# Neutrosophic Differential Equations That Translate Into Linear

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**Abstract**

In this paper, the Bernoulli's neutrosophic differential equations by a neutrosophic thick function are introduced.The main objective is defining a neutrosophic differential equations that translate into linear based on the thick function and finding solutions for this equation. Enough examples are provided to illustrate each idea.

**Keywords:** Neutrosophic differential equations, the Bernoulli's neutrosophic differential equations, neutrosophic thick function.

## 1. Introduction

As an alternative to the existing logics, Smarandache proposed the Neutrosophic Logic to represent a mathematical model of uncertainty, vagueness, ambiguity, imprecision, undefined, unknown, incompleteness, inconsistency, redundancy, contradiction, where the concept of neutrosophy is a new branch of philosophy introduced by Smarandache [4][8][18]. He presented the definition of the standard form of neutrosophic real number and conditions for the division of two neutrosophic real numbers to exist, he defined the standard form of neutrosophic complex number, and found root index $n \geq 2$ of a neutrosophic real and complex number [3][5], studying the concept of the Neutrosophic probability [4][6], the Neutrosophic statistics [5][7], and professor Smarandache entered the concept of preliminary calculus of the differential and integral calculus, where he introduced for the first time the notions of neutrosophic mereo-limit, mereo-continuity, mereoderivative, and mereo-integral [1][8]. Y.Alhasan presented the definition of the concept of neutrosophic complex numbers and its properties including the conjugate of neutrosophic complex number, division of neutrosophic complex numbers, the inverted neutrosophic complex number and the absolute value of a neutrosophic complex number and Theories related to the conjugate of neutrosophic complex numbers, the product of a neutrosophic complex number by its conjugate equals the absolute value of number[2].Madeleine Al- Taha presented results on single valued neutrosophic (weak) polygroups [10]. Edalatpanah proposed a new direct algorithm to solve the neutrosophic linear programming where the variables and right hand side represented with triangular neutrosophic numbers [11]. Chakraborty used pentagonal neutrosophic number in networking problem, and Shortest Path Problem [12][13]. Neutrophology logic provided brilliant mathematical theories as a generalization of fuzzy and crisp logic, such as a neutrosophic set theory, it followed the introduction of the neutrosophic set concepts in [14][15], Wadei Al-omeri introduce the concept of neutrosophic crisp sets, investigate the properties of continuous, open and closed maps in the neutrosophic crisp topological spaces[16]. J. Ye has studied

DOI: 10.5281/zenodo.4928396
Received:January 21, 2021, Accepted: May 11, 2021

neutrosophic number linear programming method and its application under neutrosophic number environments[17]. M. Alaswad presented A Study of the Integration of Neutrosophic Thick Function [19].

Paper consists of 4 sections. In 1th section, provides an introduction, in which neutrosophic science review has given. In 2th section, some definitions and examples of neutrosophic function and neutrosophic integration thick function are discussed. the 3th section frames the neutrosophic differential equations that translate into linear, in addition the bernoulli's neutrosophic differential equations . In 4th section, a conclusion to the paper is given.

## 2. Preliminaries

### 2.1 General neutrosophic function [18]

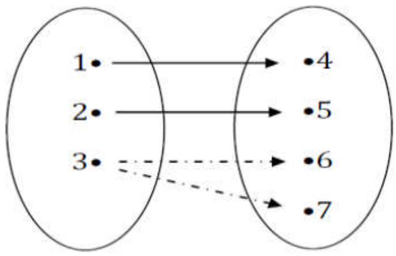
A General Neutrosophic Function is a neutrosophic relation where the vertical line test (or the vertical subset-line test) does not work. But, in this case, the general neutrosophic function coincides with the neutrosophic relation.

### 2.1.2 Neutrosophic (subset or crisp) function [18]

A neutrosophic (subset or crisp) function in general is a function that has some indeterminacy.

For example: $f:\{1,2,3\} \to \{4,5,6,7\}$ $(28)$ $f(1) = 4, f(2) = 5, but\ f(3) = 6\ or$ 7 [we are not sure].

