Application of Intuitionistic Neutrosophic Soft Sets in Decision Making Based on Game Theory

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Abstract

The main objective of the paper is to introduce intuitionistic neutrosophic soft sets(INSSs) and their properties in the study of game theory. Earlier, in 2018, Anjan et al. introduced the notion of intuitionistic fuzzy soft game theory. So, here an attempt has been made to extend the earlier concept by introducing intuitionistic neutrosophic soft game theory. We also define two-person intuitionistic neutrosophic soft games (TP-INS-games) and extend them to N-person-INS-games. Finally, with the help of the intuitionistic neutrosophic soft saddle point method, we give an application in a real decision-making problem.

Keywords: Soft set, Neutrosophic soft set, Intuitionistic neutrosophic soft set, Intuitionistic neutrosophic soft payoff.

1. Introduction

Earlier we have used the traditional mathematical tools to solve problems of various practical applications to get precise, deterministic, and exact results. Then, probability theory, decision theory are more often used successfully to describe the concept of uncertainty in some fields of knowledge. Along with the development of science and technology, the problems of our real world become more complicated and more critical, and traditional tools are not able to answer those problems. After critical and deep investigations, scientists and mathematicians reveal that the concept of uncertainty exists in various forms in various disciplines such as philosophy, social science, science and engineering, mathematics, logic, etc. Vagueness, ambiguity is the different forms of uncertainty. Vague data means they are imprecise, indeterministic, and inexact. Such issues are part of our life and we face these things in our daily practice. So, we need a powerful tool that can answer imperfect knowledge. Finally, Zadeh [39] introduced the novel concept known as the fuzzy set theory to describe the vague concept. It has been used successfully in almost all branches of mathematics and many interdisciplinary areas. A Fuzzy set(FS) is an extension of a classical set or crisp set. In a fuzzy set, every element has a membership degree and it can be measured by a membership function. Suppose X be a universal set and A be a fuzzy set over X then, the membership function is defined as $\mu_A : X \to [0,1]$. Some popular works associated with fuzzy sets and their invariants are defined in [5,14]. After that, we realize that there exist problems where along with membership degree there is a demand to consider non-

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membership degree. That is we need to explain the incomplete information of the problem. This issue has been solved by introducing an intuitionistic fuzzy set (IFS), as an extension of FS, by Atanassov [1]. In IFS, every member has a membership degree and a non-membership degree over the common universe with a restriction that the two membership degrees are dependent on each other. We give some other extensions of fuzzy sets in [2,15,33].

A fuzzy set and its variants cannot describe the parametric behavior of an attribute. To handle parametric data i.e. the data that contain uncertainty we need another mathematical tool for modeling the uncertainty. In 1999, Molodtsov [22] introduced the soft set theory to model vagueness in a parametric way. It is the more general framework to deal with uncertainty. It has several significant applications among some of them discussed in [19,38]. Later, embedding the idea of a soft set with a fuzzy set, intuitionistic fuzzy set, interval-valued fuzzy set, interval-valued intuitionistic fuzzy set respectively fuzzy soft set[27], intuitionistic fuzzy soft set[9,20], interval-valued fuzzy soft set[37],interval-valued intuitionistic fuzzy soft set[16] are introduced. In 1999, Smarandache[31] coined the term neutrosophy as a part of philosophy and logic. Afterward, in 2005, Smarandache[32] introduced a neutrosophic set(NS) as an extension of the fuzzy set, an intuitionistic fuzzy set. NS is capable to describe incomplete, indeterminate, and inconsistent knowledge in set-theoretic form. NS has three components, namely the truth-membership value, the indeterminacy-membership value, and the falsity membership value, and all these membership values are taken from the real standard or non-standard subsets of $\left]0^-,1^+\right[$. Wang et al. [36] introduced a single-valued neutrosophic set. A Single-valued neutrosophic set is more effective than a neutrosophic set for practical applications in social sciences, engineering, computer sciences, etc. In the neutrosophic set, among the three components if any one of them is zero, still it will be in a more general form than the fuzzy set and the intuitionistic fuzzy set. So, we claim that neutrosophic sets are fuzzy sets or intuitionistic fuzzy sets but the converse is not true. Neutrosophic set is emerged as the most powerful tool to date to handle different types of indeterminacies. NS and its variants have a wide range of applications, some are given in [12,13,17,21,26,30, 35,40].

Game theory is a popular branch of mathematics and it is the analysis of games of strategies in different disciplines of conflict of interest. J.V. Neumann and O. Morgenstern [25] are the pioneers of the modern game theory. In 1950, game theory has been developed extensively. In 1953, Borel introduced the concept of pure and mixed strategies. But Neumann points out that without the minimax theorem no theory of games can exist. The formal definition of games pays out the players, their preferences, the strategic actions available to them, and how these influence the outcome. It has been used widely in different studies such as economics, political science, computer science, logic, social science, etc. Two-person game is a game where the sum of the payoffs of two persons is always zero and it has been extended to an N-person game. A two-person zero-sum games are solvable if there exist strategies that are in equilibrium with each other. The payoff is the outcome of the game that depends on the selecting strategies of the player. The payoff is a value associated with a possible outcome of a game. A strategy is a rule or plan of action for playing a game. An optimal strategy provides the best payoff for a player in a game. A saddle point is an outcome of a game where it is minimum in its row and maximum in its column. In a classical game, the players know the available strategies and the payoff functions. But, classical game theory cannot solve the games with incomplete information. For this, Cagman et al.[4] introduced soft game, Deli et al.[7,10] shown the application of soft games. Also, embedding the idea of fuzzy soft sets, intuitionistic fuzzy soft sets, interval-valued intuitionistic fuzzy soft sets, interval-valued neutrosophic soft sets, game theory has been expanded respectively by introducing fuzzy soft game[8], the intuitionistic fuzzy soft game[24], the interval-valued intuitionistic fuzzy soft game[23], the interval-valued neutrosophic soft game[6]. In [18], T. Maeda has shown the characterization of equilibrium strategy of two-person zero-sum game and in[11], Deli et al. discussed the probabilistic equilibrium solution of soft game.

