Differential Equation of COVID-19 with Constraint Algebraic Equation and Sustainable health development With Applications in Neutrosophic Environment

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Abstract

The first appearance of COVID-19 in late 2019 and spread rapidly throughout the world until it became a global pandemic, and the World Health Organization announced some vaccines, and the emergence of a mutated version of COVID-19 was reported in several countries, including Iraq, and we will take care of conducting a study on the spread and dynamics of a virus, this work will be based on the study of the dynamics 3D harvesting predator (COVID-19) differential-algebraic predator-prey economic model (DA-PPM) with functional responses of Holing type-II. The appropriate and realistic description with high accuracy of this phenomenon, which may be natural and emerging as such models, has proven the sentimentality and existence of the solution to the system, and the stability of the system, was discussed in a manner similar to the stability of Matignon. The numerical results showed that the variables of stable unhappy situations have an effect, and this important study can be used as one of the methods of health science to control the spread of COVID-19 and its advanced models. One of the critical aspects of sustainable development is building resilient health systems capable of dealing with epidemics and other crises, the mathematical model (DA-PPM) was applied to analyze the sustainability of health systems under the pressure of Covid-19 and evaluate how long-term public health policies and interventions can prevent overexploitation of resources. Ensuring equitable access to care. The application of the mathematical model to understand the spread of the epidemic is discussed to observe the spread of the epidemic, the possibility of coexistence with it, its close relationship with sustainable development, and to emphasize the importance of the flexibility of the health system. In addition, we apply our results on the neutrosophic supposed data that deals with uncertainty in real-life measurements and compare it with the classical results.

Keywords: Epidemiological ecosystem; Prey-Predator model; Economic effort; Harvesting Prey-Predator; Holling type-II; Differential-algebraic system; Stability; Neutrosophic environment; Neutrosophic data; Neutrosophic variables

1. Introduction

During the previous months, many studies and research have been conducted the development of COVID-19 and its spread. These studies were concerned with the spread of the virus and expectations and suggestions on how to find various ways to treat it, as it was observed that temperature and humidity through contact or breathing are among the most common causes of transmission of the virus. In this work, we will design a new COVID-19 mathematical model and provide a dynamic behavior analytical study on how infection spreads and person-to-person transmission within a community based on Holling-type II modeling. We will also study the dynamic system equilibria and find the algebraic solutions of the system and numerical solutions. This research will help hold of positive results in develop precipitate, precise, and economic monitoring and disclosure mechanisms to reduce the risks of infection with different coronaviruses [1-6] .

A one crucial aspect of sustainable development in health systems and resilience is building resilient health systems capable of handling pandemics and other crises. The mathematical model can be applied to analyze the sustainability

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of health systems under the strain of COVID-19, assessing how long-term public health policies and interventions can prevent overexploitation of resources while ensuring equitable access to care. A study discussed the application of mathematical models to understand the spread of the pandemic, which could be linked to sustainable development by emphasizing the importance of health system resilience [4] .

The most useful method for understanding the complexity of nature is to model the dynamics of the biological ecology. The economic theory of a common -property in resources [7]. Examines the impact of harvest efforts on the ecosystem from an economic standpoint. ecological networks, predator-prey dynamics, and mathematical modeling [3]. Particularly research on biological species interactions or population increase. Numerous mathematical models illustrate population dynamics and competition [8]. following the well-known Lotka-Volterra prey-predator model [5, 9]. An equation is proposed to examine the dynamical behavior in a harvested differential-algebraic allelopathic phytoplankton system, as well as the yield of harvest effort of economic relevance [10-13].

The authors studied the differential equations and dynamical systems [14]. Bioeconomic modeling using differential-algebraic equations of a predator-prey system [15]. and singular biological economic model dynamics and stability [16]. Lotka, Volteraa with their model [17]. and semi-explicit DASs are important sub classes of non-linear DAS [18]. because of the applications in bifurcation and differential equations with economic interest, a basic model of a predator-prey population model with complex dynamics in[19] . Moreover, a predator-prey mathematical model with modified leslie-gower and Holling-type II schemes was discussed.

