Neutrosophic Multigroup Homomorphism and Some of its Properties

Memet Şahin[1,*]

[1]Department of Mathematics, Gaziantep University, Gaziantep-Turkey

* Correspondence: mesahin@gantep.edu.tr

Abstract

In a way, the notion of neutrosophic multigroup is an application of neutrosophic multisets to the theory of group. The concept of neutrosophic multigroup is an algebraic structure of neutrosophic multiset that generalizes both the theories of classical group and neutrosophic group. Neutrosophic multigroup constitutes an application of neutrosophic multiset to the elementary theory of classical group. In this paper, we propose the concept of homomorphism on neutrosophic multigroup. We define homomorphism kerlf, automorphism, homomorphic image and homomorphic preimage of neutrosophic multigroup, respectively. Some homomorphic properties of neutrosophic multigroup are explicated. Some homomorphic properties of neutrosophic multigroup are also discussed. This new concept of homomorphism as a bridge among set theory, fuzzy set theory, neutrosophic multiset theory and group theory and also shows the effect of neutrosophic multisets on a group structure. We finally derive the basic properties of neutrosophic multigroup homomorphism and give its applications to group theory.

Keywords: Neutrosophic multiset; Neutrosophic multi group; neutrosophic multigroup homomorphism.s

1.Introduction

Zadeh [1] put forward the theory of fuzzy sets in 1965, which is an effective method to deal with fuzzy information, but only limited to the truth-membership function. In actual decision-making, because of the fuzziness of people's thinking and the complexity of objective things, it is difficult for decision-makers to evaluate only through truth-membership function. Based on fuzzy set theory, Atanassov [2] proposed the intuitionistic fuzzy set which added a non-membership function. The intuitionistic fuzzy set is composed of the membership (or called truth-membership) $\mu_{\mathcal{A}}(x)$ and non-membership (or called falsity-membership) $w_{\mathcal{A}}(x)$ and satisfies the conditions $\mu_{\mathcal{A}}(x), w_{\mathcal{A}}(x) \in [0,1]$ and $0 \leq \mu_{\mathcal{A}}(x) + w_{\mathcal{A}}(x) \leq 1$. However, intuitionistic fuzzy sets can merely deal with incomplete information, but cannot do anything for the indeterminate information and inconsistent information. The indeterminacy (or called Hesitation degree) is $1 - \mu_{\mathcal{A}}(x) - w_{\mathcal{A}}(x)$ which is only given by default and cannot be solely expressed in intuitionistic fuzzy sets. With respect to this situation, Smarandache [3] developed the neutrosophic set which consists of the truth-membership $\mu_{\mathcal{A}}(x)$, falsity-membership $w_{\mathcal{A}}(x)$, and indeterminacy-membership $v_{\mathcal{A}}(x)$, and the three variables are independent completely. Neutrosophic set is a generalization of fuzzy set and intuitionistic fuzzy set. Now there are many research achievements about neutrosophic sets. Compared to other tools to model fuzzy, inconsistent and uncertain information, the neutrosophic set is more flexible and accurate, which is based on preference order. The concept of neutrosophic set and logic came into being due to neutrosophy, where

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each proposition is approximated to have the percentage of truth in a subset μ, the percentage of indeterminacy in a subset v, and the percentage of falsity in a subset w. Neutrosophic sets are the generalization to all other traditional theories of logics. Modern set theory formulated (or invented) by a German mathematician George Cantor (1845-1918) is fundamental and indispensable for the whole of mathematics. In classical set theory, a set is a well-defined collection of distinct objects. If repeated occurrences of any object are allowed in a set, then the mathematical structure is called a multiset. Thus, a multiset differs from a set in the sense that each element has a multiplicity. Sing et al. presented [4] an overview of the applications of multisets, Syropoulos introduced [5] the mathematics of multisets. Wildberger presented [6] a new look at multisets. Then Yager [7] applied the idea of multiset, which is an extension of a set with repeated elements in a collection, to propose fuzzy multiset. That is, fuzzy multiset allows repetition of membership degrees of elements in multiset framework. Therefore, on the basis of fuzzy set theory, Sebastian and Ramakrishnan [8] introduced Multi-Fuzzy Sets, Shinoj and John [9] initiated intuitionistic fuzzy multisets. Recently, the above theories have developed in many directions and found its applications in a wide variety of fields, including algebraic structures. For example, on fuzzy sets [10–12], Shinoj et al. [13] studied of various algebraic structures of fuzzy multisets, Egejwa [14] introduced some group theoretic notions in fuzzy multigroup context, on fuzzy multi sets [15–18], Adamu [19] defined homomorphic intuitionistic fuzzy multigroups homomorphism its investigates some properties, on intuitionistic fuzzy sets [20–23], on intuitionistic fuzzy multi sets [24] are some selected works.

Fuzzy multiset, intuitionistic fuzzy multi sets and neutrosophic multiset are also presented since three exist the multi occurring information taken in a different time interval which allows repeated or same membership value more than one time [7, 9, 25, 26]. Later, for the first time, Smarandache [27] defined neutrosophic set where one or more elements are repeated with the same neutrosophic components, or with different neutrosophic components. An element of a neutrosophic multiset may possess more-than-one truth-membership function in [0,1], indeterminacy membership function in [0,1] and falsity-membership function in [0,1]. In a neutrosophic multiset, the membership degrees of an element may change during the time and the membership degrees accommodates more fuzzy and vague information than the fuzzy multiset and intuitionistic fuzzy multiset. Neutrosophic multisets theory has been extensively studied and applied in real-life problems [28-33,37-44].

The concept of neutrosophic multigroups was defined in [34] as an algebraic structure of neutrosophic multisets that generalizes fuzzy and classical groups. In fact, since neutrosophic multiset is a generalization of intuitionustic fuzzy set, it then follows that neutrosphic multigroup is an extension of fuzzy group. The concept of neutrosophic multigroup constitutes an application of intuitionustic fuzzy multisets to the notion of group. Neutrosophic multigroups and fuzzy groups are different generalizations of classical groups. The main theme of this article is to present the study of neutrosophic multigroup homomorphism as a powerful extension of the existing classical theories, such as neutrosophic sets and neutrosophic multigroup. The aspiration to form a sketch of this unique technique of a neutrosophic multiset in the study of neutrosophic multiring theory served as the main motivation to propose and develop the theory of neutrosophic multirings. Another keynote of this paper is to define the neutrosophic multigroup homomorphism and examine basic properties of neutrosophic multigroup homomorphism similar to classical homomorphism. Moreover, we extend this idea to prove neutrosophic multigroup isomorphism between these specific neutrosophic multisubgroup. This concept will bring a new opportunity in research and development of neutrosophic multiset theory.

The outlines are presented as follows: Section 2 presents some foundational notions relevant to the study, whereas the main results are reported in Section 3. In Section 4, we make some concluding remarks and suggestions for future work.

2. Preliminary

In this section, we review some definitions and results for the sake of completeness and reference. Then, some essential concepts that are useful for discussions in the next sections are explained.

Definition 2.1 [1] Let E be a universe. A neutrosophic sets $\mathcal{A}$ over E is defined as:

$$\mathcal{A} = \{\langle x, (\mu_{\mathcal{A}}(x), v_{\mathcal{A}}(x), w_{\mathcal{A}}(x))\rangle : x \in E\} \quad (1)$$

where $\mu_{\mathcal{A}}(x)$, $v_{\mathcal{A}}(x)$ and $w_{\mathcal{A}}(x)$ are called truth-membership function, indeterminacy-membership function and falsity- membership function, respectively. They are respectively defined by

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$$\mu_{\mathcal{A}}: E \to \left]^{-}0,1^{+}\right[\ , \ \upsilon_{\mathcal{A}}: E \to \left]^{-}0,1^{+}\right[\ , \ w_{\mathcal{A}}: E \to \left]^{-}0,1^{+}\right[$$

such that $^{-}0 \le \mu_{\mathcal{A}}(x) + \upsilon_{\mathcal{A}}(x) + w_{\mathcal{A}}(x) \le 3^{+}$.

Definition 2.2 [36] Let E be a universe. A single valued neutrosophic set (SVN-set) over E is a neutrosophic set over E, but the truth-membership function, indeterminacy-membership function and falsity-membership function are respectively defined by

$\mu_{\mathcal{A}}: E \to [0,1]$, $\upsilon_{\mathcal{A}}: E \to [0,1]$, $w_{\mathcal{A}}: E \to [0,1]$

such that $0 \le \mu_{\mathcal{A}}(x) + \upsilon_{\mathcal{A}}(x) + w_{\mathcal{A}}(x) \le 3$.

