£ukasiewicz Intuitionistic Fuzzy Filters in Hoops and its Application in Medical Diagnosis

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Abstract

The new theory of £ukasiewicz įntuitionistic fuzzy set and £ukasiewicz įntuitionistic fuzzy filter is introduced. Some properties of £ukasiewicz įntuitionistic fuzzy filter is presented. It is explored that under what circumstances, the £ukasiewicz įntuitionistic fuzzy set can be a £ukasiewicz įntuitionistic fuzzy filter. An algorithm for diagnosing disease is developed and provided with demonstration.

Keywords: £ukasiewicz įntuitionistic fuzzy set; £ukasiewicz įntuitionistic fuzzy filter; Conjunction and disjunction of £ukasiewicz įntuitionistic fuzzy sets; Intuitionistic fuzzy filters, hoops

1. Introduction

In 1969, Bosbach introduced an algebraic structure in hoop algebra, relaxes the requirement of distributivism, which is the fundamental property of Boolean algebra. Hoop is a naturally ordered procrim, which is partially ordered commutative residuated integral monoid. L. A. Zadeh introduced the fuzzy sets in 1965.The application of fuzzy filters to the hoops was studied by Borzooei and Aaly Kologani [3]. Kologani M.A, Borzooei R.A, Kim H.S, Jun Y.B and Ahn S.S [8] studied intuitionistic fuzzy filters on hoops. On hoops, several mathematician defined the congruence relation [9]. Jan £ukasiewicz introduced £ukasiewicz logic in the early 20th century. £ukasiewicz logic is a classical many-valued logic, a logic of the £ukasiewicz t-norm. Jun and Ahn applied £ukasiewicz logic to BE-algebras [7]. Atanassov provided the concept of įntuitionistic fuzzy set in 1986. In 2024, Mohseni Takallo M, Aaly Kologani M, Jun Y. B and Borzooei R. A examined the properties of filter in hoops by applying the £ukasiewicz fuzzy set idea [11].

In real life, there may be a situation where uncertainty, imprecision and complexity is crucial for decision making, modelling and optimizing process. To contribute to the advancement of theoretical developments and in practical applications, the concept of £ukasiewicz įntuitionistic fuzzy filter is presented in this study. The properties of a £ukasiewicz įntuitionistic fuzzy filter is investigated. Using the concept and operations of £ukasiewicz įntuitionistic filter, we developed an algorithm for medical diagnosis. We provide a demonstration for the developed algorithm.

2. Preliminaries

Definition 2.1.[11]

An algebra $(\mathcal{H}, \circ, \rightsquigarrow, 1)$ is said to be ***hoop*** if it satisfies:

$(\mathcal{H}1)\ \forall \mathcal{s} \in \mathcal{H}, \mathcal{s} \rightsquigarrow \mathcal{s} = 1$

$(\mathcal{H}2)\ \forall \mathcal{s}, t \in \mathcal{H},\ \mathcal{s} \circ (\mathcal{s} \rightsquigarrow t) = t \circ (t \rightsquigarrow \mathcal{s})$

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$(\mathcal{H}3)\ \forall s,t,u \in \mathcal{H}, s \rightsquigarrow (t \rightsquigarrow u) = (s \rightsquigarrow t) \rightsquigarrow u$

$(\mathcal{H}4)$ $(\mathcal{H},\circ,1)$ is a commutative monoid

Define $s \leq t$ on a hoop $\mathcal{H}$ if and only if $s \rightsquigarrow t = 1$.

Proposition 2.2.[11]

The following assertions are true for every hoop $\mathcal{H}$.

$\forall s,t \in \mathcal{H}, s \circ t \leq u \Leftrightarrow s \leq t \rightsquigarrow u$ (1)

$\forall s,t \in \mathcal{H}, s \circ t \leq s,t$ (2)

$\forall s,t \in \mathcal{H}, s \leq t \rightsquigarrow s$ (3)

$\forall s \in \mathcal{H}, s \rightsquigarrow 1 = 1, 1 \rightsquigarrow s = s$ (4)

$\forall s,t,u \in \mathcal{H}, (s \rightsquigarrow t) \circ (t \rightsquigarrow u) \leq s \rightsquigarrow u$ (5)

$\forall s,t,u \in \mathcal{H}, s \leq t \Longrightarrow s \circ u \leq t \circ u$ (6)

$\forall s,t \in \mathcal{H}, s \circ (s \rightsquigarrow t) \leq t$ (7)

Definition 2.3.[11]

A subset $\mathcal{F}$ of $\mathcal{H}$ is called filter of hoop if for every $s,t \in \mathcal{H}$ satisfies the following two condition.

- $s,t \in \mathcal{F} \Longrightarrow s \circ t \in \mathcal{F}$
- $s \leq t\ , s \in \mathcal{F} \Longrightarrow t \in \mathcal{F}$

Proposition 2.4.[11]

A subset $\mathcal{F}$ of $\mathcal{H}$ is called filter of $\mathcal{H}$ if and only if the following are satisfied.

- $1 \in \mathcal{F}$
- $s, s \rightsquigarrow t \in \mathcal{H} \Longrightarrow t \in \mathcal{H}\ ,\quad \forall s,t \in \mathcal{H}$

Definition 2.5. (Atanassov, 1999):

Let $\mathcal{H}$ be a nonempty set. An įntuitionistic fuzzy set A in $\mathcal{H}$ is an object having the form $A = \{h, \mu(h), \nu(h) | h \in \mathcal{H}\}$, provided $0 \leq \mu(h) + \nu(h) \leq 1$.

The functions $\mu': \mathcal{H} \to [0,1]$, $\nu': \mathcal{H} \to [0,1]$ represent the degree of membership and degree of non-membership of the element $h \in \mathcal{H}$ to the set A.

Definition 2.6.[6]

Let c be a point of $\mathcal{H}$ and $\alpha \in (0,1]$, $\beta \in [0,1)$ be two real numbers such that $\alpha + \beta \leq 1$.

An įntuitionistic fuzzy subset of $\mathcal{H}$ is said to be įntuitionistic fuzzy point if it is of the form

$$C_{(\alpha,\beta)}(s) = \begin{cases} (\alpha,\beta) & if\ s = c \\ (0,1) & otherwise \end{cases}$$

Definition 2.7.[6]

For an įntuitionistic fuzzy set $\rho(s, \mu(s), \nu(s))$ in $\mathcal{H}$, $C_{(\alpha,\beta)}$ an įntuitionistic fuzzy point is contained in ρ, denoted by $C_{(\alpha,\beta)} \in (\mu,\nu)$ if it satisfies $\mu(x) \geq \alpha$ and $\nu(x) \leq \beta$.

An įntuitionistic fuzzy point is quasi-coincident with ρ denoted by $C_{(\alpha,\beta)} q\rho$ if and only if

$\alpha > \nu(c)\ or\ \beta < \mu(c)$

Definition 2.8.[8]

An įntuitionistic fuzzy set $\rho(s, \mu(s), \nu(s))$ is said to be an įntuitionistic fuzzy filter of $\mathcal{H}$ if ρ satisfies

$\mu(s) \leq \mu(1)\ and\ \nu(s) \geq \nu(1)\ ,\ \forall s \in \mathcal{H}$

$\mu(s) \geq min\{\mu(t), \mu(t \rightsquigarrow s)\}\ ,$

$\nu(s) \leq max\{\nu(t), \nu(t \rightsquigarrow s)\}\qquad \forall s,t \in \mathcal{H}$

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Definition 2.9.[14]

The conjunction and disjunction of £ukasiewicz intuitionistic fuzzy sets A and B of a hoop $\mathcal{H}$ are defined by

$$A \oplus B = \{s, min(1, \mu_A'(s) + \mu_B'(s)), max(0, \upsilon_A'(s) + \upsilon_B'(s) - 1) | s \in \mathcal{H}\}$$

$$A \odot B = \{s, max(0, \mu_A'(s) + \mu_B'(s) - 1), min(1, \upsilon_A'(s) + \upsilon_B'(s)) | s \in \mathcal{H}\}$$

3. £ukasiewicz Intuitionistic Fuzzy Filters

Definition 3.1.

