# Neutrosophic Ideal layers & Some Generalizations for GIS Topological Rules

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**ABSTRACT.**

This paper aims to introduce and study some new neutrosophic fuzzy pairwise notions via neutrosophic fuzzy ideals. We, also generalize the notion of FPL-open sets. In addition to generalizing the concept of FPL-closed sets and NPL-open function, the relationship between the above new neutrosophic Fuzzy pairwise notions and there other relevant classes are investigated. In Geographical information systems (GIS) there is a need to statistically model spatial regions with indeterminate boundary and under indeterminacy. Possible applications to GIS rules are touched upon.

**Keywords**: Neutrosophic set; Neutrosophic topology; Neutrosophic ideal open set; Neutrosophic closed set; GIS Neutrosophic Rules; Neutrosophic Statistical Model

**Introduction**

Since the world is full of indeterminacy, the neutrosophic found their place into contemporary research. The neutrosophic set was introduced by Smarandache in [3,4,7] and Salama et al. [5, 6, 8,10, 11, 12, 13, 14] introduced the neutrosophic crisp set, neutrosophic topological spaces and many applications in statistics, computer science and information systems. Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their classical and fuzzy counterparts, such as neutrosophic set theory, in this paper is to introduce and study some new neutrosophic fuzzy pairwise notion via neutrosophic fuzzy pairwise ideals. We, also generalize the notion of FPL-open sets due to Abd El-Monsef, et. al [1, 2]. In addition to generalizing the concept of FPL-closed sets. In Geographical information systems (GIS) there is a need to statistically model spatial regions with indeterminate boundary and under indeterminacy. Possible applications to GIS rules are touched upon.

**PRELIMINARIES**

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [3, 4, 7], and Salama et al. [8-13].

**2. Neutrosophic Fuzzy Pairwise Set and its Neutrosophic Fuzzy Pairwise Local Function.**

**Definition 2.1.** Given $(X,\tau_i)$, i∈{1,2} be an NFTS with neutrosophic fuzzy ideal L on X, μ, σ, u∈$I^X$. Then <μ,σ,u> is said to be :

(i) Neutrosophic fuzzy pairwise $\tau^*_i$-closed,i∈{1,2} may have two types

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Type 1: if $\mu^*\leq\mu$, $u^*\geq u$, $\sigma^*\leq\sigma$ .

Type 2: if $\mu^*\leq\mu$, $u^*\geq u$, $\sigma^*\geq\sigma$

(or PN-*closed) if $<\mu,\sigma,u>^* \leq <\mu,\sigma,u>$

(ii) Fuzzy neutrosophic pairwise NPL-dense – in – itself may have two types

Type 1: if $\mu^*\leq\mu, u^*\geq u$, $\sigma^*\leq\sigma$ .

Type 2: if $\mu^*\leq\mu, u^*\geq u$, $\sigma^*\geq\sigma$

(or PN*-dense- in – itself ) if $<\mu,\sigma,u> \subseteq <\mu,\sigma,u>^*$.

(iii) Neutrosophic fuzzy pairwise*-perfect if $<\mu,\sigma,u>$ is PN*-closed and PN*- dense – in itself.

**Theorem 2.1:** Given $(X,\tau_i)$, $i\in\{1,2\}$ be a nfbts with neutrosophic fuzzy ideal L on X , $\mu,\sigma,u\in I^X$ then $<\mu,\sigma,u>$ is

(i) PN*- closed iff $Ncl^*(<\mu,\sigma,u>) = <\mu,\sigma,u>$ .

(ii) PN*- dense – in – itself iff $Ncl^*(<\mu,\sigma,u>) = <\mu,\sigma,u>^*$.

(iii) PN*- perfect iff $Ncl^*(<\mu,\sigma,u>) = <\mu,\sigma,u>^* = <\mu,\sigma,u>$ .

**Proof:** Follows directly from the neutrosophic fuzzy pairwise closure operator Ncl* for a neutrosophic fuzzy pairwise $\tau^*_i(L)$, $i \in \{1,2\}$ in and Definition 2.1.

**Remark 2.1:** One can deduce that

(i) Every PN*-dense- in – itself is a neutrosophic fuzzy pairwise dense set.