Definition 2.3 [26] Let E be a universe. A neutrosophic multiset set (Nms) $\mathcal{A}$ on E can be defined as follows:

$$\mathcal{A} = \left\{\left\langle x, \left(\mu_{\mathcal{A}}^{1}(x), \mu_{\mathcal{A}}^{2}(x), \dots, \mu_{\mathcal{A}}^{p}(x)\right), \left(\upsilon_{\mathcal{A}}^{1}(x), \upsilon_{\mathcal{A}}^{2}(x), \dots, \upsilon_{\mathcal{A}}^{p}(x)\right), \left(w_{\mathcal{A}}^{1}(x), w_{\mathcal{A}}^{2}(x), \dots, w_{\mathcal{A}}^{p}(x)\right)\right\rangle : x \in E\right\} \quad (2)$$

where,

$$\mu_{\mathcal{A}}^{1}(x), \mu_{\mathcal{A}}^{2}(x), \dots, \mu_{\mathcal{A}}^{p}(x): E \to [0,1],$$

$$\upsilon_{\mathcal{A}}^{1}(x), \upsilon_{\mathcal{A}}^{2}(x), \dots, \upsilon_{\mathcal{A}}^{p}(x): E \to [0,1],$$

and

$$w_{\mathcal{A}}^{1}(x), w_{\mathcal{A}}^{2}(x), \dots, w_{\mathcal{A}}^{p}(x): E \to [0,1]$$

such that

$$0 \le sup\ \mu_{\mathcal{A}}^{i}(x) + sup\ v_{\mathcal{A}}^{i}(x) + sup\ w_{\mathcal{A}}^{i}(x) \le 3$$

$(i = 1,2, \dots, p)$ and

$$\left(\mu_{\mathcal{A}}^{1}(x), \mu_{\mathcal{A}}^{2}(x), \dots, \mu_{\mathcal{A}}^{p}(x)\right), \left(\upsilon_{\mathcal{A}}^{1}(x), \upsilon_{\mathcal{A}}^{2}(x), \dots, \upsilon_{\mathcal{A}}^{p}(x)\right) \text{ and } \left(w_{\mathcal{A}}^{1}(x), w_{\mathcal{A}}^{2}(x), \dots, w_{\mathcal{A}}^{p}(x)\right)$$

are the truth-membership sequence, indeterminacy-membership sequence and falsity-membership sequence of the element x, respectively. Also, p is called the dimension (cardinality) of Nms $\mathcal{A}$, denoted $d(\mathcal{A})$. We arrange the truth-membership sequence in decreasing order but the corresponding indeterminacy-membership and falsity-membership sequence may not be in decreasing or increasing order.

Definition 2.4 [34] Let $\mathcal{X}$ be a classical group $\mathcal{A} \in \mathcal{NMS}(\mathcal{X})$. Then, $\mathcal{A}$ is called a neutrosophic multi groupoid over $\mathcal{X}$ if

$$\mu_{G}^{i}(xy) \ge \mu_{G}^{i}(x) \wedge \mu_{G}^{i}(y),$$

$$\upsilon_{G}^{i}(xy) \le \upsilon_{G}^{i}(x) \vee \upsilon_{G}^{i}(y),$$

$$w_{G}^{i}(xy) \le w_{G}^{i}(x) \vee w_{G}^{i}(y),$$

for all $x, y \in \mathcal{X}$ and $i = 1,2, \dots, p$.

$\mathcal{A}$ is called a neutrosophic multi groupoid over $\mathcal{X}$ if the neutrosophic multi groupoid satisfies

$$\mu_{G}^{i}(x^{-1}) \ge \mu_{G}^{i}(x),$$

$$\upsilon_{G}^{i}(x^{-1}) \le \upsilon_{G}^{i}(x),$$

$$w_{G}^{i}(x^{-1}) \le w_{G}^{i}(x),$$

for all $x \in \mathcal{X}$ and $i = 1,2, \dots, p$.

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Definition 2.5 [33] Let Y be a subgroup of X, $B \in NM(Y)$, $B \tilde{\subseteq} A$ and $A \in NM(X)$. If $B \in NM(Y)$, then is called a neutrosophic multi subgroup of A over X and denoted by $B \tilde{\leq} A$.

Definition 2.6 [35] Let $\mathcal{X}$ and $\mathcal{Y}$ groups and $f: \mathcal{X} \to \mathcal{Y}$ be a homomorphism. Suppose $\mathcal{A}$ and $\mathcal{B}$ are fuzzy multigroups of $\mathcal{X}$ and $\mathcal{Y}$, respectively. Then, f induces a homomorphism from $\mathcal{A}$ to $\mathcal{B}$ which satisfies ('count membership' of A(CMA))

1. $CM_{\mathcal{A}}(f^{-1}(y_1 y_2)) \geq CM_{\mathcal{A}}(f^{-1}(y_1)) \wedge CM_{\mathcal{A}}(f^{-1}(y_2)) \; \forall \, y_1, y_2 \in \mathcal{Y}$
2. $CM_{\mathcal{B}}(f(x_1 x_2)) \geq CM_{\mathcal{B}}(f(x_1)) \wedge CM_{\mathcal{B}}(f(x_2)) \; \forall \, x_1, x_2 \in \mathcal{X}$.

Definition 2.7 [35] Let $f: \mathcal{X} \to \mathcal{Y}$ be a homomorphism of groups. Suppose that $\mathcal{A}$ and $\mathcal{B}$ are fuzzy multigroups of $\mathcal{X}$ and $\mathcal{Y}$, respectively. Then, $\mathcal{A}$ homomorphic to $\mathcal{B}$. The kernel of the homomorphism from $\mathcal{A}$ to $\mathcal{B}$ is defined by

$$ker f = \{x \in \mathcal{X}: CM_{\mathcal{A}}(x) = CM_{\mathcal{B}}(e'), f(e) = e'\} \quad (3)$$

where e and e' are the identities of $\mathcal{X}$ and $\mathcal{Y}$, respectively.

3. Neutrosophic Multigroup Homomorphism

In this section our main focus is to propose the concepts of neutrosophic multigroup homomorphism and to investigate various fundamental algebraic aspects of these notions.

Definition 3.1 Let $\mathcal{A} = \Big\{\Big\langle x, \big(\mu_{\mathcal{A}}^1(x), \mu_{\mathcal{A}}^2(x), \ldots, \mu_{\mathcal{A}}^p(x)\big), \big(v_{\mathcal{A}}^1(x), v_{\mathcal{A}}^2(x), \ldots, v_{\mathcal{A}}^p(x)\big)$

$$\big(w_{\mathcal{A}}^1(x), w_{\mathcal{A}}^2(x), \ldots, w_{\mathcal{A}}^p(x)\big)\Big\rangle : x \in \mathcal{X}\Big\},$$

$\{\mu_{\mathcal{A}}^i(x), v_{\mathcal{A}}^i(x), w_{\mathcal{A}}^i(x) \in [0,1]\}, (i = 1,2, \ldots, p)$ be any neutrosophic multisets on $\mathcal{X}$. Then, extending the function $f: \mathcal{X} \to \mathcal{Y}$, the neutrosophic multisets on $\mathcal{X}$ is made to correspond to neutrosophic multisets $f(\mathcal{A}) = \{\mu_{f(\mathcal{A})}^i(x), v_{f(\mathcal{A})}^i(x), w_{f(\mathcal{A})}^i(x)\}$ of $\mathcal{Y}$ may be the following ways

$$\mu_{f(\mathcal{A})}^1(y) = \begin{cases} \vee \{\mu_{\mathcal{A}}^1(x): x \in f^{-1}(y)\}, & \text{if } f^{-1}(y) \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$\mu_{f(\mathcal{A})}^2(y) = \begin{cases} \vee \{\mu_{\mathcal{A}}^2(x): x \in f^{-1}(y)\}, & \text{if } f^{-1}(y) \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

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$$\mu_{f(\mathcal{A})}^p(y) = \begin{cases} \vee \{\mu_{\mathcal{A}}^p(x): x \in f^{-1}(y)\}, & \text{if } f^{-1}(y) \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$v_{f(\mathcal{A})}^1(y) = \begin{cases} \wedge \{v_{\mathcal{A}}^1(x): x \in f^{-1}(y)\}, & \text{if } f^{-1}(y) \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$v_{f(\mathcal{A})}^2(y) = \begin{cases} \wedge \{v_{\mathcal{A}}^2(x): x \in f^{-1}(y)\}, & \text{if } f^{-1}(y) \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