Let $\rho(s, \mu(s), \nu(s))$ be an intuitionistic fuzzy set in $\mathcal{H}$ and $\theta \in (0,1)$. $L_\rho^\theta(s, \mu', \nu')$ is called £ukasiewicz intuitionistic fuzzy set of ρ in $\mathcal{H}$ if μ' and ν' are the functions given by

$$\mu': \mathcal{H} \to [0,1], s \mapsto max\{0, \mu(s) + \theta - 1\}$$

$$\nu': \mathcal{H} \to [0,1], s \mapsto max\{0, \nu(s) + \theta - 1\}$$

Definition 3.2.

£ukasiewicz intuitionistic fuzzy set L_ρ^θ of an intuitionistic fuzzy set ρ in $\mathcal{H}$ is called a £ukasiewicz intuitionistic fuzzy filter of $\mathcal{H}$ if

$\forall s, t \in \mathcal{H}, \alpha, \gamma \in (0,1], \beta, \delta \in [0,1),$

$$s_{(\alpha,\beta)} \in L_\rho^\theta, t_{(\gamma,\delta)} \in L_\rho^\theta \Rightarrow (s \circ t)_{(\alpha\wedge\gamma,\beta\vee\delta)} \in L_\rho^\theta \quad (8)$$

$\forall s, t \in \mathcal{H}, \alpha \in (0,1], \beta \in [0,1),$

$$s \le t, s_{(\alpha,\beta)} \in L_\rho^\theta \Rightarrow t_{(\alpha,\beta)} \in L_\rho^\theta \quad (9)$$

Example 3.3.

Assume that the set $\mathcal{H} = \{s, t, u\}$ has binary operations " $\circ$ " and " $\rightsquigarrow$ " as in Tables 3.3.1. and 3.3.2.

Table 3.3.1

$\circ$	s	t	u
s	s	s	s
t	s	s	t
u	s	t	u

Table 3.3.2

$\rightsquigarrow$	s	t	u
s	u	u	u
t	t	u	u
u	s	t	u

Clearly $\mathcal{H}$ is hoop.

Define an intuitionistic fuzzy set $\rho: \mathcal{H} \longrightarrow [0,1]$ by

$$\rho(x) = \{(s, 0.4, 0.5), (t, 0.4, 0.5), (u, 0.7, 0.1)\}$$

For $\theta = 0.8$, the £ukasiewicz intuitionistic fuzzy set L_ρ^θ of ρ in $\mathcal{H}$ as follows

$$L_\rho^\theta(x) = \{(s, 0.2, 0.3), (t, 0.2, 0.3), (u, 0.5, 0)\}$$

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The reader can now confirm that L_ρ^θ is a £ukasiewicz įntuitionistic fuzzy filter of $\mathcal{H}$.

Theorem 3.4.

£ukasiewicz įntuitionistic fuzzy set $L_\rho^\theta(\mathscr{s},\mu',\nu')$ of an įntuitionistic fuzzy set $\rho(\mathscr{s},\mu(\mathscr{s}),\nu(\mathscr{s}))$ in $\mathcal{H}$ is a £ukasiewicz įntuitionistic fuzzy filter of $\mathcal{H}$ if and only if the following are satisfied.

- $\mu'(1)$ is an ŭpper bound of $\{\mu'(\mathscr{s})|\mathscr{s}\in\mathcal{H}\}$
  $\nu'(1)$ is a lower bound of $\{\nu'(\mathscr{s})|\mathscr{s}\in\mathcal{H}\}$ (10)
- $\forall \mathscr{s},\mathscr{t}\in\mathcal{H}, \forall\alpha,\gamma\in(0,1],\ \beta,\delta\in[0,1)$
  $$\mathscr{s}_{(\alpha,\beta)}\in L_\rho^\theta, (\mathscr{s}\rightsquigarrow\mathscr{t})_{(\gamma,\delta)}\in L_\rho^\theta \quad\Rightarrow \mathscr{t}_{(\alpha\wedge\gamma,\beta\vee\delta)}\in L_\rho^\theta \qquad (11)$$

Proof Let us assume that L_ρ^θ be a £ukasiewicz įntuitionistic fuzzy filter of $\mathcal{H}$.

If $\mu'(1)$ is not an ŭpper bound of $\{\mu'(\mathscr{s})|\mathscr{s}\in\mathcal{H}\}$ and $\nu'(1)$ is not a lower bound of $\{\nu'(\mathscr{s})|\mathscr{s}\in\mathcal{H}\}$, then $\mu'(1)<\mu'(z)$ & $\nu'(1)>\nu'(z)$ for some $z\in\mathcal{H}$.

Because of $z\le 1$ and $z_{(\mu'(z),\nu'(z))}\in L_\rho^\theta$, by (9) we have $1_{(\mu'(z),\nu'(z))}\in L_\rho^\theta$.

That is $\mu'(1)\ge\mu'(z)$ & $\nu'(1)\le\nu'(z)$ which contradicts the assumption. Thus $\mu'(1)$ is an ŭpper bound of $\{\mu'(\mathscr{s})|\mathscr{s}\in\mathcal{H}\}$ and $\nu'(1)$ is a lower bound of $\{\nu'(\mathscr{s})|\mathscr{s}\in\mathcal{H}\}$.

Now assume that for $\mathscr{s},\mathscr{t}\in\mathcal{H}$ and $\alpha,\gamma\in(0,1],\ \beta,\delta\in[0,1)$,

$\mathscr{s}_{(\alpha,\beta)}\in L_\rho^\theta$ and $(\mathscr{s}\rightsquigarrow\mathscr{t})_{(\gamma,\delta)}\in L_\rho^\theta$.

Then by (8), we have $((\mathscr{s}\rightsquigarrow\mathscr{t})\circ\mathscr{s})_{(\alpha\wedge\gamma,\beta\vee\delta)}\in L_\rho^\theta$. Since $(\mathscr{s}\rightsquigarrow\mathscr{t})\circ\mathscr{s}\le\mathscr{t}$, it follows from (9) that $\mathscr{t}_{(\alpha\wedge\gamma,\beta\vee\delta)}\in L_\rho^\theta$.

Conversely assume that L_ρ^θ satisfies (10) and (11). Let $\mathscr{s},\mathscr{t}\in\mathcal{H}$ and $\alpha\in(0,1],\ \beta\in[0,1)$ be such that $\mathscr{s}\le\mathscr{t}$ and $\mathscr{s}_{(\alpha,\beta)}\in L_\rho^\theta$.

Then by definition of hoop and (10), we have $\mu'(\mathscr{s}\rightsquigarrow\mathscr{t})=\mu'(1)\ge\mu'(\mathscr{s})\ge\alpha$ and $\nu'(\mathscr{s}\rightsquigarrow\mathscr{t})=\nu'(1)\le\nu'(\mathscr{s})\le\beta$. That is $(\mathscr{s}\rightsquigarrow\mathscr{t})_{(\alpha,\beta)}\in L_\rho^\theta$. Using (11), we have $\mathscr{t}_{(\alpha,\beta)}\in L_\rho^\theta$ which proves (9).