In this paper, a generalized mathematical model of the harvesting effort predator-prey differential-algebraic system in the present economic interest with the functional response of Holling-type II has been represented as a differential-algebraic mathematical model. The solvability of n-dimensional differential-algebraic system have presented with the necessary assumptions and proofs. A stability of 3D harvesting economic differential-algebraic predator-prey models (DA-PPM) with functional response of Holling-type II was developed with numerical results.

2. Epidemiological Ecosystem

Epidemiological environmental models can represent their temporal evolution through two independent first-order continuous-time differential equations as in this study. Natural pandemic ecosystems contain all the necessary components that are non-linear, high-dimensional, etc. the ability to support induction, predator harvesting and stability is discussed in [20 ,5] .

Harvested prey models with economic equation in predator prey model (PPM) have been examined asing a variety of analytical methods to study the stability, regularity and continuity of these PPMs in two types of epidemiological ecosystems [1, 21] .

3. Predator Prey Model (PPM)

The mathematical model in its general form, which describes the dynamics between the types of interaction between predator and prey, has the following structure [1, 6].

$$\frac{dx}{dt} = xg(x) - yf(x.y) \qquad (1a)$$

$$\frac{dy}{dt} = \sigma y\, f(x.y) - yd(y) \qquad (1b)$$

Where: x: the prey density.

y: predator density.

$g(x)$: denotes the prey growth rate in the absence of a predator.

σ : the conversion rate of prey eaten into a new predator.

$d(y)$: the predator in the absence of its natural prey death rate.

$f(x, y)$: the prey equation.

4. Mathematical Modeling of a Predator Prey Model

Significant interest has been shown in the mathematical modeling of PPM interactions. Given that the Lutka and Volterra origins worked in the last century in the 1920s, with equations of functional response that are strictly prey dependent, such as the Holing family [21]. They predominate in the literatures. In this, work the implementation of the Holing type-II functional response, in which the individual consumption rate predator's increased at a decreasing rate with prey density until it became constant at satiation level. It is a hyperbola; the maximal value asymptotically approaches $a = \frac{1}{h}$ asymptotically, and is defined as.

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$f(x) = \frac{Ax}{1+Ahx} = \frac{ax}{b+x}$, where a search rate A, the time spent on the handling of one prey is h, the maximum attack rate is a and b is half the saturation level $\left(f(b) = \frac{a}{2}\right)$. Their dynamics have been extensively studied for the processes that influence the dynamics of PPM and to know which models could best represent the interactions of species with their economy, depending on how static they might be, which is modeling with economic interest (profit) [1, 21].

Remark 2.1 [22]**:**

A pair of differential equations modeling the interaction between the predator (COVID-19) and prey (human) can be considered as a special

$$\begin{cases} \frac{d\bar{N}}{d\bar{t}} = r\bar{N} \left(1 - \frac{\bar{N}}{K}\right) - \frac{a\,\bar{N}}{b+\bar{N}}\,\bar{P} \\ \frac{d\bar{P}}{d\bar{t}} = \bar{\sigma}\,\frac{a\,\bar{N}}{b+\bar{N}}\,\bar{P} - \bar{d}\bar{P} - \bar{E}\,\bar{P} \end{cases} \tag{2}$$

where: $\bar{N}$ and $\bar{P}$: prey and predator respectively, and r, a , k, $\bar{\sigma}$ and $\bar{d}$ are positive constants.

The prey grows logistically with an inherent growth rate of r and a carrying capacity of K when there is no predation. When a predator (COVID-19) is present, the rate of predation is a, and the prey species N declines proportionately to the functional response.

The operator $\bar{\sigma}$ indicates predation efficiency, which separates the maximum birth rate per capita from predators by the maximum consumption rate per master. Without prey, there was no assessment of predation and predatory species decreased significantly with mortality ($\bar{d}$).