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$$v_{f(\mathcal{A})}^p(y) = \begin{cases} \wedge \{v_{\mathcal{A}}^p(x): x \in f^{-1}(y)\}, & \text{if } f^{-1}(y) \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

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$$w^1_{f(\mathcal{A})}(y) = \begin{cases} \wedge \{w^1_{\mathcal{A}}(x): x \in f^{-1}(y)\}, & \text{if } f^{-1}(y) \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$w^2_{f(\mathcal{A})}(y) = \begin{cases} \wedge \{w^2_{\mathcal{A}}(x): x \in f^{-1}(y)\}, & \text{if } f^{-1}(y) \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

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$$w^p_{f(\mathcal{A})}(y) = \begin{cases} \wedge \{w^p_{\mathcal{A}}(x): x \in f^{-1}(y)\}, & \text{if } f^{-1}(y) \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

Definition 3.2 Let $\mathcal{X}$ be a group and $\mathcal{A} \in \mathcal{NM}\mathrm{G}(\mathcal{X})$. Then there exists a natural homomorphism $f: \mathcal{X} \to \mathcal{X}/\mathcal{A}$ defined by

$$f(x) = x\mathcal{A} \qquad \forall\, x \in \mathcal{X}. \tag{4}$$

Proof: Let $f: \mathcal{X} \to \mathcal{X}/\mathcal{A}$ be a mapping defined by $f(x) = x\mathcal{A}$ for all $x \in \mathcal{X}$. We show that f is homomorphism i.e.

$$f(xy) = f(x)f(y)$$

for all $x \in \mathcal{X}$.

$$(xy)\mathcal{A} = (x\mathcal{A})(y\mathcal{A})$$

for all $x, y \in \mathcal{X}$. Since $\mathcal{A}$ is $\mathcal{NM}$-group of group $\mathcal{X}$, therefore we have

$$\mu^i_{\mathcal{A}}(m^{-1}xm) = \mu^i_{\mathcal{A}}(x)$$

$$v^i_{\mathcal{A}}(m^{-1}xm) = v^i_{\mathcal{A}}(x)$$

$$w^i_{\mathcal{A}}(m^{-1}xm) = w^i_{\mathcal{A}}(x)$$

$\forall\, x \in \mathcal{A}$ and $m \in \mathcal{X}$ or equivalently,

$$\mu^i_{\mathcal{A}}(xy) = \mu^i_{\mathcal{A}}(yx)$$

$$v^i_{\mathcal{A}}(xy) = v^i_{\mathcal{A}}(yx)$$

$$w^i_{\mathcal{A}}(xy) = w^i_{\mathcal{A}}(yx)$$

for all $x, y \in \mathcal{X}$, also

$$\begin{aligned}
(x\,\mathcal{A})(m) &= \left(\mu^i_{(x\,\mathcal{A})(m)}, v^i_{(x\,\mathcal{A})(m)}, w^i_{(x\,\mathcal{A})(m)}\right) \\
&= \left(\mu^i_{\mathcal{A}}(x^{-1}m), v^i_{\mathcal{A}}(x^{-1}m), w^i_{\mathcal{A}}(x^{-1}m)\right) \quad \forall m \in \mathcal{X} \\
(y\,\mathcal{A})(m) &= \left(\mu^i_{(y\,\mathcal{A})(m)}, v^i_{(y\,\mathcal{A})(m)}, w^i_{(y\,\mathcal{A})(m)}\right) \\
&= \left(\mu^i_{\mathcal{A}}(y^{-1}m), v^i_{\mathcal{A}}(y^{-1}m), w^i_{\mathcal{A}}(y^{-1}m)\right) \quad \forall m \in \mathcal{X} \\
(xy\,\mathcal{A})(m) &= \left(\mu^i_{(xy\,\mathcal{A})(m)}, v^i_{(xy\,\mathcal{A})(m)}, w^i_{(xy\,\mathcal{A})(m)}\right) \\
&= \left(\mu^i_{\mathcal{A}}((xy)^{-1}m), v^i_{\mathcal{A}}((xy)^{-1}m), w^i_{\mathcal{A}}((xy)^{-1}m)\right) \quad \forall m \in \mathcal{X}
\end{aligned}$$

now

$$(x\,\mathcal{A})(y\,\mathcal{A})(m) = \Bigg(\bigvee_{m=rs} \left[\mu^i_{x\mathcal{A}}(r) \wedge \mu^i_{y\mathcal{A}}(s)\right], \bigwedge_{m=rs} \left[v^i_{x\mathcal{A}}(r) \vee v^i_{y\mathcal{A}}(s)\right],$$

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$$\left.\bigwedge_{m=rs}\left[w^{i}_{x\mathcal{A}}(r)\vee w^{i}_{y\mathcal{A}}(s)\right]\right),\forall m\in\mathcal{X}$$

$$=\left(\bigvee_{m=rs}\left[\mu^{i}_{x\mathcal{A}}(r)\wedge\mu^{i}_{y\mathcal{A}}(s)\right],\ \bigwedge_{m=rs}\left[v^{i}_{x\mathcal{A}}(r)\vee v^{i}_{y\mathcal{A}}(s)\right],\right.$$

$$\left.\bigwedge_{m=rs}\left[w^{i}_{x\mathcal{A}}(r)\vee w^{i}_{y\mathcal{A}}(s)\right]\right),\forall m\in\mathcal{X}$$

we claim that

$$\mu^{i}_{\mathcal{A}}((xy)^{-1}m)=\bigvee_{m=rs}\left[\mu^{i}_{\mathcal{A}}(x^{-1}r)\wedge\mu^{i}_{\mathcal{A}}(y^{-1}s)\right]$$

$$v^{i}_{\mathcal{A}}((xy)^{-1}m)=\bigwedge_{m=rs}\left[v^{i}_{\mathcal{A}}(x^{-1}r)\vee v^{i}_{\mathcal{A}}(y^{-1}s)\right]$$

$$w^{i}_{\mathcal{A}}((xy)^{-1}m)=\bigwedge_{m=rs}\left[w^{i}_{\mathcal{A}}(x^{-1}r)\vee w^{i}_{\mathcal{A}}(y^{-1}s)\right]$$

Now

$$\begin{aligned}\mu^{i}_{\mathcal{A}}((xy)^{-1}m)=&\ \mu^{i}_{\mathcal{A}}(y^{-1}x^{-1}m)\\ =&\ \mu^{i}_{\mathcal{A}}(y^{-1}x^{-1}rs)\\ =&\ \mu^{i}_{\mathcal{A}}\left(y^{-1}(x^{-1}rsy^{-1})\right)\\ =&\ \mu^{i}_{\mathcal{A}}(x^{-1}rsy^{-1})\\ \geq&\ \mu^{i}_{\mathcal{A}}(x^{-1}r)\wedge\mu^{i}_{\mathcal{A}}(sy^{-1})\\ =&\ \mu^{i}_{\mathcal{A}}(x^{-1}r)\wedge\mu^{i}_{\mathcal{A}}(y^{-1}s);\forall m=rs\in\mathcal{X}\end{aligned}$$

thus

$$\mu^{i}_{\mathcal{A}}((xy)^{-1}m)=\bigvee_{m=rs}\left[\mu^{i}_{\mathcal{A}}(x^{-1}r)\wedge\mu^{i}_{\mathcal{A}}(y^{-1}s)\right]$$

similarly we can show that

$$\begin{aligned}v^{i}_{\mathcal{A}}((xy)^{-1}m)=&\ v^{i}_{\mathcal{A}}(y^{-1}x^{-1}m)\\ =&\ v^{i}_{\mathcal{A}}(y^{-1}x^{-1}rs)\\ =&\ v^{i}_{\mathcal{A}}\left(y^{-1}(x^{-1}rsy^{-1})\right)\\ =&\ v^{i}_{\mathcal{A}}(x^{-1}rsy^{-1})\\ \leq&\ v^{i}_{\mathcal{A}}(x^{-1}r)\vee v^{i}_{\mathcal{A}}(sy^{-1})\\ =&\ v^{i}_{\mathcal{A}}(x^{-1}r)\vee v^{i}_{\mathcal{A}}(y^{-1}s);\forall m=rs\in\mathcal{X}\end{aligned}$$

thus

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$$v^i_{\mathcal{A}}((xy)^{-1}m) = \bigwedge_{m=rs} \left[v^i_{\mathcal{A}}(x^{-1}r) \vee v^i_{\mathcal{A}}(y^{-1}s)\right]$$

$$\begin{aligned} w^i_{\mathcal{A}}((xy)^{-1}m) = &\ w^i_{\mathcal{A}}(y^{-1}x^{-1}m) \\ = &\ w^i_{\mathcal{A}}(y^{-1}x^{-1}rs) \\ = &\ w^i_{\mathcal{A}}\big(y^{-1}(x^{-1}rsy^{-1})\big) \\ = &\ w^i_{\mathcal{A}}(x^{-1}rsy^{-1}) \\ \leq &\ w^i_{\mathcal{A}}(x^{-1}r) \vee w^i_{\mathcal{A}}(sy^{-1}) \\ = &\ w^i_{\mathcal{A}}(x^{-1}r) \vee w^i_{\mathcal{A}}(y^{-1}s); \forall m = rs \in \mathcal{X} \end{aligned}$$

thus

$$w^i_{\mathcal{A}}((xy)^{-1}m) = \bigwedge_{m=rs} \left[w^i_{\mathcal{A}}(x^{-1}r) \vee w^i_{\mathcal{A}}(y^{-1}s)\right]$$

thus

$$\begin{aligned} (xy)\mathcal{A} = &\ (x\mathcal{A})(y\mathcal{A});\ \forall m \in \mathcal{X} \\ \Rightarrow (xy)\mathcal{A} = &\ (x\mathcal{A})(y\mathcal{A}) \\ \Rightarrow\ f(xy) = &\ f(x)f(y). \end{aligned}$$

hence f is neutrosophic homomorphism.