(ii) Every neutrosophic fuzzy pairwise closed (resp. neutrosophic fuzzy pairwise open) set is PN*-closed (resp. $PN\tau^*_i -$ open, $i \in \{1,2\}$).

**Corollary 2.1:** Given $(X,\tau_i)$, $i\in\{1,2\}$ be a nfbts with neutrosophic fuzzy ideal L on X , $<\mu,\sigma,u> \in\tau_i$ then we have :

(i) If $<\mu,\sigma,u>$ is PN*-closed then $<\mu,\sigma,u>^* \leq Nint(<\mu,\sigma,u>) \leq Ncl(<\mu,\sigma,u>)$.

(ii) If $<\mu,\sigma,u>$ is PN*-dense- itself then $Nint(<\mu,\sigma,u>) \leq <\mu,\sigma,u>^*$.

(iii) If $<\mu,\sigma,u>$ is PN*- perfect then $Nint(<\mu,\sigma,u>) = Ncl(<\mu,\sigma,u>) = <\mu,\sigma,u>^*$.

**Proof:** Obvious.

**Theorem 2.2:** Given $(X,\tau_i)$, $i\in\{1,2\}$ be a nfbts with neutrosophic fuzzy ideal $L_n$ on X , $\mu,\sigma,u\in I^X$ then we have the following: $<\mu,\sigma,u>$ is neutrosophic fuzzy pairwise α - closed iff $<\mu,\sigma,u>$ is PN*- closed.

**Proof:** It's clear.

**Corollary 2.2:** For a nbts $(X,\tau_i)$, $i\in\{1,2\}$ with neutrosophic fuzzy ideal L on X , $\mu,\sigma,u\in I^X$, the following holds:

(i) If $<\mu,\sigma,u> \in PNC(X)$ then $<\mu,\sigma,u>$ is PN*- closed.

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(ii) If <μ,σ,u> ∈PNβC(X) then Nint (Nint(<μ,σ,u>*)) ≤ <μ,σ,u>.

(iii) If <μ,σ,u> ∈PNSC(X) then Nint(<μ,σ,u>*) ≤ <μ,σ,u>.

**Proof:** Obvious.

**3. Neutrosophic Fuzzy Pairwise L-Open and Neutrosophic Fuzzy Pairwise L- Closed Sets.**

**Definition 3.1**. Given $(X,\tau_i)$, $i\epsilon\{1,2\}$ be a NFBTS with neutrosophic fuzzy ideal L on X , $\mu, \sigma, u\epsilon I^x$ and <μ,σ,u> is called a neutrosophic fuzzy pairwise $N$L -open set iff there exists $<\xi, \rho, \theta > \epsilon\tau_i, i\epsilon\{1,2\}, P =< p_1, p_2, p_3 >$ such that $< \mu, \sigma, u > \subseteq < \xi, \rho, \theta > \subseteq P(< \mu, \sigma, u >^*)(L, \tau_i), i\epsilon\{1,2\}$.

We will denote the family of all neutrosophic fuzzy pairwise NL –open $(X,\tau_i) = \mu, \sigma, u\epsilon I^x\colon < \mu, \sigma, u > \subseteq \tau_1 - \mathrm{Nint}[P(< \mu, \sigma, u >^*)(L, \tau_1)\ ]\text{and} < \mu, \sigma, u > \subseteq \tau_2 - \mathrm{Nint}[P(< \mu, \sigma, u >^*)(L, \tau_2)], i\epsilon\{1,2\}$ (simplify NPLO(X)) when there is no chance for confusion.

**Theorem 3.1:** Let $(X,\tau_i), i\epsilon\{1,2\}$ be a nfbts with neutrosophic fuzzy ideal L, then $< \mu, \sigma, u > \epsilon\mathrm{NPOL}(X)$

iff $< \mu, \sigma, u > \subseteq \tau_i - \mathrm{int}\big(P(< \mu, \sigma, u >^*)(L, \tau_i)\big)$for $i\epsilon\{1,2\}, P =< p_1, p_2, p_3 >$.