5. Harvesting and Economic of Predator Prey system

We must consider prey (humans) and predators (COVID-19) at a new harvest rate. Study ideas provide a rate of harvest derived from the ability of viral therapies studied in dynamic models [1, 3, 23]. Thus economic recovery and resource management: Sustainable development also emphasizes the efficient use of resources, which in the case of COVID-19, translates to managing healthcare resources, economic impacts, and the deployment of vaccines. Using the predator-prey model, we could examine how economic efforts (such as investments in healthcare infrastructure) influence both the rate of virus spread and economic recovery. The equation modeling economic interest in your system mirrors real-world efforts to balance economic and public health objectives, and managing COVID-19's spread while considering economic ramifications aligns well with sustainable economic recovery [3].

Assumption: Using the two-dimensional normal differential equation system (2), we assume that harvesting take place, however in this predatory (COVID-19) segment it is under harvesting and the introduction of the predator virus (COVID-19) harvest function of $\bar{E}\bar{P}$ into the predatory prey system (2). The goal of the economic theory of common property resource's is to examine how harvesting affects the ecosystem from an economic perspective [1]. According to the equation proposed to investigate the economic harvest effort crop benefit, we have $\bar{E}(\bar{t})$ is the harvest effort, $\bar{N}(\bar{t})$ and $\bar{P}(\bar{t})$ represents the prey and predator harvested population densities respectively. Thus, the total revenue of predator (COVID-19) is

$$TR = \bar{w}\,\bar{E}(\bar{t})\,\bar{P}(\bar{t}) \tag{3}$$

where $\bar{w}$: is the unit price of the harvest (fixed), and the total cost is as follows

$$TC = \bar{c}\bar{E}(\bar{t}) \tag{4}$$

where $\bar{c}$: is the harvested effort unit cost (constant).

Combined with (3), the algebraic equation that consider the economic benefit $\bar{m}$ from the harvest effort $\bar{E}(\bar{t})$, and can be expressed in relation to the predator (COVID-19) as follows:

$$\bar{E}(\bar{t})[w\,\bar{P}(\bar{t}) - \bar{c}] = \bar{m} \tag{5}$$

6. Differential-Algebraic Predator-Prey Model

The general form of two- dimensional PPM (2) is ordinary differential equations to transform it to DAS, and the following assumption is very important:

Assumption: The transformation of the ordinary differential equations (2) to the differential algebraic equations, which is possible by the effect of harvesting effort only, by adding the economic interest equation as an algebraic (constraint) equation, then the system will become as follows:

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$$\frac{d\bar{N}}{d\bar{t}} = r\bar{N} \left(1 - \frac{\bar{N}}{K}\right) - \frac{a\,\bar{N}}{b+\bar{N}}\,\bar{P}$$
$$\frac{d\bar{P}}{d\bar{t}} = \sigma\,\frac{a\,\bar{N}}{b+\bar{N}}\,\bar{P} - \bar{d}\bar{P} - \bar{E}\,\bar{P} \qquad (6)$$
$$0 = \bar{E}(\bar{w}\,\bar{P} - \bar{c}) - \bar{m}$$

The above system is called the Differential-Algebraic Prey-Predator economic system with a Holing type-II functional response and harvesting of a predator (DA-PPS).

7. Non-Dimensional Transformation

The plan starts by determining the non-dimensional form of the DA-PPS, and non-dimensional transformation of DA-PP (6) can be determine by the following lemma:

Lemma 3.1 Consider DA-PPS (6), let the linear: $\boldsymbol{t = r\,\bar{t}}$, $\boldsymbol{N = \frac{\bar{N}}{K}}$, $\boldsymbol{\alpha = \frac{a}{r}}$, $\boldsymbol{\beta = \frac{b}{k}}$, $\boldsymbol{P = \frac{\bar{p}}{K}}$, $\boldsymbol{d = \frac{\bar{d}}{r}}$, $\boldsymbol{E = \frac{\bar{E}}{r}}$, $\boldsymbol{w = r\bar{w}K}$, $\boldsymbol{c = r\bar{c}}$, $\boldsymbol{\mu = \bar{m}}$. Then, the non-dimensional form of the DA-PPS (6) is