Bhowmik et al. [3] introduce an intuitionistic neutrosophic set and Broumi et al. [28] extend it by developing an intuitionistic neutrosophic soft set. In this work, we concentrate upon the intuitionistic neutrosophic set and its properties. We use this concept in game-theoretic strategy and apply it to decision-making problems. We also give some definitions and properties of intuitionistic neutrosophic soft sets based on game theory. Our main motivation

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behind organizing this paper is to extend the earlier concept proposed by Anjan et al. in [24]. The present article is arranged in the following manner :

Section-2 is comprised of some basic definitions that are relevant to the topic. In section-3, we have defined two-person intuitionistic neutrosophic games and their properties to find the optimal strategy by using the saddle point method, .In section-4, the two-person intuitionistic neutrosophic soft game has been extended to N-person intuitionistic neutrosophic game and study some of its properties. In section-5, we give an application of the two-person intuitionistic neutrosophic saddle point method in real decision-making. In section-6, a comparative analysis of different types of soft games has been illustrated. In the last section(in section-7), the main purpose and the scope of the paper are discussed briefly.

2. Preliminaries:

Here we define some basic definitions and examples that are relevant for the subsequent discussions.

Definition 2.1 [1] Let X be a fixed non-empty set. An intuitionistic fuzzy set A in X can be defined as,

$A=\left\{\left(x,\mu_A\left(x\right),\gamma_A\left(x\right)\right):x\in X\right\}$, where μ_A and γ_A denote respectively the membership function and the non-membership function such that $\mu_A,\gamma_A:X\rightarrow\left[0,1\right]$ and $0\le\mu_A\left(x\right)+\gamma_A\left(x\right)\le 1$. If $\gamma_A\left(x\right)$=0, then IFS reduces to FS.

Definition 2.2 [22] Let U be the initial universe and E be the set of parameters. Again, let $A\subseteq E$ and $P\left(U\right)$ denotes the power set of U. Then the pair $\left(F,A\right)$ is called the soft set over U, where $F:A\rightarrow P\left(U\right)$.

Definition 2.3 [20] Let U be the initial universe and E be the set of parameters. Let $A\subseteq E$ and the collection of all intuitionistic fuzzy subsets of U be denoted by IF^U. Then the pair $\left(F,A\right)$ is called the intuitionistic fuzzy soft set over U, where $F:A\rightarrow IF^U$.

Definition 2.4 [31] A neutrosophic set A in X is defined as $A=\left\{\left\langle x,\mathrm{T}_A\left(x\right),\mathrm{I}_A\left(x\right),\mathrm{F}_A\left(x\right)\right\rangle:x\in X\right\}$, where $T_A,I_A,F_A:X\rightarrow\left]^{-}0,1^{+}\right[$ and $^{-}0\le T_A\left(x\right)+I_A\left(x\right)+F_A\left(x\right)\le 3^{+}$. From a philosophical point of view, the neutrosophic set takes the value from the real standard or non-standard subsets of $\left]^{-}0,1^{+}\right[$. For the use of the real scientific application, we use the value from the subset of $\left[0,1\right]$.

Definition 2.5 [36] A single-valued neutrosophic set A in Y is defined as an object of the form:

$$A=\left\{\left\langle y,\mathrm{T}_A\left(y\right),\mathrm{I}_A\left(y\right),\mathrm{F}_A\left(y\right)\right\rangle:\mathrm{y}\in Y\right\}\text{ Where, }T_A,I_A,F_A:Y\rightarrow\left[0,1\right]$$

,and $0\le T_A\left(x\right)+I_A\left(x\right)+F_A\left(x\right)\le 3.$

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Definition 2.6 [21] Let U be the universe of discourse and E be the set of parameters. Let $A \subseteq E$ and N^U denotes the collection of neutrosophic subsets of U. Then the pair (F, A) is called the neutrosophic soft set over U, where $F : A \to N^U$.

Definition 2.7 [3] An element y of the set of the universe U is called significant in respect of a neutrosophic set A of U if $T_A(y)$ or $I_A(y)$ or $F_A(y) \leq 0.5$. Otherwise, we call it insignificant. Since in the neutrosophic set T, I, F can be any numbers in $[0,1]$, i.e. for example $(0.1, 0.1, 0.1), (0.6, 0.7, 0.8)$ etc. We define an intuitionistic neutrosophic set by

$A = \{\langle y, T_A(y), I_A(y), F_A(y)\rangle : y \in U\}$, where

$$\min\{T_A(y), F_A(y)\} \leq 0.5$$
$$\min\{T_A(y), I_A(y)\} \leq 0.5$$
$$\min\{F_A(y), I_A(y)\} \leq 0.5$$

with the condition $0 \leq T_A(y) + I_A(y) + F_A(y) \leq 2.$

Definition 2.8 [28] Let U denote the set of the universe and $A \subseteq E$ denote the set of parameters. Also, let IN^U denote the collection of intuitionistic neutrosophic subsets of U. Then the pair (F, A) is termed as the intuitionistic neutrosophic soft set over U, where $F : A \to IN^U$.