**Proof:** Assume that $< \mu, \sigma, u > \epsilon\mathrm{NPOL}(X)$ then Definition 3.1.1. there exists $<\xi, \rho, \theta > \epsilon\tau_i$

such that $< \mu, \sigma, u > \subseteq < \xi, \rho, \theta > \subseteq P(< \mu, \sigma, u >^*)(L, \tau_i), i\epsilon\{1,2\}$. But $\mathrm{Nint}\big(P(< \mu, \sigma, u >^*)\big) \subseteq P(< \mu, \sigma, u >^*)$,

put $<\xi, \rho, \theta >= \mathrm{Nint}\big(P(< \mu, \sigma, u >^*)\big)$. Hence $< \mu, \sigma, u > \subseteq \mathrm{Nint}\big(P(< \mu, \sigma, u >^*)\big)$.

Conversely $< \mu, \sigma, u > \subseteq \mathrm{Nint}(P(< \mu, \sigma, u >^*)) \subseteq P(< \mu, \sigma, u >^*)$.

Then there exists $<\xi, \rho, \theta >= \mathrm{Nint}(P(< \mu, \sigma, u >^*))\epsilon\tau_i$. Hence $< \mu, \sigma, u >$ in NPLO(X).

**Definition 3.2.** The largest $\tau_i - \mathrm{NPL} - \mathrm{open}(\text{simply } \tau_i - \mathrm{NPLO}(X))$ set contained in $< \mu, \sigma, u >$ is called a $\tau_i - \mathrm{NPL} - \text{neutrosophic interior of } < \mu, \sigma, u >$. The complement of the neutrosophic fuzzy pairwise NL-open subset of X is a neutrosophic fuzzy pairwise NL-closed subset of X (simply NPLC(X)).

We denoted by NPL-Nint($< \mu, \sigma, u >$).

**Theorem 3.2.** Let $(X,\tau_i), i\epsilon\{1,2\}$ be a nfbts with neutrosophic fuzzy ideal L, $\mu, \sigma, u\epsilon I^x$ and j is an arbitrary set then

i- The union of neutrosophic fuzzy pairwise NL-open subsets may be neutrosophic fuzzy neutrosophic pairwise NL-open.

ii- If $\nu =< \nu_1, \nu_2, \nu_3 >$ is neutrosophic fuzzy pairwise open and <μ,σ,u> may be neutrosophic fuzzy pairwise NL-open subset of X. Then $< \mu, \sigma, u > \cap\ \nu$ may be pairwise NL-open subset.

**Proof.** (i) Let $\{< \mu, \sigma, u >_J : J\epsilon j\}$ be a family of NPLO(X). Then for each

$J\epsilon j, < \mu, \sigma, u >_J \subseteq \tau_i - \mathrm{Nint}(p < \mu, \sigma, u >^*{}_J)$

and so $Y_J < \mu, \sigma, u >_J \subseteq Y_J\tau_i - \mathrm{Nint}\left(p < \mu, \sigma, u >^*{}_J\right) \subseteq \tau_i - \mathrm{Nint}(Y_J p < \mu, \sigma, u >^*{}_J)^*$.

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(ii) Assume that $\nu = <\nu_1, \nu_2, \nu_3>$ is neutrosophic fuzzy pairwise open and <μ,σ,u> may be neutrosophic fuzzy pairwise NL-open subsets of X.

Then $<\mu,\sigma,u> \cap \nu \subseteq \nu \cap \left(\tau_i - \mathrm{Nint}\left(P(<\mu,\sigma,u>^*)\right)\right) \subseteq \tau_i - \mathrm{Nint}\left(\nu \cap P(<\mu,\sigma,u>^*)\right) \subseteq$

$\tau_i - \mathrm{Nint}P(\nu \cap <\mu,\sigma,u>)^*$.

**Definition 3.3.** Let $(X,\tau_i)$, $i\in\{1,2\}$ be a nfbts with neutrosophic fuzzy ideal L on X, $\mu, \sigma, u \in I^X$. Then <μ,σ,u> is said to be neutrosophic fuzzy.

i- $\tau^*{}_i$ - closed iff $\tau^*{}_i - N\mathrm{cl}^*(<\mu,\sigma,u>) = <\mu,\sigma,u>$.

ii- $\tau^*{}_i - \mathrm{dense} - \mathrm{in} - \mathrm{itself}$ if $<\mu,\sigma,u> \subseteq P(<\mu,\sigma,u>^*)(L,\tau_i)$.

iii- $\tau^*{}_i - \mathrm{perfect}$ if $<\mu,\sigma,u>$ is $\tau^*{}_i -$ closed and $\tau^*{}_i - \mathrm{dense} - \mathrm{in\ itself}$.