$$\frac{dN}{dt} = N\left(1 - N - \frac{\alpha P}{\beta+N}\right)$$
$$\frac{dP}{dt} = P\left(\frac{\sigma\alpha N}{\beta+N} - d - E\right) \qquad (7)$$
$$0 = E(wP - c) - \mu$$

where $N\ and\ P$ represents the prey and predator harvested population densities respectively, σ is the conversion rate of eaten prey into a new predator, d is the natural death rate of the predator in the absence of its prey, α is the maximum attack rate, β is half the saturation level, E is the harvest effort, w is the unit price of the harvest, c is the unit cost of harvest effort, μ is the economic benefit from harvest effort [21, 22] .

8. The Solvability of the DA-PPS

To study the solvability of DA-PPS (7), consider a system described by the semi-explicit description with set $N_1 = (N,P)^T \in R^{n_1}$ as:

$$\dot{N} = F_1(N_1, E; \mu) = \begin{pmatrix} f_1(N,P,E;\mu) \\ f_2(N,P,E;\mu) \end{pmatrix} \qquad (8a)$$
$$0 = F_2(N, E; \mu) \qquad (8b)$$

Assume that: $x_1 = N_1 = (N,P)^T \in R^{n_1}$, $x_2 = E \in R^{n_2}$ with parameter μ. Then, using the problem formulations in [16, 20], the system (8) becomes:

$$\dot{N} = F_1(N_1, E; \mu)$$
$$0 = F_2(N, E; \mu)$$

where, $F_1(N_1, E; \mu) \in C^1(D \times R^{n_2} : R^{n_1})$, $F_2(N, E; \mu) \in C^2(D \times R^{n_2} : R^{n_2})$,

$(N_1; \mu) \in D \subset R^{n_1+1}$, D is an open subset, $N \in R^{n_1}$, $E \in R^{n_2}$ and $\mu \in R$ with $n_1 + n_2 = n$. Therefore, system (7) is solvable and has a unique solution locally.

9. Linearization of DA-PPM

The linearization of DAS (7) about an equilibrium point of the class of equilibrium points in general form determined by the Taylor expansion as follows [14, 22] :

$$\dot{N} - \dot{N}^* = F(N^*, P^*, E^*) + \frac{\partial F}{\partial N}(N^*, P^*, E^*)(\mathbb{N} - N^*) \ + H.O.T$$

$$\dot{N} = F(N^*, P^*, E^*) + \begin{bmatrix} \frac{\partial f_1}{\partial N} & \frac{\partial f_1}{\partial P} & \frac{\partial f_1}{\partial E} \\ \frac{d f_2}{\partial N} & \frac{\partial f_2}{\partial P} & \frac{\partial f_2}{\partial E} \\ \frac{\partial F_2}{\partial N} & \frac{\partial F_2}{\partial P} & \frac{\partial F_2}{\partial E} \end{bmatrix}_{(N^*,P^*,E^*)} . \begin{bmatrix} N - N^* \\ p - p^* \\ E - E^* \end{bmatrix} + H.O.T$$

Because (N^*, P^*, E^*) is the equilibrium point, then $F(N^*, P^*, E^*) = 0$. Let $N - N^* = x_1$, $P - P^* = x_2$, $E - E^* = x_3$ and $\dot{N} = \dot{N}^* = \dot{X}$, since N^* is constant $\Rightarrow \dot{N}^* = 0$

$$F(N, P, E) = F(x_1 + N^*, x_2 + P^*, x_3 + E^*) = G_1(x_1, x_2, x_3).$$

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Therefore, the linearization system will be as following

$$\bar{E}\,\dot{X} = \bar{A}\,X \qquad (9)$$

such that $\bar{E} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $\bar{A}$ is the Jacobian matrix, and $X = (x_1, x_2, x_3)$. Then we see that $n = 3$ and hence $rank(\bar{E}) = 2 < n = 3$. This is because system (7) has one algebraic equation and two ordinary differential equations.

