let $Q(I)$ be a NNA over a NF $X(I)$. $Q_0(I) = \{y \in Q(I) : y0 = 0\}$ is entitled the zero symmetric part of $Y(I)$. $Y(I)$ is entitled zero symmetric NNA if $Q(I) = Q_0(I)$.

**Definition: 3.5.** A subset $\aleph(I)$ of a NNA $Q(I)$ over a NF $X(I)$ is held to be a NSNA of $Q(I)$, if the subsequent criteria are met:

(i) $\aleph(I)$ is a linear subspace of $Q(I)$,

(ii) $(\aleph(I), .)$ is a semi group.

**Example: 3.6**. Let $V(I)$ be a NLS over a NF $X(I)$. Consider all operators $T(V(I)) = \{\varrho/\varrho : V(I) \to V(I)\}$, forms a NNA involves addition and multiplication of scalars at the point level



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$(\varrho+\zeta)(g+rI)=\varrho(g+rI)+\zeta(g+rI), \forall\, \varrho,\zeta \in T(V(I)),\ g+rI \in V(I)$,
$(\lambda.\varrho)(g+rI)=\lambda(\varrho(g+rI)), \forall\, \varrho \in T(V(I)),\ g+rI \in V(I),\ \lambda \in X(I)$ and function composition as the multiplication by $(\varrho\zeta)(g+rI)=(\varrho o \zeta)(g+rI)=\varrho\left(\zeta((g+rI))\right), \forall\, \varrho,\zeta \in T(V(I)),\ g+rI \in V(I)$. Then $T_0(V(I))=\{\varrho/\varrho \in T(V(I)),\ \varrho(0)=0\}$.

**Proposition 3.7:** Let $Q(I)$be a NNA over a NF $X(I)$Then
$(i)$ $0\,.\,g = g, \forall\ g \in Q(I)$ and 0 is the additive identity in $Q(I)$,
$(ii)$ $(-r)\,.\,g =-(gr), \forall\, g, r \in Q(I)$.

**Theorem: 3.8.** A subset $\aleph(I)$ of a NNA $Q(I)$ over a NF $X(I)$ is a NSNA of $Q(I)$ if and only if the ensuing situations holds:
(i) $\varsigma-\tau \in \aleph(I)$
(ii) $\varsigma\tau \in \aleph(I),\ \lambda\varsigma \in \aleph(I), \forall\, \varsigma,\tau \in \aleph(I), \lambda \in X(I)$.
**Proof:** Let $Q(I)$ be a NNA over a NF $X(I)$ and $\aleph(I)$ be a subset of $Y(I)$.
$\aleph(I)$ is a NSNA of $Q(I)$ gives $\varsigma-\tau \in \aleph(I), \lambda\varsigma \in \aleph(I), \forall\, \varsigma,\tau \in \aleph(I), \lambda \in X(I)$ and
$\varsigma\tau \in \aleph(I), \forall\, \varsigma,\tau \in \aleph(I)$.
Conversely, presume that $\aleph(I)$ satisfies the two conditions of the hypothesis.
Let $\varsigma,\tau \in \aleph(I), \lambda,\varpi \in X(I)$ so $-\varpi \in X(I)$. Then $\lambda\varsigma, -\varpi\tau \in \aleph(I)$, so that $\lambda\varsigma-(-\varpi\tau) \in \aleph(I)$, thus $\lambda\varsigma+\varpi\tau \in \aleph(I)$. Therefore $\aleph(I)$ is a NLS of $Y(I)$.
Meanwhile we take $\aleph(I) \subseteq Q(I)$. Then we can write $(\varsigma\tau)r=\varsigma(\tau r), \forall\, \varsigma,\tau,r \in \aleph(I)$.
By the hypothesis, we have $\varsigma\tau \in \aleph(I), \forall\, \varsigma,\tau \in \aleph(I)$. Thus $(\aleph(I), .)$ is a neutrosophic semigroup.