Example 2.9 Let U denote the set of students and E be the set of parameters (qualities). Here each parameter is an intuitionistic neutrosophic word or phrase or sentence. Consider $E = \{ e_1$=self-disciplined, e_2=diligent, e_3=punctual, e_4=courteous, e_5=confident, e_6=responsible, e_7=intelligent}. In this case, to define intuitionistic neutrosophic soft set is to point out self-disciplined student, diligent student, punctual student and so on. Suppose $U = \{s_1, s_2, s_3, s_4, s_5\}$ and $E = \{e_1, e_2, e_4, e_5, e_6\}$, where each parameter signifies a special quality of a student.

Consider that,

$$F(e_1) = \{\langle s_1, 0.4, 0.7, 0.5\rangle, \langle s_2, 0.6, 0.3, 0.4\rangle, \langle s_3, 0.8, 0.4, 0.3\rangle, \langle s_4, 0.4, 0.2, 0.6\rangle, \langle s_5, 0.4, 0.5, 0.7\rangle\}$$
$$F(e_2) = \{\langle s_1, 0.3, 0.5, 0.6\rangle, \langle s_2, 0.6, 0.2, 0.4\rangle, \langle s_3, 0.7, 0.5, 0.4\rangle, \langle s_4, 0.1, 0.3, 0.5\rangle, \langle s_5, 0.6, 0.5, 0.4\rangle\}$$
$$F(e_4) = \{\langle s_1, 0.5, 0.6, 0.5\rangle, \langle s_2, 0.4, 0.2, 0.6\rangle, \langle s_3, 0.6, 0.4, 0.3\rangle, \langle s_4, 0.7, 0.2, 0.5\rangle, \langle s_5, 0.6, 0.2, 0.3\rangle\}$$
$$F(e_5) = \{\langle s_1, 0.1, 0.6, 0.3\rangle, \langle s_2, 0.8, 0.1, 0.3\rangle, \langle s_3, 0.4, 0.7, 0.2\rangle, \langle s_4, 0.5, 0.2, 0.6\rangle, \langle s_5, 0.3, 0.7, 0.5\rangle\}$$
$$F(e_6) = \{\langle s_1, 0.5, 0.3, 0.7\rangle, \langle s_2, 0.9, 0.3, 0.5\rangle, \langle s_3, 0.5, 0.6, 0.4\rangle, \langle s_4, 0.5, 0.6, 0.3\rangle, \langle s_5, 0.5, 0.6, 0.3\rangle\}$$

Thus we can represent the intuitionistic neutrosophic soft set (F, A) as a collection of approximation given by:

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$$(F,A)=\begin{cases}\text{self-disciplined students}=\langle s_1,0.4,0.7,0.5\rangle,\langle s_2,0.6,0.3,0.4\rangle,\langle s_3,0.8,0.4,0.3\rangle,\langle s_4,0.4,0.2,0.6\rangle,\langle s_5,0.4,0.5,0.7\rangle\\ \text{diligent students}=\langle s_1,0.3,0.5,0.6\rangle,\langle s_2,0.6,0.2,0.4\rangle,\langle s_3,0.7,0.5,0.4\rangle,\langle s_4,0.1,0.3,0.5\rangle,\langle s_5,0.6,0.5,0.4\rangle\\ \text{courteous students}=\langle s_1,0.5,0.6,0.5\rangle,\langle s_2,0.4,0.2,0.6\rangle,\langle s_3,0.6,0.4,0.3\rangle,\langle s_4,0.7,0.2,0.5\rangle,\langle s_5,0.6,0.2,0.3\rangle\\ \text{confident students}=\langle s_1,0.5,0.6,0.5\rangle,\langle s_2,0.4,0.2,0.6\rangle,\langle s_3,0.6,0.4,0.3\rangle,\langle s_4,0.7,0.2,0.5\rangle,\langle s_5,0.6,0.2,0.3\rangle\\ \text{responsible students}=\langle s_1,0.5,0.3,0.7\rangle,\langle s_2,0.9,0.3,0.5\rangle,\langle s_3,0.5,0.6,0.4\rangle,\langle s_4,0.5,0.6,0.3\rangle,\langle s_5,0.5,0.6,0.3\rangle\end{cases}$$

For algebraic operation, we represent (F,A) in the tabular form given by:

U	self-disciplined	Diligent	Courteous	confident	Responsible
s_1	(0.4,0.7,0.5)	(0.3,0.5,0.6)	(0.5,0.6,0.5)	(0.5,0.6,0.5)	(0.5,0.3,0.7)
s_2	(0.6,0.3,0.4)	(0.6,0.2,0.4)	(0.4,0.2,0.6)	(0.4,0.2,0.6)	(0.9,0.3,0.5)
s_3	(0.8,0.4,0.3)	(0.7,0.5,0.4)	(0.6,0.4,0.3)	(0.6,0.4,0.3)	(0.5,0.6,0.4)
s_4	(0.4,0.2,0.6)	(0.1,0.3,0.5)	(0.7,0.2,0.5)	(0.7,0.2,0.5)	(0.5,0.6,0.3)
s_5	(0.4,0.5,0.7)	(0.6,0.5,0.4)	(0.6,0.2,0.3)	(0.6,0.2,0.3)	(0.5,0.6,0.3)

Table 1: Tabular representation of INSS (F,A)

3. Two-Person intuitionistic neutrosophic soft games

In this section, first, we give some basic definitions and then construct two-person intuitionistic neutrosophic soft games with intuitionistic neutrosophic soft payoffs. Here, we extend the concept of various types of game theory defined in [4,8,23,24,29] to intuitionistic neutrosophic soft games.

Definition 3.1 Let G be a set of strategies and $P,Q\subseteq G$. Then the collection of all the ordered pairs of the set $P\times Q$ are called the available action pairs.