If we consider a neutrosophic diagram representation of this neutrosophic function, we have graph 2.1:




Graph 2.1

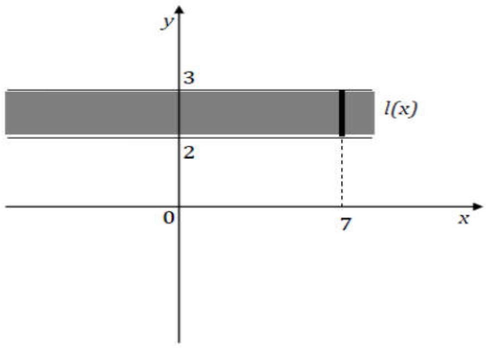
### 2.1.3 A constant neutrosophic function (or thick function): [18]

$l: R \to P(R)$ ; where $P(R)$ is the set of all subsets of $R$

$l(x) = [2,3]\ for\ any\ x \in \mathbb{R}$,

For example: $l(7)$ is the vertical segment of line [2, 3]. Shown in graph 2.2:

DOI: 10.5281/zenodo.4928396




Graph 2.2

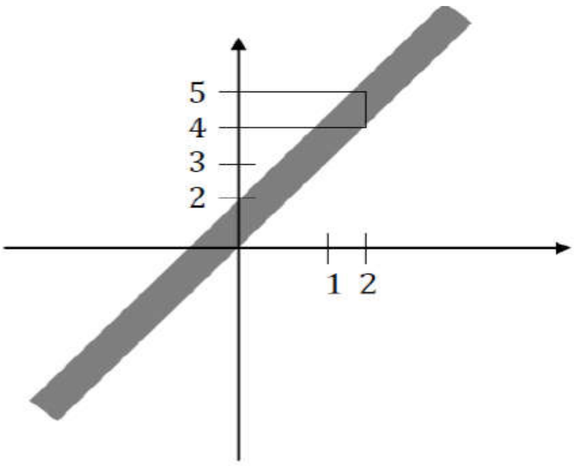
### 2.1.4 A non-constant neutrosophic thick function [18]

$k: R \to P(R)$

For example:

$k(x) = [2x, 2x + 1] \Rightarrow k(2) = [2(2), 2(2) + 1] = [4, 5].$

whose graph is (shown in graph 2.3):



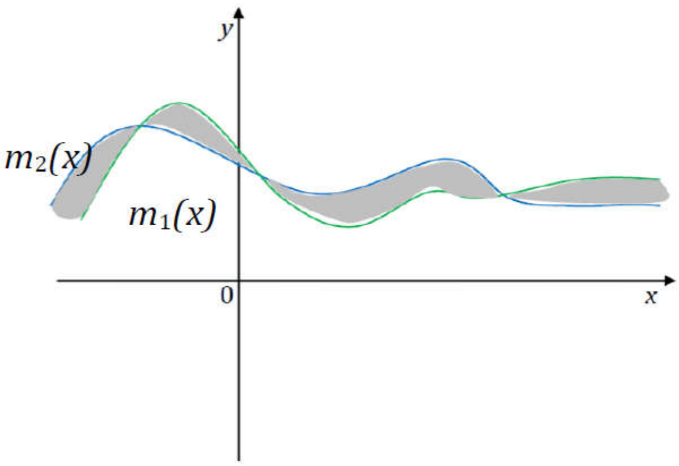

Graph 2.3

### 2.1.5 The general state of neutrosophic thick function [18]

In general, we may define a neutrosophic thick function as:

$$m: R \to P(R);\ m(x) = [m_1(x), m_2(x)]$$

shown in graph 2.4:




Graph 2.4

and, of course, instead of brackets we may have an open interval $\left(m_1(x), m_2(x)\right)$ , or semi-open/semi-close inter-vals $(m_1(x), m_2(x)]$, or $[m_1(x), m_2(x)]$.

## 2.2 A neutrosophic integration [19]

As in Euclidean integration, integration is the opposite of differentiation.

In other words, the anti-thesis of the derivative of the neutrosophic function $f(x)$is also a neutrosophic function $F(x)$.

That the definition of neutrosophic integral is:

$$F(x) = \int f(x, I)\, dx$$

Where I represents the indeterminacy and constant integration is $a + bI$.