Let $\mathscr{s},\mathscr{t}\in\mathcal{H}$ and $\alpha,\gamma\in(0,1],\ \beta,\delta\in[0,1)$ be such that $\mathscr{s}_{(\alpha,\beta)}\in L_\rho^\theta$ and $\mathscr{t}_{(\gamma,\delta)}\in L_\rho^\theta$. Since, we have $\mathscr{s}\rightsquigarrow(\mathscr{t}\rightsquigarrow\mathscr{s}\circ\mathscr{t})=\mathscr{s}\circ\mathscr{t}\rightsquigarrow\mathscr{s}\circ\mathscr{t}=1$ by the definition of hoop, we have

$\mu'(\mathscr{s}\rightsquigarrow(\mathscr{t}\rightsquigarrow\mathscr{s}\circ\mathscr{t}))=\mu'(1)\ge\mu'(\mathscr{t})\ge\gamma$ and

$\nu'(\mathscr{s}\rightsquigarrow(\mathscr{t}\rightsquigarrow\mathscr{s}\circ\mathscr{t}))=\nu'(1)\le\nu'(\mathscr{t})\le\delta$.

Therefore, we have $(\mathscr{s}\rightsquigarrow(\mathscr{t}\rightsquigarrow\mathscr{s}\circ\mathscr{t}))_{(\gamma,\delta)}\in L_\rho^\theta$. By using (11), we have $(\mathscr{t}\rightsquigarrow\mathscr{s}\circ\mathscr{t})_{(\alpha\wedge\gamma,\beta\vee\delta)}\in L_\rho^\theta$. Again using (11), we have $(\mathscr{s}\circ\mathscr{t})_{(\alpha\wedge\gamma,\beta\vee\delta)}\in L_\rho^\theta$ which proves (8).

Therefore L_ρ^θ in $\mathcal{H}$ is a £ukasiewicz įntuitionistic fuzzy filter of $\mathcal{H}$.

Theorem 3.5.

£ukasiewicz įntuitionistic fuzzy set L_ρ^θ of an įntuitionistic fuzzy set ρ in $\mathcal{H}$ is a £ukasiewicz įntuitionistic fuzzy filter of $\mathcal{H}$ if and only if

- $\forall\mathscr{s}\in\mathcal{H}, \forall\alpha\in(0,1],\ \beta\in[0,1), \mathscr{s}_{(\alpha,\beta)}\in L_\rho^\theta\Rightarrow 1_{(\alpha,\beta)}\in L_\rho^\theta$ (12)
- $\forall\mathscr{s},\mathscr{t}\in\mathcal{H}, \mu'(\mathscr{t})\ge min\{\mu'(\mathscr{s}),\mu'(\mathscr{s}\rightsquigarrow\mathscr{t})\}$ and
  $$\nu'(\mathscr{t})\le max\{\nu'(\mathscr{s}),\nu'(\mathscr{s}\rightsquigarrow\mathscr{t})\} \qquad (13)$$

Proof Suppose L_ρ^θ is a £ukasiewicz įntuitionistic fuzzy filter of $\mathcal{H}$.

Let $\mathscr{s}\in\mathcal{H}$ and $\alpha\in(0,1]$, $\beta\in[0,1)$ be such that $\mathscr{s}_{(\alpha,\beta)}\in L_\rho^\theta$. From (10), it follows that $\mu'(1)\ge\mu'(\mathscr{s})\ge\alpha$ and $\nu'(1)\le\nu'(\mathscr{s})\le\beta$ and so $1_{(\alpha,\beta)}\in L_\rho^\theta$ which proves (12). Note that $\mathscr{s}_{(\mu'(\mathscr{s}),\nu'(\mathscr{s}))}\in L_\rho^\theta$ and $(\mathscr{s}\rightsquigarrow\mathscr{t})_{(\mu'(\mathscr{s}\rightsquigarrow\mathscr{t}),\nu'(\mathscr{s}\rightsquigarrow\mathscr{t}))}\in L_\rho^\theta$, $\forall\mathscr{s},\mathscr{t}\in\mathcal{H}$. By (11), we have

$$\mathscr{t}_{(\mu'(\mathscr{s})\wedge\mu'(\mathscr{s}\rightsquigarrow\mathscr{t}),\nu'(\mathscr{s})\vee\mu'(\mathscr{s}\rightsquigarrow\mathscr{t}))}\in L_\rho^\theta$$

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$\Rightarrow \mu'(t) \geq min\{\mu'(s), \mu'(s \rightsquigarrow t)\}$ and $\upsilon'(t) \leq max\{\upsilon'(s), \upsilon'(s \rightsquigarrow t)\}, \forall s, t \in \mathcal{H}$.

Conversely, suppose that L_ρ^θ satisfies (12) and (13). Since $s_{(\mu'(s),\upsilon'(s))} \in L_\rho^\theta$, $\forall s \in \mathcal{H}$ and by (12), we have $1_{(\mu'(s),\upsilon'(s))} \in L_\rho^\theta$ and so $\mu'(1) \geq \mu'(s), \upsilon'(1) \leq \upsilon'(s)$, $\forall s \in \mathcal{H}$.

$\therefore \mu'(1)$ is an ũpper bound of $\{ \mu'(s) | s \in \mathcal{H} \}$ and $\nu'(1)$ is a lower bound of $\{ \nu'(s) | s \in \mathcal{H} \}$. Let $s, t \in \mathcal{H}$ and $\alpha, \gamma \in (0,1]$, $\beta, \delta \in [0,1)$ be such that $s_{(\alpha,\beta)} \in L_\rho^\theta$ and $(s \rightsquigarrow t)_{(\gamma,\delta)} \in L_\rho^\theta$. Then $\mu'(s) \geq \alpha$, $\upsilon'(s) \leq \beta$ and $\mu'(s \rightsquigarrow t) \geq \gamma$, $\upsilon'(s \rightsquigarrow t) \leq \delta$. By using (13), we have,

$\mu'(t) \geq min\{\mu'(s), \mu'(s \rightsquigarrow t)\} \geq min\{\alpha, \gamma\}$ and

$\upsilon'(t) \leq max\{\upsilon'(s), \upsilon'(s \rightsquigarrow t)\} \leq max\{\beta, \delta\}$

Thus $t_{(\alpha\wedge\gamma, \beta\vee\delta)} \in L_\rho^\theta$. By Theorem 3.4, L_ρ^θ is a Łukasiewicz ịntuitionistic fuzzy filter of $\mathcal{H}$.

Proposition 3.6.

