Single Valued Trapezoidal Neutrosophic Travelling Salesman Problem with Novel Greedy Method: The Dhouib-Matrix-TSP1 (DM-TSP1)

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Abstract

Travelling salesman problem (TSP) is a prominent computational problem where trail technique is used to calculate all the possible travel and choose the best one. Since there is no branching or back tracking in greedy algorithms, determining the run time is much easier than the existing methods and hence, in this paper, a novel greedy method called Dhouib-Matrix-TSP1 is proposed as the first resolution of TSP to get the optimal solution using single valued trapezoidal neutrosophic numbers with several numerical examples. Also, results have been analyzed with graphical solutions.

Keywords: Neutrosophic Optimization, Neutrosophic graphs, Travelling Salesman Problem, Operational Research, Combinatorial Problems, Heuristic, Dhouib-Matrix, Dhouib-Matrix-TSP1.

1.Introduction

The Travelling Salesman Problem (TSP) portrays the salesman who is travelling between *n* cities where the salesman knows the transmits and cost of the cities. This helps the salesman to identify the possible shortest route that visits each city exactly once and returns to the starting city. TSP is an NP-hard problem and the aim of this problem is to calculate a shortest possible tour that visits every city exactly once. TSP is related with daily activities in uncertain and indeterminate parameters namely distance, cost and time. These parameters are unstable and not exactly known and hence, neutrosophic concept has been widely used in TSP.

Fuzzy set deals with only membership function and intuitionistic fuzzy set handles both membership and non-membership function. But real-world problems have indeterminacy too in nature and hence Smrandache introduced the neutrosophic concept in 1995 concerning the chance of reflecting the human thinking using the three membership functions namely Truth (T), Indeterminacy (I) and Falsity (F) which belongs to neutrosophic set. These three components are independent and hence neutrosophic set plays a vital role and has been a major focus of the research field. This field a new branch of philosophy called Neutrosophy as a generalization of intuitionistic, inconsistent, and fuzzy logic. It deals with not only ambiguity; also analyze the relation and absolute truth [1].

Shortest path problems have been studied and made an overview by the researchers under single valued and trapezoidal neutrosophic information [6-7, 13, 23]. TSP has been solved using various methods under different set environments as follows. TSP is solved using combinational Evolutionary Algorithm [2], a study was carried out on TSP using fuzzy self-organizing map [3], two methods have been proposed to solve TSP namely Ant Colony Systems to understand the operations and applied Hopfield Neural Network to TSP as it has fully connected neurons generally used in optimization tasks [4], intuitionistic fuzzy modeling [5], metaheuristic algorithm [8]. Also, shortest path problem is solved under triangular fuzzy neutrosophic environment by considering the edge weight as the triangular fuzzy neutrosophic numbers. Here, the length of the shortest path has been calculated using ranking function [9], using the first iteration of modified Vogel method to calculate the best starting city for the nearest neighbor algorithm [10].

Also, TSP is examined using Genetic Algorithm which supports the growth of life [11], using Heart Algorithm which handles the action of the heart and circulatory setup in human beings [12], improved Genetic Algorithm [14], Multi-element Genetic Algorithm [15], E-commerce website evaluation is done under single valued neutrosophic environment using a novel integrated decision system which contain three modules namely, information acquisition, SVTN-DEMATEL module and the integration module [16], operations on single valued trapezoidal neutrosophic numbers have been proposed and applied in a group decision making problem [17], using Hungarian method under intuitionistic fuzzy environment [18].

In [19], metaheuristic method is used to solve TSP by finding the near optimal solution, implemented fuzzy intuitionistic algorithm is used to solve TSP [20], Kidney inspired algorithm in which the Kidney process in the human body is used [21], the method of Graphical Processing Units is applied [22], using Genetic Algorithm under intuitionistic fuzzy environment [24]. Moreover, EMF-CE algorithm which uses a negative exponent function to achieve critical value as the feedback regulation of the implementation of algorithm [25], using Branch and Bound method [26], Quasi optimization with time dependent [27], fuzzy environment [28], using intuitionistic fuzzy approach [29], time dependent TSP where edge costs depend on their position in the tour is solve under interval valued intuitionistic fuzzy environment [30].

TSP has been solved using Ising model where the data given by ISING solver as text in matrix market format with simulated bifurcation where the nearly optimal solution can be obtain quickly [31], solved transportation problem using ECCT and standard deviation under fuzzy environment [40], modified grey wolf optimizer [42], using modified Grey Wolf optimizer where the leadership ranking and hunting mechanism of grey wolves in nature is simulated [43], and review has been done with TSP using GA-ACO hybrid approach.

Though, there are various types of neutrosophic numbers, we used trapezoidal neutrosophic numbers as it is the combination of trapezoidal fuzzy numbers and neutrosophic set. According to the literature survey, so far, TSP is not yet been studied under single valued trapezoidal neutrosophic numbers. Hence the motivation of this present work is to enhance the novel greedy method called Dhouib-Matrix-TSP1 (DM-TSP1) to solve this kind of indeterminant TSP.