Definition 3.2 Let U be the set of alternatives and IN^U denotes the collection of all intuitionistic neutrosophic subsets over U. Again, let G be a set of strategies and $P,Q\subseteq G$. Then, the set-valued function $\rho_{P\times Q}:P\times Q\to IN^U$ is called an intuitionistic neutrosophic soft payoff function and it is defined as

$\rho_{P\times Q}(p,q)=(\mu_T(p,q),\delta_I(p,q),\gamma_F(p,q))$, where the first element is the truth-membership value, the second element is the indeterminacy-membership value and the third element is the falsity-membership value.

Definition 3.3 Let $P\times Q$ be a set of available action pairs. Then, an action $(p^*,q^*)\in P\times Q$ is called an optimal action if $\rho_{P\times Q}(p^*,q^*)\supseteq\rho_{P\times Q}(p\ ,q\ )$ i.e. $\mu_T(p^*,q^*)\geq\mu_T(p\ ,q\ ),\delta_I(p^*,q^*)\geq\delta_I(p\ ,q\ )$, and $\gamma_F(p^*,q^*)\leq\gamma_F(p\ ,q\ )$, $\forall\ (p,q)\in P\times Q$.

Definition 3.4 Let $P\times Q$ be a set of available action pairs and $(p_i,q_j),(p_k,q_l)\in P\times Q$. Then,

a) if $\rho_{P\times Q}\left(p_i,q_j\right)\supseteq\rho_{P\times Q}\left(p_k,q_l\right)$,then a player either prefers $\left(p_i,q_j\right)$ over $\left(p_k,q_l\right)$ or is indifferent between the two actions.

b) if $\rho_{P\times Q}\left(p_i,q_j\right)\supset\rho_{P\times Q}\left(p_k,q_l\right)$, then a player strictly prefers $\left(p_i,q_j\right)$ over $\left(p_k,q_l\right)$.

Definition 3.5 Let P and Q be the set of strategies of player 1 and player 2 respectively. Again, let U be a set of alternatives and $\rho_{P\times Q}:P\times Q\to IN^U$ be an intuitionistic neutrosophic soft payoff function for player k=1,2. Then, for each player k, the two-person intuitionistic neutrosophic soft game(tp-ins-game)is denoted and defined by an intuitionistic neutrosophic set over U as

$$\Upsilon^k_{P\times Q}=\left\{\left(\left(p,q\right),\rho^k_{P\times Q}\left(p,q\right)\right):\left(p,q\right)\in P\times Q\right\}$$

Now we describe the tp-ins-game as follows: at a particular time player 1 chooses a strategy $p_i\in P$, simultaneously player 2 chooses another strategy $q_i\in Q$. Then, each player k=1,2 receives the intuitionistic neutrosophic payoff $\rho^k_{P\times Q}\left(p_i,q_j\right)$.

If $P=\left\{p_1,p_2,........,p_r\right\}$, and $Q=\left\{q_1,q_2,........,q_s\right\}$, then the intuitionistic neutrosophic soft payoffs $\rho^k_{P\times Q}$ can be presented in the form of a $r\times s$ matrix as shown in table 2 given by

$\Upsilon^k_{P\times Q}$	q_1	q_2	$\cdots$	q_s
p_1	$\rho^k_{P\times Q}\left(p_1,q_1\right)$	$\rho^k_{P\times Q}\left(p_1,q_2\right)$	$\cdots$	$\rho^k_{P\times Q}\left(p_1,q_s\right)$
p_2	$\rho^k_{P\times Q}\left(p_2,q_1\right)$	$\rho^k_{P\times Q}\left(p_2,q_2\right)$	$\cdots$	$\rho^k_{P\times Q}\left(p_2,q_s\right)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
p_r	$\rho^k_{P\times Q}\left(p_r,q_1\right)$	$\rho^k_{P\times Q}\left(p_r,q_2\right)$	$\cdots$	$\rho^k_{P\times Q}\left(p_r,q_s\right)$

Table 2: The two-person intuitionistic neutrosophic soft game

Example 3.6 Let $U=\left\{u_1,u_2,u_3,u_4,u_5,u_6,u_7\right\}$ be a set of alternatives and $E=\left\{x_1,x_2,x_3,x_4,x_5,x_6\right\}$ be a set of strategies. Let IN^U denotes the set of all intuitionistic neutrosophic sets over U and $X=\left\{x_1,x_3,x_5\right\}$ and $Y=\left\{x_3,x_5\right\}$ be a set of strategies of Player 1 and player 2 respectively.

If player 1 construct the tp-ins-game as follows,

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$$\Upsilon^1_{X\times Y}=\left\{\begin{matrix}\left((x_1,x_3),\{(0.5,0.4,0.3)/u_1,(0.3,0.7,0.4)/u_3,(0.2,0.6,0.4)/u_5\}\right),\left((x_1,x_5),\{(0.5,0.6,0.3)/u_2,(0.4,0.6,0.3)/u_3,(0.5,0.1,0.6)/u_5\}\right)\\ \left((x_3,x_3),\{(0.6,0.4,0.4)/u_1,(0.1,0.6,0.3)/u_4,(0.5,0.5,0.6)/u_6\}\right),\left((x_3,x_5),\{(0.8,0.4,0.6)/u_2,(0.3,0.9,0.5)/u_3,(0.8,0.5,0.4)/u_4\}\right)\\ \left((x_5,x_3),\{(0.3,0.7,0.5)/u_2,(0.2,0.8,0.5)/u_3,(0.1,0.3,0.8)/u_5\}\right),\left((x_5,x_5),\{(0.7,0.3,0.5)/u_1,(0.2,0.5,0.7)/u_2,(0.4,0.8,0.5)/u_4\}\right)\end{matrix}\right\}$$

Then the intuitionistic neutrosophic soft payoffs of the game can be presented in a matrix form given in Table 3.