10. Stability Study of DA-PPS with Harvested Predator and Holling Type-II Functional Response

Consider the 3D Differential-Algebraic Prey-Predator system in its non-dimensional form, which includes a harvested predator and a Holing type-II functional response (7) with the same region of the system (5.9) as well as the equilibrium points location. The equilibrium points class of system (7) will consider $\mu < 0$,then the system as follows:

$$\begin{aligned} \dot{N} &= N(1 - N - \tfrac{\alpha P}{\beta + N}) \\ \dot{P} &= P(\sigma \tfrac{\alpha N}{\beta + N} - d - E) \qquad (10) \\ 0 &= E(wP - c) + \mu \end{aligned}$$

which has the equilibrium points: $P_1(0,0,\frac{\mu}{c})$ and the general positive equilibrium point

$P_2^*(1 - \frac{\alpha P}{\beta + N}, \frac{cE - \mu}{wE}, \sigma \frac{\alpha N}{\beta + N} - d)$.

Theorem 3.1: The equilibrium point $P(0,0,\frac{\mu}{c})$ of (10) is saddle, when the rate between the economic benefit from the harvest effort and the harvest effort unit costs more than the natural death rate of the predator in the absence of prey.

Proof: Starting with linearization form (9) of the DA-PPS (10), the equilibrium point's Jacobian matrix $P(0,0,\frac{\mu}{c})$:
$\bar{A}(P_1) = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, where

$a_{11} = \frac{\partial f_1}{\partial N} = 1 - 2N - \frac{\alpha P(\beta + N) - \alpha NP}{(\beta + N)^2}$, $a_{12} = -\sigma N(\beta + N)$, $a_{13} = 0$, $a_{21} = \frac{\sigma \alpha P(\beta + N) - \sigma \alpha NP}{(\beta + N)}$, $a_{22} = \sigma \alpha N(\beta + N) - 0 - d - E$, $a_{23} = -P$

$a_{31} = 0$, $a_{32} = wE$, $a_{33} = wP - c$

$$\therefore \bar{A} = \begin{bmatrix} 1 - 2N - \frac{\alpha P(\beta + N) - \alpha NP}{(\beta + N)^2} & -\sigma N(\beta + N) & 0 \\ \frac{\sigma \alpha P(\beta + N) - \sigma \alpha NP}{(\beta + NB)^2} & \sigma \alpha N(\beta + N) - d - E & -P \\ 0 & wE & wP - c \end{bmatrix} \Bigg|_{(0,0,\frac{\mu}{c})}$$

$$\bar{A}(P) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\left(d + \frac{\mu}{c}\right) & 0 \\ 0 & \frac{w\mu}{c} & -c \end{bmatrix}$$

The regularity is determined as follows:

$$\det\left(\lambda \bar{E} - \bar{A}(P_1)\right) = \begin{vmatrix} \lambda - 1 & 0 & 0 \\ 0 & \lambda + \left(d + \frac{\mu}{c}\right) & 0 \\ 0 & -\frac{w\mu}{c} & c \end{vmatrix}$$

$$= c(\lambda - 1)\left(\lambda + \left(d + \frac{\mu}{c}\right)\right) = 0$$

The classes of regularity are: Regularity of

$$\bar{A}(P_1) = \left\{(x_1, x_2, x_3) \,\middle|\, since\ \mu < 0\ with\ \frac{\mu}{c} = d\right\}$$

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Not Regularity of

$$\bar{A}(P_1) = \left\{(x_1, x_2, x_3) \,\middle|\, c \neq 0 \;, \frac{\mu}{c} \neq d\right\}, \text{ where } P_1 \in \overline{EQ}$$

Therefore, the system is not regular with some parameters and constants, so we can determine the stability by regularity, the eigenvalues are $\lambda_1 = -1 \;\; and \;\; \lambda_2 = -(d + \frac{\mu}{c})$. Because $\mu < 0$ in this case $\lambda_2 = \frac{\mu}{c} - d$. Therefor $\lambda_1 < 0$, and if $\frac{\mu}{c} > d$ this implies that $\lambda_2 > 0$ then P is a saddle (unstable (see figure 1).