The rest of the paper is presented as follows. In section 2, basic concepts are described related to the present work. In section 3, the novel greedy method entitled DM-TSP1 is proposed under trapezoidal neutrosophic environment. In section 4, the first resolution of the TSP are carried out with step-wise application to test the validity. Also, graphical representation is given. In section 5, conclusion and future direction of the present work are given.

2. Mathematical definition for trapezoidal and neutrosophic numbers

Definition 1[23]. Let us consider a space X composed of universal elements denoted by x. The neutrosophic set A is a phenomenon having the following construction $N=\{(T_N(x), I_N(x), F_N(x))/x\in X\}$ where the three grades of memberships are from X to $]^{-}0,1^{+}[$ of the element $x \in X$ to the set N, with the criterion:

$^{-}0 \le T_N(x)+I_N(x)+F_N(x) \le 3^{+}$. The functions $T_N(x), I_N(x)$ and $F_N(x)$ are the truth, indeterminate and falsity grades lies in $]-0, 1^{+}[$.

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Definition 2 [6]. For the space X of objects contains a global elements x. A single valued neutrosophic number represented by three degrees of membership grades $T_A(x)$, $I_A(x)$ $F_A(x)$ ∈[0, 1]. A single valued neutrosophic set is defined by $N = \{(T_N(x), I_N(x), F_N(x)) / x \in X\}$

Definition 3[41]. The three independent membership functions Truth (*T*), Indeterminacy (*I*), Falsity (*F*) for the single valued trapezoidal neutrosophic number (SVTpNN) $N = \langle [N_a, N_b, N_c, N_d](T_N, I_N, F_N) \rangle$ are defined by:

$$\gamma(X) = \begin{cases} \dfrac{(x-a)T_N}{b-a} & , \quad a \le x \le b \\ T_N & , \quad b \le x \le c \\ \dfrac{(d-x)T_N}{d-c} & , \quad c \le x \le d \\ 0 & , \quad otherwise \end{cases}$$

$$\varsigma(X) = \begin{cases} \dfrac{b-x+(x-a)I_N}{b-a} & , \quad a \le x \le b \\ I_N & , \quad b \le x \le c \\ \dfrac{x-c+(d-x)I_N}{d-c} & , \quad c \le x \le d \\ 0 & , \quad otherwise \end{cases}$$

$$\xi(X) = \begin{cases} \dfrac{b-x+(x-a)F_N}{b-a} & , \quad a \le x \le b \\ F_N & , \quad b \le x \le c \\ \dfrac{x-c+(d-x)F_N}{d-c} & , \quad c \le x \le d \\ 0 & , \quad otherwise \end{cases}$$

Definition 4 [30]. Let $N_1 = \langle [N_{1a}, N_{1b}, N_{1c}, N_{1d}](T_{N1}, I_{N1}, F_{N1}) \rangle$ and $N_2 = \langle [N_{2a}, N_{2b}, N_{2c}, N_{2d}](T_{N2}, I_{N2}, F_{N2}) \rangle$ be two SVTpNNs and $\rho > 0$ be any real number. Then

$$N_1 \oplus N_2 = \langle [N_{1a} + N_{2a}, N_{1b} + N_{2b}, N_{1c} + N_{2c}, N_{1d} + N_{2d}](T_{N_1} + T_{N_2} - T_{N_1}T_{N_2}, I_{N_1}I_{N_2}, F_{N_1}F_{N_2}) \rangle$$

$$N_1 \otimes N_2 = \langle [N_{1a}N_{2a}, N_{1b}N_{2b}, N_{1c}N_{2c}, N_{1d}N_{2d}](T_{N_1}T_{N_2}, I_{N_1} + I_{N_2} - I_{N_1}I_{N_2}, F_{N_1} + F_{N_2} - F_{N_1}F_{N_2}) \rangle$$

$$\rho N_1 = \langle [\rho N_{1a}, \rho N_{1b}, \rho N_{1c}, \rho N_{1d}](1 - (1 - T_{N_1})^\rho, I_{N_1}^\rho, F_{N_1}^\rho) \rangle$$

$$N_1^\rho = \langle [N_{1a}^\rho, N_{1b}^\rho, N_{1c}^\rho, N_{1d}^\rho](T_{N_1}{}^\rho, 1 - (1 - I_{N_1}^\rho), 1 - (1 - F_{N_1}^\rho)) \rangle$$

3. Novel greedy method: The Dhouib-Matrix-TSP1 (DM-TSP1)

The Travelling Salesman Problem (TSP) is one of the most important combinatorial problems. It deals with generating a minimal cycle between all cities (see Equation 1) where d_{ij} represents the distance between city *i* and city *j* and x_{ij} denotes a binary decision variable (x_{ij}= 1 if city *i* is related to city *j* otherwise x_{ij} = 0).