### 2.2.1 Neutrosophic integration thick function [19]

**Definition 2.2.1**

Let $m(x) = [m_1(x), m_2(x)]$ be a neutrosophic thick function. Then we define the integration of this

function as:

$$\int m(x)\, dx = \int [m_1(x), m_2(x)] dx = \left[\int m_1(x) dx + C_1 , \int m_2(x) dx + C_2\right] = [A, B]$$

### 2.2.2 Neutrosophic linear differential equation [19]

**Definition 2.2.2**

We define the Neutrosophic identical linear differential equation by a neutrosophic thick function form:

$$\acute{y} + m(x) y = 0 \quad ; m(x) = [m_1(x), m_2(x)]$$

DOI: 10.5281/zenodo.4928396

### 2.2.3 Neutrosophic non-homogeneous linear differential equation [19]

**Definition 2.2.3**

We define the Neutrosophic non- homogeneous linear differential equation by a neutrosophic thick function which takes one of the following forms:

$$\acute{y} + [m_1(x), m_2(x)]y = q(x)$$

$$\acute{y} + p(x)y = [f_1(x), f_2(x)]$$

$$\acute{y} + [m_1(x), m_2(x)]y = [f_1(x), f_2(x)]$$

**3. Neutrosophic differential equations that translate into linear**

**3.1 The Bernoulli's neutrosophic differential equations**

**Definition 3.1.1:**

The Bernoulli's neutrosophic differential equations by a neutrosophic thick function takes the one of the following forms:

$$\acute{y} + [m_1(x), m_2(x)]y = q(x)\, y^n \qquad (1)$$

$$\acute{y} + p(x)y = [f_1(x), f_2(x)]\, y^n \qquad (2)$$

$$\acute{y} + [m_1(x), m_2(x)]y = [f_1(x), f_2(x)]\, y^n \qquad (3)$$

Where $n \neq 0, n \neq 1$ .

To solve this equations we follow the following steps on the equation (1), likewise, we solve equations (2) and (3):

1) we'll first divide the differential equation by $y^n$ to get:

$$\frac{\acute{y}}{y^n} + \frac{[m_1(x), m_2(x)]y}{y^n} = \frac{q(x)\, y^n}{y^n}$$

$$y^{-n}\, \acute{y} + [m_1(x), m_2(x)]\, y^{1-n} = q(x) \qquad (*)$$

2) Assume that: $w = y^{1-n} \quad \Rightarrow \quad \acute{w} = (1-n)\, y^{-n}$

$$\Rightarrow\ y^{-n} = \frac{1}{1-n}\acute{w}$$

by substitution into equation (*), we get:

$$\frac{1}{1-n}\acute{w} + [m_1(x), m_2(x)]w = q(x)$$

$$\Rightarrow\ \acute{w} + (1-n)[m_1(x), m_2(x)]w = (1-n)q(x) \qquad (**)$$

3) The complement factor of the equation (**) is:

DOI: 10.5281/zenodo.4928396

$$\mu(x) = e^{(1-n)[\int m_1(x)dx, \int m_2(x)dx]} = \left[e^{(1-n)\int m_1(x)dx}, e^{(1-n)\int m_2(x)dx}\right]$$

4) By multiplying equation (**) by the complement factor, we get:

$$\acute{w}\,\mu(x) + (1-n)[m_1(x), m_2(x)]\,\mu(x)\,w = (1-n)\,q(x)\,\mu(x)$$

$$\acute{w}\,\mu(x) + \left[(1-n)m_1(x)\,e^{(1-n)\int m_1(x)dx}, (1-n)\,m_2(x)e^{(1-n)\int m_2(x)dx}\right] w = (1-n)\,q(x)\,\mu(x)$$

$$\big(w\,\mu(x)\big)' = (1-n)\,q(x)\,\left[e^{(1-n)\int m_1(x)dx}, e^{(1-n)\int m_2(x)dx}\right]$$

$$\big(w\,\mu(x)\big)' = \left[(1-n)\,q(x)\,e^{(1-n)\int m_1(x)dx}, (1-n)\,q(x)e^{(1-n)\int m_2(x)dx}\right]$$