**Theorem: 3.9.** Let $\aleph_1(I)$ and $\aleph_2(I)$ be two NSNA of a NNA $Q(I)$ over a NF $X(I)$. Then $\aleph_1(I) \cap \aleph_2(I)$ is a NSNA of $Q(I)$ over a NF $X(I)$.
**Proof:** Let $\varsigma,\tau \in \aleph_1(I) \cap \aleph_2(I)$ and $\lambda \in X(I)$. Then $\varsigma,\tau \in \aleph_1(I), \varsigma,\tau \in \aleph_2(I)$, and hence $\varsigma-\tau \in \aleph_1(I),\ \varsigma\tau \in \aleph_1(I),\ \lambda\varsigma \in \aleph_1(I),\ \varsigma-\tau \in \aleph_2(I), \varsigma\tau \in \aleph_2(I),\ \lambda\varsigma \in \aleph_2(I), \forall\, \varsigma,\tau \in \aleph_1(I), \varsigma,\tau \in \aleph_2(I), \lambda \in X(I)$. So that $\varsigma-\tau$, $\varsigma\tau,\ \lambda\varsigma \in \aleph_1(I) \cap \aleph_2(I), \forall\, \varsigma,\tau \in \aleph_1(I) \cap \aleph_2(I)$ and $\lambda \in X(I)$. Hence $\aleph_1(I) \cap \aleph_2(I)$ is a NSNA of $Y(I)$ over a NF $X(I)$.

**Theorem: 3.10.** Let $\aleph_1(I)$ and $\aleph_2(I)$ be two NSNA of a NNA $Y(I)$ over a NF $X(I)$. Then $\aleph_1(I) \cup \aleph_2(I)$ is a NSNA of $Q(I)$ over a NF $X(I)$ if and only if $\aleph_1(I) \subseteq \aleph_2(I)$ or $\aleph_2(I) \subseteq \aleph_1(I)$.
**Proof:** If possible, infer that $\aleph_1(I) \nsubseteq \aleph_2(I)$ or $\aleph_2(I) \nsubseteq \aleph_1(I)$.
Then $\aleph_1(I) \nsubseteq \aleph_2(I)$ implies there exist $\varsigma \in \aleph_1(I)$ and $\varsigma \notin \aleph_2(I)$. ... (1)
In the same way $\aleph_2(I) \nsubseteq \aleph_1(I)$ gives $\tau \in \aleph_2(I)$ and $\tau \notin \aleph_1(I)$. ... (2)
Therefore $\varsigma \in \aleph_1(I) \cup \aleph_2(I),\ \tau \in \aleph_1(I) \cup \aleph_2(I)$, thus $\varsigma+\tau \in \aleph_1(I) \cup \aleph_2(I)$.
So $\varsigma+\tau \in \aleph_1(I)$ or $\varsigma+\tau \in \aleph_2(I)$. Now, $\varsigma+\tau,\ \varsigma \in \aleph_1(I)$, then $\varsigma+\tau-\varsigma \in \aleph_1(I)$, hence $\tau \in \aleph_1(I)$. ... (3)
And, $\varsigma+\tau,\ \tau \in \aleph_2(I)$, then $\varsigma+\tau-\tau \in \aleph_2(I)$, hence $\varsigma \in \aleph_2(I)$ ... (4)
Thus equations (3) and (4) contradicts equations (1) and (2).
Hence $\aleph_1(I) \subseteq \aleph_2(I)$ or $\aleph_2(I) \subseteq \aleph_1(I)$.
Conversely, infer that $\aleph_1(I) \subseteq \aleph_2(I)$ or $\aleph_2(I) \subseteq \aleph_1(I)$. Then $\aleph_1(I) \cup \aleph_2(I)=\aleph_2(I)$ or $\aleph_1(I) \cup \aleph_2(I)=\aleph_1(I)$.
Hence $\aleph_1(I) \cup \aleph_2(I)$ is NSNA of $Y(I)$ over a NF $X(I)$.

**Theorem: 3.11**. Let $Q(I)$ and $Q'(I)$ be two NNA over a NF $X(I)$. Then the direct product $Q(I)\ X\ Q'(I)=\{(\daleth,\daleth')/\daleth \in Q(I),\ \daleth' \in Q'(I)\}$ is a NNA over a NF $X(I)$, where addition and multiplication is demarcated as
$(\daleth,\daleth')+(\tau,\tau')=(\daleth+\tau,\ \daleth'+\tau')$,
$(\daleth,\daleth').(\tau,\tau')=(\daleth.\tau, \daleth'.\tau')$,
$\lambda\,(\daleth,\daleth')=(\lambda\daleth,\ \lambda\daleth'), \forall\, \daleth,\tau \in Q(I)\ , \daleth',\tau' \in Q'(I), \lambda \in X(I)$.