Minimize:

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$$\sum_{i=1}^{n}\sum_{j=1}^{n} t_{ij} x_{ij} \quad (1)$$

Subject to:

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1,...,n$$
$$\sum_{i=1}^{n} x_{ij} = 1, \quad j = 1,...,n \qquad (2)$$
$$x_{ij} = 0 \; or \; 1, \; i = 1,...,n \; , \; \; j = 1,...,n$$

In order to obtain an initial basic feasible solution for the TSP, [32] designed a novel greedy method named Dhouib-Matrix-TSP1 (DM-TSP1). Then, this method is enhanced to be a stochastic technique entitled Dhouib-Matrix-TSP2 (DM-TSP2) in [33]. Hence, DM-TSP1 and DM-TSP2 are applied on several instances in [34]. Then, an application of DM-TSP1 on TSP under fuzzy environment is depicted in [35, 36, 37] and its application on neutrosophic domain is illustrated in [38,39]. Two new metaheuristics are recently developed and inspired from DM-TSP1 and DM-TSP2: the iterated DM3 in [45] and the multi-start DM4 in [46].

The greedy heuristic DM-TSP1 is subdivized into four steps (see Figure 1). DM-TSP1 is independent to the statistical metrics (Min, Max, Average, … etc.). In this paper, we will use the range function.

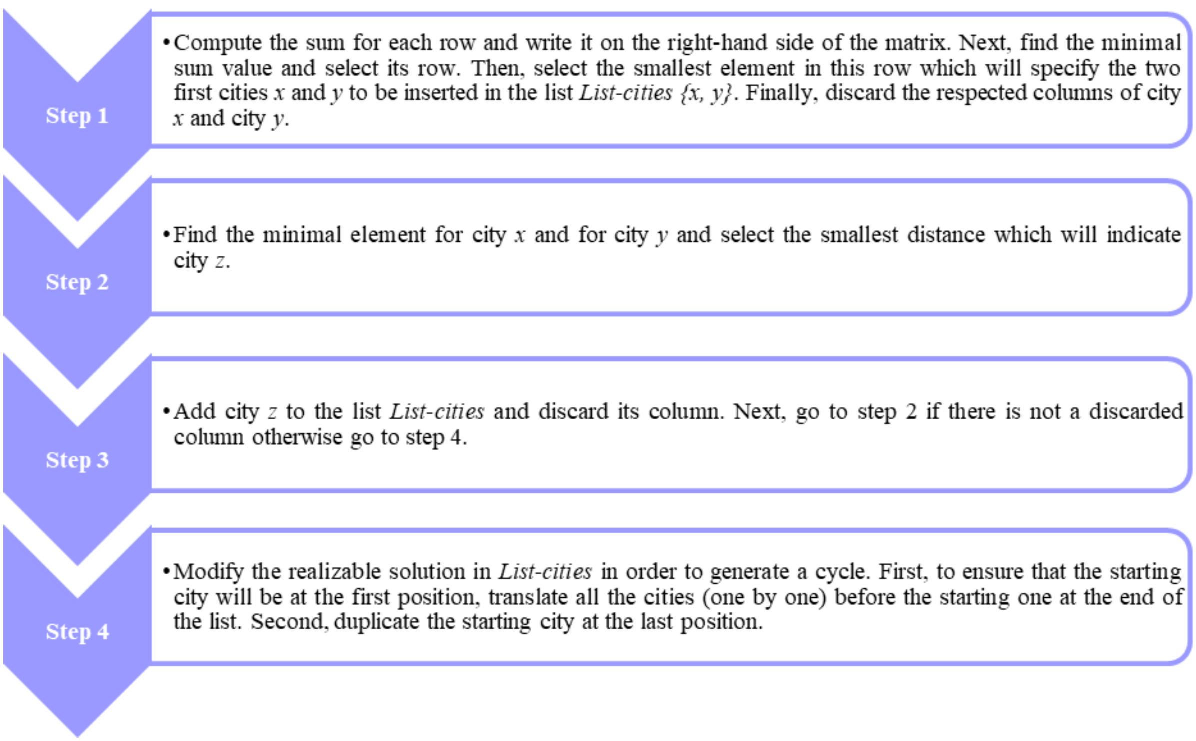

Figure 1 Flow chart of the proposed new heuristic DM-TSP1

The score function developed in [11] is applied to transform the single value trapezoidal neutrosophic number to crisp number (see Equation 1). Let $N = \langle [N_a, N_b, N_c, N_d](T_N, I_N, F_N) \rangle$ be a neutrosophic number so its score function will be calculated as:

$$Sc(N) = \left(N_a + 2N_b + 2N_c + N_d\right) \times \left(2 + T_N - I_N - F_N\right) \Big/ 18 \tag{1}$$

4. Application of DM-TSP1 on the neutrosophic Travelling Salesman Problem

This section aims to carry out the first optimization of the TSP under single valued trapezoidal neutrosophic domain. The greedy method DM-TSP1 is proposed to generate the initial basic feasible solution. Moreover, a step-wise application was conducted to test its validity. However, there are no instances for this variant of TSP, so we create three numerical examples (presented in the next subsection).