Any Łukasiewicz ịntuitionistic fuzzy filter L_ρ^θ of $\mathcal{H}$ satisfies:

- $\forall s, t, u \in \mathcal{H}, \forall \alpha, \gamma \in (0,1], \beta, \delta \in [0,1)$,

  $u \leq s \rightsquigarrow t, s_{(\alpha,\beta)} \in L_\rho^\theta, u_{(\gamma,\delta)} \in L_\rho^\theta \Rightarrow t_{(\alpha\wedge\gamma,\beta\vee\delta)} \in L_\rho^\theta$ (14)
- $\forall s, t, u \in \mathcal{H}, \forall \alpha, \gamma \in (0,1], \beta, \delta \in [0,1)$,

  $s \leq t \rightsquigarrow u, s_{(\alpha,\beta)} \in L_\rho^\theta, t_{(\gamma,\delta)} \in L_\rho^\theta \Rightarrow u_{(\alpha\wedge\gamma,\beta\vee\delta)} \in L_\rho^\theta$ (15)
- $\forall s, t, u \in \mathcal{H}, \forall \alpha, \gamma \in (0,1], \beta, \delta \in [0,1)$,

  $(s \rightsquigarrow t)_{(\alpha,\beta)} \in L_\rho^\theta, (t \rightsquigarrow u)_{(\gamma,\delta)} \in L_\rho^\theta \Rightarrow (s \rightsquigarrow u)_{(\alpha\wedge\gamma,\beta\vee\delta)} \in L_\rho^\theta$ (16)
- $\forall s, t, u \in \mathcal{H}, \forall \alpha, \gamma \in (0,1], \beta, \delta \in [0,1), (s \rightsquigarrow t)_{(\alpha,\beta)} \in L_\rho^\theta$,

  $(s \circ u)_{(\gamma,\delta)} \in L_\rho^\theta \Rightarrow (t \circ u)_{(\alpha\wedge\gamma,\beta\vee\delta)} \in L_\rho^\theta$ (17)

Proof Let $s, t, u \in \mathcal{H}$ and $\alpha, \gamma \in (0,1], \beta, \delta \in [0,1)$ be such that $u \leq s \rightsquigarrow t$, $s_{(\alpha,\beta)} \in L_\rho^\theta$ and $u_{(\gamma,\delta)} \in L_\rho^\theta$. Since $u \leq s \rightsquigarrow t$, then $u \rightsquigarrow (s \rightsquigarrow t) = 1$, $\mu'(s) \geq \alpha, \upsilon'(s) \leq \beta$ and $\mu'(u) \geq \gamma$, $\upsilon'(u) \leq \delta$. Hence by (13) we have,

$$\mu'(t) \geq min\{ \mu'(s) , \mu'(s \rightsquigarrow t) \}$$
$$\geq min\left\{ \mu'(s)", min\{ \mu'(u) , \mu'\left(u \rightsquigarrow (s \rightsquigarrow t)\right) \}\right\}$$
$$= min\{\mu'(s), min\{\mu'(u), \mu'(1)\}\}$$
$$= min\{\mu'(s), \mu'(u)\}$$
$$\geq min\{\alpha, \gamma\} \text{ and}$$
$$\upsilon'(t) \leq max\{\upsilon'(s), \upsilon'(s \rightsquigarrow t)\}$$
$$\leq max\left\{\upsilon'(s), max\{\upsilon'(u), \upsilon'\left(u \rightsquigarrow (s \rightsquigarrow t)\right)\}\right\}$$
$$= max\{\upsilon'(s), max\{\mu'(u), \upsilon'(1)\}\}$$
$$= max\{\upsilon'(s), \upsilon'(u)\}$$
$$\leq max\{\beta, \delta\}$$

and so $t_{(\alpha\wedge\gamma,\beta\vee\delta)} \in L_\rho^\theta$. Therefore (14) is true.

Let $s, t, u \in \mathcal{H}$ and $\alpha, \gamma \in (0,1], \beta, \delta \in [0,1)$ be such that $s \leq t \rightsquigarrow u$, $s_{(\alpha,\beta)} \in L_\rho^\theta$ and $t_{(\gamma,\delta)} \in L_\rho^\theta$. Since $s \leq t \rightsquigarrow u$, using (1) we have, $s \circ t \leq u$ and by (8) we have $(s \circ t)_{(\alpha\wedge\gamma,\beta\vee\delta)} \in L_\rho^\theta$ and so by (9)we have $u_{(\alpha\wedge\gamma,\beta\vee\delta)} \in L_\rho^\theta$ which proves (15).

Let $s, t, u \in \mathcal{H}$ and $\alpha, \gamma \in (0,1], \beta, \delta \in [0,1)$ be such that $(s \rightsquigarrow t)_{(\alpha,\beta)} \in L_\rho^\theta$ and $(t \rightsquigarrow u)_{(\gamma,\delta)} \in L_\rho^\theta$. Using (8) we have,

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$$((s \rightsquigarrow t) \circ (t \rightsquigarrow u))_{(\alpha\wedge\gamma,\beta\vee\delta)} \in L_\rho^\theta$$

Since $(s \rightsquigarrow t) \circ (t \rightsquigarrow u) \leq s \rightsquigarrow u$, we have from (9) that,$(s \rightsquigarrow u)_{(\alpha\wedge\gamma,\beta\vee\delta)} \in L_\rho^\theta$. This proves (16).

Let $s, t, u \in \mathcal{H}$ and $\alpha, \gamma \in (0,1], \beta, \delta \in [0,1)$ be such that $(s \rightsquigarrow t)_{(\alpha,\beta)} \in L_\rho^\theta$, and $(s \circ u)_{(\gamma,\delta)} \in L_\rho^\theta$. By using (8) we have, $(s \circ u \circ (s \rightsquigarrow t))_{(\alpha\wedge\gamma,\beta\vee\delta)} \in L_\rho^\theta$ and by commutativity of $\circ$, we have $(u \circ s \circ (s \rightsquigarrow t))_{(\alpha\wedge\gamma,\beta\vee\delta)} \in L_\rho^\theta$. Since $s \circ (s \rightsquigarrow t) \leq t$, we have $u \circ s \circ (s \rightsquigarrow t) \leq u \circ t = t \circ u$ by (6).

By (9), we have $(t \circ u)_{(\alpha\wedge\gamma,\beta\vee\delta)} \in L_\rho^\theta$ which proves (17).

Theorem 3.7.

Let ρ be an ịntuitionistic fuzzy filter of $\mathcal{H}$. Then the £ukasiewicz ịntuitionistic fuzzy set L_ρ^θ of ρ in $\mathcal{H}$ is a £ukasiewicz ịntuitionistic fuzzy filter of $\mathcal{H}$.

Proof L_ρ^θ be £ukasiewicz ịntuitionistic fuzzy set of an ịntuitionistic fuzzy filter ρ in $\mathcal{H}$. Then

$$\begin{aligned}\mu'(1) &= max\{\ 0\ ,\ \mu(1) + \theta - 1\ \} \\ &\geq max\{\ 0\ ,\ \mu(s) + \theta - 1\ \} \\ &= \mu'(s) \quad \text{and}\end{aligned}$$

$$\begin{aligned}v'(1) &= max\{0, v(1) + \theta - 1\} \\ &\leq max\{0, v(s) + \theta - 1\} \\ &= v'(s)\ \ \forall s \in \mathcal{H}.\end{aligned}$$

Therefore, $\mu'(1)$ is an ữpper bound of $\{\ \mu'(s) | s \in \mathcal{H}\ \}$

$v'(1)$ is a lower bound of $\{\ v'(s) | s \in \mathcal{H}\ \}$.