**Proof:** Let $(\daleth,\daleth'), (\tau,\tau') \in Q(I)\ X\ Q'(I), \forall\, \daleth,\tau \in Q(I), \daleth',\tau' \in Q'(I)$,
then $\daleth+\tau \in Q(I),\ \daleth'+\tau' \in Q'(I)$. Thus $(\daleth,\daleth')+(\tau,\tau')=(\daleth+\tau,\ \daleth'+\tau') \in Q(I)\ X\ Q'(I)$. Let $(\daleth,\daleth'), (\tau,\tau'), (r,r') \in Q(I)\ X\ Q'(I), \forall\, \daleth,\tau,r \in Q(I), \daleth',\tau',r' \in Q'$.

$$\begin{aligned}\text{Then } [(\daleth,\daleth')+(\tau,\tau')]+(r,r') &= (\daleth+\tau,\ \daleth'+\tau')+(r,r')\\ &= \big((\daleth+\tau)+r, (\daleth'+\tau')+r'\big)\\ &= \big(\daleth+(\tau+r), \daleth'+(\tau'+r')\big)\\ &= (\daleth,\daleth')+(\tau+r,\tau'+r')\\ &= (\daleth,\daleth')+[(\tau,\tau')+(r,r')].\end{aligned}$$


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Let $(\daleth,\daleth') \in Q(I)\ X\ Q'(I)$, there exist $(0,0') \in Q(I)\ X\ Q'(I)$ such that
$(\daleth,\daleth') + (0,0') = (0,0') + (\daleth,\daleth') = (\daleth,\daleth')$. Here $(0,0')$ is the additive identity in $Q(I)\ X\ Q'(I)$.
For each $(\daleth,\daleth') \in Q(I)\ X\ Q'(I)$, there exists $-(\daleth,\daleth') \in Q(I)\ X\ Q'(I)$ such that
$(\daleth,\daleth') + [-(\daleth,\daleth')] = [-(\daleth,\daleth')] + (\daleth,\daleth') = (0,0')$.
Here $-(\daleth,\daleth')$ is the additive inverse of $(\daleth,\daleth')$.
Let $(\daleth,\daleth'), (\tau,\tau') \in Q(I)\ X\ Q'(I)$. Then $(\daleth,\daleth') + (\tau,\tau') = (\daleth+\tau,\ \daleth+\tau')$
$= (\tau+\daleth, \tau'+\daleth') \quad = (\tau,\tau') + (\daleth,\daleth')$. Hence $(Q(I)\ X\ Q'(I), +)$ is a neutrosophic abelian group.
Let $(\daleth,\daleth') \in Q(I)\ X\ Q'(I), \forall\, \daleth \in Q(I), \daleth' \in Q'(I)$ and $\lambda \in X(I)$. Then $(\daleth,\daleth') = (\lambda\daleth,\ \lambda\daleth') \in Q(I)\ X\ Q'(I)$.
Let $(\daleth,\daleth'), (\tau,\tau') \in Q(I)\ X\ Q'(I)$ and $\lambda, \varpi \in X(I)$. Then

$$\begin{aligned}
\lambda\big((\daleth,\daleth') + (\tau,\tau')\big) &= \lambda(\daleth+\tau, \daleth'+\tau') = \big(\lambda(\daleth+\tau), \lambda(\daleth'+\tau')\big)\\
&= (\lambda\daleth+\lambda\tau, \lambda\daleth'+\lambda\tau') = (\lambda\daleth,\lambda\daleth') + (\lambda\tau,\lambda\tau')\\
&= \lambda(\daleth,\daleth') + \lambda(\tau,\tau').
\end{aligned}$$

$$\begin{aligned}
\text{Now } (\lambda+\varpi)(\daleth,\daleth') &= \big((\lambda+\varpi)\daleth, (\lambda+\varpi)\daleth'\big)\\
&= (\lambda\daleth+\varpi\daleth, \lambda\daleth'+\varpi\daleth')\\
&= (\lambda\daleth,\lambda\daleth') + (\varpi\daleth,\varpi\daleth')\\
&= \lambda(\daleth,\daleth') + \varpi(\daleth,\daleth'),\\
(\lambda.\varpi)(\daleth,\daleth') &= \big((\lambda\varpi)\daleth, (\lambda\varpi)\daleth'\big)\\
&= \big(\lambda(\varpi\daleth), \lambda(\varpi\daleth')\big)\\
&= \lambda(\varpi\daleth, \varpi\daleth')\\
&= \lambda\big(\varpi(\daleth,\daleth')\big).
\end{aligned}$$