4.1. Example 1

Let us consider an example of 5 cities where the distances are denoted by single valued trapezoidal neutrosophic numbers (see Figure 2).

$$\begin{bmatrix}
\infty & \begin{matrix}<(1,4,9,18);\\0.8,0.1,0.1>\end{matrix} & \begin{matrix}<(5,6,9,19);\\0.8,0.2,0.1>\end{matrix} & \begin{matrix}<(3,9,11,14);\\0.9,0.1,0.1>\end{matrix} & \begin{matrix}<(5,7,14,15);\\0.7,0.4,0.2>\end{matrix} \\
\begin{matrix}<(1,4,9,18);\\0.8,0.1,0.1>\end{matrix} & \infty & \begin{matrix}<(1,5,7,11);\\0.8,0.3,0.2>\end{matrix} & \begin{matrix}<(3,8,16,21);\\0.8,0.3,0.2>\end{matrix} & \begin{matrix}<(3,9,15,19);\\0.8,0.2,0.3>\end{matrix} \\
\begin{matrix}<(5,6,9,19);\\0.8,0.2,0.1>\end{matrix} & \begin{matrix}<(1,5,7,11);\\0.8,0.3,0.2>\end{matrix} & \infty & \begin{matrix}<(1,5,11,15);\\0.8,0.4,0.3>\end{matrix} & \begin{matrix}<(2,6,13,15);\\0.7,0.1,0.1>\end{matrix} \\
\begin{matrix}<(3,9,11,14);\\0.9,0.1,0.1>\end{matrix} & \begin{matrix}<(3,8,16,21);\\0.8,0.3,0.2>\end{matrix} & \begin{matrix}<(1,5,11,15);\\0.8,0.4,0.3>\end{matrix} & \infty & \begin{matrix}<(3,9,11,14);\\0.8,0.2,0.2>\end{matrix} \\
\begin{matrix}<(5,7,14,15);\\0.7,0.4,0.2>\end{matrix} & \begin{matrix}<(3,9,15,19);\\0.8,0.2,0.3>\end{matrix} & \begin{matrix}<(2,6,13,15);\\0.7,0.1,0.1>\end{matrix} & \begin{matrix}<(3,9,11,14);\\0.8,0.2,0.2>\end{matrix} & \infty
\end{bmatrix}$$

Figure 2 Neutrosophic matrix

By applying the score function described by Equation 1, the crisp matrix is presented as follows (see Figure 3).

$$\begin{bmatrix}
\infty & 6.50 & 7.50 & 8.55 & 7.23 \\
6.50 & \infty & 4.60 & 9.20 & 8.94 \\
7.50 & 4.60 & \infty & 5.60 & 7.64 \\
8.55 & 9.20 & 5.60 & \infty & 7.60 \\
7.23 & 8.94 & 7.64 & 7.60 & \infty
\end{bmatrix}$$

Figure 3 Crisp matrix

Now, compute the sum of each row and find the smallest which is equal to 25.34 at row 3 (see Figure 4).

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$$\begin{bmatrix} \infty & 6.50 & 7.50 & 8.55 & 7.23 \\ 6.50 & \infty & 4.60 & 9.20 & 8.94 \\ 7.50 & \boxed{4.60} & \infty & 5.60 & 7.64 \\ 8.55 & 9.20 & 5.60 & \infty & 7.60 \\ 7.23 & 8.94 & 7.64 & 7.60 & \infty \end{bmatrix} \begin{matrix} 29.78 \\ 29.24 \\ \boxed{25.34} \\ 30.95 \\ 31.41 \end{matrix}$$

Figure 4 Select the smallest element in row 3

Then, select the smallest value in row 3 which is equal to 4.60 at position d_{32}, insert the corresponding city 3 and city 2 into *List-cities* {3-2} and remove their corresponding columns (see Figure 5).

$$\begin{bmatrix} \infty & 6.50 & 7.50 & 8.55 & 7.23 \\ \boxed{6.50} & \infty & 4.60 & 9.20 & 8.94 \\ 7.50 & \boxed{4.60} & \infty & \boxed{5.60} & 7.64 \\ 8.55 & 9.20 & 5.60 & \infty & 7.60 \\ 7.23 & 8.94 & 7.64 & 7.60 & \infty \end{bmatrix}$$

Figure 5 Discard columns 3 and 2

Hence, choose the smallest element between row 2 and row 3 (equal to 5.60) at position d_{34}.Then, add city 4 to *List-cities* {4-3-2} just before city 3 and discard its column (see Figure 6).

$$\begin{bmatrix} \infty & 6.50 & 7.50 & 8.55 & 7.23 \\ \boxed{6.50} & \infty & 4.60 & 9.20 & 8.94 \\ 7.50 & \boxed{4.60} & \infty & 5.60 & 7.64 \\ 8.55 & 9.20 & 5.60 & \infty & \boxed{7.60} \\ 7.23 & 8.94 & 7.64 & 7.60 & \infty \end{bmatrix}$$

Figure 6 Discard column 4

Similarly, find the smallest element between rows 2 and 4 that is equal to 6.50 is at position d_{21}. Thus, add city 1 to the *List-cities* {4-3-2-1} just after city 2 and discard its column (see Figure 7).

