Neutrosophic Soft Block Matrices And Some Of Its Properties

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Abstract

In real life situations, there are many issues in which there are uncertainties, vagueness, complexities and unpredictability. Neutrosophic sets are a mathematical tool to address some issues which cannot be met using the existing methods. Neutrosophic soft matrices play a crucial role in handling indeterminant and inconsistent information during decision making process. The main focus of this article is to discuss the concept of neutrosophic sets, neutrosophic soft sets, neutrosophic soft matrices theory and finally to discuss about neutrosophic soft block matrics which are very useful and applicable in various situations involving uncertainties and imprecisions. In this article, neutrosophic soft block matrices, various types of neutrosophic soft block matrices, some operations on it along with some properties associated with it are discussed in details.

Keywords: Fuzzy sets, soft sets, soft matrix, neutrosophic sets, neutrosophic soft sets, neutrosophic soft matrix.

1.Introduction

The theory of fuzzy sets introduced by Zadeh[1], showed applications in many field of studies. This idea of fuzzy sets is welcome because it handles uncertainty and vagueness which cannot be met with classical set theory. Fuzzy set has membership function which assigns to each element of the Universe of discourse, a number from the unit interval [0, 1], to indicate the degree of belongingness of the set under consideration. In the fuzzy set theory, the membership of an element to a fuzzy set is a single value between zero and one. But in reality, it may not always be true that the degree of non membership of an element in a fuzzy set is equal to one minus the membership degree because there may be some hesitation degree as well. Therefore, a generalization of fuzzy set was realized by Atanassv [2], as intuitionistic fuzzy set in which the elements have degrees of membership and non membership which belong to the real unit interval [0,1] and the sum of these two functions belongs to the same interval.

Intuitionistic fuzzy sets as generalization of fuzzy sets is useful in some situation when the description of a problem by linguistic variable, given in terms of membership function only seems to be too difficult to handle. For example, in decision making problem particularly in the case of medical diagnosis, sales analysis, new product marketing, financial services etc, there is a fair chance of a non null hesitation part in each moment of evaluation of an unknown project.

In real life situations, most of the problems in economics, social sciences, environment etc, have various uncertainties. However, most of the existing mathematical tools for formal modeling, reasoning and computation are crisp deterministic and precise in character. There are theories namely, theory of probability, evidence, fuzzy set, intuitionistic fuzzy sets, rough sets etc for dealing with uncertainties.

These theories have their own difficulties as pointed out by Molodtsov[3], and as such the novel concept of soft set theory was initiated. Soft set theory has rich potential for application in solving practical problems in

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economics, social science, medical science etc. Maji *.et.al* ([4], [5]) have studied the theory of fuzzy soft set. Maji. *et. al* [6], have extended the theory of fuzzy soft set to intuitionistic fuzzy soft sets.

Intuitionistic fuzzy sets can only handle the incomplete information considering both the truth membership and falsity membership values. It does not handle the interminants and inconsistants information which exist in belief system.

Smarandache [7], introduced the concept of neutrosiphic sets as a mathematical tool to deal with some situations which involves impreciseness, inconsistencies and interminancy. It is expected that neutrosophic sets will produce more accurate result than those obtained by using fuzzy sets or intuitionistic fuzzy sets. Maji. *et. al* [8], have extended the theory of neutrosophic set to neutrosophic soft set. Later on Maji. *et. al* [9], have used the theory of neutrosophic soft set to decision making process.

Using these concepts, several mathematicians have initiated many research works in different mathematical structures, for instance Deli *et.al* [10, 11,12],. Later, this concept has been modified by Deli and Broumi [13] to develop the idea of neutrosophic soft matrices and its successful utilization in decision making process. Broumi and Smarandache [14] introduced the concept of intuitionistic neutrosophic sets and related properties. Accordingly, Bera and Mahapatra [15] introduce some view on algebraic structure on neutrosophic soft set.

Many researchers extended the concept of neutrosophic soft sets to neutrosophic parameterized soft sets, neutrosophic parameterized soft relations, interval neutrosophic soft sets, interval valued neutrosophic soft sets, interval valued parameterized neutrosophic soft sets, single valued neutrosophic soft sets, linear optimization of single valued neutrosophic soft sets, linear optimization of single valued neutrosophic soft sets, neutrosophic parameterized neutrosophic soft sets, interval valued neutrosophic parameterized interval valued neutrosophic soft sets which can be found in the references ([16],[17], [18],[19],[20], [21], [22]).
Further researches on the extension of neutrosophic soft sets to many other direction are going on and these are visible in valuable works ([23], [24], [25], [26])

In this article, the main aim is to introduce the concept of neutrosophic soft block matrices and thereafter to discuss about various types of neutrosophic block matrices. The transpose of neutrosophic soft block matrix will also be defined. In the process some operations on neutrosophic soft block matrices are defined and accordingly some properties will be discussed.