Let $s, t \in \mathcal{H}$ and $\alpha, \gamma \in (0,1], \beta, \delta \in [0,1)$ be such that $s_{(\alpha,\beta)} \in L_\rho^\theta$ and $(s \rightsquigarrow t)_{(\gamma,\delta)} \in L_\rho^\theta$. Then $\mu'(s) \geq \alpha$, $v'(s) \leq \beta$ and $\mu'(s \rightsquigarrow t) \geq \gamma$, $v'(s \rightsquigarrow t) \leq \delta$ which imply that

$$\begin{aligned}\mu'(t) &= max\{0, \mu(t) + \theta - 1\} \\ &\geq max\{0, min(\mu(s), \mu(s \rightsquigarrow t)) + \theta - 1\} \\ &= max\{0, min\{\mu(s) + \theta - 1, \mu(s \rightsquigarrow t) + \theta - 1\}\} \\ &= min\{max\{0, \mu(s) + \theta - 1\}, max\{0, \mu(s \rightsquigarrow t) + \theta - 1\}\} \\ &= min\{\mu'(s), \mu'(s \rightsquigarrow t)\} \\ &\geq min\{\alpha, \gamma\} \text{ and}\end{aligned}$$

$$\begin{aligned}v'(t) &= max\{0, v(t) + \theta - 1\} \\ &\leq max\{0, max(v(s), v(s \rightsquigarrow t)) + \theta - 1\} \\ &= max\{0, max\{v(s) + \theta - 1, v(s \rightsquigarrow t) + \theta - 1\}\} \\ &= max\{max\{0, v(s) + \theta - 1\}, max\{0, v(s \rightsquigarrow t) + \theta - 1\}\} \\ &= max\{v'(s), v'(s \rightsquigarrow t)\} \\ &\leq max\{\beta, \delta\}\end{aligned}$$

Therefore $t_{(\alpha\wedge\gamma,\beta\vee\delta)} \in L_\rho^\theta$ and so L_ρ^θ is a £ukasiewicz ịntuitionistic fuzzy filter of $\mathcal{H}$.

The converse need not be true as in the example given below.

Example 3.8.

In the Example 3.3., L_ρ^θ is a £ukasiewicz ịntuitionistic fuzzy filter of $\mathcal{H}$.

Since $\rho(u) = 0.4 \ngeq 0.7 = min\{\rho(u), \rho(t \longrightarrow u)\}$, ρ fails to be an ịntuitionistic fuzzy filter of $\mathcal{H}$.

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Theorem 3.9.

A Łukasiewicz ịntuitionistic fuzzy set L_ρ^θ in $\mathcal{H}$ satisfying (10) and (15) is a Łukasiewicz ịntuitionistic fuzzy filter of $\mathcal{H}$.

Proof For $s,t \in \mathcal{H}$ and $\alpha,\gamma \in (0,1], \beta,\delta \in [0,1)$,

$s_{(\alpha,\beta)} \in L_\rho^\theta$ and $(s \rightsquigarrow t)_{(\gamma,\delta)} \in L_\rho^\theta$.

Since $s \rightsquigarrow t \leq s \rightsquigarrow t$, using (15) we have $t_{(\alpha\wedge\gamma,\beta\vee\delta)} \in L_\rho^\theta$.f

Hence by Theorem (3.4), L_ρ^θ is a Łukasiewicz ịntuitionistic fuzzy filter of $\mathcal{H}$.

Theorem 3.10.

A Łukasiewicz ịntuitionistic fuzzy set L_ρ^θ in $\mathcal{H}$ satisfying (10) and (16) is a Łukasiewicz ịntuitionistic fuzzy ilter of $\mathcal{H}$.

Proof For $t,u \in \mathcal{H}$ and $\alpha,\gamma \in (0,1], \beta,\delta \in [0,1)$,

$t_{(\alpha,\beta)} \in L_\rho^\theta$ and $(t \rightsquigarrow u)_{(\gamma,\delta)} \in L_\rho^\theta$.

Since $t_{(\alpha,\beta)} = (1 \rightsquigarrow t)_{(\alpha,\beta)} \in L_\rho^\theta$. Using (16) we have, $(1 \rightsquigarrow u)_{(\alpha\wedge\gamma,\beta\vee\delta)} \in L_\rho^\theta$ and it follows from (4) that $u_{(\alpha\wedge\gamma,\beta\vee\delta)} \in L_\rho^\theta$.

Hence by Theorem(3.4), L_ρ^θ is a Łukasiewicz ịntuitionistic fuzzy filter of $\mathcal{H}$.

Theorem 3.11.

A Łukasiewicz ịntuitionistic fuzzy set L_ρ^θ in $\mathcal{H}$ satisfying (10) and (17) is a Łukasiewicz ịntuitionistic fuzzy filter of $\mathcal{H}$.

Proof For $t,u \in \mathcal{H}$ and $\alpha,\gamma \in (0,1], \beta,\delta \in [0,1)$,

$t_{(\alpha,\beta)} \in L_\rho^\theta$ and $u_{(\gamma,\delta)} \in L_\rho^\theta$.

Then $t_{(\alpha,\beta)} = (1 \rightsquigarrow t)_{(\alpha,\beta)} \in L_\rho^\theta$ and $u_{(\gamma,\delta)} = (1 \circ u)_{(\gamma,\delta)} \in L_\rho^\theta$. Using (17) we have $(t \circ u)_{(\alpha\wedge\gamma,\beta\vee\delta)} \in L_\rho^\theta$.

Let $s,t \in \mathcal{H}$ and $\sigma \in (0,1], \tau \in [0,1)$ be such that $s \leq t$ and $s_{(\sigma,\tau)} \in L_\rho^\theta$. The condition (10) leads to

$\mu'(s \rightsquigarrow t) = \mu'(1) \geq \mu'(s) \geq \sigma$ and $v'(s \rightsquigarrow t) = v'(1) \leq v'(s) \leq \tau$

Hence $(s \rightsquigarrow t)_{(\sigma,\tau)} \in L_\rho^\theta$. Since $s_{(\sigma,\tau)} = (s \circ 1)_{(\sigma,\tau)} \in L_\rho^\theta$, (3.10) implies that $(t \circ 1)_{(\sigma,\tau)} = t_{(\sigma,\tau)} \in L_\rho^\theta$. Therefore L_ρ^θ is a Łukasiewicz ịntuitionistic fuzzy filter of $\mathcal{H}$.

Theorem 3.12.

L_ρ^θ be Łukasiewicz ịntuitionistic fuzzy set in $\mathcal{H}$ which satisfies the condition(10). If L_ρ^θ satisfies

$$\forall s,t,u \in \mathcal{H}, \forall \alpha,\gamma \in (0,0.5], \forall \beta,\delta \in [0,0.5),\ s \leq t \rightsquigarrow u,\ s_{(\alpha,\beta)} \in L_\rho^\theta,\ t_{(\gamma,\delta)} \in L_\rho^\theta$$
$$\Rightarrow u_{(\alpha\wedge\gamma,\beta\vee\delta)} q L_\rho^\theta. \quad (18)$$

(or)

$$\forall s,t,u \in \mathcal{H}, \forall \alpha,\gamma \in (0.5,1], \forall \beta,\delta \in [0.5,1),\ s \leq t \rightsquigarrow u,\ s_{(\alpha,\beta)} q L_\rho^\theta,\ t_{(\gamma,\delta)} q L_\rho^\theta$$
$$\Rightarrow u_{(\alpha\wedge\gamma,\beta\vee\delta)} \in L_\rho^\theta. \quad (19)$$

then L_ρ^θ is a Łukasiewicz ịntuitionistic fuzzy filter of $\mathcal{H}$.

Proof L_ρ^θ be Łukasiewicz ịntuitionistic fuzzy set in $\mathcal{H}$ and satisfies the condition(10). Assume that L_ρ^θ satisfies (18). For $s,t \in \mathcal{H}$ and $\alpha,\gamma \in (0,0.5] \subseteq (0,1], \beta,\delta \in [0,0.5) \subseteq [0,1)$ $s_{(\alpha,\beta)} \in L_\rho^\theta$ and $(s \rightsquigarrow t)_{(\gamma,\delta)} \in L_\rho^\theta$.