$$\begin{bmatrix} \infty & 6.50 & 7.50 & 8.55 & \boxed{7.23} \\ 6.50 & \infty & 4.60 & 9.20 & 8.94 \\ 7.50 & \boxed{4.60} & \infty & 5.60 & 7.64 \\ 8.55 & 9.20 & 5.60 & \infty & \boxed{7.60} \\ 7.23 & 8.94 & 7.64 & 7.60 & \infty \end{bmatrix}$$

Figure 7 Discard column 1

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Next select the smallest element between row 1 and row 4 that is equal to 7.23 at position d_{15}. So, add city 5 to *List-cities* {4-3-2-1-5} and remove its coresponding column (see Figure 8).

$$\begin{bmatrix} \infty & 6.50 & 7.50 & 8.55 & 7.23 \\ 6.50 & \infty & 4.60 & 9.20 & 8.94 \\ 7.50 & 4.60 & \infty & 5.60 & 7.64 \\ 8.55 & 9.20 & 5.60 & \infty & 7.60 \\ 7.23 & 8.94 & 7.64 & 7.60 & \infty \end{bmatrix}$$

Figure 8 Discard column 5

To finish, generate a cycle from *List-cities* {4-3-2-1-5} by translating all cities before city 1 to the last position and adding city 1 at the end {1-5-4-3-2-1}.

The crisp optimal solution for this problem is equal to 31.53 in which the neutrosophic optimal solution is equal to $N = \langle (11, 30, 52, 73); 0.7, 0.4, 0.3 \rangle$ (see Figure 9).

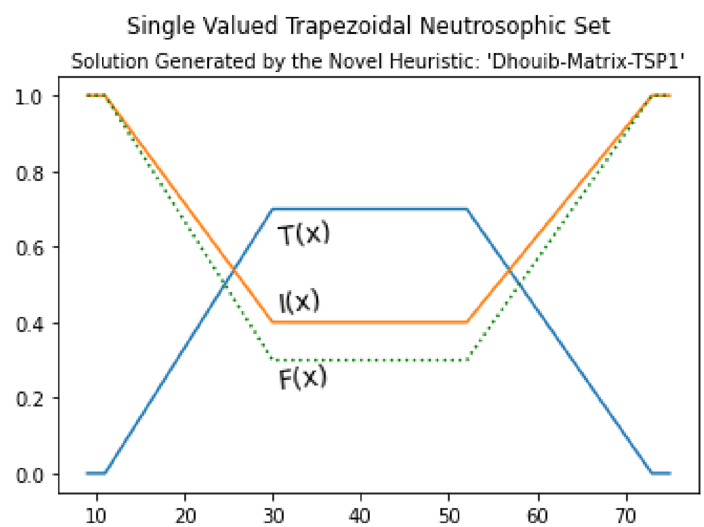

Figure 9 Graphical representation of the single valued trapezoidal neutrosophic optimal solution

4.2. Example 2

The second example deals with 5x5 TSP matrix with single valued trapezoidal neutrosophic TSP (see Figure 10).

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$$
\begin{bmatrix}
\infty & <(1,5,8,9); & <(2,7,9,14); & <(5,6,9,15); & <(7,8,11,13); \\
 & 0.7,0.2,0.3> & 0.6,0.1,0.4> & 0.9,0.1,0.1> & 0.9,0.1,0.1> \\
<(1,5,8,9); & \infty & <(2,4,6,8); & (3,6,8,14); & <(5,7,8,13); \\
0.7,0.2,0.3> & & 0.5,0.3,0.4> & 0.7,0.1,0.2 & 0.7,0.1,0.2> \\
<(2,7,9,14); & <(2,4,6,8); & \infty & (1,3,7,15); & <(7,8,9,13); \\
0.6,0.1,0.4> & 0.5,0.3,0.4> & & 0.9,0.1,0.2 & 0.6,0.3,0.1> \\
<(5,6,9,15); & <(3,6,8,14); & (1,3,7,15); & \infty & <(3,5,7,12); \\
0.9,0.1,0.1> & 0.7,0.1,0.2> & 0.9,0.1,0.2 & & 0.8,0.3,0.1> \\
<(7,8,11,13); & <(5,7,8,13); & <(7,8,9,13); & <(3,5,7,12); & \infty \\
0.9,0.1,0.1> & 0.7,0.1,0.2> & 0.6,0.3,0.1> & 0.8,0.3,0.1> &
\end{bmatrix}
$$

Figure 10 Neutrosophic matrix

By applying the score function described by Equation 1, the crisp matrix is presented as follows (see Figure 11).

$$
\begin{bmatrix}
\infty & 4.40 & 5.60 & 7.50 & 8.70 \\
4.40 & \infty & 3.00 & 6.00 & 6.40 \\
5.60 & 3.00 & \infty & 5.20 & 6.60 \\
7.50 & 6.00 & 5.20 & \infty & 5.20 \\
8.70 & 6.40 & 6.60 & 5.20 & \infty
\end{bmatrix}
$$

Figure 11 Crisp matrix

Figure 12, depicts the stepwise application of DM-TSP1 on 5x5 distance matrix. DM-TSP1 needs only n (n=5) iterations to solve this problem.