Theorem 3.3 Let $\mathcal{A} = \Big\{\Big\langle x, \big(\mu^1_{\mathcal{A}}(x), \mu^2_{\mathcal{A}}(x), \dots, \mu^p_{\mathcal{A}}(x)\big), \big(v^1_{\mathcal{A}}(x), v^2_{\mathcal{A}}(x), \dots, v^p_{\mathcal{A}}(x)\big)$

$$\big(w^1_{\mathcal{A}}(x), w^2_{\mathcal{A}}(x), \dots, w^p_{\mathcal{A}}(x)\big)\Big\rangle : x \in \mathcal{X}\Big\},$$

such that $\mu^i_{\mathcal{A}}(x) = \mu^i_{\mathcal{A}}(e), v^i_{\mathcal{A}}(x) = v^i_{\mathcal{A}}(e), w^i_{\mathcal{A}}(x) = w^i_{\mathcal{A}}(e)$ be $\mathcal{NMG}$ of group $\mathcal{X}$ and $\mathcal{B}$ be a $\mathcal{NMS}$ of group $\mathcal{X}$ and $f: \mathcal{X} \to \mathcal{X}/\mathcal{A}$ $(or\ f(\mathcal{A}) = \{(\mu^i_{\mathcal{A}}), (v^i_{\mathcal{A}}), (w^i_{\mathcal{A}})\}$ be natural homomorphism defined by $f(x) = x\mathcal{A} \quad \forall x \in \mathcal{X}$, then

$$f^{-1}\big(f(\mathcal{B})\big) = \mathcal{A} \circ \mathcal{B}. \tag{5}$$

Proof Let $x \in \mathcal{X}$ be any element, then

$$(\mathcal{A} \circ \mathrm{B})(x) = \begin{cases} \bigvee_{x=yz} \left[\mu^i_{\mathcal{B}}(y) \wedge \mu^i_{\mathcal{A}}(z)\right], & \bigwedge_{x=yz} \left[v^i_{\mathcal{B}}(y) \vee v^i_{\mathcal{A}}(z)\right], \quad \bigwedge_{x=yz} \left[w^i_{\mathcal{B}}(y) \vee w^i_{\mathcal{A}}(z)\right] \\ (0,0,1) & \text{otherwise} \end{cases}$$

$$= \begin{cases} \bigvee_{z=xy^{-1}\in\mathcal{A}} \left[\mu^i_{\mathcal{B}}(y) \wedge \mu^i_{\mathcal{A}}(z)\right], & \bigwedge_{z=xy^{-1}\in\mathcal{A}} \left[v^i_{\mathcal{B}}(y) \vee v^i_{\mathcal{A}}(z)\right], \bigwedge_{z=xy^{-1}\in\mathcal{A}} \left[w^i_{\mathcal{B}}(y) \vee w^i_{\mathcal{A}}(z)\right] \\ (0,0,1) & \text{otherwise} \end{cases}$$

$$= \begin{cases} \bigvee_{z=xy^{-1}\in\mathcal{A}} \left[\mu^i_{\mathcal{B}}(y) \wedge \mu^i_{\mathcal{A}}(xy^{-1})\right], & \bigwedge_{z=xy^{-1}\in\mathcal{A}} \left[v^i_{\mathcal{B}}(y) \vee v^i_{\mathcal{A}}(xy^{-1})\right], \bigwedge_{z=xy^{-1}\in\mathcal{A}} \left[w^i_{\mathcal{B}}(y) \vee w^i_{\mathcal{A}}(xy^{-1})\right] \\ (0,0,1) & \text{otherwise} \end{cases}$$

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$$= \begin{cases} \bigvee_{z=xy^{-1}\in\mathcal{A}} [\mu_{\mathcal{B}}^i(y) \wedge \mu_{\mathcal{A}}^i(e)], \bigwedge_{z=xy^{-1}\in\mathcal{A}} [v_{\mathcal{B}}^i(y) \vee v_{\mathcal{A}}^i(e)], \bigwedge_{z=xy^{-1}\in\mathcal{A}} [w_{\mathcal{B}}^i(y) \vee w_{\mathcal{A}}^i(e)] \\ (0,0,1) \quad \text{otherwise} \end{cases}$$

$$= \begin{cases} \bigvee_{z=xy^{-1}\in\mathcal{A}} [\mu_{\mathcal{B}}^i(y)], \bigwedge_{z=xy^{-1}\in\mathcal{A}} [v_{\mathcal{B}}^i(y)], \bigwedge_{z=xy^{-1}\in\mathcal{A}} [w_{\mathcal{B}}^i(y)] \\ (0,0,1) \quad \text{otherwise} \end{cases}$$

$$= \begin{cases} \bigvee_{y=z^{-1}x\in\mathcal{B}} [\mu_{\mathcal{B}}^i(y)], \bigwedge_{y=z^{-1}x\in\mathcal{B}} [v_{\mathcal{B}}^i(y)], \bigwedge_{y=z^{-1}x\in\mathcal{B}} [w_{\mathcal{B}}^i(y)] \\ (0,0,1) \quad \text{otherwise} \end{cases}$$

$$= \begin{cases} \bigvee_{y=z^{-1}x\in\mathcal{B}} [\mu_{\mathcal{B}}^i(z^{-1}x)], \bigwedge_{y=z^{-1}x\in\mathcal{B}} [v_{\mathcal{B}}^i(z^{-1}x)], \bigwedge_{y=z^{-1}x\in\mathcal{B}} [w_{\mathcal{B}}^i(z^{-1}x)] \\ (0,0,1) \quad \text{otherwise} \end{cases}$$

also

$$f^{-1}(f(\mathcal{B}))(x) = \left(\left(\mu_{f^{-1}(f(\mathcal{B}))}^i(x)\right), \left(v_{f^{-1}(f(\mathcal{B}))}^i(x)\right), \left(w_{f^{-1}(f(\mathcal{B}))}^i(x)\right)\right)$$

$$= \left(\left(\mu_{f(\mathcal{B})}^i f(x)\right), \left(v_{f(\mathcal{B})}^i f(x)\right), \left(w_{f(\mathcal{B})}^i f(x)\right)\right)$$

$$= \begin{cases} \bigvee_{f(x)=f(y)} [\mu_{\mathcal{B}}^i(y)], \bigwedge_{f(x)=f(y)} [v_{\mathcal{B}}^i(y)], \bigwedge_{f(x)=f(y)} [w_{\mathcal{B}}^i(y)] \\ (0,0,1) \quad \text{otherwise} \end{cases}$$

$$= \begin{cases} \bigvee_{xy^{-1}\in\mathcal{A}} [\mu_{\mathcal{B}}^i(y)], \bigwedge_{xy^{-1}\in\mathcal{A}} [v_{\mathcal{B}}^i(y)], \bigwedge_{xy^{-1}\in\mathcal{A}} [w_{\mathcal{B}}^i(y)] \\ (0,0,1) \quad \text{otherwise} \end{cases}$$

$$= \begin{cases} \bigvee_{y=z^{-1}x\in\mathcal{B}} [\mu_{\mathcal{B}}^i(y)], \bigwedge_{y=z^{-1}x\in\mathcal{B}} [v_{\mathcal{B}}^i(y)], \bigwedge_{y=z^{-1}x\in\mathcal{B}} [w_{\mathcal{B}}^i(y)] \\ (0,0,1) \quad \text{otherwise, where } z = xy^{-1} \end{cases}$$

$$= \begin{cases} \bigvee_{z^{-1}x\in\mathcal{B}} [\mu_{\mathcal{B}}^i(y)], \bigwedge_{z^{-1}x\in\mathcal{B}} [v_{\mathcal{B}}^i(y)], \bigwedge_{z^{-1}x\in\mathcal{B}} [w_{\mathcal{B}}^i(y)] \\ (0,0,1) \quad \text{otherwise,} \end{cases}$$

Thus $f^{-1}(f(\mathcal{B}))(x) = (\mathcal{A} \circ \mathcal{B})(x), \forall\, x \in \mathcal{X}$. Hence $f^{-1}(f(\mathcal{B})) = \mathcal{A} \circ \mathcal{B}$.