By integrating the tow side, we get:

$$w = \frac{1}{\mu(x)}\left(\left[\int (1-n)\,q(x)\,e^{(1-n)\int m_1(x)dx}\,dx, \int (1-n)\,q(x)e^{(1-n)\int m_2(x)dx}\,dx\right] + a + bI\right)$$

5) Back to the primary variable $y$, we get:

$$y^{1-n} = \frac{1}{\mu(x)}\left(\left[\int (1-n)\,q(x)\,e^{(1-n)\int m_1(x)dx}\,dx, \int (1-n)\,q(x)e^{(1-n)\int m_2(x)dx}\,dx\right] + a + bI\right)$$

$$\Rightarrow \quad y^{n-1} = \frac{\mu(x)}{\left[\int (1-n)\,q(x)\,e^{(1-n)\int m_1(x)dx}\,dx, \int (1-n)\,q(x)e^{(1-n)\int m_2(x)dx}\,dx\right] + a + bI}$$

**Example 3.1.1:**

Find the general solution of the following Bernoulli's neutrosophic differential equation:

$$\acute{y} - \left[2x, \frac{1}{x}\right] y = -x\,y^3$$

**Solution:**

$$\frac{\acute{y}}{y^3} - \left[2x, \frac{1}{x}\right]\frac{y}{y^3} = -x$$

$$y^{-3}\acute{y} - \left[2x, \frac{1}{x}\right] y^{-2} = -x \qquad (*)$$

$$w = y^{-2} \quad \Rightarrow \quad \acute{w} = -2\,y^{-3}\acute{y}$$

$$\Rightarrow \quad y^3\,\acute{y} = \frac{-1}{2}\,\acute{w}$$

By substitution in (*), we get:

$$\frac{-1}{2}\,\acute{w} - \left[2x, \frac{1}{x}\right] w = -x$$

DOI: 10.5281/zenodo.4928396

$$\acute{w} + 2\left[2x, \frac{1}{x}\right] w = 2x$$

$$\acute{w} + \left[4x, \frac{2}{x}\right] w = 2x \qquad (**)$$

the complement factor is:

$$\mu(x) = e^{\left[\int 4x\, dx, \int \frac{2}{x} dx\right]}$$

$$= \left[e^{\int 4x\, dx}, e^{\int \frac{2}{x} dx}\right] = \left[e^{2x^2}, x^2\right]$$

By multiplying equation (**) by the complement factor, we get:

$$\acute{w}\, \mu(x) + \left[4x, \frac{2}{x}\right] w\, \mu(x) = 2x\, \mu(x)$$

$$\acute{w}\, \mu(x) + \left[4x\, e^{2x^2}, 2x\right] w = 2x \left[e^{2x^2}, x^2\right]$$

$$\left(w\, \mu(x)\right)' = \left[2x\, e^{2x^2}, 2x^3\right]$$

By integrating the tow side, we get:

$$w\, \mu(x) = \int \left[2x\, e^{2x^2}, 2x^3\right] dx$$

$$w = \frac{1}{\mu(x)}\left(\left[\int 2x\, e^{2x^2} dx\, , \int 2x^3\, dx\right] + a + bI\right)$$

$$w = \frac{1}{\mu(x)}\left(\left[\frac{1}{2} e^{2x^2}, \frac{1}{2} x^4\right] + a + bI\right)$$

Back to the primary variable $y$, we get:

$$y^{-2} = \frac{1}{\mu(x)}\left(\left[\frac{1}{2} e^{2x^2}, \frac{1}{2} x^4\right] + a + bI\right)$$

$$y^2 = \frac{\mu(x)}{\left[\frac{1}{2} e^{2x^2}, \frac{1}{2} x^4\right] + a + bI}$$

$$y^2 = \frac{\left[e^{2x^2}, x^2\right]}{\left[\frac{1}{2} e^{2x^2}, \frac{1}{2} x^4\right] + a + bI}$$

**Example 3.1.2:**

Find the general solution of the following Bernoulli's neutrosophic differential equation:

DOI: 10.5281/zenodo.4928396

$$\acute{y} - Cotx\ y = [Sin(x), Cos(x)]\ y^2$$

**Solution:**

$$\frac{\acute{y}}{y^2} - \frac{Cotx\ y}{y^2} = [Sin(x), Cos(x)]$$

$$\acute{y}\ y^{-2} - y^{-1}Cotx = [Sin(x), Cos(x)] \qquad (*)$$

$$w = y^{-1} \quad \Rightarrow \quad \acute{w} = -\ y^{-2}\acute{y}$$

$$\Rightarrow \quad y^{-2}\ \acute{y} = -\ \acute{w}$$

By substitution in (*), we get:

$$-\ \acute{w} - w\ Cotx\ = [Sin(x), Cos(x)]$$

$$\acute{w} + w\ Cotx\ = [-Sin(x), -Cos(x)] \qquad (**)$$

the complement factor is:

$$\mu(x) = e^{\int\ Cotx\ dx}$$

$$= e^{\ln(Sinx)} = Sin(x)$$

By multiplying equation (**) by the complement factor, we get:

$$\acute{w}\mu(x) + w\ \mu(x)\ Cotx\ = \mu(x)[-Sin(x), -Cos(x)]$$

$$\acute{w}\mu(x) + w\ Sin(x)\ Cotx\ = Sin(x)[-Sin(x), -Cos(x)]$$

$$\acute{w}\mu(x) + w\ Cosx = [-Sin^2(x), -Sin(x)Cos(x)]$$

$$\big(w\ \mu(x)\big)' = [-Sin^2(x), -Sin(x)Cos(x)]$$

By integrating the tow side, we get:

$$w\ \mu(x) = \int [-Sin^2(x), -Sin(x)Cos(x)]dx$$

$$w = \frac{1}{\mu(x)}\left(\left[\int -Sin^2(x)dx\,, \int -Sin(x)Cos(x)\ dx\right] + a + bI\right)$$

$$w = \frac{1}{Sinx}\left(\left[\int -\left(\frac{1}{2} - \frac{1}{2}Cosx\right) dx\,, \int -\frac{1}{2}Sin(2x)\ dx\right] + a + bI\right)$$

$$w = \frac{1}{Sinx}\left(\left[-\frac{1}{2}x + \frac{1}{4}Sinx, \frac{1}{4}Cos2x\right] + a + bI\right)$$

Back to the primary variable $y$, we get:

DOI: 10.5281/zenodo.4928396

$$y^{-1} = \frac{1}{Sinx}\left(\left[-\frac{1}{2}x + \frac{1}{4}Sinx, \frac{1}{4}Cos2x\right] + a + bI\right)$$

$$y = \frac{Sinx}{\left[-\frac{1}{2}x + \frac{1}{4}Sinx, \frac{1}{4}Cos2x\right] + a + bI}$$

**Example 3.1.3:**

Find the general solution of the following Bernoulli's neutrosophic differential equation:

$$\acute{y} + \left[Secx\ Tanx, \frac{1}{x}\right] y = \left[e^{\frac{-1}{2}Secx}, x^3\right]\sqrt{y}$$

**Solution:**

$$\frac{\acute{y}}{\sqrt{y}} + \left[Secx\ Tanx, \frac{1}{x}\right]\frac{y}{\sqrt{y}} = \left[e^{\frac{-1}{2}Secx}, x^3\right]$$

$$y^{-1/2}\ \acute{y} + \left[Secx\ Tanx, \frac{1}{x}\right] y^{1/2} = \left[e^{\frac{-1}{2}Secx}, x^3\right] \qquad (*)$$

$$w = y^{1/2} \quad \Rightarrow \quad \acute{w} = \frac{1}{2}\ y^{-1/2}\acute{y}$$

$$\Rightarrow \quad y^{-1/2}\ \acute{y} = 2\ \acute{w}$$

By substitution in (*), we get:

$$2\ \acute{w} + \left[Secx\ Tanx, \frac{1}{x}\right] w = \left[e^{\frac{-1}{2}Secx}, x^3\right]$$

$$\acute{w} + \frac{1}{2}\left[Secx\ Tanx, \frac{1}{x}\right] w = \frac{1}{2}\left[e^{\frac{-1}{2}Secx}, x^3\right] \qquad (**)$$

the complement factor is:

$$\mu(x) = e^{\left[\int \frac{1}{2}Secx\ Tanx\ dx, \int \frac{1}{2x}dx\right]}$$

$$= \left[e^{\int \frac{1}{2}Secx\ Tanx\ dx}, e^{\int \frac{1}{2x}\ dx}\right] = \left[e^{\frac{1}{2}Secx}, \sqrt{x}\right]$$