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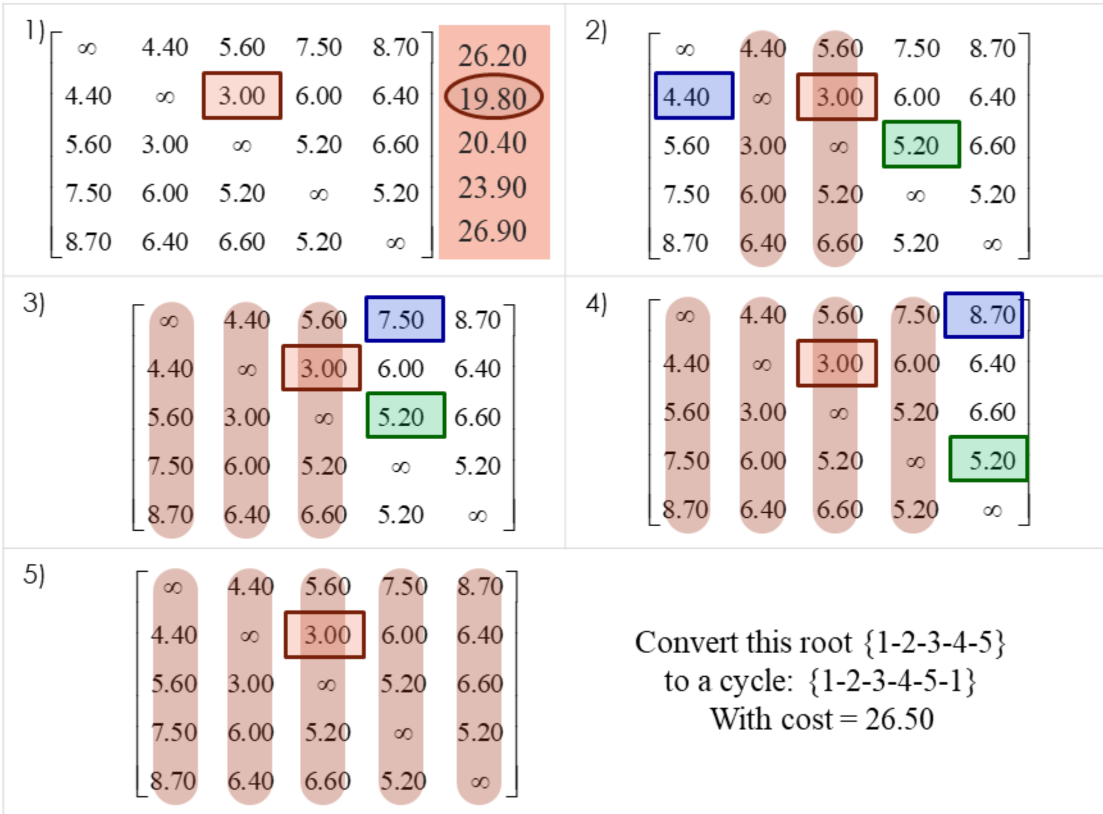

Figure 12 Step by step application of the heuristic DM-TSP1 on 5x5 matrix

The crisp optimal solution for this problem is equal to 26.50 in which the neutrosophic optimal solution is equal to $N = \langle (14, 25, 39, 57); 0.5, 0.3, 0.4 \rangle$ (See Figure 13).

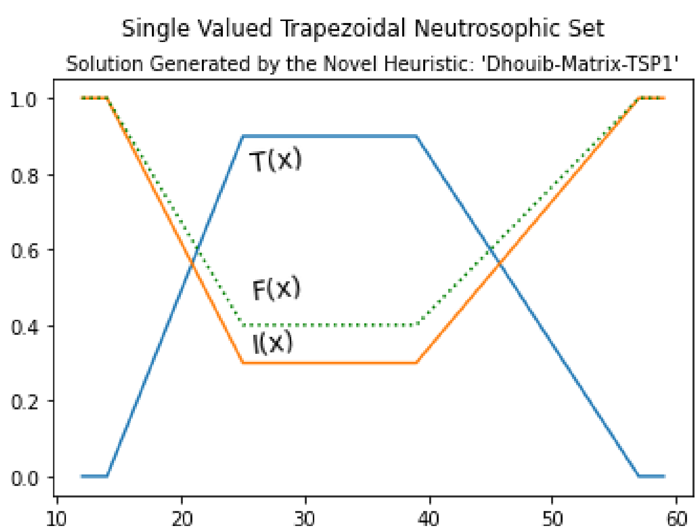

Figure 13 Graphical representation of the single valued trapezoidal neutrosophic optimal solution

4.3. Example 3

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The third example presents a 6*6 TSP matrix with single valued trapezoidal neutrosophic numbers (see Figure 14).