Since $s \rightsquigarrow t \leq s \rightsquigarrow t$, the condition (18) leads to $t_{(\alpha\wedge\gamma,\beta\vee\delta)} q L_\rho^\theta$. Since $\alpha\wedge\gamma \leq 0.5$, $\beta\vee\delta < 0.5$, it follows that $\mu'(t) > \beta\vee\delta \geq \alpha\wedge\gamma$ and $v'(t) \leq \beta\vee\delta$. i.e., $t_{(\alpha\wedge\gamma,\beta\vee\delta)} \in L_\rho^\theta$. Therefore L_ρ^θ is a Łukasiewicz ịntuitionistic fuzzy filter of $\mathcal{H}$.

Now, let L_ρ^θ satisfies (19) and $s,t \in \mathcal{H}$ and $\alpha,\gamma \in (0.5,1] \subseteq (0,1]$, $\beta,\delta \in [0.5,1) \subseteq [0,1)$ be such that $s_{(\alpha,\beta)} \in L_\rho^\theta$ and $(s \rightsquigarrow t)_{(\gamma,\delta)} \in L_\rho^\theta$. Then $\mu'(s) \geq \alpha > 1-\alpha$, $v'(s) \leq \beta < 1-\beta$ and $\mu'(s \rightsquigarrow t) \geq \gamma > 1-\gamma$,

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$\upsilon'(s \rightsquigarrow t) \leq \delta < 1-\delta$. Therefore, $s_{(\alpha,\beta)} q L_\rho^\theta$ and $(s \rightsquigarrow t)_{(\gamma,\delta)} q L_\rho^\theta$. Using the condition (19), we have $t_{(\alpha\wedge\gamma,\beta\vee\delta)} \in L_\rho^\theta$ since $s \rightsquigarrow t \leq s \rightsquigarrow t$.

Hence L_ρ^θ is a Łukasiewicz ịntuitionistic fuzzy filter of $\mathcal{H}$.

Theorem 3.13.

Let L_ρ^θ be a Łukasiewicz ịntuitionistic fuzzy set in $\mathcal{H}$ and satisfy the condition(10). If L_ρ^θ satisfies

$$\forall s,t,u \in \mathcal{H}, \forall \alpha,\gamma \in (0,0.5], \forall \beta,\delta \in [0,0.5),\ (s \rightsquigarrow t)_{(\alpha,\beta)} \in L_\rho^\theta,$$

$$(t \rightsquigarrow u)_{(\gamma,\delta)} \in L_\rho^\theta \quad \Rightarrow (s \rightsquigarrow u)_{(\alpha\wedge\gamma,\beta\vee\delta)} q L_\rho^\theta. \tag{20}$$

(or)

$$\forall s,t,u \in \mathcal{H}, \forall \alpha,\gamma \in (0.5,1], \forall \beta,\delta \in [0.5,1),\ (s \rightsquigarrow t)_{(\alpha,\beta)} q L_\rho^\theta,$$

$$(t \rightsquigarrow u)_{(\gamma,\delta)} q L_\rho^\theta \quad \Rightarrow (s \rightsquigarrow u)_{(\alpha\wedge\gamma,\beta\vee\delta)} \in L_\rho^\theta. \tag{21}$$

then L_ρ^θ is a Łukasiewicz ịntuitionistic fuzzy filter of $\mathcal{H}$.

Proof L_ρ^θ be Łukasiewicz ịntuitionistic fuzzy set in $\mathcal{H}$ satisfying (10). Assume that L_ρ^θ satisfies (20). Assume that for $t,u \in \mathcal{H}$ and $\alpha,\gamma \in (0,0.5] \subseteq (0,1]$, $\beta,\delta \in [0,0.5) \subseteq [0,1)$,

$t_{(\alpha,\beta)} \in L_\rho^\theta$ and $(t \rightsquigarrow u)_{(\gamma,\delta)} \in L_\rho^\theta$.

Then the condition (4) implies that $t_{(\alpha,\beta)} = (1 \rightsquigarrow t)_{(\alpha,\beta)} \in L_\rho^\theta$ and (20) implies $(1 \rightsquigarrow u)_{(\alpha\wedge\gamma,\beta\vee\delta)} = u_{(\alpha\wedge\gamma,\beta\vee\delta)} q L_\rho^\theta$. Since $\alpha\wedge\gamma \leq 0.5$, $\beta\vee\delta < 0.5$, it follows that $\mu'(1 \rightsquigarrow u) > \beta\vee\delta \geq \alpha\wedge\gamma$ or $\upsilon'(1 \rightsquigarrow u) \leq \beta\vee\delta$. i.e., $(1 \rightsquigarrow u)_{(\alpha\wedge\gamma,\beta\vee\delta)} = u_{(\alpha\wedge\gamma,\beta\vee\delta)} \in L_\rho^\theta$. Thus L_ρ^θ is a Łukasiewicz ịntuitionistic fuzzy filter of $\mathcal{H}$.

Suppose that L_ρ^θ satisfy (21) and for $t,u \in \mathcal{H}$ and $\alpha,\gamma \in (0.5,1] \subseteq (0,1]$, $\beta,\delta \in [0.5,1) \subseteq [0,1)$,

$t_{(\alpha,\beta)} \in L_\rho^\theta$ and $(t \rightsquigarrow u)_{(\gamma,\delta)} \in L_\rho^\theta$.

Since $\alpha,\gamma > 0.5$ and $\beta,\delta > 0.5$, it follows that $\mu'(t) = \mu'(1 \rightsquigarrow t) > \beta$, $\upsilon'(t) = \upsilon'(1 \rightsquigarrow t) < \alpha$ and

$\mu'(t \rightsquigarrow u) > \delta$, $\upsilon'(t \rightsquigarrow u) < \gamma$. Therefore, $(1 \rightsquigarrow t)_{(\alpha,\beta)} q L_\rho^\theta$ and $(t \rightsquigarrow u)_{(\gamma,\delta)} \in L_\rho^\theta$. Hence $(1 \rightsquigarrow u)_{(\alpha\wedge\gamma,\beta\vee\delta)} = u_{(\alpha\wedge\gamma,\beta\vee\delta)} \in L_\rho^\theta$ since (21) . Thus L_ρ^θ is a Łukasiewicz ịntuitionistic fuzzy filter of $\mathcal{H}$.

Theorem 3.14.

Let L_ρ^θ be a Łukasiewicz ịntuitionistic fuzzy set in $\mathcal{H}$ which satisfies (10). If L_ρ^θ satisfies

$$\forall s,t,u \in \mathcal{H}, \forall \alpha,\gamma \in (0,0.5], \forall \beta,\delta \in [0,0.5),\ (s \rightsquigarrow t)_{(\alpha,\beta)} \in L_\rho^\theta,$$

$$(s \circ u)_{(\gamma,\delta)} \in L_\rho^\theta \quad \Rightarrow (t \circ u)_{(\alpha\wedge\gamma,\beta\vee\delta)} q L_\rho^\theta. \tag{22}$$

(or)

$$\forall s,t,u \in \mathcal{H}, \forall \alpha,\gamma \in (0.5,1], \forall \beta,\delta \in [0.5,1),\ (s \rightsquigarrow t)_{(\alpha,\beta)} q L_\rho^\theta,$$

$$(s \circ u)_{(\gamma,\delta)} q L_\rho^\theta \quad \Rightarrow (t \circ u)_{(\alpha\wedge\gamma,\beta\vee\delta)} \in L_\rho^\theta. \tag{23}$$

then L_ρ^θ is a Łukasiewicz ịntuitionistic fuzzy filter of $\mathcal{H}$.