**Proof.** Follows directly from the neutrosophic fuzzy closure operator for $\tau^*{}_i$ and Definition 3.1.

**Theorem 3.3:** Given $(X,\tau_i)$, $i\in\{1,2\}$ a nfbts with neutrosophic fuzzy ideal L on X, $\mu, \sigma, u \in I^X$, then the following holds:

(i) If <μ,σ,u> is both neutrosophic fuzzy pairwise NL-open and $\tau^*{}_i$ -perfect then <μ,σ,u> may be neutrosophic fuzzy pairwise open.

(ii) If <μ,σ,u> is both neutrosophic fuzzy pairwise open and $\tau^*{}_i$ dense – in – itself then <μ,σ,u> may be neutrosophic fuzzy pairwise NL-open.

**Proof.** Follows from the definitions.

**Corollary 3.1.** For a neutrosophic fuzzy subset <μ,σ,u> of a nfbts $(X,\tau_i)$, $i\in\{1,2\}$ with neutrosophic fuzzy ideal L on X, we have:

(i) If <μ,σ,u> is $\tau^*{}_i -$ closed and NPL $-$ open then $\mathrm{Nint}(<\mu,\sigma,u>) = \mathrm{Nint}(P(<\mu,\sigma,u>))^*$.

(ii) If <μ,σ,u> is $\tau^*{}_i -$ perfect and NPL $-$ open then $<\mu,\sigma,u> = \mathrm{Nint}(P(<\mu,\sigma,u>^*))$.

**Theorem 3.4:** If $(X,\tau_i)$, $i\in\{1,2\}$ a nfbts with neutrosophic fuzzy ideal L and $\mu, \sigma, u \in I^X$ then

(i) $<\mu,\sigma,u> \cap \mathrm{Nint}(P(<\mu,\sigma,u>^*))$ may be a neutrosophic fuzzy NL-open set.

(ii) NPL $\tau_i - \mathrm{Nint}(<\mu,\sigma,u>) = 0_N$ iff $\mathrm{Nint}(P(<\mu,\sigma,u>^*)) = 0_N$.

**Proof.** (i) Since $\mathrm{Nint}\left(P(<\mu,\sigma,u>^*)\right) = P(<\mu,\sigma,u>^*) \cap \mathrm{Nint}\left(P(<\mu,\sigma,u>^*)\right)$, then

$\mathrm{Nint}(P(<\mu,\sigma,u>^*)) = P(<\mu,\sigma,u>^*) \cap \mathrm{Nint}\,P(<\mu,\sigma,u>^*) \subseteq P((<\mu,\sigma,u>) \cap (<\mu,\sigma,u>^*))^*$. Thus

$<\mu,\sigma,u> \cap P(<\mu,\sigma,u>^*) \subseteq (<\mu,\sigma,u> \cap (<\mu,\sigma,u> \cap \mathrm{Nint}\,P\left(P(<\mu,\sigma,u>^*)\right)^* \subseteq \mathrm{Nint}\,P(<\mu,\sigma,u>) \cap \mathrm{Nint}\,P(P(<\mu,\sigma,u>^*))^*$

Hence $<\mu,\sigma,u> \cap \mathrm{Nint}\left(P(<\mu,\sigma,u>^*)\right)$ in NPLO(X).

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(ii) Let $NPL\tau_i - Nint(<\mu,\sigma,u>) = 0_N$, then $<\mu,\sigma,u> \cap P(<\mu,\sigma,u>^*) = 0_N$, implies

$Ncl\big(<\mu,\sigma,u> \cap Nint\,P(<\mu,\sigma,u>^*)\big) = 0_N$ and so

$<\mu,\sigma,u> \cap Nint\,P(<\mu,\sigma,u>^*) = 0_N$. Conversely assume that $Nint\,P(<\mu,\sigma,u>^*) = 0_N$,

then $<\mu,\sigma,u> \cap Nint\,P(<\mu,\sigma,u>^*) = 0_N$ . Hence $NPL\tau_i - Nint(<\mu,\sigma,u>) = 0_N$.