2. Definition and Preliminaries

Some basic definitions that are useful in subsequent sections of this article are discussed in this section.

Definition 1: Soft set(Molodsov, 1999)

Suppose that U is an initial universe of discourse and E is the set of parameters, let P (U) denote the power set of U. A pair (E, F) is called a soft set over U where F is a mapping given by $F : E \rightarrow P(U)$. Clearly soft set is a mapping from parameters to P (U).

Example: Let U= {u_1, u_2, u_3, u_4}be a set of four types of ornaments and E={costly(e_1), Medium(e_2), Cheap(e_3)} be the set of parameters. If $A = \{e_1, e_3\} \subseteq E$. Let F(e_1)={u_1, u_4} and F(e_3)={ u_2, u_3}. Then the soft set can be described as
(F, E)={(e_1,{u_1, u_4}), (e_3, { u_2, u_3})} over U which describes the " Quality of Ornaments" which MR. Z is going to buy.

This soft set can be represented in the following table 1:

U	Costly(e_1)	Medium(e_2)	Cheap(e_3)
u_1	1	0	0
u_2	0	0	1
u_3	0	0	1
u_4	1	0	0

Table1

Definition 2: Fuzzy soft set (Maji. *et. al.*, 2001)

Suppose that U is an initial universe of discourse and E is the set of parameters. Let $A \subseteq E$. A pair (F_A, E) is called fuzzy soft set over U where F_A is a mapping given by $F_A : E \to P(U)$.where $P(U)$ denotes the collection of all fuzzy subsets of U.

Definition 3: Intuitionistic fuzzy sets (Atanasav, 1986)

Let U be an universe of discourse. Then the intuitionistic fuzzy set A is an object having the form $A = \{< x, \mu_A(x), \nu_A(x) >, x \in U\}$, where the function $\mu_A(x), \nu_A(x) : U \to [0,1]$ define the degree of membership and the degree of non membership of the element $x \in X$ to the set A with the condition $0 \le \mu_A(x) + \nu_A(x) \le 1$

Definition 4: Intuitionistic fuzzy soft set (Maji, *et. al*, 2001)

Suppose that U is an initial universe of discourse and E is the set of parameters. Let P(U) denotes the collection of all intuitionistic fuzzy subsets of U. Let $A \subseteq E$. A pair (F_A,E) is called an intuitionistic fuzzy soft set over U where F_A is a mapping given by $F_A : E \to P(U)$

Definition 5: Neutrosophic sets (Smarandache, 2005)

Let U be the universe of discourse, The neutrosophic set A on the universe of discourse U is defined as $A = \{< T_A(x), I_A(x), F_A(x) >: x \in U\}$, where the characteristic functions $T, I, F : U \to [0,1]$ and $^-0 \le T + I + F \le 3^+$; T,I,F are neutrosophic components which defines the degree of membership, the degree of interminancy and the the degree of non membership respectively.

Definition 6: Neutrosophic soft set (Maji, 2013)

Suppose that U is an initial universe set and E is the set of parameters. Let P(U) denotes the collection of all neutrosophic subsets of U. Let $A \subseteq E$. A pair (F_A,E) is called neutrosophic soft set over U where F_A is a mapping given by $F_A : E \to P(U)$

Let us consider the following example for illustration purpose

Let U be the set of houses under consideration and E be the set of parameters where each parameters includes neutrosophic words.

Example: Let U={u_1, u_2, u_3, u_4, u_5,u_6}is a universal set of ornaments and E={ e_1, e_2, e_3, e_4} be the set of parameters where e_1, e_2, e_3, e_4 stands for costly, medium, cheap and very cheap.Let E={ e_1, e_2, e_4} Let us consider the following case:

$F_A(e_1)$={(u_1, 0.3, 0.4,0.2), (u_2, 0.5, 0.4, 0.1),(u_3,0.4, 0.4, 0.2), (u_4, 0.5, 0.2,0.1), (u_5, 0.6, 0.2,0.2), (u_6, 0.5,0.2, 0.2)}
$F_A(e_2)$={(u_1, 0.4, 0.5,0.1), (u_2, 0.4, 0.2,0.3),(u_3,0.1, 0.6,0.2), (u_4, 0.6, 0.2,0.1), (u_5, 0.3, 0.4,0.2), (u_6, 0.5, 0.3,0.1)}
$F_A(e_4)$={(u_1, 0.5, 0.2,0.2), (u_2,0.4, 0.5,0.1), (u_3, 0.5, 0.2,0.2), (u_4, 0.4, 0.2,0.3), (u_5, 0.4, 0.4,0.2), (u_6, 0.5, 0.3,0.1) }

The tabular representation of NSS (F_A,E) is

U	Costly(e_1)	Medium(e_2)	Very cheap(e_4)
u_1	(0.3, 0.4 ,0.2)	(0.4, 0.5,0.1)	(0.5, 0.2,0.2)
u_2	(0.5, 0.4, 0.1)	0.4, 0.2,0.3)	(0.4, 0.5,0.1)
u_3	(0.4, 0.4, 0.2)	(0.1, 0.6,0.2)	(0.5, 0.2,0.2)
u_4	(0.5, 0.2,0.1)	(0.6, 0.2,0.1)	(0.4, 0.2,0.3)
u_5	(0.6, 0.2,0.2)	(0.3, 0.4,0.2)	(0.4, 0.4,0.2)
u_6	(0.5,0.2, 0.2)	(0.5, 0.3,0.1)	(0.5, 0.3,0.1)

Definition 7: Neutrosophic Soft Matrix (Deli. *et. al*, 2015)

Let (F_A,E) is a neutrosophic soft set over U where F_A is a mapping given by $F_A : E \to P(U)$ where P(U) is the collection of all neutrosophic subsets of U. Then the subset of UXE is uniquely defined by $R_A = \{(u,e) : e \in A, u \in F_A(e)\}$ which is called a relation form of (F_A,E) .Now the relation R_A characterized by truth membership function $T_A : U \times E \to [0,1]$, interminancy membership function $I_A : U \times E \to [0,1]$ and falsity membership function $F_A : U \times E \to [0,1]$ where T_A(u,e) is the truth membership value, I_A(u,e) is the interminancy membership value and F_A(u,e) is the falsity membership value of the object u associated with the parameter e.

Let $U = \{u_1, u_2, u_3, ...u_m\}$ be the universe set and $E = \{x_1, x_2, x_3, ...x_n\}$ be the set of parameters. Then R_A can be represented by tabular form as follows:

R_N	e_1	e_2	……	e_n
u_1	$(T_{A_{11}}, I_{A_{11}}, F_{A_{11}})$	$(T_{A_{12}}, I_{A_{12}}, F_{A_{12}})$	……	$(T_{A_{1n}}, I_{A_{1n}}, F_{A_{1n}})$
u_2	$(T_{A_{21}}, I_{A_{21}}, F_{A_{21}})$	$(T_{A_{22}}, I_{A_{22}}, F_{A_{22}})$	……	$(T_{A_{2n}}, I_{2n}, F_{A_{2n}})$
:			……	
u_m	$(T_{A_{m2}}, I_{A_{m2}}, F_{A_{m2}})$	$(T_{A_{m2}}, I_{A_{m2}}, F_{A_{m2}})$	⋯	$(T_{A_{mn}}, I_{A_{mn}}, F_{A_{mn}})$

Where

$(T_{A_{mn}}, I_{A_{mn}}, F_{A_{mn}})$=($T_A$($u_m$,$e_n$), I_A(u_m,e_n), F_A(u_m,e_n)). If a_{ij}=(T_A(u_i,e_j), I_A(u_i,e_j), F_A(u_i,e_j)) we can define a matrix

$$a_{ij} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ : & : & : & : \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

This is called neutrosophic soft matrix of order mxn corresponding to the neutrosophioc soft set (F_A,E) over U.

Example: Let U={u_1, u_2, u_3, u_4, u_5,u_6}is a universal set. and E={ e_1, e_2, e_3, e_4} be the set of parameters and let
E={ e_1, e_2, e_3}

$F_A(e_1)$={(u_1, 0.3, 0.4,0.2), (u_2, 0.5, 0.4,0.3),(u_3,0.4, 0.5,0.4), (u_4, 0.6, 0.3,0.2), (u_5, 0.8, 0.1,0.3), (u_6, 0.7, 0.2,0.1)}
$F_A(e_2)$={(u_1, 0.4, 0.5,0.2), (u_2, 0.6, 0.2,0.3),(u_3,1, 0,0.4), (u_4, 0.6, 0.2,0.5), (u_5, 0.3, 0.4,0.3), (u_6, 0.5, 0.4,0.4)}
$F_A(e_3)$={(u_1, 0.6, 0.2,0.3), (u_2,0.4, 0.3,0.3), (u_3, 0.5, 0.1,0.4), (u_4, 0.4, 0.2,0.3), (u_5, 0.6, 0.4,0.2), (u_6, 0.7, 0.3,0.2) }