Proposition 3.4 Let $\mathcal{A}, \mathcal{B} \in \mathcal{NMG}(\mathcal{X})$ and $\mathcal{A}$ be natural neutrosphic multisubgroup of $\mathcal{B}$. Let $\mathcal{Y}$ be a group and f a homomorphism from $\mathcal{X}$ into $\mathcal{Y}$. Then $f(\mathcal{A}) = \{(\mu_{\mathcal{A}}^i), (v_{\mathcal{A}}^i), (w_{\mathcal{A}}^i)\}$ is a normal neutrosophic multisubgroup of $f(\mathcal{B}) = \{(\mu_{\mathcal{B}}^i), (v_{\mathcal{B}}^i), (w_{\mathcal{B}}^i)\}$.

Proof $f(\mathcal{A}), f(\mathcal{B}) \in \mathcal{NMG}(\mathcal{Y})$ and $f(\mathcal{A}) \subseteq f(\mathcal{B})$. Now

$$(\mu_{f(\mathcal{A})}^i)(xyx^{-1}) = \vee\{(\mu_{f(\mathcal{A})}^i)(z) : z \in \mathcal{X}, (\mu_{f(\mathcal{A})}^i)(z) = xyx^{-1}\}$$
$$\geq \vee\{(\mu_{f(\mathcal{A})}^i)(uvu^{-1}) : u, v \in \mathcal{X}, (\mu_{f(\mathcal{A})}^i)(u) = x, (\mu_{f(\mathcal{A})}^i)(v) = y\}$$

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$$
\begin{aligned}
&\geq \vee\{(\mu^i_{f(\mathcal{A})})(u) \wedge (\mu^i_{f(\mathcal{A})})(v): u, v \in \mathcal{X}, (\mu^i_{f(\mathcal{A})})(u) = x, (\mu^i_{f(\mathcal{A})})(v) = y\} \\
&= (\vee\{(\mu^i_{f(\mathcal{B})})(v): v \in \mathcal{X}, (\mu^i_{f(\mathcal{B})})(v) = y\}) \\
&\qquad \wedge (\vee\{(\mu^i_{f(\mathcal{A})})(u): u \in \mathcal{X}, (\mu^i_{f(\mathcal{A})})(u) = x\}) \\
&= (\mu^i_{f(\mathcal{A})})(y) \wedge (\mu^i_{f(\mathcal{B})})(x)
\end{aligned}
$$

$$
\begin{aligned}
(v^i_{f(\mathcal{A})})(xyx^{-1}) &= \wedge\{(v^i_{f(\mathcal{A})})(z): z \in \mathcal{X}, (v^i_{f(\mathcal{A})})(z) = xyx^{-1}\} \\
&\leq \wedge\{(v^i_{f(\mathcal{A})})(uvu^{-1}): u, v \in \mathcal{X}, (v^i_{f(\mathcal{A})})(u) = x, (v^i_{f(\mathcal{A})})(v) = y\} \\
&\leq \wedge\{(v^i_{f(\mathcal{A})})(u) \vee (v^i_{f(\mathcal{A})})(v): u, v \in \mathcal{X}, (v^i_{f(\mathcal{A})})(u) = x, (v^i_{f(\mathcal{A})})(v) = y\} \\
&= (\wedge\{(v^i_{f(\mathcal{B})})(v): v \in \mathcal{X}, (v^i_{f(\mathcal{B})})(v) = y\}) \\
&\qquad \vee (\wedge\{(v^i_{f(\mathcal{A})})(u): u \in \mathcal{X}, (v^i_{f(\mathcal{A})})(u) = x\}) \\
&= (v^i_{f(\mathcal{A})})(y) \vee (v^i_{f(\mathcal{B})})(x) \\
(w^i_{f(\mathcal{A})})(xyx^{-1}) &= \wedge\{(w^i_{f(\mathcal{A})})(z): z \in \mathcal{X}, (w^i_{f(\mathcal{A})})(z) = xyx^{-1}\} \\
&\leq \wedge\{(w^i_{f(\mathcal{A})})(uvu^{-1}): u, v \in \mathcal{X}, (w^i_{f(\mathcal{A})})(u) = x, (w^i_{f(\mathcal{A})})(v) = y\} \\
&\leq \wedge\{(w^i_{f(\mathcal{A})})(u) \vee (w^i_{f(\mathcal{A})})(v): u, v \in \mathcal{X}, (w^i_{f(\mathcal{A})})(u) = x, (w^i_{f(\mathcal{A})})(v) = y\} \\
&= (\wedge\{(w^i_{f(\mathcal{B})})(v): v \in \mathcal{X}, (w^i_{f(\mathcal{B})})(v) = y\}) \\
&\qquad \vee (\wedge\{(w^i_{f(\mathcal{A})})(u): u \in \mathcal{X}, (w^i_{f(\mathcal{A})})(u) = x\}) \\
&= (w^i_{f(\mathcal{A})})(y) \vee (w^i_{f(\mathcal{B})})(x)
\end{aligned}
$$

$\forall\, x, y \in \mathcal{Y}$. Hence $f(\mathcal{A})$ is a normal neutrosophic multisubgroup of $f(\mathcal{B})$.

Theorem 3.5 Let $\mathcal{Y}$ be a group. Let $\mathcal{A}, \mathcal{B} \in \mathcal{NMG}(\mathcal{Y})$ and $\mathcal{A}$ be a normal neutrosophic multisubgroup of $\mathcal{B}$. Let f be a homomorphism from $\mathcal{X}$ into $\mathcal{Y}$. Then $f^{-1}(\mathcal{A}) = \{((\mu^{-1})^i_{f(\mathcal{A})}), ((v^{-1})^i_{f(\mathcal{A})}), ((w^{-1})^i_{f(\mathcal{A})})\}$ is a normal neutrosophic multisubgroup of $f^{-1}(\mathcal{B}) = \{((\mu^{-1})^i_{f(\mathcal{B})}), ((v^{-1})^i_{f(\mathcal{B})}), ((w^{-1})^i_{f(\mathcal{B})})\}$.

Proof Clearly, $f^{-1}(\mathcal{A}), f^{-1}(\mathcal{B}) \in \mathcal{NMG}(\mathcal{X})$. ıt follows easily that $f^{-1}(\mathcal{A}) \subseteq f^{-1}(\mathcal{B})$. Now

$$
\begin{aligned}
((\mu^{-1})^i_{f(\mathcal{A})})(xyx^{-1}) &= f(\mathcal{A})\left((\mu^i_{f(\mathcal{A})})(xyx^{-1})\right) \\
&= f(\mathcal{A})\left((\mu^i_{f(\mathcal{A})})(x)(\mu^i_{f(\mathcal{A})})(y)(\mu^i_{f(\mathcal{A})})(x^{-1})\right) \\
&\geq f(\mathcal{A})(\mu^i_{f(\mathcal{A})})(y) \wedge f(\mathcal{B})(\mu^i_{f(\mathcal{A})})(x) \\
&= ((\mu^{-1})^i_{f(\mathcal{A})})(y) \wedge ((\mu^{-1})^i_{f(\mathcal{B})})(x)
\end{aligned}
$$

$$
\begin{aligned}
((v^{-1})^i_{f(\mathcal{A})})(xyx^{-1}) &= f(\mathcal{A})\left((v^i_{f(\mathcal{A})})(xyx^{-1})\right) \\
&= f(\mathcal{A})\left((v^i_{f(\mathcal{A})})(x)(v^i_{f(\mathcal{A})})(y)(v^i_{f(\mathcal{A})})(x^{-1})\right) \\
&\leq f(\mathcal{A})(v^i_{f(\mathcal{A})})(y) \vee f(\mathcal{B})(v^i_{f(\mathcal{A})})(x)
\end{aligned}
$$

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$$= \left((v^{-1})^{i}_{f(\mathcal{A})}\right)(y) \vee \left((v^{-1})^{i}_{f(\mathcal{B})}\right)(x)$$

$$\begin{aligned}
\left((w^{-1})^{i}_{f(\mathcal{A})}\right)(xyx^{-1}) &= f(\mathcal{A})\left(\left(w^{i}_{f(\mathcal{A})}\right)(xyx^{-1})\right)\\
&= f(\mathcal{A})\left(\left(w^{i}_{f(\mathcal{A})}\right)(x)\left(w^{i}_{f(\mathcal{A})}\right)(y)\left(w^{i}_{f(\mathcal{A})}\right)(x^{-1})\right)\\
&\leq f(\mathcal{A})\left(w^{i}_{f(\mathcal{A})}\right)(y) \vee f(\mathcal{B})\left(w^{i}_{f(\mathcal{A})}\right)(x)\\
&= \left((w^{-1})^{i}_{f(\mathcal{A})}\right)(y) \vee \left((w^{-1})^{i}_{f(\mathcal{B})}\right)(x)
\end{aligned}$$

$\forall\, x, y \in \mathcal{X}$. Hence $f^{-1}(\mathcal{A})$ is a normal neutrosophic multisubgroup of $f^{-1}(\mathcal{B})$.