For $1 \in X(I)$, we have $1(\daleth,\daleth') = (1.\daleth, 1.\daleth') = (\daleth,\daleth')$. Therefore $Q(I)\ X\ Q'(I)$ is a NLS over a NF $X(I)$.
Let $(\daleth,\daleth'), (\tau,\tau') \in Q(I)\ X\ Q'(I), \forall\, \daleth,\tau \in Q(I), \daleth',\tau' \in Q'(I)$, then $\daleth.\tau \in Q(I),\ \daleth'.\tau' \in Q'(I)$. Thus
$(\daleth,\daleth').(\tau,\tau') = (\daleth.\tau,\ \daleth'.\tau') \in Q(I)\ X\ Q'(I)$.
Let $(\daleth,\daleth'), (\tau,\tau'), (r,r') \in Q(I)\ X\ Q'(I), \forall\, \daleth,\tau,r \in Q(I), \daleth',\tau',r' \in Q'(\mathrm{I})$.

$$\begin{aligned}
\text{Then } [(\daleth,\daleth').(\tau,\tau')].(r,r') &= (\daleth.\tau,\ \daleth'.\tau').(r,r')\\
&= \big((\daleth.\tau).r, (\daleth'.\tau').r'\big)\\
&= \big(\daleth.(\tau.r), \daleth'.(\tau'.r')\big)\\
&= (\daleth,\daleth').(\tau.r, \tau'.r')\\
&= \big((\daleth,\daleth')\big).[(\tau,\tau').(r,r')].
\end{aligned}$$

Therefore $Q(I)\ X\ Q'(I)$ is a neutrosophic semi group under multiplication.
Let $(\daleth,\daleth'), (\tau,\tau'), (r,r') \in Q(I)\ X\ Q'(I), \forall\, \daleth,\tau,r \in Q(I),\ \daleth',\tau',r' \in Q'(I)$, then

$$\begin{aligned}
[(\daleth,\daleth') + (\tau,\tau')].(r,r') &= (\daleth+\tau,\ \daleth'+\tau')(r,r')\\
&= \big((\daleth+\tau)r,\ (\daleth'+\tau')r'\big)\\
&= (\daleth r+\tau r, \daleth' r'+\tau' r')\\
&= (\daleth r, \daleth' r') + (\tau r, \tau' r')\\
&= (\daleth,\daleth')(r,r') + (\tau,\tau')(r,r').
\end{aligned}$$

Finally, for any $(\daleth,\daleth'), (\tau,\tau') \in Q(I)\ X\ Q'(I)$ and $\lambda \in X(I), \forall\, \daleth,\tau \in Q(I), \daleth',\tau' \in Q'(I)$, we get

$$\begin{aligned}
\big(\lambda(\daleth,\daleth')\big)(\tau,\tau') &= (\lambda\daleth, \lambda\daleth')(\tau,\tau')\\
&= \big((\lambda\daleth)\tau, (\lambda\daleth')\tau'\big)\\
&= \big(\lambda(\daleth\tau), \lambda(\daleth'\tau')\big)\\
&= \lambda(\daleth\tau, \daleth'\tau')\\
&= \lambda\big((\daleth,\daleth')(\tau,\tau')\big).
\end{aligned}$$

Hence $Q(I)\ X\ Q'(I)$ is a NNA over a NF $X(I)$.

## 4. Conclusion

In this study, we got to know the idea of NNA, NSNA and discussed some algebraic properties. We have proved that intersection of two NSNA is a NNA over a NF. Also investigated the direct product of any two NNA is a NNA over a NF. We have also introduced the union of two NSNA over a NF. The ideal of NNA and gamma neutrosophic near algebra can both be studied further.

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