$\Upsilon^1_{X\times Y}$	x_3	x_5
x_1	$\{(0.5,0.4,0.3)/u_1,(0.3,0.7,0.4)/u_3,(0.2,0.6,0.4)/u_5\}$	$\{(0.5,0.6,0.3)/u_2,(0.4,0.6,0.3)/u_3,(0.5,0.1,0.6)/u_5\}$
x_3	$\{(0.6,0.4,0.4)/u_1,(0.1,0.6,0.3)/u_4,(0.5,0.5,0.6)/u_6\}$	$\{(0.8,0.4,0.6)/u_2,(0.3,0.9,0.5)/u_3,(0.8,0.5,0.4)/u_4\}$
x_5	$\{(0.3,0.7,0.5)/u_2,(0.2,0.8,0.5)/u_3,(0.1,0.3,0.8)/u_5\}$	$\{(0.7,0.3,0.5)/u_1,(0.2,0.5,0.7)/u_2,(0.4,0.8,0.5)/u_4\}$

Table 3: Intuitionistic neutrosophic soft payoffs of the game of Player 1

From Table 3, we discuss some elements of the game. If player 1 chooses x_3 and player 2 chooses x_5, then the value of the game will be $\{(0.8,0.4,0.6)/u_2,(0.3,0.9,0.5)/u_3,(0.8,0.5,0.4)/u_4\}$ i.e., the entry at the intersection of the row along x_3 and column along x_5.

Similarly, if player 2 construct the tp-ins-game as follows,

$$\Upsilon^2_{X\times Y}=\left\{\begin{matrix}\left((x_1,x_3),\{(0.6,0.5,0.4)/u_3,(0.7,0.3,0.5)/u_5\}\right),\left((x_1,x_5),\{(0.6,0.4,0.3)/u_2,(0.3,0.8,0.4)/u_4,(0.2,0.1,0.4)/u_5\}\right)\\ \left((x_3,x_3),\{(0.5,0.4,0.4)/u_3,(0.3,0.7,0.5)/u_5,(0.6,0.5,0.4)/u_6\}\right),\left((x_3,x_5),\{(0.4,0.8,0.4)/u_1,(0.3,0.9,0.5)/u_3,(0.7,0.5,0.4)/u_5\}\right)\\ \left((x_5,x_3),\{(0.2,0.6,0.5)/u_2,(0.2,0.6,0.5)/u_4,(0.2,0.8,0.4)/u_5\}\right),\left((x_5,x_5),\{(0.8,0.4,0.5)/u_4,(0.4,0.5,0.8)/u_6\}\right)\end{matrix}\right\}$$

Then the intuitionistic neutrosophic soft payoffs of the game can be arranged in a matrix form given in Table 4.

$\Upsilon^1_{X\times Y}$	x_3	x_5
x_1	$\{(0.6,0.5,0.4)/u_3,(0.7,0.3,0.5)/u_5\}$	$\{(0.6,0.4,0.3)/u_2,(0.3,0.8,0.4)/u_4,(0.2,0.1,0.4)/u_5\}$
x_3	$\{(0.5,0.4,0.4)/u_3,(0.3,0.7,0.5)/u_5,(0.6,0.5,0.4)/u_6\}$	$\{(0.4,0.8,0.4)/u_1,(0.3,0.9,0.5)/u_3,(0.7,0.5,0.4)/u_5\}$
x_5	$\{(0.2,0.6,0.5)/u_2,(0.2,0.6,0.5)/u_4,(0.2,0.8,0.4)/u_5\}$	$\{(0.8,0.4,0.5)/u_4,(0.4,0.5,0.8)/u_6\}$

Table 4: Intuitionistic neutrosophic soft payoffs of the game of Player 2

Definition 3.7 A tp-ins-game is called a two-person empty intersection intuitionistic neutrosophic soft game if the intersection of the ins-payoffs of players is an empty set for each ins-action pairs.

Definition 3.8 Let $\rho^k_{X\times Y}$ be an intuitionistic neutrosophic soft payoff function of a tp-ins-game $\Upsilon^k_{X\times Y}$, where k=1,2. Then, if the following properties hold:

(i) $\bigcup_{i=1}^{m}\rho^k_{X\times Y}(x_i,y_j)=\rho^k_{X\times Y}(x,y)=\left\{\max\mu^k_T(x_i,y_j),\max\delta^k_I(x_i,y_j),\min\gamma^k_F(x_i,y_j)\right\}/u_t$

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(ii) $\bigcap_{j=1}^{n} \rho^{k}_{X\times Y}\left(x_i, y_j\right) = \rho^{k}_{X\times Y}\left(x, y\right) = \left\{\min \mu^{k}_{T}\left(x_i, y_j\right), \min \delta^{k}_{I}\left(x_i, y_j\right), \max \gamma^{k}_{F}\left(x_i, y_j\right)\right\} / u_t$

Then $\rho^{k}_{X\times Y}\left(x, y\right)$ is called an intuitionistic neutrosophic soft saddle point value and $\left(x, y\right)$ is called an intuitionistic neutrosophic soft saddle point of player k's in the tp-ins-game. Hence the intuitionistic neutrosophic soft saddle point is a value of the tp-ins-game.