Definition 3.6 Let $\mathcal{X}$ and $\mathcal{Y}$ be groups and $\mathcal{A} \in \mathcal{NMG}(\mathcal{X})$ and $\mathcal{B} \in \mathcal{NMG}(\mathcal{Y})$.

A homomorphism $f = \{(\mu^{i}_{f}), (v^{i}_{f}), (w^{i}_{f})\}$ of $\mathcal{X}$ onto $\mathcal{Y}$ is called a weak neutrosophic homomorphism of $\mathcal{A}$ into $\mathcal{B}$ if $f(\mathcal{A}) \subseteq \mathcal{B}$. If f is a weak neutrosophic homomorphism of $\mathcal{A}$ into $\mathcal{B}$ then we say that $\mathcal{A}$ is a weakly neutrosophic homomorphic to $\mathcal{B}$ and we write $\mathcal{A} \sim \mathcal{B}$.

An isomorphism $f = \{(\mu^{i}_{f}), (v^{i}_{f}), (w^{i}_{f})\}$ of $\mathcal{X}$ onto $\mathcal{Y}$ is called a weak neutrosophic isomorphism of $\mathcal{A}$ into $\mathcal{B}$ if $f(\mathcal{A}) \subseteq \mathcal{B}$. If f is a weak neutrosophic isomorphism of $\mathcal{A}$ into $\mathcal{B}$ then we say that $\mathcal{A}$ is a weakly neutrosophic isomorphism to $\mathcal{B}$ and we write $\mathcal{A} \simeq \mathcal{B}$.

A homomorphism $f = \{(\mu^{i}_{f}), (v^{i}_{f}), (w^{i}_{f})\}$ of $\mathcal{X}$ onto $\mathcal{Y}$ is called a neutrosophic homomorphism of $\mathcal{A}$ into $\mathcal{B}$ if $f(\mathcal{A}) = \mathcal{B}$. If f is a neutrosophic homomorphism of $\mathcal{A}$ into $\mathcal{B}$ then we say that $\mathcal{A}$ is a neutrosophic homomorphic to $\mathcal{B}$ and we write $\mathcal{A} \approx \mathcal{B}$.

An isomorphism $f = \{(\mu^{i}_{f}), (v^{i}_{f}), (w^{i}_{f})\}$ of $\mathcal{X}$ onto $\mathcal{Y}$ is called a neutrosophic isomorphism of $\mathcal{A}$ into $\mathcal{B}$ if $f(\mathcal{A}) = \mathcal{B}$. If f is a neutrosophic isomorphism of $\mathcal{A}$ into $\mathcal{B}$ then we say that $\mathcal{A}$ is a neutrosophic isomorphism to $\mathcal{B}$ and we write $\mathcal{A} \cong \mathcal{B}$.

Example 3.7 Let $\mathcal{X} = \{i, -i,\ 1, -1\}$ and $\mathcal{Y} = \{1, -1\}$ be groups. Then, there exists a homomorphism f be a homomorphism from $\mathcal{X}$ into $\mathcal{Y}$ defined by $f(x) = x^2\ \forall x \in \mathcal{X}$. Assume

$$\begin{aligned}
\mathcal{A} = &\{\langle i, (0.6,0.4, \ldots,0.8), (0.6,0.7, \ldots,0.8), (0.3,0.5, \ldots,0.8)\rangle,\\
&\langle -i, (0.1,0.4, \ldots,0.5), (0.2,0.2, \ldots,0.7), (0.1,0.3, \ldots,0.9)\rangle\\
&\langle 1, (0.4,0.5, \ldots,0.6), (0.1,0.1, \ldots,0.5), (0.2,0.5, \ldots,0.6)\rangle\\
&\langle -1, (0.1,0.3, \ldots,0.7), (0.3,0.3, \ldots,0.9), (0.2,0.4, \ldots,0.8)\rangle\}
\end{aligned}$$

and

$$\begin{aligned}
\mathcal{B} = &\{\langle 1, (0.4,0.5, \ldots,0.6), (0.1,0.1, \ldots,0.5), (0.1,0.3, \ldots,0.6)\rangle,\\
&\langle -1, (0.1,0.3, \ldots,0.7), (0.3,0.3, \ldots,0.9), (0.2,0.4, \ldots,0.8)\rangle\}
\end{aligned}$$

are neutrosphic multigroups of $\mathcal{X}$ and $\mathcal{Y}$, respectively. Then,

$$\begin{aligned}
f(\mathcal{A}) = &\{\langle 1, (0.1,0.4, \ldots,0.5), (0.2,0.2, \ldots,0.7), (0.2,0.5, \ldots,0.9)\rangle,\\
&\langle -1, (0.1,0.3, \ldots,0.7), (0.6,0.7, \ldots,0.9), (0.3,0.5, \ldots,0.8)\rangle\}
\end{aligned}$$

and

$$\begin{aligned}
f^{-1}(\mathcal{B}) = &\{\langle i, (0.1,0.3, \ldots,0.7), (0.6,0.7, \ldots,0.9), (0.3,0.5, \ldots,0.8)\rangle,\\
&\langle -i, (0.1,0.3, \ldots,0.5), (0.3,0.3, \ldots,0.9), (0.2,0.4, \ldots,0.9)\rangle
\end{aligned}$$

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$$\langle 1, (0.4,0.5, \dots ,0.6), (0.1,0.1, \dots ,0.5), (0.2,0.5, \dots ,0.6)\rangle$$
$$\langle -1, (0.4,0.5, \dots ,0.6), (0.1,0.1, \dots ,0.5), (0.1,0.3, \dots ,0.6)\rangle\}$$

Clearly, $f(\mathcal{A}) \subseteq \mathcal{B}$, that is $\mathcal{A} \subseteq f^{-1}(\mathcal{B})$.

Definition 3.8 Let $f: \mathcal{X} \to \mathcal{Y}$ be a homomorphism of groups. Suppose that $\mathcal{A}$ and $\mathcal{B}$ are neutrosophic multigroups of $\mathcal{X}$ and $\mathcal{Y}$, respectively. Then, $\mathcal{A}$ homomorphic to $\mathcal{B}$. The kernel of the homomorphism from $\mathcal{A}$ to $\mathcal{B}$ is defined by

$$kerf = \{x \in \mathcal{X}: (\mu^i_{\mathcal{A}})(x) = (\mu^i_{\mathcal{B}})(e'), (v^i_{\mathcal{A}})(x) = (v^i_{\mathcal{B}})(e'), (w^i_{\mathcal{A}})(x) = (w^i_{\mathcal{B}})(e'), f(e) = e'\} \quad (6)$$

where e and e' are the identities of $\mathcal{X}$ and $\mathcal{Y}$, respectively.