Proof Suppose that L_ρ^θ is a Łukasiewicz ịntuitionistic fuzzy set in $\mathcal{H}$ satisfying(10). Assume that L_ρ^θ satisfies the condition(22). Assume that for $t,u \in \mathcal{H}$ and $\alpha,\gamma \in (0,0.5] \subseteq (0,1]$, $\beta,\delta \in [0,0.5) \subseteq [0,1)$,

$t_{(\alpha,\beta)} \in L_\rho^\theta$ and $u_{(\gamma,\delta)} \in L_\rho^\theta$.

Then $1 \rightsquigarrow t_{(\alpha,\beta)} = t_{(\alpha,\beta)} \in L_\rho^\theta$ and $1 \circ u_{(\gamma,\delta)} = u_{(\gamma,\delta)} \in L_\rho^\theta$. The condition (22) leads to $(t \circ u)_{(\alpha\wedge\gamma,\beta\vee\delta)} q L_\rho^\theta$. Since $\alpha\wedge\gamma \leq 0.5$, $\beta\vee\delta < 0.5$, it follows that $\upsilon'(t \circ u) \leq \beta\vee\delta$ and $\mu'(t \circ u) \geq \alpha\wedge\gamma$. i.e., $(t \circ u)_{(\alpha\wedge\gamma,\beta\vee\delta)} \in L_\rho^\theta$. Let $s,t \in \mathcal{H}$ and $\alpha \in (0,0.5] \subseteq (0,1]$, $\beta \in [0,0.5) \subseteq [0,1)$ be such that $s \leq t$ and $s_{(\alpha,\beta)} \in L_\rho^\theta$. It follows from the condition (10) that $\mu'(s \rightsquigarrow t) = \mu'(1) \geq \mu'(s) \geq \alpha$ and $\upsilon'(s \rightsquigarrow t) = \upsilon'(1) \leq \upsilon'(s) \leq \beta$. i.e., $(s \rightsquigarrow t)_{(\alpha,\beta)} \in L_\rho^\theta$. Since $s_{(\alpha,\beta)} = (s \circ 1)_{(\alpha,\beta)} \in L_\rho^\theta$, using the condition (22) we have $t_{(\alpha,\beta)} = (t \circ 1)_{(\alpha,\beta)} q L_\rho^\theta$. Since $\alpha \leq 0.5$ and $\beta < 0.5$, $\mu'(t) \geq \alpha$ and $\upsilon'(t) \leq \beta$. Therefore, $t_{(\alpha,\beta)} \in L_\rho^\theta$. Hence L_ρ^θ is a Łukasiewicz ịntuitionistic fuzzy filter of $\mathcal{H}$.

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Now assume L_ρ^θ satisfies(23). For $t, u \in \mathcal{H}$, $\alpha, \gamma \in (0.5,1] \subseteq (0,1]$, $\beta, \delta \in [0.5,1) \subseteq [0,1)$,

$t_{(\alpha,\beta)} \in L_\rho^\theta$ and $u_{(\gamma,\delta)} \in L_\rho^\theta$.

Then $1 \rightsquigarrow t_{(\alpha,\beta)} = t_{(\alpha,\beta)} \in L_\rho^\theta$ and $1 \circ u_{(\gamma,\delta)} = u_{(\gamma,\delta)} \in L_\rho^\theta$. Since $\alpha, \gamma \in (0.5,1]$, $\beta, \delta \in [0.5,1)$, $\mu'(1 \rightsquigarrow t) > \beta$, $v'(1 \rightsquigarrow t) < \alpha$ and $\mu'(1 \circ u) > \delta$, $v'(1 \circ u) < \gamma$. That is $(1 \rightsquigarrow t)_{(\alpha,\beta)} q L_\rho^\theta$ and $(1 \circ u)_{(\gamma,\delta)} q L_\rho^\theta$. The condition (23) induces that $(t \circ u)_{(\alpha\wedge\gamma, \beta\vee\delta)} \in L_\rho^\theta$. Let $s, t \in \mathcal{H}$ and $\alpha \in (0.5,1] \subseteq (0,1]$, $\beta \in [0.5,1) \subseteq [0,1)$ be such that $s \leq t$ and $s_{(\alpha,\beta)} \in L_\rho^\theta$. It follows from the condition (10) that

$$\mu'(s \rightsquigarrow t) = \mu'(1) \geq \mu'(s) \geq \alpha \text{ and } v'(s \rightsquigarrow t) = v'(1) \leq v'(s) \leq \beta$$

which imply that $v'(s \rightsquigarrow t) < \alpha$ or $\mu'(s \rightsquigarrow t) > \beta$. i.e., $(s \rightsquigarrow t)_{(\alpha,\beta)} q L_\rho^\theta$. Since $\mu'(s \circ 1) = \mu'(s) \geq \alpha$, $v'(s \circ 1) = v'(s) \leq \beta$, we have $\mu'(s \circ 1) > \beta$, $v'(s \circ 1) < \alpha$. Therefore $(s \circ 1)_{(\alpha,\beta)} q L_\rho^\theta$. Using (23), we get $(t \circ 1)_{(\alpha,\beta)} = t_{(\alpha,\beta)} \in L_\rho^\theta$. Hence L_ρ^θ is a Łukasiewicz ịntuitionistic fuzzy filter of $\mathcal{H}$.

4. Application in Medical Diagnosis

The application of Łukasiewicz ịntuitionistic fuzzy set spans various fields such as complex decision-making, mathematical modelling, financial forecasting etc…

Now, we will focus on the application of Łukasiewicz ịntuitionistic fuzzy sets in medical diagnosis. We will diagnose the diseases using the following algorithm.

4.1. Algorithm

- Consider the set of patients $P = \{\dot{P}_i\}$.
- Consider the factors such as set of symptoms and test results as $S = \{\dot{S}_i\}$
- From the data collected from the patients, define the ịntuitionistic fuzzy set for each criteria $\dot{S}_i = \{p_i, \mu(p_i), v(p_i)\}$ by

$$\mu(p_i), v(p_i) : P \longrightarrow [0,1]$$

- For an arbitrary value of $\theta \in (0,1)$, find the Łukasiewicz ịntuitionistic fuzzy set.
- Fix the rules or conditions of a disease to be diagnosed using the operations of Łukasiewicz ịntuitionistic fuzzy set.
- Computing according to the condition, we get the resulting Łukasiewicz ịntuitionistic fuzzy set.
- Calculate score by subtracting the non-membership value from the membership value of the resulting Łukasiewicz ịntuitionistic fuzzy set.
- By the value of score, we get the possibility of the patients having that disease.

4.2. Demonstration

Suppose if we are to diagnose the possibility of a person having heart disease.

Let us the consider the Kaggle data set (https://www.kaggle.com/datasets/rishidamarla/heart-disease-prediction). From the above dataset, we consider set of 14 patients $P = \{\dot{P}_1, \dot{P}_2, \dot{P}_3, \dot{P}_4, \dot{P}_5, \dot{P}_6, \dot{P}_7, \dot{P}_8, \dot{P}_9, \dot{P}_{10}, \dot{P}_{11}, \dot{P}_{12}, \dot{P}_{13}, \dot{P}_{14}\}$ going to be diagnosed.