Then the NSS (F_A, E) is a parameterized family {$F_A(e_1)$, $F_A(e_2)$, $F_A(e_3)$ }of all NSS over U and gives an approximate description of the object.
Hence neutrosophic soft matrix can be represented by

$$A = \begin{bmatrix} (0.3,0.4,0.2) & (0.4,0.5,0.2) & (0.6,0.2,0.3) \\ (0.5,0.4,0.3) & (0.6,0.2,0.3) & (0.4,0.3,0.3) \\ (0.4,0.5,0.4) & (0.1,0.0,0.4) & (0.5,0.1,0.4) \\ (0.6,0.3,0.2) & (0.6,0.2,0.5) & (0.4,0.2,0.3) \\ (0.8,0.1,0.3) & (0.3,0.4,0.3) & (0.6,0.4,0.2) \\ (0.7,0.2,0.1) & (0.5,0.4,0.4) & (0.7,0.3,0.2) \end{bmatrix}$$

3. **Operations on Neutrosophic Soft matrices**

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3.1 Addition of neutrosophic soft matrices

Let $A=[(T_{A_{ij}},I_{A_{ij}},F_{A_{ij}})]$, $B=[(T_{B_{ij}},I_{B_{ij}},F_{B_{ij}})]$ be two neutrosophic soft matrices. Then the max-min product of the two neutrosophic soft matricesA and B is denoted as $A+B$ is defined as

$A+B=[\max(T_{A_{ij}},T_{B_{ij}}),\min(I_{A_{ij}},I_{B_{ij}}),\min(F_{A_{ij}},F_{B_{ij}})]$ for all i and j.

3.2 Max-min product of neutrosophic soft matrices

Let $A=[(T_{A_{ij}},I_{A_{ij}},F_{A_{ij}})]$, $B=[(T_{B_{ij}},I_{B_{ij}},F_{B_{ij}})]$ be two neutrosophic soft matrices. Then the max-min product of the two neutrosophic soft matricesA and B is denoted as $A*B$ is defined as

$A*B=[\max\min(T_{A_{ij}},T_{B_{ij}}),\min\max(I_{A_{ij}},I_{B_{ij}}),\min\max(F_{A_{ij}},F_{B_{ij}})]$ for all i and j.

3.3 Transpose of neutrosophic soft matrices

Let $A=[(T_{A_{ij}},I_{A_{ij}},F_{A_{ij}})]$ be a neutrosophic soft matrix. Then the transpose of this neutrosophic soft matrix will be defined by denoted by A^T and is defined by $A^T=[(T_{A_{ji}},I_{A_{ji}},F_{A_{ji}})]$

4. Neutrosophic soft block matrices

In this section, neutrosophic soft block matrices and its related properties will be discussed.

4.1 Neutrosofic soft block matrix

A matrix may be subdivided into sub-matrices by drawing lines parallel to its rows and columns. These sub-matrices may be considered as the elements of the original matrices.

For example

$$A=\begin{bmatrix} (T_{A_{11}},I_{A_{11}},F_{A_{11}}) & (T_{A_{12}},I_{A_{12}},F_{A_{12}}) & : & (T_{A_{13}},I_{A_{13}},F_{A_{13}}) & (T_{A_{14}},I_{A_{14}},F_{A_{14}}) \\ \cdots & \cdots & & \cdots & \cdots \\ (T_{A_{21}},I_{A_{21}},F_{A_{21}}) & (T_{A_{22}},I_{A_{22}},F_{A_{22}}) & : & (T_{A_{23}},I_{A_{23}},F_{A_{23}}) & (T_{A_{24}},I_{A_{24}},F_{A_{24}}) \\ (T_{A_{31}},I_{A_{31}},F_{A_{31}}) & (T_{A_{32}},I_{A_{32}},F_{A_{32}}) & : & (T_{A_{33}},I_{A_{33}},F_{A_{33}}) & (T_{A_{34}},I_{A_{34}},F_{A_{34}}) \end{bmatrix}$$