Example 3.9 Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ be a set of alternatives, $X = \{x_1, x_2, x_3\}$ and $Y = \{y_1, y_2\}$ be the set of strategies of Player 1 and Player 2 respectively. If we construct the tp-ins-game of player 1, then the tp-ins payoff of player 1 is shown in Table 5 given as,

$\Upsilon^{1}_{X\times Y}$	y_1	y_2
x_1	$\{(0.3, 0.4, 0.7)/u_1, (0.5, 0.4, 0.3)/u_2, (0.3, 0.6, 0.4)/u_3, (0.2, 0.6, 0.4)/u_5\}$	$\{(0.3, 0.4, 0.6)/u_1, (0.5, 0.6, 0.3)/u_2, (0.4, 0.6, 0.3)/u_3, (0.3, 0.6, 0.3)/u_5\}$
x_2	$\{(0.2, 0.4, 0.7)/u_1, (0.5, 0.4, 0.6)/u_2\}$	$\{(0.3, 0.4, 0.6)/u_2, (0.3, 0.9, 0.5)/u_3, (0.8, 0.5, 0.4)/u_4\}$
x_3	$\{(0.3, 0.3, 0.7)/u_1, (0.1, 0.3, 0.8)/u_5\}$	$\{(0.7, 0.3, 0.5)/u_1, (0.2, 0.5, 0.7)/u_2, (0.4, 0.8, 0.5)/u_4\}$

Table 5: The tp-ins-game of player 1

Now, $\bigcup_{i=1}^{3} \rho^{1}_{X\times Y}\left(x_i, y_1\right) = \{(0.3,0.4,0.7)/u_1, (0.5,0.4,0.3)/u_2, (0.3,0.6,0.4)/u_3, (0.2,0.6,0.4)/u_5\}$

$$\bigcup_{i=1}^{3} \rho^{1}_{X\times Y}\left(x_i, y_2\right) = \{(0.7,0.4,0.5)/u_1, (0.5,0.6,0.3)/u_2, (0.4,0.9,0.3)/u_3, (0.8,0.8,0.4)/u_4, (0.3,0.6,0.3)/u_5\}$$

Similarly,

$$\bigcap_{j=1}^{2} \rho^{1}_{X\times Y}\left(x_1, y_j\right) = \{(0.3,0.4,0.7)/u_1, (0.5,0.4,0.3)/u_2, (0.3,0.6,0.4)/u_3, (0.2,0.6,0.4)/u_5\}$$

$$\bigcap_{j=1}^{2} \rho^{1}_{X\times Y}\left(x_2, y_j\right) = \{(0.5,0.4,0.6)/u_2\}$$

$$\bigcap_{j=1}^{2} \rho^{1}_{X\times Y}\left(x_3, y_j\right) = \{(0.3,0.3,0.7)/u_1\}$$

Clearly, $\bigcup_{i=1}^{3} \rho^{1}_{X\times Y}\left(x_i, y_1\right) = \bigcap_{j=1}^{2} \rho^{1}_{X\times Y}\left(x_1, y_j\right) = \{(0.3, 0.4, 0.7)/u_1, (0.5, 0.4, 0.3)/u_2, (0.3, 0.6, 0.4)/u_3, (0.2, 0.6, 0.4)/u_5\}$

Since, the intersection of the first row is equal to the union of the first column therefore $\{(0.3,0.4,0.7)/u_1,(0.5,0.4,0.3)/u_2,(0.3,0.6,0.4)/u_3,(0.2,0.6,0.4)/u_5\}$ is an intuitionistic neutrosophic soft saddle point value of the tp-ins-game. Hence $\{(0.3,0.4,0.7)/u_1,(0.5,0.4,0.3)/u_2,(0.3,0.6,0.4)/u_3,(0.2,0.6,0.4)/u_5\}$ is the value of the tp-ins-game.

It is to be noted that every tp-ins-game has not an intuitionistic neutrosophic soft saddle point value. For example, in Table 5 if we replace $\{(0.2,0.4,0.7)/u_1,(0.5,0.4,0.6)/u_2\}$ by $\{(0.2,0.4,0.7)/u_1,(0.6,0.3,0.5)/u_2\}$, then an intuitionistic neutrosophic soft saddle point value does not exist. Then, the value of the game cannot be found.

Definition 3.10 Let $\Upsilon_{X\times Y}$ be a tp-ins-game with its corresponding ins-payoff function $\rho_{X\times Y}$, where $\rho_{X\times Y}(x,y)=\{(\mu_T(x,y),\delta_I(x,y),\gamma_F(x,y)):\forall(x,y)\in X\times Y\}$. Then,

(i) an intuitionistic neutrosophic soft upper value of the tp-ins-game is denoted and defined by

$$\upsilon^*=\left\{\bigcap_{y\in Y}^{n}\left(\bigcup_{x\in X}^{n}\rho_{X\times Y}(x,y)\right)\right\}=\left\{\left(\bigcap_{y\in Y}^{n}\left(\bigcup_{x\in X}^{n}\mu_T(x,y)\right),\bigcap_{y\in Y}^{n}\left(\bigcup_{x\in X}^{n}\delta_I(x,y)\right),\bigcup_{y\in Y}^{n}\left(\bigcap_{x\in X}^{n}\gamma_F(x,y)\right)\right)\right\}$$

(ii) an intuitionistic neutrosophic soft lower value of the tp-ins-game is denoted and defined as

$$\upsilon_*=\left\{\bigcup_{x\in X}^{n}\left(\bigcap_{y\in Y}^{n}\rho_{X\times Y}(x,y)\right)\right\}=\left\{\left(\bigcup_{x\in X}^{n}\left(\bigcap_{y\in Y}^{n}\mu_T(x,y)\right),\bigcup_{x\in X}^{n}\left(\bigcap_{y\in Y}^{n}\delta_I(x,y)\right),\bigcap_{x\in X}^{n}\left(\bigcup_{y\in Y}^{n}\gamma_F(x,y)\right)\right)\right\}$$

(iii) If $\upsilon^*=\upsilon_*$ i.e. intuitionistic neutrosophic soft upper value and the intuitionistic neutrosophic soft lower value of a tp-ins-game are equal, then they are called the value of the tp-ins-game and it is denoted by υ i.e. $\upsilon^*=\upsilon_*=\upsilon$.