**Theorem 3.5:** If $(X,\tau_i)$, $i\in\{1,2\}$ a nfbts with neutrosophic fuzzy ideal L on X, $\mu,\sigma,u \in I^X$,

then $NPL\tau_i - Nint(<\mu,\sigma,u>) = <\mu,\sigma,u> \wedge Nint\,P(<\mu,\sigma,u>^*)$.

**Proof.** The first implication follows from Theorem 3.1.1 that is

$$<\mu,\sigma,u> \wedge p <\mu,\sigma,u>^* \leq NPL - Nint(<\mu,\sigma,u>) \quad (1)$$

For the reverse inclusion, if $<\xi,\rho,\theta>$ in NPLO(X) and $<\xi,\rho,\theta> \leq <\mu,\sigma,u>$ then $P(<\xi,\rho,\theta>^*) \leq P(<\mu,\sigma,u>^*)$ and hence $Nint\,P(<\xi,\rho,\theta>^*) \leq Nint\,P(<\mu,\sigma,u>^*)$. This implies $<\xi,\rho,\theta> = <\xi,\rho,\theta> \wedge Nint\,P(<\xi,\rho,\theta>^*) \leq <\mu,\sigma,u> \wedge P(<\mu,\sigma,u>^*)$.

Thus $NPL\tau_i - Nint(<\mu,\sigma,u>) \leq <\mu,\sigma,u> \wedge P(<\mu,\sigma,u>^*)$ (2)

From (1) and (2) we have the result.

**Definition 3.4:** Given $(X,\tau_i)$, $i\in\{1,2\}$ a nfbts with neutrosophic fuzzy ideal L and $\xi,\rho,\theta \in I^X$, $<\xi,\rho,\theta>$ is called neutrosophic fuzzy pairwise NL-closed set if its complement is neutrosophic fuzzy NL-open set. We will denote the family of neutrosophic fuzzy NL-closed sets by NPLC(X).

**Theorem 3.6:** Given $(X,\tau_i)$, $i\in\{1,2\}$ a nfbts with neutrosophic fuzzy ideal L and $\xi,\rho,\theta \in I^X$, $<\xi,\rho,\theta>$ $\eta$ is neutrosophic fuzzy $NL-closed$, then $P((Nint <\xi,\rho,\theta>)^*) \leq <\xi,\rho,\theta>$ .

**Proof.** It' s clear.

**Theorem 3.7:** Given $(X,\tau)$ be a nfbts with neutrosophic fuzzy ideal L on X and $\xi,\rho,\theta \in I^X$ such that

$P((Nint <\xi,\rho,\theta>)^{*c}) = Nint\,P\left((<\xi,\rho,\theta>^{c^*})\right)$,

then $<\xi,\rho,\theta>$ in NPLC(X) iff $P((Nint <\xi,\rho,\theta>)^*) \leq <\xi,\rho,\theta>$.

**Proof.** (Necessity) Follows immediately from the above theorem. (Sufficiency).

Let $P((Nint <\xi,\rho,\theta>)^*) \leq <\xi,\rho,\theta>$, then $<\xi,\rho,\theta>^c \leq (P(Nint <\xi,\rho,\theta>))^{*c} = Nint <\xi,\rho,\theta>^{c^*}$, from the hypothesis.

Hence $<\xi,\rho,\theta>^c$ in NPLO(X). Thus $<\xi,\rho,\theta>$ in NPLC(X).

**Corollary 3.2:** For a nfbts $(X,\tau_i)$, $i\in\{1,2\}$ with neutrosophic fuzzy ideal L on X the following holds:

(i) The union of neutrosophic fuzzy NPL-closed set and neutrosophic fuzzy NP-closed set may be a neutrosophic fuzzy NPL-closed set.

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(ii) The union of neutrosophic fuzzy NPL-closed and neutrosophic fuzzy NPL-closed may be neutrosophic fuzzy NPL-closed.

**Conclusions**

The notions of the sets and functions in neutrosophic fuzzy bitopological spaces are highly developed and several characterizations have already been obtained. These are used extensively in many practical and engineering problems, the computational topology for geometric design, computer-aided geometric design, engineering design research, Geographic Information System (GIS) and mathematical sciences. We are in the process of preparing a statistical model for neutrosophic bitopological layers for geographic information systems.

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