We may write the above matrix as

$$A=\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$$

Where

$$P_{11}=\left[(T_{A_{11}},I_{A_{11}},F_{A_{11}}) \quad (T_{A_{12}},I_{A_{12}},F_{A_{12}})\right]$$

$$P_{12}=\left[(T_{A_{13}},I_{A_{13}},F_{A_{13}}) \quad (T_{A_{14}},I_{A_{14}},F_{A_{14}})\right]$$

$$P_{21}=\begin{bmatrix} (T_{A_{21}},I_{A_{21}},F_{A_{21}}) & (T_{A_{22}},I_{A_{22}},F_{A_{22}}) \\ (T_{A_{31}},I_{A_{31}},F_{A_{31}}) & (T_{A_{32}},I_{A_{32}},F_{A_{32}}) \end{bmatrix}, \quad P_{22}=\begin{bmatrix} (T_{A_{23}},I_{A_{23}},F_{A_{23}}) & (T_{A_{24}},I_{A_{24}},F_{A_{24}}) \\ (T_{A_{33}},I_{A_{33}},F_{A_{33}}) & (T_{A_{34}},I_{A_{34}},F_{A_{34}}) \end{bmatrix}$$

Then

$A=\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$ is an example of neutrosophic soft block matrix.

So the matrix A is partitioned. The dotted lines divided the matrix into sub-matrices P_{11}, P_{12}, P_{21}, P_{22} are the sub matrices. The matrix A can be partitioned in several ways.

4.2 Square neutrosophic soft block matrix

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If the number of rows and the number of columns of blocks are equal then the matrix is said to be square fuzzy block matrix.

For example

$$A=\begin{bmatrix} (T_{A_{11}},I_{A_{11}},F_{A_{11}}) & (T_{A_{12}},I_{A_{12}},F_{A_{12}}) & : & (T_{A_{13}},I_{A_{13}},F_{A_{13}}) & (T_{A_{14}},I_{A_{14}},F_{A_{14}}) & : & (T_{A_{15}},I_{A_{15}},F_{A_{15}}) & (T_{A_{16}},I_{A_{16}},F_{A_{16}}) \\ (T^A_{21},I^A_{21},F^A_{21}) & (T^A_{22},I^A_{22},F^A_{22}) & : & (T^A_{23},I^A_{23},F^A_{23}) & (T^A_{24},I^A_{24},F^A_{24}) & : & (T_{A_{25}},I_{A_{25}},F_{A_{25}}) & (T_{A_{26}},I_{A_{26}},F_{A_{26}}) \\ \cdots & \cdots & .. & \cdots & \cdots & \cdots & \cdots & \cdots \\ (T_{A_{31}},I_{A_{31}},F_{A_{31}}) & (T_{A_{32}},I_{A_{32}},F_{A_{32}}) & : & (T_{A_{33}},I_{A_{33}},F_{A_{33}}) & (T_{A_{34}},I_{A_{34}},F_{A_{34}}) & : & (T_{A_{35}},I_{A_{35}},F_{A_{35}}) & (T_{A_{36}},I_{A_{36}},F_{A_{36}}) \\ (T_{A_{41}},I_{A_{41}},F_{A_{41}}) & (T_{A_{42}},I_{A_{42}},F_{A_{42}}) & : & (T_{A_{43}},I_{A_{43}},F_{A_{43}}) & (T_{A_{44}},I_{A_{44}},F_{A_{44}}) & : & (T^A_{45},I^A_{45},F^A_{45}) & (T^A_{46},I^A_{46},F^A_{46}) \end{bmatrix}$$

or

$$A=\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \end{bmatrix}$$

is a square fuzzy block matrix since all A_{ij} 's are square blocks.

Numerical Example:

$$A=\begin{bmatrix} (0.5,0.4,0.3) & (0.6,0.7,0.2) & : & (0.4,0.6,0.5) & (0.5,0.4,0.5) & : & (0.5,0.4,0.3) & (0.6,0.4,0.4) \\ (0.4,0.5,0.5) & (0.7,0.2,0.3) & : & (0.8,0.3,0.3) & (0.5,0.4,0.3) & : & (0.8,0.2,0.1) & (0.6,0.2,0.3) \\ \cdots & \cdots & .. & \cdots & \cdots & \cdots & \cdots & \cdots \\ (0.6,0.4,0.3) & (0.7,0.2,0.4) & : & (0.5,0.1,0.3) & (0.6,0.3,0.3) & : & (0.4,0.5,0.4) & (0.7,0.2,0.4) \\ (0.5,0.4,0.5) & (0.6,0.4,0.5) & : & (0.5,0.5,0.3) & (0.6,0.4,0.3) & : & (0.5,0.4,0.5) & (0.6,0.5,0.4) \end{bmatrix}$$

Thus A is an example of square fuzzy block matrix since all the blocks considered here are themselves square neutrosophic soft matrices.