Proposition 3.9 If $f: \mathcal{X} \to \mathcal{Y}$ be a homomorphism and $\mathcal{A} \in \mathcal{NMG}(\mathcal{X})$. Then,

$$f(\mathcal{A}^{-1}) = (f(\mathcal{A}))^{-1},$$

$$f^{-1}(f(\mathcal{A}^{-1})) = f\left((f(\mathcal{A}))^{-1}\right).$$

Proof Let $y \in \mathcal{Y}$. Then we have

$$\begin{aligned} \mu^i_{f(\mathcal{A}^{-1})}(y) &= \mu^i_{(\mathcal{A}^{-1})}(f^{-1}(y)) \\ &= \mu^i_{(\mathcal{A})}(f^{-1}(y)) \\ &= \mu^i_{f(\mathcal{A})}(y) \\ &= \mu^i_{(f(\mathcal{A}))^{-1}}(y) \end{aligned}$$

$$\begin{aligned} v^i_{f(\mathcal{A}^{-1})}(y) &= v^i_{(\mathcal{A}^{-1})}(f^{-1}(y)) \\ &= v^i_{(\mathcal{A})}(f^{-1}(y)) \\ &= v^i_{f(\mathcal{A})}(y) \\ &= v^i_{(f(\mathcal{A}))^{-1}}(y) \end{aligned}$$

$$\begin{aligned} w^i_{f(\mathcal{A}^{-1})}(y) &= w^i_{(\mathcal{A}^{-1})}(f^{-1}(y)) \\ &= w^i_{(\mathcal{A})}(f^{-1}(y)) \\ &= w^i_{f(\mathcal{A})}(y) \\ &= w^i_{(f(\mathcal{A}))^{-1}}(y) \end{aligned}$$

$\forall y \in \mathcal{Y}$. Hence $f(\mathcal{A}^{-1}) = (f(\mathcal{A}))^{-1}$.

ii. It can be done similar to (i).

Proposition 3.10 Let $\mathcal{X}$ and $\mathcal{Y}$ be groups such that $f: \mathcal{X} \to \mathcal{Y}$ is an isomorphic mapping. If $\mathcal{A} \in \mathcal{NMG}(\mathcal{X})$ and $\mathcal{B} \in \mathcal{NMG}(\mathcal{Y})$, respectively, then

$$(f^{-1}(\mathcal{B}))^{-1} = f^{-1}(\mathcal{B}^{-1}),$$

$$f^{-1}(f(\mathcal{A})) = f^{-1}\left(f(f^{-1}(\mathcal{B}))\right).$$

Proof Recall that, if f is an isomorphism, then $f(x) = y \ \forall x \in \mathcal{X}, \forall y \in \mathcal{Y}$.

Consequently, $f(\mathcal{A}) = \mathcal{B}$.

$$\begin{aligned} \mu^i_{(f^{-1}(\mathcal{B}))^{-1}}(x) &= \mu^i_{(f^{-1}(\mathcal{B}))}(x^{-1}) \\ &= \mu^i_{f^{-1}(\mathcal{B})}(x) \\ &= \mu^i_{\mathcal{B}}(f(x)) \end{aligned}$$

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i.

$$= \mu^i_{(\mathcal{B})^{-1}}\left(\left(f(x)\right)^{-1}\right)$$
$$= \mu^i_{f^{-1}(\mathcal{B}^{-1})}(x)$$

$$v^i_{\left(f^{-1}(\mathcal{B})\right)^{-1}}(x) = v^i_{\left(f^{-1}(\mathcal{B})\right)}(x^{-1})$$
$$= v^i_{f^{-1}(\mathcal{B})}(x)$$
$$= v^i_{\mathcal{B}}\left(f(x)\right)$$
$$= v^i_{(\mathcal{B})^{-1}}\left(\left(f(x)\right)^{-1}\right)$$
$$= v^i_{f^{-1}(\mathcal{B}^{-1})}(x)$$

$$w^i_{\left(f^{-1}(\mathcal{B})\right)^{-1}}(x) = w^i_{\left(f^{-1}(\mathcal{B})\right)}(x^{-1})$$
$$= w^i_{f^{-1}(\mathcal{B})}(x)$$
$$= w^i_{\mathcal{B}}\left(f(x)\right)$$
$$= w^i_{(\mathcal{B})^{-1}}\left(\left(f(x)\right)^{-1}\right)$$
$$= w^i_{f^{-1}(\mathcal{B}^{-1})}(x)$$

hence, $\left(f^{-1}(\mathcal{B})\right)^{-1} = f^{-1}(\mathcal{B}^{-1})$.

ii. It can be done similar to (i).

Theorem 3.11 Let $\mathcal{X}$ be group and $f: \mathcal{X} \to \mathcal{X}$ be an automorphism. If $\mathcal{A} \in \mathcal{NMG}(\mathcal{X})$, then, $f(\mathcal{A}) = \mathcal{A}$ if and only if $f^{-1}(\mathcal{A}) = \mathcal{A}$. Consequently, $f(\mathcal{A}) = f^{-1}(\mathcal{A})$.

Proof Let $f(x) = x, \forall x \in \mathcal{X}$ since f is an automorphism. Assume that $f(\mathcal{A}) = \mathcal{A}$, we have

$$\mu^i_{f(\mathcal{A})}(x) = \mu^i_{\mathcal{A}}\left(f(x)\right)$$
$$= \mu^i_{\mathcal{A}}(x)$$
$$= \mu^i_{\mathcal{A}}\left(f^{-1}(x)\right)$$
$$= \mu^i_{f^{-1}(\mathcal{A})}(x)$$

$$v^i_{f(\mathcal{A})}(x) = v^i_{\mathcal{A}}\left(f(x)\right)$$
$$= v^i_{\mathcal{A}}(x)$$
$$= v^i_{\mathcal{A}}\left(f^{-1}(x)\right)$$
$$= v^i_{f^{-1}(\mathcal{A})}(x)$$

implies that

$$w^i_{f(\mathcal{A})}(x) = w^i_{\mathcal{A}}\left(f(x)\right)$$
$$= w^i_{\mathcal{A}}(x)$$
$$= w^i_{\mathcal{A}}\left(f^{-1}(x)\right)$$
$$= w^i_{f^{-1}(\mathcal{A})}(x),$$

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$f^{-1}(\mathcal{A}) = \mathcal{A}$. Conversely, let $f^{-1}(\mathcal{A}) = \mathcal{A}$, we get

Hence, $f(\mathcal{A}) =$

$$\mu^i_{f^{-1}(\mathcal{A})}(x) = \mu^i_{\mathcal{A}}(f(x)) = \mu^i_{\mathcal{A}}(x) = \mu^i_{\mathcal{A}}(f^{-1}(x)) = \mu^i_{f(\mathcal{A})}(x)$$

$$v^i_{f^{-1}(\mathcal{A})}(x) = v^i_{\mathcal{A}}(f(x)) = v^i_{\mathcal{A}}(x) = v^i_{\mathcal{A}}(f^{-1}(x)) = v^i_{f(\mathcal{A})}(x)$$

$$w^i_{f^{-1}(\mathcal{A})}(x) = w^i_{\mathcal{A}}(f(x)) = w^i_{\mathcal{A}}(x) = w^i_{\mathcal{A}}(f^{-1}(x)) = w^i_{f(\mathcal{A})}(x).$$

$\mathcal{A}$.

Therefore $f(\mathcal{A}) = \mathcal{A} \Leftrightarrow f^{-1}(\mathcal{A}) = \mathcal{A}$.

Theorem 3.12 Let $\mathcal{X}$ and $\mathcal{Y}$ be groups and $f: \mathcal{X} \to \mathcal{Y}$ be an isomorphism. Then, $\mathcal{A} \in \mathcal{NMG}(\mathcal{X})$ if and only if $f(\mathcal{A}) \in \mathcal{NMG}(\mathcal{Y})$.

Proof Assume $\mathcal{A} \in \mathcal{NMG}(\mathcal{X})$. Let $x, y \in \mathcal{Y}$, then, $\exists\, f(a) = x$ and $f(\mathfrak{b}) = y$ since f is an isomorphism $\forall\, a, \mathfrak{b} \in \mathcal{X}$. We know that

$$\mu^i_{\mathcal{B}}(x) = \mu^i_{\mathcal{A}}(f^{-1}(x)) = \bigvee_{a \in f^{-1}(x)} \mu^i_{\mathcal{B}}(a)$$

$$v^i_{\mathcal{B}}(x) = v^i_{\mathcal{A}}(f^{-1}(x)) = \bigwedge_{a \in f^{-1}(x)} v^i_{\mathcal{B}}(a)$$

$$w^i_{\mathcal{B}}(x) = w^i_{\mathcal{A}}(f^{-1}(x)) = \bigwedge_{a \in f^{-1}(x)} w^i_{\mathcal{B}}(a)$$

and

$$\mu^i_{\mathcal{B}}(y) = \mu^i_{\mathcal{A}}(f^{-1}(y)) = \bigvee_{\mathfrak{b} \in f^{-1}(y)} \mu^i_{\mathcal{A}}(\mathfrak{b})$$

$$v^i_{\mathcal{B}}(y) = v^i_{\mathcal{A}}(f^{-1}(y)) = \bigwedge_{\mathfrak{b} \in f^{-1}(y)} v^i_{\mathcal{A}}(\mathfrak{b})$$

$$w^i_{\mathcal{B}}(y) = w^i_{\mathcal{A}}(f^{-1}(y)) = \bigwedge_{\mathfrak{b} \in f^{-1}(y)} w^i_{\mathcal{A}}(\mathfrak{b})$$

Clearly, $a \in f^{-1}(x) \neq \emptyset$ and $\mathfrak{b} \in f^{-1}(y) \neq \emptyset$. For $a \in f^{-1}(x)$ and $\mathfrak{b} \in f^{-1}(y) \Rightarrow y = f(\mathfrak{b})$ and $x = f(a)$.