By multiplying equation (**) by the complement factor, we get:

$$\acute{w}\ \mu(x) + \left[\frac{1}{2}Secx\ Tanx, \frac{1}{2x}\right]\mu(x)\ w = \mu(x)\left[\frac{1}{2}e^{\frac{-1}{2}Secx}, \frac{1}{2}x^3\right]$$

$$\acute{w}\ \mu(x) + \left[\frac{1}{2}Secx\ Tanx, \frac{1}{2x}\right]\left[e^{\frac{1}{2}Secx}, \sqrt{x}\right]\ w = \left[e^{\frac{1}{2}Secx}, \sqrt{x}\right]\left[\frac{1}{2}e^{\frac{-1}{2}Secx}, \frac{1}{2}x^3\right]$$

$$\acute{w}\ \mu(x) + \left[\frac{1}{2}Secx\ Tanx\ e^{\frac{1}{2}Secx}, \frac{1}{2\sqrt{x}}\right]\ w = \left[e^{\frac{1}{2}Secx}, \sqrt{x}\right]\left[\frac{1}{2}e^{\frac{-1}{2}Secx}, \frac{1}{2}x^3\right]$$

$$\acute{w}\,\mu(x) + \left[\frac{1}{2}Secx\,Tanx\,e^{\frac{1}{2}Secx}, \frac{1}{2\sqrt{x}}\right] w = \left[\frac{1}{2}, \frac{1}{2}x^{7/2}\right]$$

$$\left(w\,\mu(x)\right)' = \left[\frac{1}{2}, \frac{1}{2}x^{7/2}\right]$$

By integrating the tow side, we get:

$$w\,\mu(x) = \int \left[\frac{1}{2}, \frac{1}{2}x^{7/2}\right] dx$$

$$w = \frac{1}{\mu(x)}\left(\left[\int \frac{1}{2}dx\,, \int \frac{1}{2}x^{7/2}\,dx\right] + a + bI\right)$$

$$w = \frac{1}{\mu(x)}\left(\left[\frac{1}{2}x, \frac{1}{9}x^{9/2}\right] + a + bI\right)$$

Back to the primary variable $y$, we get:

$$y^{1/2} = \frac{1}{\mu(x)}\left(\left[\frac{1}{2}x, \frac{1}{9}x^{9/2}\right] + a + bI\right)$$

$$y^{1/2} = \frac{\mu(x)}{\left[\frac{1}{2}x, \frac{1}{9}x^{9/2}\right] + a + bI}$$

$$y^{1/2} = \frac{\left[e^{\frac{1}{2}Secx}, \sqrt{x}\right]}{\left[\frac{1}{2}x, \frac{1}{9}x^{9/2}\right] + a + bI}$$

**3.2 Neutrosophic differential equations by a neutrosophic thick function takes the one of the following forms**

$$\acute{f}(y)\frac{dy}{dx} + [m_1(x), m_2(x)]f(y) = q(x) \qquad (4)$$

$$\acute{f}(y)\frac{dy}{dx} + p(x)f(y) = [f_1(x), f_2(x)] \qquad (5)$$

$$\acute{f}(y)\frac{dy}{dx} + [m_1(x), m_2(x)]f(y) = [f_1(x), f_2(x)] \qquad (6)$$

To solve this equations we follow the following steps on the equation (4), likewise, we solve equations (5) and (6):

1) Assume that: $z = f(y) \quad \Rightarrow \quad \acute{z} = (\acute{f}(y)\frac{dy}{dx}$

by substitution into equation (4), we get:

$$\acute{z} + [m_1(x), m_2(x)]z = q(x) \qquad (*)$$

DOI: 10.5281/zenodo.4928396

2) The complement factor of the equation (*) is:

$$\mu(x) = e^{[\int m_1(x)dx, \int m_2(x)dx]} = \left[e^{\int m_1(x)dx}, e^{\int m_2(x)dx}\right]$$