4.3 Rectangular neutrosophic soft block matrix

If the number of rows and the number of columns of blocks are unequal then the matrix is said to be rectangular neutrosophic soft block matrix.

Numerical Example:

$$A=\begin{bmatrix} (0.8,0.3,0.2) & (0.6,0.3,0.3) & : & (0.7,0.3,0.4) & (0.6,0.5,0.3) \\ \cdots & \cdots & & \cdots & \cdots \\ (0.5,0.3,0.4) & (0.6,0.4,0.4) & : & (0.5,0.1,0.4) & (0.8,0.3,0.1) \\ (0.6,0.2,0.4) & (0.5,0.3,0.2) & : & (0.5,0.3,0.5) & (0.6,0.3,0.2) \end{bmatrix}$$

The above neutrosophic block matrix is rectangular neutrosophic soft block matrix because each block is not of the same order.

5. Operations on neutrosophic soft block matrices

5.1 Addition of two neutrosophic soft block matrices

Let $A=\begin{bmatrix} A_{11} & : & A_{12} \\ ..... & : & ..... \\ A_{21} & : & A_{22} \end{bmatrix}$ and $B=\begin{bmatrix} B_{11} & : & B_{12} \\ ..... & : & ..... \\ B_{21} & : & B_{22} \end{bmatrix}$ be two matrices of the same order and are

partitioned identically, then the addition of two neutrosophic soft block matrices can be defined as

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$$A+B=\begin{bmatrix} A_{11}+B_{11} & : & A_{12}+B_{12} \\ ..... & : & ..... \\ A_{21}+B_{21} & : & A_{22}+B_{22} \end{bmatrix}$$

5.2 Properties of addition of fuzzy block matrices

If three neutrosophic soft block matrices are represented as

$$A=\begin{bmatrix} A_{11} & : & A_{12} \\ ..... & : & ..... \\ A_{21} & : & A_{22} \end{bmatrix},\ B=\begin{bmatrix} B_{11} & : & B_{12} \\ ..... & : & ..... \\ B_{21} & : & B_{22} \end{bmatrix} \text{ and } C=\begin{bmatrix} C_{11} & : & C_{12} \\ ..... & : & ..... \\ C_{21} & : & C_{22} \end{bmatrix}$$

Then

i. A+B=B+A

ii. A+(B+C)=(A+B+C)

The addition of the above two block matrices will be as

$$A+B=\begin{bmatrix} A_{11}+B_{11} & : & A_{12}+B_{12} \\ ..... & : & ..... \\ A_{21}+B_{21} & : & A_{22}+B_{22} \end{bmatrix}$$

$$B+A=\begin{bmatrix} B_{11}+A_{11} & : & B_{12}+A_{12} \\ ..... & : & ..... \\ B_{21}+A_{21} & : & B_{22}+A_{22} \end{bmatrix}$$

From the above it can be concluded that A+B=B+A.

Similarly it can be proved that A+(B+C)=(A+B)+C

5.3 Max-min operations on neutrosophic soft block matrix

If A and B be two neutrosophic soft block matrices are represented as

$$A=\begin{bmatrix} A_{11} & : & A_{12} \\ ..... & : & ..... \\ A_{21} & : & A_{22} \end{bmatrix},\ B=\begin{bmatrix} B_{11} & : & B_{12} \\ ..... & : & ..... \\ B_{21} & : & B_{22} \end{bmatrix}$$

Then the product of two neutrosophic soft block matrices will be represented by

$$AB=\begin{bmatrix} A_{11}B_{11}+A_{12}B_{21} & : & A_{11}B_{12}+A_{12}B_{22} \\ ..... & : & ..... \\ A_{21}B_{11}+A_{22}B_{21} & : & A_{21}B_{12}+A_{22}B_{22} \end{bmatrix}$$

Where each block is conformable for multiplication i.e the number of columns of one block should be equal to the number of rows of the other block which are taken into consideration.