$$\begin{bmatrix}
\infty & \begin{matrix}<(2,7,9,13);\\0.9,0.1,0.1>\end{matrix} & \begin{matrix}<(3,5,8,9);\\0.9,0.3,0.2>\end{matrix} & \begin{matrix}<(5,6,8,15);\\0.9,0.1,0.1>\end{matrix} & \begin{matrix}<(8,9,16,18);\\0.7,0.4,0.5>\end{matrix} & \begin{matrix}<(4,7,8,17);\\0.9,0.1,0.1>\end{matrix} \\
\begin{matrix}<(2,7,9,13);\\0.9,0.1,0.1>\end{matrix} & \infty & \begin{matrix}<(4,8,11,18);\\0.9,0.1,0.1>\end{matrix} & \begin{matrix}<(3,9,12,19);\\0.9,0.4,0.1>\end{matrix} & \begin{matrix}<(8,9,15,16);\\0.7,0.4,0.3>\end{matrix} & \begin{matrix}<(2,8,16,18);\\0.6,0.5,0.4>\end{matrix} \\
\begin{matrix}<(3,5,8,9);\\0.9,0.3,0.2>\end{matrix} & \begin{matrix}<(4,8,11,18);\\0.9,0.1,0.1>\end{matrix} & \infty & \begin{matrix}<(9,10,13,15);\\0.7,0.4,0.5>\end{matrix} & \begin{matrix}<(4,7,8,11);\\0.9,0.1,0.1>\end{matrix} & \begin{matrix}<(1,6,9,17);\\0.8,0.2,0.2>\end{matrix} \\
\begin{matrix}<(5,6,8,15);\\0.9,0.1,0.1>\end{matrix} & \begin{matrix}<(3,9,12,19);\\0.9,0.4,0.1>\end{matrix} & \begin{matrix}<(9,10,13,15);\\0.7,0.4,0.5>\end{matrix} & \infty & \begin{matrix}<(1,4,12,15);\\0.9,0.1,0.1>\end{matrix} & \begin{matrix}<(4,7,9,18);\\0.7,0.3,0.1>\end{matrix} \\
\begin{matrix}<(8,9,16,18);\\0.7,0.4,0.5>\end{matrix} & \begin{matrix}<(8,9,15,16);\\0.7,0.4,0.3>\end{matrix} & \begin{matrix}<(4,7,8,11);\\0.9,0.1,0.1>\end{matrix} & \begin{matrix}<(1,4,12,15);\\0.9,0.1,0.1>\end{matrix} & \infty & \begin{matrix}<(1,2,12,15);\\0.9,0.1,0.1>\end{matrix} \\
\begin{matrix}<(4,7,8,17);\\0.9,0.1,0.1>\end{matrix} & \begin{matrix}<(2,8,16,18);\\0.6,0.5,0.4>\end{matrix} & \begin{matrix}<(1,6,9,17);\\0.8,0.2,0.2>\end{matrix} & \begin{matrix}<(4,7,9,18);\\0.7,0.3,0.1>\end{matrix} & \begin{matrix}<(1,2,12,15);\\0.9,0.1,0.1>\end{matrix} & \infty
\end{bmatrix}$$

Figure 14 Neutrosophic matrix

By applying the score function described by Equation 1, the crisp matrix is presented as follows (see Figure 15).

$$\begin{bmatrix}
\infty & 7.05 & 8.00 & 7.20 & 7.60 & 7.65 \\
7.05 & \infty & 9.00 & 8.53 & 8.00 & 6.42 \\
8.00 & 9.00 & \infty & 7.00 & 6.75 & 6.40 \\
7.20 & 8.53 & 7.00 & \infty & 7.20 & 6.90 \\
7.60 & 8.00 & 6.75 & 7.20 & \infty & 6.60 \\
7.65 & 6.42 & 6.40 & 6.90 & 6.60 & \infty
\end{bmatrix}$$

Figure 15 Crisp matrix

Figure 16, depicts the stepwise application of DM-TSP1 on 6x6 distance matrix. DM-TSP1 needs only *n* (*n*=6) iterations to solve this problem.