Theorem 3.11 If υ^* and υ_* be an intuitionistic neutrosophic soft upper and intuitionistic neutrosophic soft lower value respectively, then $\upsilon_*\subseteq\upsilon^*$.

Proof: Let us consider that υ^* and υ_* are the intuitionistic neutrosophic soft upper value and the intuitionistic neutrosophic soft lower value respectively. Also, let $X=\{x_1,x_2,....,x_m\}$ and $Y=\{y_1,y_2,....,\mathrm{y}_n\}$ are the sets of strategies for Player 1 and Player 2 respectively. If we choose $x_i^*\in X$ and $y_j^*\in Y$, then

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$$\upsilon_* = \bigcup_{x\in X}\left(\bigcap_{y\in Y}\left(\rho_{X\times Y}(x,y)\right)\right)$$
$$=\left\{\left(\bigcup_{x\in X}\left(\bigcap_{y\in Y}\mu_T(x,y)\right),\bigcup_{x\in X}\left(\bigcap_{y\in Y}\delta_I(x,y)\right),\bigcap_{x\in X}\left(\bigcup_{y\in Y}\gamma_F(x,y)\right)\right)\right\}$$
$$\subseteq\left\{\bigcap_{y\in Y}\mu_T(x^*,y),\bigcap_{y\in Y}\delta_I(x^*,y),\bigcup_{y\in Y}\gamma_F(x^*,y)\right\}$$
$$\subseteq\bigcap_{y\in Y}\left(\rho_{X\times Y}(x^*,y)\right)$$
$$\subseteq\rho_{X\times Y}(x^*,y^*)$$
$$\subseteq\bigcup_{x\in X}\left(\rho_{X\times Y}(x\ ,y^*)\right)$$
$$\subseteq\bigcap_{y\in Y}\left(\bigcup_{x\in X}\left(\rho_{X\times Y}(x,y)\right)\right)=\upsilon^*$$

Thus, $\upsilon_*\subseteq\upsilon^*$.

Definition 3.12 Let $\Upsilon^k_{X\times Y}$ be a tp-ins-game with its intuitionistic neutrosophic soft payoff function $\rho^k_{X\times Y}$ for k=1, 2. If the following properties hold true

(i) $\rho^1_{X\times Y}(x^*,y^*)\supseteq\rho^1_{X\times Y}(x\ ,y^*)$, for each $x\in X$

(ii) $\rho^2_{X\times Y}(x^*,y^*)\supseteq\rho^2_{X\times Y}(x^*,y\ )$, for each $y\in Y$

then, $(x^*,y^*)\in X\times Y$ is called an intuitionistic neutrosophic soft Nash equilibrium of a tp-ins-game.

4. N-person intuitionistic neutrosophic soft games

There are several instances where intuitionistic neutrosophic soft games are often played between more than two players. So, there is a need to extend tp-ins-games to N-person-ins-games in many real-life problems.

Here we use $X_n^{\otimes}$ for $X_1\times X_2\times ....\times X_n$.

Definition 4.1 Let Ω be a set of alternatives and IN^{Ω} denote the set of all intuitionistic neutrosophic subsets over Ω. Again, let E be a set of strategies and $X_1,X_2,.....,X_n\subseteq E$ and X_k is the set of strategies of Player k, where $k=1,2,...,n$. Then, for each Player k, an N-person intuitionistic neutrosophic soft games is defined by an intuitionistic neutrosophic soft set over Ω as

$\Upsilon^k_{X_n^{\otimes}}=\left\{\left((x_1,x_2,....,x_n),\rho^k_{X_n^{\otimes}}(x_1,x_2,....,x_n)\right):(x_1,x_2,....,x_n)\in X_n^{\otimes}\right\}$, where $\rho^k_{X_n^{\otimes}}$ is an intuitionistic neutrosophic soft payoff function of Player k.

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The N-person ins-game is played as follows: at a certain time Player 1 chooses a strategy $x_1 \in X_1$ and simultaneously each player k =2, 3,…., n chooses a strategy $x_k \in X_k$ and once this is done, each player k receives the ins-payoff $\rho^k_{X^{\otimes}_n}(x_1,x_2,\ldots,x_n)$.

Definition 4.2 Let $\Upsilon^k_{X^{\otimes}_n}$ be an N-person ins-game along with its ins-payoff function $\rho^k_{X^{\otimes}_n}$, for k =1,2,…,n. Then, a strategy $x_k \in X_k$ is called an intuitionistic neutrosophic soft dominated to another strategy $x \in X_k$, if

$\rho^k_{X^{\otimes}_n}(x_1,x_2,\ldots,x_{k-1},x_k,x_{k+1},\ldots,x_n) \supseteq \rho^k_{X^{\otimes}_n}(x_1,x_2,\ldots,x_{k-1},x,x_{k+1},\ldots,x_n)$ i.e.,

$\mu^k_{T\,X^{\otimes}_n}(x_1,x_2,\ldots,x_{k-1},x_k,x_{k+1},\ldots,x_n) \geq \rho^k_{X^{\otimes}_n}(x_1,x_2,\ldots,x_{k-1},x,x_{k+1},\ldots,x_n)$,

$\delta^k_{I\,X^{\otimes}_n}(x_1,x_2,\ldots,x_{k-1},x_k,x_{k+1},\ldots,x_n) \geq \delta^k_{I\,X^{\otimes}_n}(x_1,x_2,\ldots,x_{k-1},x,x_{k+1},\ldots,x_n)$ and

$\gamma^k_{F\,X^{\otimes}_n}(x_1,x_2,\ldots,x_{k-1},x_k,x_{k+1},\ldots,x_n) \leq \gamma^k_{F\,X^{\otimes}_n}(x_1,x_2,\ldots,x_{k-1},x,x_{k+1},\ldots,x_n)$, for each strategy $x_i \in X_i$ of player $i = 1,2,\ldots,k-1,k+1,\ldots.,n$ respectively.