Lemma 3.1: If the rate between the economic benefit from the harvest effort and the harvest effort unit cost is less than that of predator in the absence of its prey's natural death rate, then, the equilibrium point $P(0,0,\frac{\mu}{c})$ of (10) is stable (see figure 2).

11. Numerical Results

In this section we will discuss some of the COVID-19 data acquired for Iraq country by the World Health Organization, According to which 2,464,997 confirmed cases of COVID-19 and 25,369 fatalities occurred between January 3, 2020, and December 23, 2022, at 4:54 p.m. CET. On December 10, 2022, 19,534,812 doses of the vaccine were administered.

The simulation results are summarized along with the initial conditions used in Figures 1 and 2. The descriptor system (10) is numerically integrated with the previously stated model parameter values, and the natural predator death rate without its prey is d=0.09, 0.04, and less. The conversion rate of eaten prey into new predator is $\sigma = 6.1541$, ehere the maximum attack rate is $\alpha = 0.02$, half of the saturation level is $\beta = 0.04$, E=0.01 is the harvest effort, w=0.001 is the harvest price, c=0.02 is the harvest cost, and μ=0.002 is the economic gain of the harvest effort.

According to the research results, descriptor system (10) outperforms its integer-order equivalent in terms of convergence to the coexistence equilibrium point (see Fig. 2). The dynamics of the suggested Holing type-II of COVID-19 are better described by the differential-algebraic predator-predator system with a harvested predator and Holing type-II functional response (10). The simulation findings also show that, the injury curves can be flattened using these parameters as control units. Therefore, until the conditions and causes of infection are established, these models may offer better public health policies to either coexist with the lethal epidemic or attenuate or slow its rapid expansion.

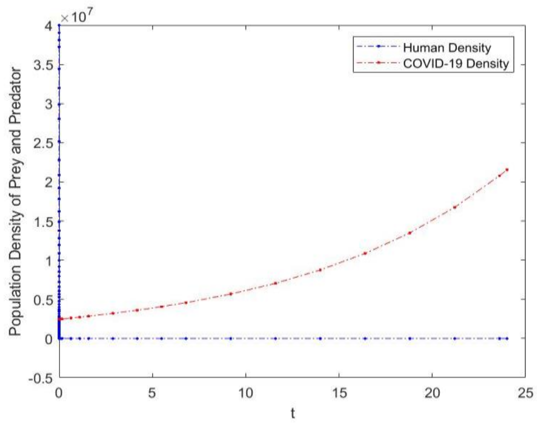

Figure 1. Phase plots of system (10) with $\frac{\mu}{c} > d$.

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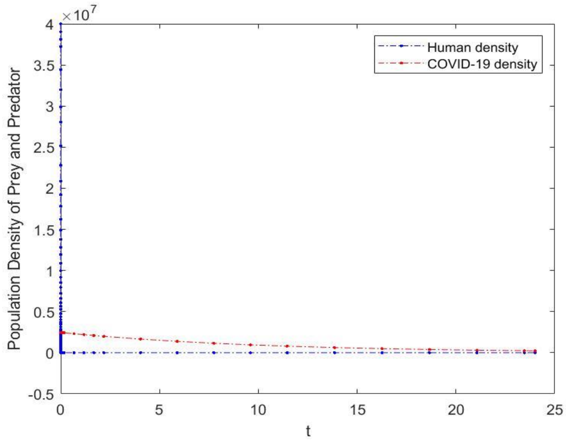

Figure 2. Phase plots of system (10) with $\frac{\mu}{c} < d$.

12. Applications in neutrosophic environment

The differential-algebraic system is one of the strong and modern systems to be applied in many areas of life. In this study, we applied the above system to one of the vital topics (Covid-19) to determine the extent of people's coexistence with the virus as well as access to fundamental ideas such as treatment for infected cases or elimination of the virus. With environmental sustainability and pandemic response, environmental sustainability is another element of sustainable development. The global pandemic highlighted how human behavior affects the environment, and COVID-19 provided a brief example of reduced human activity leading to environmental improvements. Integrating this into your mathematical model might allow for the exploration of how pandemic responses can protect public health and contribute to or detract from environmental sustainability.