3) By multiplying equation (*) by the complement factor, we get:

$$\acute{z}\,\mu(x) + [m_1(x), m_2(x)]z\,\mu(x) = q(x)\,\mu(x)$$

$$\acute{z}\,\mu(x) + \left[m_1(x)\,e^{\int m_1(x)dx}, m_2(x)e^{\int m_2(x)dx}\right] z\,\mu(x) = q(x)\,\mu(x)$$

$$\left(z\,\mu(x)\right)' = q(x)\ \left[e^{\int m_1(x)dx}, e^{\int m_2(x)dx}\right]$$

$$\left(z\,\mu(x)\right)' = \left[\,q(x)\,e^{\int m_1(x)dx},\ q(x)e^{\int m_2(x)dx}\right]$$

By integrating the tow side, we get:

$$z = \frac{1}{\mu(x)}\left(\left[\int q(x)\,e^{\int m_1(x)dx}\,dx, \int q(x)e^{\int m_2(x)dx}\,dx\right] + a + bI\right)$$

4) Back to the primary variable $y$, we get:

$$f(y) = \frac{1}{\mu(x)}\left(\left[\int q(x)\,e^{\int m_1(x)dx}\,dx, \int q(x)e^{\int m_2(x)dx}\,dx\right] + a + bI\right)$$

**Example 3.2.1:**

Find the general solution of the following neutrosophic differential equation:

$$e^y\frac{dy}{dx} + \left[-2x, \frac{2}{x}\right]e^y = 5x$$

**Solution:**

$$z = e^y \quad \Longrightarrow \quad \acute{z} = e^y.\frac{dy}{dx}$$

By substitution in the equation, we get:

$$\acute{z} + \left[-2x, \frac{2}{x}\right]z = 5x \qquad (***)$$

the complement factor is:

$$\mu(x) = e^{\left[\int -2x\,dx, \int\frac{2}{x}dx\right]}$$

$$= \left[\mathrm{e}^{\int -2x\,dx}, \mathrm{e}^{\int\frac{2}{x}dx}\right] = \left[\mathrm{e}^{-x^2}, x^2\right]$$

By multiplying equation (***) by the complement factor, we get:

DOI: 10.5281/zenodo.4928396

$$\acute{z}\,\mu(x) + \left[-2x, \frac{2}{x}\right] z\,\mu(x) = 5x\,\mu(x)$$

$$\acute{z}\,\mu(x) + \left[-2x\mathrm{e}^{-x^2}, 2x\right] z = 5x\left[\mathrm{e}^{-x^2}, x^2\right]$$

$$\left(z\,\mu(x)\right)' = \left[5x\mathrm{e}^{-x^2}, 5x^3\right]$$

By integrating the tow side, we get:

$$z\,\mu(x) = \int \left[5x\mathrm{e}^{-x^2}, 5x^3\right] dx$$

$$z = \frac{1}{\mu(x)}\left(\left[\int 5x\mathrm{e}^{-x^2} dx\,, \int 5x^3\,dx\right] + a + bI\right)$$

$$z = \frac{1}{\mu(x)}\left(\left[\frac{-5}{2}\mathrm{e}^{-x^2}, \frac{5}{4}x^4\right] + a + bI\right)$$

Back to the primary variable $y$, we get:

$$e^y = \frac{1}{\mu(x)}\left(\left[\frac{-5}{2}\mathrm{e}^{-x^2}, \frac{5}{4}x^4\right] + a + bI\right)$$