Definition 4.3 Let $\Upsilon^k_{X^{\otimes}_n}$ be an N-person ins-game along with its ins-payoff function $\rho^k_{X^{\otimes}_n}$. If for each player $k = 1,2,\ldots,n$, the following property holds true

$\rho^k_{X^{\otimes}_n}(x^*_1,x^*_2,\ldots,x^*_{k-1},x^*_k,x^*_{k+1},\ldots,x^*_n) \supseteq \rho^k_{X^{\otimes}_n}(x^*_1,x^*_2,\ldots,x^*_{k-1},x,x^*_{k+1},\ldots,x^*_n)$, for each $x \in X_k$, then $(x^*_1,x^*_2,\ldots,x^*_{k-1},x^*_k,x^*_{k+1},\ldots,x^*_n) \in X^{\otimes}_n$ is called an intuitionistic neutrosophic soft Nash equilibrium of an N-person ins-game.

5. An Application

A mobile manufacturing company appointed two decision-makers(say Player 1 and Player 2) to make a solid strategic policy regarding the production of high quality android mobile phones so that the said company can compete in the market with the high selling rates of mobile phones as compared to previous years. Suppose the company produces six different types of android mobiles and it is denoted by the set $U = \{u_1,u_2,u_3,u_4,u_5,u_6\}$. Again, let $X = \{x_1,x_2,x_3\}$ and $Y = \{y_1 = x_1, y_2 = x_2\}$ be the strategies adopted by Player 1 and Player 2 respectively so that they together can select the best strategy to increase the selling of the company. Here, the strategies x_i (for i=1,2,3) stand for "high storage capacity", "high megapixel camera", "strong phone security".

By revisiting the example 3.9 above, the tp-ins-game of Player 1 can be presented in the form of Table 5.

From Table 5, we have

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$$\bigcup_{i=1}^{3} \rho^{1}_{X\times Y}\left(x_i, y_1\right) = \bigcap_{j=1}^{2} \rho^{1}_{X\times Y}\left(x_1, y_j\right) = \left\{\left(0.3, 0.4, 0.7\right)/u_1, \left(0.5, 0.4, 0.3\right)/u_2, \left(0.3, 0.6, 0.4\right)/u_3, \left(0.2, 0.6, 0.4\right)/u_5\right\}$$

Therefore, the value of the tp-ins-game is $\left\{\left(0.3, 0.4, 0.7\right)/u_1, \left(0.5, 0.4, 0.3\right)/u_2, \left(0.3, 0.6, 0.4\right)/u_3, \left(0.2, 0.6, 0.4\right)/u_5\right\}$.

6. Comparative Analysis on Different Types of Soft Games

Name of the game	Based on	Decision-makers View
Soft game (Deli et al., 2013)	Soft sets	Each element has crisp value(0 or 1)
Fuzzy soft game (Deli et al., 2014)	Fuzzy soft sets	Each element has a membership value and it belongs to [0,1] (0.4,0.6, etc)
Intuitionistic fuzzy soft game (Anjan et al., 2018)	Intuitionistic fuzzy soft sets	Each element has the membership and the non-membership values with the condition that their sum is less or equal to 1. (<0.4,0.5>,<0.5,0.4> ,etc)
Neutrosophic fuzzy soft game (Selvakumari et al.,2018)	Neutrosophic fuzzy soft sets	Each element has the truth-membership,falsity-membership and indeterminacy-membership and their sum is less or equal to 3. (<0.3,0.6,0.7>,<0.8,0.7,0.9>, etc)
Interval-valued neutrosophic soft game (Debnath,(in press))	Interval-valued neutrosophic soft sets	Each element has the truth-membership,indeterminacy-membership, falsity-membership, and each membership value is a subset of [0,1] and the sum of the supremum of three membership values is less or equal to 3. (<[0.3,0.5][0.7,0.9][0.6,0.8]>)
Intuitionistic neutrosophic soft game (Proposed study)	Intuitionistic neutrosophic soft sets	Each element has truth-membership,indeterminacy-membership, and the falsity-membership with the condition that their sum is less or equal to 2. (<0.5,0.6,0.4>)

7. Conclusion and Scope

In this paper, we have used the notion of an intuitionistic neutrosophic soft set [34]. After that, we have constructed tp-ins-games using the ins-payoff function and study some properties with examples. We then define ins-lower values and ins-upper values of a tp-ins-game and establish a relation between them. Also, the concept of tp-ins-games has been extended to N-person-ins-games and studies some of its properties. Finally, we give an application where the intuitionistic neutrosophic soft saddle point method is used to solve the financial problem.

In future work, the intuitionistic neutrosophic soft games may be applied in many fields such as computer science, decision-making, and so on to solve the related problems. Also, as the soft set was generalized to the hypersoft set[34] by Smarandache in 2018, therefore by using the notion of the hypersoft set, the proposed game theory can be further extended by introducing the intuitionistic neutrosophic hypersoft game and apply it to decision-making problem.

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