For applying our results in neutrosophic environment, we must do a neutrosophication to real variables to be neutrosophic variables as follows:

Neutrosophic Differential-Algebraic Predator-Prey Model

Assumption: The transformation of the ordinary differential equations (2) to the neutrosophic differential algebraic equations, which is possible by the effect of harvesting effort only, by adding the economic interest equation as an algebraic (constraint) equation, where we substitute the real variables $\bar{N}, P, t$ with generalized neutrosophic versions $(\bar{N}) = m + nI, (P) = v + wI, (t) = x + yI$

then the system will become as follows:

$$\frac{d\overline{(N)}}{d\overline{(t)}} = (r)(\bar{N}) \quad \left(1 - \frac{\bar{N}}{K}\right) - \frac{a\,\bar{N}}{b+\bar{N}}\,\overline{(P)}$$

$$\frac{d\overline{(P)}}{d\overline{(t)}} = \sigma\,\frac{a\,\bar{N}}{b+\bar{N}}\,\overline{(P)} - \bar{d}\overline{(P)} - \bar{E}\,(\bar{P})$$

$$0 = \bar{E}(\bar{w}\,\bar{P} - \bar{c}) - \bar{m}$$

The above system is called the neutrosophic Differential-Algebraic Prey-Predator economic system.

Neutrosophic Non-Dimensional Transformation

Consider the previous system, let the linear: $(N) = m + nI, (P) = v + wI, (t) = x + yI$

$(\boldsymbol{t}) = \boldsymbol{r}\,(\bar{\boldsymbol{t}})\,, (\boldsymbol{N}) = \frac{(\bar{\boldsymbol{N}})}{\boldsymbol{K}}, \boldsymbol{\alpha} = \frac{\boldsymbol{a}}{\boldsymbol{r}},\quad \boldsymbol{\beta} = \frac{\boldsymbol{b}}{\boldsymbol{k}}, (\boldsymbol{P}) = \frac{(\bar{\boldsymbol{p}})}{\boldsymbol{K}}, \boldsymbol{d} = \frac{\bar{\boldsymbol{d}}}{\boldsymbol{r}},\quad \boldsymbol{E} = \frac{\bar{\boldsymbol{E}}}{\boldsymbol{r}}, \boldsymbol{w} = \boldsymbol{r}\bar{\boldsymbol{w}}\boldsymbol{K},\quad \boldsymbol{c} = \boldsymbol{r}\bar{\boldsymbol{c}}, \boldsymbol{\mu} = \bar{\boldsymbol{m}}.$ With the neutrosophication of variables

Hence: $$\frac{d(N)}{d(t)} = (N)\left(1 - (N) - \frac{\alpha P}{\beta + (N)}\right)$$

$$\frac{d(P)}{d(t)} = P\left(\frac{\sigma\alpha(N)}{\beta + (N)} - d - E\right)$$

$$0 = E(wP - c) - \mu$$

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The Solvability of The neutrosophic system

Consider a neutrosophic system described by the semi-explicit description with the previous neutrosophication of variables:

$(N_1) = ((N), (P))^T \in R^{n_1}$ as:

$$(\dot{N}) = F_1((N_1), E; \mu) = \begin{pmatrix} f_1((N), P, E; \mu) \\ f_2((N), P, E; \mu) \end{pmatrix}$$

$$0 = F_2((N), E; \mu)$$

Assume that: $(x_1) = (N_1) = ((N), (P))^T \in R^{n_1}, x_2 = E \in R^{n_2}$ with parameter μ. Then, the system becomes:

$$(\dot{N}) = F_1((N_1), E; \mu)$$

$$0 = F_2((N), E; \mu)$$

where, $F_1((N_1), E; \mu) \in C^1(D \times R^{n_2}: R^{n_1})$, $F_2((N), E; \mu) \in C^2(D \times R^{n_2}: R^{n_2})$,

$((N_1); \mu) \in D \subset R^{n_1+1}$, D is an open subset, $(N) \in R^{n_1}, E \in R^{n_2}$ and $\mu \in R$ with $n_1 + n_2 = n$.