**Example 3.2.2:**

Find the general solution of the following neutrosophic differential equation:

$$Cosy\frac{dy}{dx} + Cotx\;Siny = [-2x, Sinx]$$

**Solution:**

$$z = Siny \quad \Longrightarrow \quad \acute{z} = Cosy\;\frac{dy}{dx}$$

By substitution in (*), we get:

$$\acute{z} + Cotx\;z = [-2x, Sinx] \qquad (\acute{*})$$

the complement factor is:

$$\mu(x) = e^{\int Cotx\,dx}$$

$$= e^{\ln(Sinx)} = Sinx$$

By multiplying equation $(\acute{*})$ by the complement factor, we get:

$$\acute{z}\,\mu(x) + Cotx\;\mu(x)\;z = \mu(x)[-2x, Sinx]$$

$$\acute{z}\,\mu(x) + Cotx\;Sin(x)\;z = Sinx[-2x, Sinx]$$

$$\acute{z}\,\mu(x) + Cosx\;z = [-2x\;Sinx, Sin^2x]$$

DOI: 10.5281/zenodo.4928396

$$\left(z\,\mu(x)\right)' = [-2x\,Sinx, Sin^2x]$$

By integrating the tow side, we get:

$$z\,\mu(x) = \int [-2x\,Sinx, Sin^2x]dx$$

$$z = \frac{1}{\mu(x)}\left(\left[\int -2x\,Sinx\,dx\,, \int Sin^2x\,dx\right] + a + bI\right)$$

$$z = \frac{1}{Sinx}\left(\left[\int -2x\,Sinx\,dx\,, \int \left(\frac{1}{2} - \frac{1}{2}Cosx\right)dx\right] + a + bI\right)$$

$$z = \frac{1}{Sinx}\left(\left[2x\,Cosx + 2Sinx, \frac{1}{2}x - \frac{1}{4}Sinx\right] + a + bI\right)$$

Back to the primary variable $y$, we get:

$$Siny = \frac{1}{Sinx}\left(\left[-\frac{1}{2}x + \frac{1}{4}Sinx, \frac{1}{4}Cos2x\right] + a + bI\right)$$

**Example 3.2.3:**

Find the general solution of the following neutrosophic differential equation:

$$Sec^2y\,\frac{dy}{dx} + \left[3x^2, \frac{1}{x}\right]Tany = [x^2, x^4]$$

**Solution:**

$$z = Tany \quad \Rightarrow \quad \acute{z} = Sec^2y\,\frac{dy}{dx}$$

By substitution in the equation, we get:

$$\acute{z} + \left[3x^2, \frac{1}{x}\right]z = [x^2, x^4] \qquad (\acute{**})$$

the complement factor is:

$$\mu(x) = e^{\left[\int 3x^2\,dx, \int \frac{1}{x}dx\right]}$$

$$= \left[e^{\int 3x^2 dx}, e^{\int \frac{1}{x}dx}\right] = \left[e^{x^3}, x\right]$$

By multiplying equation $(\acute{**})$ by the complement factor, we get:

$$\acute{z}\,\mu(x) + \left[3x^2, \frac{1}{x}\right]z\,\mu(x) = [x^2, x^4]\,\mu(x)$$

$$\acute{z}\,\mu(x) + \left[3x^2e^{x^3}, 1\right]z = \left[x^2e^{x^3}, x^5\right]$$

$$\left(z\,\mu(x)\right)' = \left[x^2e^{x^3}, x^5\right]$$

DOI: 10.5281/zenodo.4928396

By integrating the tow side, we get:

$$z\,\mu(x) = \int \left[x^2 e^{x^3}, x^5\right] dx$$

$$z = \frac{1}{\mu(x)}\left(\left[\int x^2 e^{x^3} dx\,, \int\ x^5\, dx\right] + a + bI\right)$$

$$z = \frac{1}{\mu(x)}\left(\left[3e^{x^3}, \frac{1}{6}x^6\right] + a + bI\right)$$

Back to the primary variable $y$, we get:

$$Tany\ = \frac{1}{\mu(x)}\left(\left[3e^{x^3}, \frac{1}{6}x^6\right] + a + bI\right)$$

**4. Conclusions**

In this paper, we introduced and solved a set of neutrosophic differential equations that transform to linear depending on the thick function. The most important of them is , the Bernoulli's neutrosophic differential equations by a neutrosophic thick function. This paper is considered one of the important research papers in neutrosophic differential equations.

**Acknowledgments:** This publication was supported by the Deanship of Scientific Research at Prince Sattam bin Abdulaziz University, Alkharj, Saudi Arabia.

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