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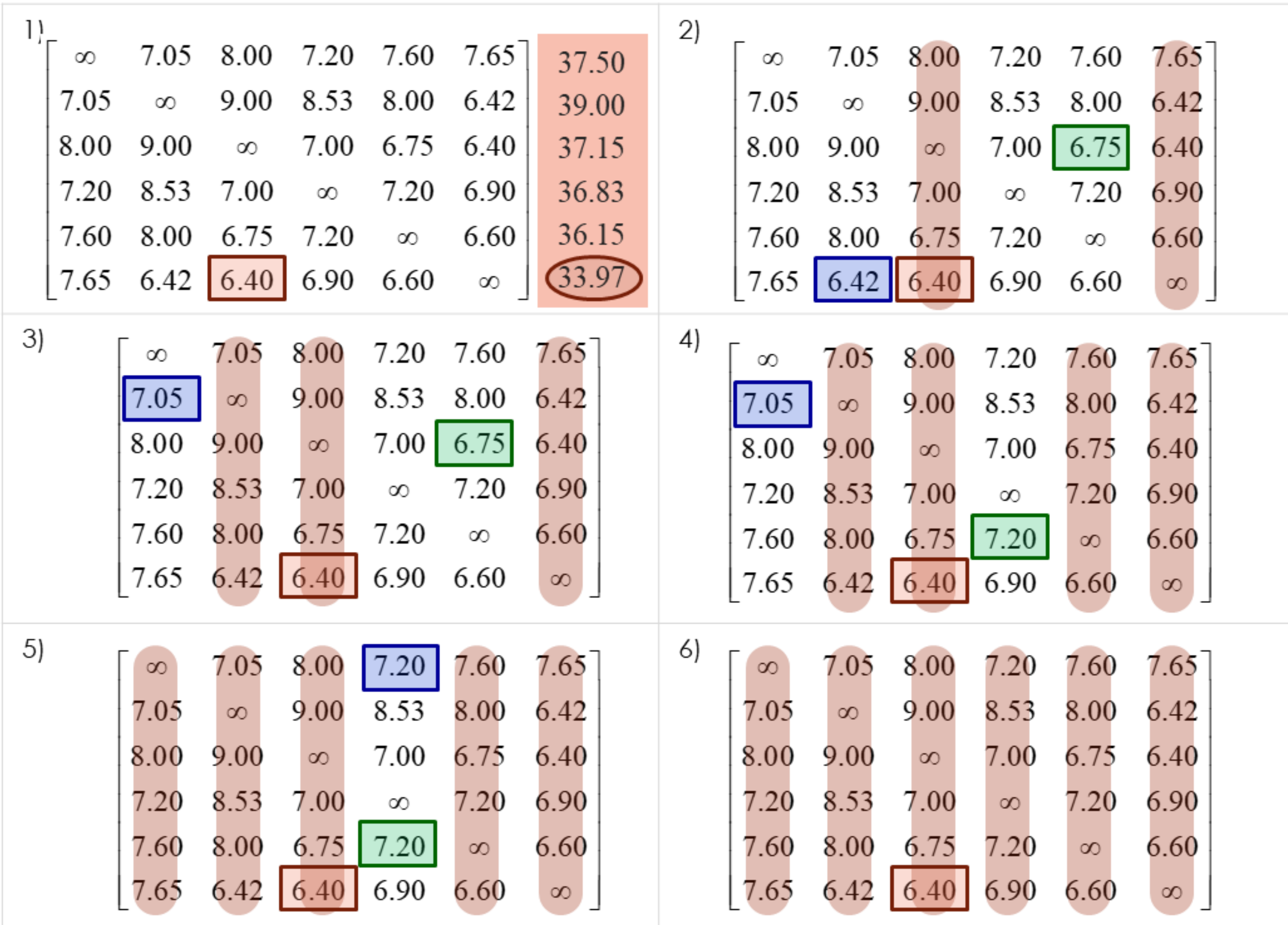

Figure 16 Step by step application of the heuristic DM-TSP1 on 6x6 matrix

The crisp optimal solution for this problem is equal to 41.02 in which the neutrosophic optimal solution is equal to $N = \langle (15, 38, 62, 89); 0.6, 0.5, 0.4 \rangle$ (See Figure 17).

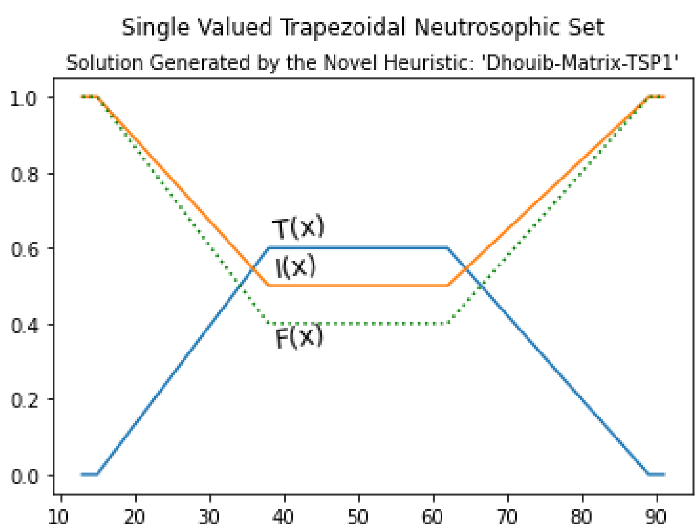

Figure 17 Graphical representation of the single valued trapezoidal neutrosophic optimal solution

5. Conclusions

This article presents the first optimization of the Travelling Salesman Problem (TSP) under single valued trapezoidal neutrosophic number. Several numerical examples are generated and the novel greedy method Dhouib-Matrix-TSP1 (DM-TSP1) is applied in order to find an initial basic feasible solution after only *n* iterations (*n* is the number of nodes). Acquired results are analyzed and graphical solutions are presented. As an extension of this research work, we look to solve other combinatorial problems under neutrosophic domain.

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