Neutrosophic numerical Results

We will apply our study on some supposed data written with neutrosophic variables and coefficients.

we will discuss some of the data acquired , According to which 2,464,997+I confirmed cases of COVID-19 and 25,369+I fatalities.

The descriptor system is numerically integrated with the previously stated model parameter values, and the natural predator death rate without its prey is d=0.09+0.01I, $0.04 + 0.02I$, and less. The conversion rate of eaten prey into new predator is $\sigma = 6.1541 + I$, ehere the maximum attack rate is $\alpha = 0.02 + 0.01I$, half of the saturation level is $\beta = 0.04 + 0.02I$, E=$0.01 + I$ is the harvest effort, $w = 0.001 + I$ is the harvest price, c=0.02+0.01I is the harvest cost, and μ=0.002+0.001I is the economic gain of the harvest effort.

According to the research results, descriptor system outperforms its integer-order equivalent in terms of convergence to the coexistence equilibrium point.

The dynamics of the suggested Holing type-II of COVID-19 are better described by the differential-algebraic predator-predator system with a harvested predator and Holing type-II functional response.

We show the results in two figures. The figure (3) shows the results on the real part of the neutrosophic variable, and figure (4) shows the results on the neutrosophic part (coefficients of the indeterminacy I).

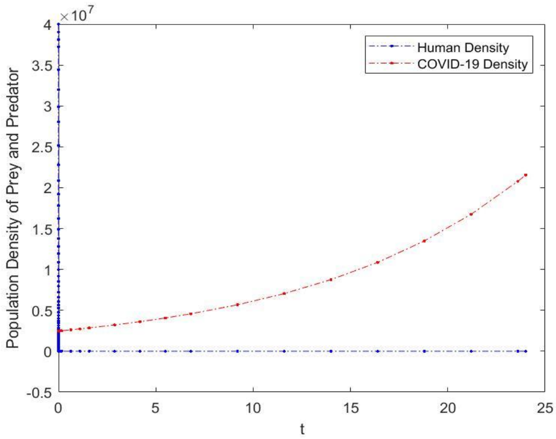

Figure 3. Phase plots of system for the real part with $\frac{\mu}{c} + I > d + I$.

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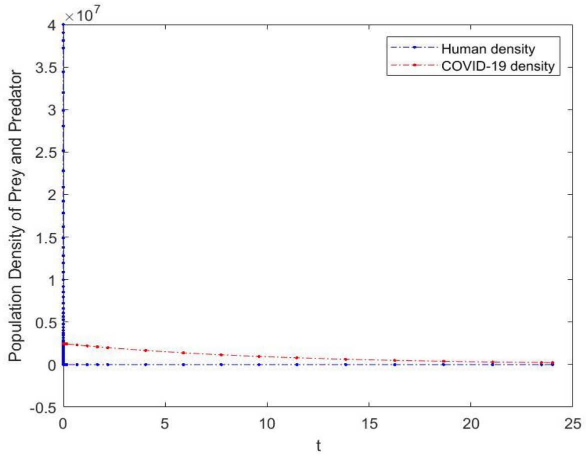

Figure 4. Phase plots of system for the neutrosophic part with $\frac{\mu}{c} + I < d + I$.

13. Conclusion

This work is based on the study of chromosomes dynamics in the three-dimensional harvesting predator (COVID-19) differential-algebraic prey-predator economic model (DA-PPM) with Holing type-II functional responses. The appropriate and realistic description with high accuracy of this phenomenon and mathematical modeling of the original predator-prey model, harvesting predator, and economic theory as a constraint equation with non-dimensional form has proven, the sentimentality and existence of the solution to the system, and numerical modeling results have demonstrated stability.

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