Let us consider the two criteria. $S = \{\dot{S}_1, \dot{S}_2\}$ be the set of test results of the patients for diagnosing heart disease, where $\dot{S}_1 = ST\ depression$, $\dot{S}_2 = Number\ of\ vessels\ fluro$.

To predict the heart disease, let us use the fuzzy rule:

If ST depression is moderate and Number of vessels fluro is moderate then heart disease is present.

Consider the data set,

Patients	ST depression	Number of vessels fluro
$\dot{P}_1$	2.4	3
$\dot{P}_2$	1.6	0
$\dot{P}_3$	0.3	0

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$\dot{P}_4$	0.2	1
$\dot{P}_5$	0.2	1
$\dot{P}_6$	0.4	0
$\dot{P}_7$	0.6	1
$\dot{P}_8$	1.2	1
$\dot{P}_9$	1.2	2
$\dot{P}_{10}$	4	3
$\dot{P}_{11}$	0.5	0
$\dot{P}_{12}$	0	0
$\dot{P}_{13}$	0	0
$\dot{P}_{14}$	2.6	2

From the above data, the įntuitionistic fuzzy sets $S_i : P \longrightarrow [0,1]$ are defined by the membership function.

Membership function for ST depression is moderate:

$$\mu_{moderate} = \begin{cases} 0 & if\ x \leq 1 \\ \frac{x-1}{1.5} & if\ 1 < x \leq 2.5 \\ \frac{4-x}{1.5} & if\ 2.5 < x \leq 4 \\ 0 & if\ x > 4 \end{cases}$$

Membership function for Number of vessels fluro:

$$\mu_{moderate} = \begin{cases} 0 & for\ x < 2 \\ 3-x & for\ 2 \leq x \leq 3 \\ 0 & for\ x > 3 \end{cases}$$

$$\dot{S}_1 = \left\{ \begin{array}{c} (\dot{P}_1, 0.93, 0.07), (\dot{P}_2, 0.4, 0.6), (\dot{P}_3, 0,1), (\dot{P}_4, 0,1), (\dot{P}_5, 0,1), \\ (\dot{P}_6, 0,1), (\dot{P}_7, 0,1), (\dot{P}_8, 0.13, 0.87), (\dot{P}_9, 0.13, 0.87), (\dot{P}_{10}, 0,1), \\ (\dot{P}_{11}, 0,1), (\dot{P}_{12}, 0,1), (\dot{P}_{13}, 0,1), (\dot{P}_{14}, 0.93, 0.07) \end{array} \right\}$$

$$\dot{S}_2 = \left\{ \begin{array}{c} (\dot{P}_1, 0,1), (\dot{P}_2, 0,1), (\dot{P}_3, 0,1), (\dot{P}_4, 0,1), (\dot{P}_5, 0,1), \\ (\dot{P}_6, 0,1), (\dot{P}_7, 0,1), (\dot{P}_8, 0,1), (\dot{P}_9, 1,0), (\dot{P}_{10}, 0,1), \\ (\dot{P}_{11}, 0,1), (\dot{P}_{12}, 0,1), (\dot{P}_{13}, 0,1), (\dot{P}_{14}, 1,0) \end{array} \right\}$$

For $\theta = 0.9$, the Łukasiewicz įntuitionistic fuzzy sets $L^{\theta}_{\dot{S}_i}$ of $\dot{S}_i$ as follows

$$L^{\theta}_{\dot{S}_1} = \left\{ \begin{array}{c} (\dot{P}_1, 0.83, 0), (\dot{P}_2, 0.3, 0.5), (\dot{P}_3, 0, 0.9), (\dot{P}_4, 0, 0.9), (\dot{P}_5, 0, 0.9), \\ (\dot{P}_6, 0, 0.9), (\dot{P}_7, 0, 0.9), (\dot{P}_8, 0.03, 0.77), (\dot{P}_9, 0.03, 0.77), (\dot{P}_{10}, 0, 0.9), \\ (\dot{P}_{11}, 0, 0.9), (\dot{P}_{12}, 0, 0.9), (\dot{P}_{13}, 0, 0.9), (\dot{P}_{14}, 0.83, 0) \end{array} \right\}$$

$$L^{\theta}_{\dot{S}_2} = \left\{ \begin{array}{c} (\dot{P}_1, 0, 0.9), (\dot{P}_2, 0, 0.9), (\dot{P}_3, 0, 0.9), (\dot{P}_4, 0, 0.9), (\dot{P}_5, 0, 0.9), (\dot{P}_6, 0, 0.9), (\dot{P}_7, 0, 0.9), \\ (\dot{P}_8, 0, 0.9), (\dot{P}_9, 0.9, 0), (\dot{P}_{10}, 0, 0.9), (\dot{P}_{11}, 0, 0.9), (\dot{P}_{12}, 0, 0.9), (\dot{P}_{13}, 0, 0.9), (\dot{P}_{14}, 0.9, 0) \end{array} \right\}$$

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$$L_{\dot{S}_1}^{\theta} \oplus L_{\dot{S}_2}^{\theta} = \left\{ \begin{array}{c} (\dot{P}_1, 0.83,0), (\dot{P}_2, 0.3,0.4), (\dot{P}_3, 0,0.8), (\dot{P}_4, 0,0.8), (\dot{P}_5, 0,0.8), \\ (\dot{P}_6, 0,0.8), (\dot{P}_7, 0.9,0), (\dot{P}_8, 0.03,0.67), (\dot{P}_9, 0.93,0), (\dot{P}_{10}, 0,0.8), \\ (\dot{P}_{11}, 0,0.8), (\dot{P}_{12}, 0,0.8), (\dot{P}_{13}, 0,0.8), (\dot{P}_{14}, 1,0) \end{array} \right\}$$

Patients	Score
$\dot{P}_1$	0.83
$\dot{P}_2$	-0.1
$\dot{P}_3$	-0.8
$\dot{P}_4$	-0.8
$\dot{P}_5$	-0.8
$\dot{P}_6$	-0.8
$\dot{P}_7$	0.9
$\dot{P}_8$	-0.64
$\dot{P}_9$	0.93
$\dot{P}_{10}$	-0.8
$\dot{P}_{11}$	-0.8
$\dot{P}_{12}$	-0.8
$\dot{P}_{13}$	-0.8
$\dot{P}_{14}$	1

Patients $\dot{P}_1$,$\dot{P}_7$, $\dot{P}_9$, $\dot{P}_{14}$ are more likely to have heart disease and $\dot{P}_2$, $\dot{P}_3$, $\dot{P}_4$, $\dot{P}_5$, $\dot{P}_6$, $\dot{P}_8$, $\dot{P}_{10}$, $\dot{P}_{11}$, $\dot{P}_{12}$, $\dot{P}_{13}$ are unlikely to have heart disease.

The accuracy depends on the rules we determine. Here we provide a demonstration with 80% accuracy. To enhance the accuracy, work on a more generalized fuzzy rule for diagnosing the disease.

6. Conclusion

In this paper, we introduced the notion of £ukasiewicz ịntuitionistic fuzzy filter based on ịntuitionistic fuzzy point. The properties of £ukasiewicz ịntuitionistic fuzzy filter is investigated. An algorithm is developed for medical diagnosis using the concept of £ukasiewicz ịntuitionistic fuzzy set and its operation. A demonstration is provided for our developed algorithm. In future, we want to make our algorithm applicable for large data sets using programming languages with better accuracy.

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