5.4 Transpose of neutrosophic soft block matrix

If $A=\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$

be a neutrosophic soft block matrix, then the transpose of that neutrosophic soft block matrix is defined as

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$$A^T = \begin{bmatrix} P_{11}{}^T & P_{12}{}^T \\ P_{21}{}^T & P_{22}{}^T \end{bmatrix}$$

Where

$$P_{11}{}^T = \begin{bmatrix} (T_{11}^A, I_{11}^A, F_{11}^A) \\ (T_{12}^A, I_{12}^A, F_{12}^A) \end{bmatrix},$$

$$P_{12}{}^T = \begin{bmatrix} (T_{13}^A, I_{13}^A, F_{13}^A) \\ (T_{14}^A, I_{14}^A, F_{14}^A) \end{bmatrix}$$

$$P_{21}{}^T = \begin{bmatrix} (T_{21}^A, I_{21}^A, F_{21}^A) & (T_{31}^A, I_{31}^A, F_{31}^A) \\ (T_{22}^A, I_{22}^A, F_{22}^A) & (T_{32}^A, I_{32}^A, F_{32}^A) \end{bmatrix}$$

And

$$P_{22}{}^T = \begin{bmatrix} (T_{23}^A, I_{23}^A, F_{23}^A) & (T_{33}^A, I_{33}^A, F_{33}^A) \\ (T_{24}^A, I_{24}^A, F_{24}^A) & (T_{34}^A, I_{34}^A, F_{34}^A) \end{bmatrix}$$

5.5 Properties of transpose of neutrosophic soft block matrix

If A and B be two fuzzy block matrices then the following properties hold:

i. $(A+B)^T = A^T + B^T$

ii. $(A^T)^T = A$

iii. $(kA)^T = kA^T$

Proof of (i)

Let us consider two neutrosophic soft partition matrices as

$$A = \begin{bmatrix} (T^A{}_{11}, I^A{}_{11}, F_{11}{}^A) & (T^A{}_{12}, I^A{}_{12}, F_{12}{}^A) & : & (T^A{}_{13}, I^A{}_{13}, F_{13}{}^A) & (T^A{}_{14}, I^A{}_{14}, F_{14}{}^A) \\ \dots & \dots & & \dots & \dots \\ (T^A{}_{21}, I^A{}_{21}, F_{21}{}^A) & (T^A{}_{22}, I^A{}_{22}, F_{22}{}^A) & : & (T^A{}_{23}, I^A{}_{23}, F_{23}{}^A) & (T^A{}_{24}, I^A{}_{24}, F_{24}{}^A) \\ (T^A{}_{31}, I^A{}_{31}, F_{31}{}^A) & (T^A{}_{32}, I^A{}_{32}, F_{32}{}^A) & : & (T^A{}_{33}, I^A{}_{33}, F_{33}{}^A) & (T^A{}_{34}, I^A{}_{34}, F_{34}{}^A) \end{bmatrix}$$

and if it is denoted by

$A = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$ then $A^T = \begin{bmatrix} P_{11}{}^T & P_{12}{}^T \\ P_{21}{}^T & P_{22}{}^T \end{bmatrix}$ and so $kA^T = \begin{bmatrix} kP_{11}{}^T & kP_{12}{}^T \\ kP_{21}{}^T & kP_{22}{}^T \end{bmatrix}$

Again

$kA = \begin{bmatrix} kP_{11} & kP_{12} \\ kP_{21} & kP_{22} \end{bmatrix}$ and so $(kA)^T = \begin{bmatrix} kP^T{}_{11} & kP_{12}^T \\ kP_{21}^T & kP_{22}^T \end{bmatrix}$

Hence the proposition $(kA)^T = kA^T$

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$(A^T)^T = \begin{bmatrix} (P_{11}{}^T)^T & (P_{12}{}^T)^T \\ (P_{21}{}^T)^T & (P_{22}{}^T)^T \end{bmatrix}$ and if we find the transposes of the elements which sub-matrices $(P^T_{11})^T$, $(P^T_{12})^T$, $(P^T_{21})^T$, $(P^T_{22})^T$ of this partitioned neutrosophic matrix then it can easily seen that

$(P^T_{11})^T = P_{11}, (P^T_{12})^T = P_{12}, (P^T_{21})^T = P_{21}$ and $(P^T_{22})^T = P_{22}$

Hence the proposition $(A^T)^T = A$ holds true.