Thus,

$$f(a\mathfrak{b}^{-1}) = f(a)f(\mathfrak{b}^{-1}) = x.y^{-1}.$$

Let

$c = a\mathfrak{b}^{-1} \Rightarrow c \in f^{-1}(x.y^{-1})$.

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Now,

$$\begin{aligned}\mu^i_{\mathcal{B}}(x.y^{-1}) &= \bigvee_{c\in f^{-1}(x.y^{-1})} \mu^i_{\mathcal{A}}(c)\\ &= \mu^i_{\mathcal{A}}(a\mathfrak{b}^{-1})\\ &\geq \mu^i_{\mathcal{A}}(a)\wedge\mu^i_{\mathcal{A}}(\mathfrak{b})\\ &= \mu^i_{f^{-1}(\mathcal{B})}(a)\wedge\mu^i_{f^{-1}(\mathcal{B})}(\mathfrak{b})\\ &= \mu^i_{\mathcal{B}}(f(a))\wedge\mu^i_{\mathcal{B}}(f(\mathfrak{b}))\\ &= \mu^i_{\mathcal{B}}(x)\wedge\mu^i_{\mathcal{B}}(y), \qquad \forall x,y\in\mathcal{Y}\end{aligned}$$

$$\begin{aligned}v^i_{\mathcal{B}}(x.y^{-1}) &= \bigwedge_{c\in f^{-1}(x.y^{-1})} v^i_{\mathcal{A}}(c)\\ &= v^i_{\mathcal{A}}(a\mathfrak{b}^{-1})\\ &\leq v^i_{\mathcal{A}}(a)\vee v^i_{\mathcal{A}}(\mathfrak{b})\\ &= v^i_{f^{-1}(\mathcal{B})}(a)\vee v^i_{f^{-1}(\mathcal{B})}(\mathfrak{b})\\ &= v^i_{\mathcal{B}}(f(a))\vee v^i_{\mathcal{B}}(f(\mathfrak{b}))\\ &= v^i_{\mathcal{B}}(x)\vee v^i_{\mathcal{B}}(y), \qquad \forall x,y\in\mathcal{Y}\end{aligned}$$

$$\begin{aligned}w^i_{\mathcal{B}}(x.y^{-1}) &= \bigwedge_{c\in f^{-1}(x.y^{-1})} w^i_{\mathcal{A}}(c)\\ &= w^i_{\mathcal{A}}(a\mathfrak{b}^{-1})\\ &\leq w^i_{\mathcal{A}}(a)\vee w^i_{\mathcal{A}}(\mathfrak{b})\\ &= w^i_{f^{-1}(\mathcal{B})}(a)\vee w^i_{f^{-1}(\mathcal{B})}(\mathfrak{b})\\ &= w^i_{\mathcal{B}}(f(a))\vee w^i_{\mathcal{B}}(f(\mathfrak{b}))\\ &= w^i_{\mathcal{B}}(x)\vee w^i_{\mathcal{B}}(y),\ \forall x,y\in\mathcal{Y}.\end{aligned}$$

Hence, $f(\mathcal{A})\in \mathcal{NMG}(\mathcal{Y})$.

Conversely, let $\forall\, a,\mathfrak{b}\in\mathcal{X}$ and suppose $f(\mathcal{A})\in \mathcal{NMG}(\mathcal{Y})$. Then,

$$\begin{aligned}\mu^i_{\mathcal{A}}(a\mathfrak{b}^{-1}) &= \mu^i_{f^{-1}(\mathcal{B})}(a\mathfrak{b}^{-1})\\ &= \mu^i_{\mathcal{B}}(f(a\mathfrak{b}^{-1}))\\ &= \mu^i_{\mathcal{B}}(f(a)f(\mathfrak{b}^{-1}))\\ &= \mu^i_{\mathcal{B}}\left(f(a)(f(\mathfrak{b}))^{-1}\right)\\ &\geq \mu^i_{\mathcal{B}}(f(a))\wedge\mu^i_{\mathcal{B}}(f(\mathfrak{b}))\\ &= \mu^i_{f^{-1}(\mathcal{B})}(a)\wedge\mu^i_{f^{-1}(\mathcal{B})}(\mathfrak{b})\end{aligned}$$

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$$= \mu^i_{\mathcal{A}}(a) \wedge \mu^i_{\mathcal{A}}(\mathfrak{b})$$

$$\begin{aligned} v^i_{\mathcal{A}}(a\mathfrak{b}^{-1}) &= v^i_{f^{-1}(\mathcal{B})}(a\mathfrak{b}^{-1}) \\ &= v^i_{\mathcal{B}}(f(a\mathfrak{b}^{-1})) \\ &= v^i_{\mathcal{B}}(f(a)f(\mathfrak{b}^{-1})) \\ &= v^i_{\mathcal{B}}\left(f(a)\left(f(\mathfrak{b})\right)^{-1}\right) \\ &\leq v^i_{\mathcal{B}}(f(a)) \vee v^i_{\mathcal{B}}(f(\mathfrak{b})) \\ &= v^i_{f^{-1}(\mathcal{B})}(a) \vee v^i_{f^{-1}(\mathcal{B})}(\mathfrak{b}) \\ &= v^i_{\mathcal{A}}(a) \vee v^i_{\mathcal{A}}(\mathfrak{b}) \end{aligned}$$

$$\begin{aligned} w^i_{\mathcal{A}}(a\mathfrak{b}^{-1}) &= w^i_{f^{-1}(\mathcal{B})}(a\mathfrak{b}^{-1}) \\ &= w^i_{\mathcal{B}}(f(a\mathfrak{b}^{-1})) \\ &= w^i_{\mathcal{B}}(f(a)f(\mathfrak{b}^{-1})) \\ &= w^i_{\mathcal{B}}\left(f(a)\left(f(\mathfrak{b})\right)^{-1}\right) \\ &\leq w^i_{\mathcal{B}}(f(a)) \vee w^i_{\mathcal{B}}(f(\mathfrak{b})) \\ &= w^i_{f^{-1}(\mathcal{B})}(a) \vee w^i_{f^{-1}(\mathcal{B})}(\mathfrak{b}) \\ &= w^i_{\mathcal{A}}(a) \vee w^i_{\mathcal{A}}(\mathfrak{b}), \ \forall\, a, \mathfrak{b} \in \mathcal{X} \end{aligned}$$

Hence, $\mathcal{A} \in \mathcal{NMG}(\mathcal{X})$.

4. Conclusions

The aim of this paper was to highlight the function between neutrosophic multisets and algebraic structures from other a point of view. It is well known that the concept of neutrosophic multiset is well established in dealing with many real life problems. So, the algebraic structure defined concerning them in this paper would help to approach these problems with a different perspective. The benefit of the paper is the in order to study effectively an object with a given algebraic structure, it is necessary to study as well the functions that preserve the given algebraic structure found between algebraic structures and neutrosophic multisets by introducing neutrosophic multigroup homomorphism of neutrosophic multigroup.

In this paper, we have defined the notion of neutrosophic multigroup homomorphism and some properties were developed. We have shown that, homomorphism kerlf, automorphism, homomorphic image and homomorphic preimage of neutrosophic multigroup. The various some fundamental operations, definitions and theorems related to neutrosophic multigroup homomorphism have been discussed. The results in this paper can be considered as a generalization of the results known for homomorphism of groups. Moreover, our results are considered as a generalization for multigroup homomorphism of fuzzy multigroup. This is because every group is a neutrosophic multigroup. This concept could be extended to analogous of isomorphism theorems (the homomorphism of cuts of neutrosophic multigroup) in neutrosophic multigroup setting. The foundations which we made through this paper can be used to get an insight into the higher order structures of neutrosophic group theory. For further research one can handle cyclic(respectively, symmetric, Abelian) neutrosophic multigroup structure and some other algebraic structures such as ideal, ring, field etc. as well the neutrosophic topological structures. We hope that our work would help advance the researchers interested in these topics.

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