$$B = \begin{bmatrix} (T^B{}_{11}, I^B{}_{11}, F_{11}{}^B) & (T^B{}_{12}, I^B{}_{12}, F_{12}{}^B) & : & (T^B{}_{13}, I^B{}_{13}, F_{13}{}^B) & (T^B{}_{14}, I^B{}_{14}, F_{14}{}^B) \\ \dots & \dots & & \dots & \dots \\ (T^B{}_{21}, I^B{}_{21}, F_{21}{}^B) & (T^B{}_{22}, I^B{}_{22}, F_{22}{}^B) & : & (T^B{}_{23}, I^B{}_{23}, F_{23}{}^B) & (T^B{}_{24}, I^B{}_{24}, F_{24}{}^B) \\ (T^B{}_{31}, I^B{}_{31}, F_{31}{}^B) & (T^B{}_{32}, I^B{}_{32}, F_{32}{}^B) & : & (T^B{}_{33}, I^B{}_{33}, F_{33}{}^B) & (T^B{}_{34}, I^B{}_{34}, F_{34}{}^B) \end{bmatrix}$$

$$B = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix}$$

$A + B = \begin{bmatrix} P_{11} + Q_{11} & : & P_{12} + Q_{12} \\ \dots\dots & : & \dots\dots \\ P_{21} + Q_{21} & : & P_{22} + Q_{22} \end{bmatrix}$ and so $(A+B)^T = \begin{bmatrix} (P_{11} + Q_{11})^T & : & (P_{12} + Q_{12})^T \\ \dots\dots & : & \dots\dots \\ (P_{21} + Q_{21})^T & : & (P_{22} + Q_{22})^T \end{bmatrix}$

$P_{11} = \left[(T^A{}_{11}, I^A{}_{11}, F^A_{11}) \quad (T^A{}_{12}, I^A{}_{12}, F^A_{12}) \right]$ and $Q_{11} = \left[(T^B{}_{11}, I^B{}_{11}, F^B_{11}) \quad (T^B{}_{12}, I^B{}_{12}, F^B_{12}) \right]$

$P_{11} + Q_{11} = [\{\max(T^A{}_{11}, T^B{}_{11}), \min(I^A{}_{11}, I^B{}_{11}), \min(F^A_{11}, F^B{}_{11})\},$
$\{\max(T^A{}_{12}, T^B{}_{12}), \min(I^A{}_{12}, I^B{}_{12}), \min(F^A_{12}, F^B{}_{12})\}]$

$$(P_{11} + Q_{11})^T = \begin{bmatrix} \{\max(T^A{}_{11}, T^B{}_{11}), \min(I^A{}_{11}, I^B{}_{11}), \min(F^A_{11}, F^B{}_{11})\} \\ \{\max(T^A{}_{12}, T^B{}_{12}), \min(I^A{}_{12}, I^B{}_{12}), \min(F^A_{12}, F^B{}_{12})\} \end{bmatrix}$$

$P^T_{11} = \begin{bmatrix} (T^A{}_{11}, I^A{}_{11}, F^A_{11}) \\ (T^A{}_{12}, I^A{}_{12}, F^A_{12}) \end{bmatrix}$ and $Q^T_{11} = \begin{bmatrix} (T^B{}_{11}, I^B{}_{11}, F^B_{11}) \\ (T^B{}_{12}, I^B{}_{12}, F^B_{12}) \end{bmatrix}$

$$(P_{11})^T + (Q_{11})^T = \begin{bmatrix} \{\max(T^A{}_{11}, T^B{}_{11}), \min(I^A{}_{11}, I^B{}_{11}), \min(F^A_{11}, F^B{}_{11})\} \\ \{\max(T^A{}_{12}, T^B{}_{12}), \min(I^A{}_{12}, I^B{}_{12}), \min(F^A_{12}, F^B{}_{12})\} \end{bmatrix}$$

Hence it can be seen that $(P_{11} + Q_{11})^T = P_{11}{}^T + Q_{11}{}^T$

Similarly for the others.

Then we have

$A^T = \begin{bmatrix} P_{11}{}^T & P_{12}{}^T \\ P_{21}{}^T & P_{22}{}^T \end{bmatrix}$ and $B^T = \begin{bmatrix} Q^T_{11} & Q_{12}{}^T \\ Q_{21}{}^T & Q_{22}{}^T \end{bmatrix}$

And so

$$A^T + B^T = \begin{bmatrix} P_{11}{}^T + Q_{11}{}^T & P_{12}{}^T + Q_{12}{}^T \\ P_{21}{}^T + Q_{21}{}^T & P_{22}{}^T + Q_{22}{}^T \end{bmatrix}$$

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6. Conclusions

In this article, the concept of neutrosophic soft block matrices are developed and accordingly various types of neutrosophic soft block matrices are studied. Some operations on neutrosophic soft block matrices are also discussed. Thereafter some properties of neutrosophic soft block matrices are studied and it is found that neutrosophic soft block matrices behave in the same way as those of other block matrices that exist in the literature.

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