Kamal Transform for Neutrosophic Initial Value Problem

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Abstract

The manuscript dealt with the problem of the initial value, especially in second-order differential equations with three degrees of Neutrosophic conditions, which are truth, falsity, and indeterminacy. In addition, we exploited the Kamal transformation to solve it.

Keywords: Fuzzy logic; Neutrosophic logic; Kamal transform; Neutrosophic initial conditions

1. Introduction

Natural phenomena are described by differential equations under certain initial conditions that control the solutions of these equations. The classical solution of the differential equation as a function may not be as accurate as it should be. This has been overcome by using fuzzy sets, which are functions that are independent of the true and false memberships of an element. Since this logic does not carry the term of indeterminacy, this obstacle has been dealt with by developing the Neutrosophic set.

Florentin Smarandache tackles several problems using a novel idea called "neutrosophic set theory," which is drawn from the domain of uncertainty. Underlined searchable fields include description, cause, factual belongingness, false values, and values of unknown [1]. As a result, fuzzy logic is thought to be generalized by neutrosophic logic, a new logic in mathematics that is based on the idea of indeterminacy [2]. It has been established that neoclassical logic is among the most crucial and useful modeling tools in almost all engineering and research application areas. Differential equations may be used to explain a wide range of real-world events, allowing the approach to be applied to them (see [3], [4], [5], [6], and [7]).

The Kamal transform (KT), first presented by Abdelilah and Hassan in 2016[14], is employed to address an ordinary differential equation. In this work, we work through an ordinary differential equation of second order (2^{nd} ODE) in a Neutrosophic environment using the Kamal transform technique, with many cut-point computations made in the solution.

2. Second order differential equation with Fuzzy Kamal transform

Fuzzy Kamal transform for first order differential equation was defined in [8] by:

Assume that $ų(ş)e^{-\frac{ş}{\mathscr{v}}}$ is an improper Riemann integrable on $[0,\infty)$ and let $ų(t)$ be a continuous fuzzy – valued function, therefore $\int_0^\infty ų(ş)e^{-\frac{ş}{\mathscr{v}}}dş$ is known as the fuzzy Kamal transform and is represented by:

$$\widehat{\mathcal{K}}[ų(ş)] = \int_0^\infty ų(ş)e^{-\frac{ş}{\mathscr{v}}}dş\ ,\ (\mathscr{v} > 0\ and\ integer).$$

Thus,

$$\int_0^\infty ų(ş)e^{-\frac{ş}{\mathscr{v}}}dş = \left(\int_0^\infty \underline{ų}(ş,\alpha)e^{-\frac{ş}{\mathscr{v}}}dş\, , \int_0^\infty \overline{ų}(ş,\alpha)e^{-\frac{ş}{\mathscr{v}}}dş\right)$$

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By the classical definition of KT:

$\mathcal{K}\left[\underline{ų}(ş,\alpha)\right] = \int_0^\infty \underline{ų}(ş,\alpha)e^{-\frac{ş}{v}}dş$ and $\mathcal{K}[\overline{ų}(ş,\alpha)] = \int_0^\infty \overline{ų}(ş,\alpha)e^{-\frac{ş}{v}}dş$,

then

$$\hat{\mathcal{K}}[ų(ş,\alpha)] = \left(\mathcal{K}\left[\underline{ų}(ş,\alpha)\right], \mathcal{K}[\overline{ų}(ş,\alpha)]\right).$$

Theorem 2.1[10] Assume that $ų: [a,b] \to [0,1]$,be a function such that $ų(ş) = \left(\underline{ų}(ş,\alpha), \overline{ų}(ş,\alpha)\right)$,for all α in [0,1] then :

1. İf ų is a first form – differentiable(i-d) then $\underline{ų}(ş,\alpha)$ and $\overline{ų}(ş,\alpha)$ and $ų'(ş,\alpha) = \left(\underline{ų}'(ş,\alpha), \overline{ų}'(ş,\alpha)\right)$ are differentiable function.
2. İf ų is a second- form differentiable (ii-d) then $\underline{ų}(ş,\alpha)$ and $\overline{ų}(ş,\alpha)$ and $ų'(ş,\alpha) = \left(\overline{ų}'(ş,\alpha), \underline{ų}'(ş,\alpha)\right)$ are differentiable function

The fuzzy Kamal transform for second-order ordinary differential equation will be defined as follows in this work:

Definition 2.1 [12] Suppose ų be a continuous function with fuzzy values that has the following property: $ų(ş_0): (a,b) \to R$ and $ş_0 \in (a,b)$. If an element $ų''(ş_0) \in R$ exists, we say that a mapping ų is strongly generalized differentiable at $ş_0$, such that:

i. $\forall ŧ > 0$,sufficiently small,$\exists\, ų'(ş_0+ŧ) \ominus ų'(ş_0), ų'(ş_0) \ominus ų'(ş_0-ŧ)$ where $\lim_{ŧ\to 0}\frac{ų'(ş_0+ŧ)\ominus ų'(ş_0)}{ŧ} = \lim_{ŧ\to 0}\frac{ų'(ş_0)\ominus ų'(ş_0-ŧ)}{ŧ} = ų''(ş_0)$ or
ii. $\forall ŧ > 0$,sufficiently small,$\exists\, ų'(ş_0) \ominus ų'(ş_0+ŧ), ų'(ş_0-ŧ) \ominus ų'(ş_0)$ where $\lim_{ŧ\to 0}\frac{ų'(ş_0)\ominus ų'(ş_0+ŧ)}{ŧ} = \lim_{ŧ\to 0}\frac{ų'(ş_0-ŧ)\ominus ų'(ş_0)}{ŧ} = ų''(ş_0)$ or
iii. $\forall ŧ > 0$,sufficiently small,$\exists\, ų'(ş_0+ŧ) \ominus ų'(ş_0), ų'(ş_0-ŧ) \ominus ų'(ş_0)$ where $\lim_{ŧ\to 0}\frac{ų'(ş_0+ŧ)\ominus ų'(ş_0)}{ŧ} = \lim_{ŧ\to 0}\frac{ų'(ş_0-ŧ)\ominus ų'(ş_0)}{ŧ} = ų''(ş_0)$ or
iv. $\forall ŧ > 0$,sufficiently small,$\exists\, ų'(ş_0) \ominus ų'(ş_0+ŧ), ų'(ş_0) \ominus ų'(ş_0-ŧ)$ where $\lim_{ŧ\to 0}\frac{ų'(ş_0)\ominus ų'(ş_0+ŧ)}{ŧ} = \lim_{ŧ\to 0}\frac{ų'(ş_0)\ominus ų'(ş_0-ŧ)}{ŧ} = ų''(ş_0)$

Theorem 2.2 [11] Given that $ų(ş) = \left(\underline{ų}(ş,\alpha), \overline{ų}(ş,\alpha)\right)$ $\forall\alpha \in [0,1]$, ,and that $ų(ş)$ and $ų'(ş)$ are two fuzzy-valued differentiable functions, then:

i. Let $ų(ş)$ and $ų'(ş)$are i-d ,or let $ų(ş)$ and $ų'(ş)$ are ii-d, then $\underline{ų}(ş,\alpha)$ *and* $\overline{ų}(ş,\alpha)$ possess 1st - and 2nd - order derivatives such that $ų''(ş) = \left(\underline{ų}''(ş,\alpha), \overline{ų}''(ş,\alpha)\right)$.
ii. Let $ų(ş)$ is i-d and $ų'(ş)$ is the ii-d , or let $ų(ş)$ be ii-d and $ų'(ş)$ be a i-d , then $\underline{ų}(ş,\alpha)$ *and* $\overline{ų}(ş,\alpha)$ possess 1st - and 2nd -order derivatives such that $ų''(ş) = \left(\overline{ų}''(ş,\alpha), \underline{ų}''(ş,\alpha)\right)$.

Theorem 2.3 [13] Given a continuous Neutrosophic valued function $ų(ş)$ and $ų'(ş)$ on $[0,\infty)$ and a piecewise continuous function with Neutrosophic values $ų''(ş)$ on $[0,\infty)$ then,

a. $\hat{\mathcal{K}}[ų''(ş)] = \left\{\frac{1}{v^2}\hat{\mathcal{K}}[ų(ş)] \ominus \frac{1}{v}ų(0)\right\} \ominus ų'(0)$, where $ų(ş)$ and $ų'(ş)$ are i-d.
b. $\hat{\mathcal{K}}[ų''(ş)] = \frac{-1}{v}ų(0) \ominus \left\{\frac{-1}{v^2}\hat{\mathcal{K}}[ų(ş)]\right\} \ominus ų'(0)$, where $ų(ş)$ be i-d and $ų'(ş)$ be ii-d.
c. $\hat{\mathcal{K}}[ų''(ş)] = \frac{-1}{v}ų(0) \ominus \left\{\frac{-1}{v^2}\hat{\mathcal{K}}[ų(ş)]\right\} \ominus ų'(0)$, where $ų'(ş)$ i-d and $ų(ş)$ be ii-d.
d. $\hat{\mathcal{K}}[ų''(ş)] = \frac{1}{v^2}\hat{\mathcal{K}}[ų(ş)] \ominus \frac{1}{v}ų(0) \ominus ų'(0)$, where $ų(ş)$ and $ų'(ş)$ are ii-d

Proof: a. when $ų(ş)$ and $ų'(ş)$ are i-d.

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$$\left\{\frac{1}{\upsilon^2}\widehat{\mathcal{K}}[\mathrm{ų}(\text{ş})]\ominus\frac{1}{\upsilon}\mathrm{ų}(0)\right\}\ominus\mathrm{ų}'(0)$$
$$=\left(\frac{1}{\upsilon^2}\underline{\mathrm{ų}}(\text{ş},\alpha)-\frac{1}{\upsilon}\underline{\mathrm{ų}}(0,\alpha)-\underline{\mathrm{ų}}'(0,\alpha),\frac{1}{\upsilon^2}\mathcal{K}[\overline{\mathrm{ų}}(\text{ş},\alpha)]-\frac{1}{\upsilon}\overline{\mathrm{ų}}(0,\alpha)-\overline{\mathrm{ų}}'(0,\alpha)\right)$$

Since $\mathcal{K}\left[\underline{\mathrm{ų}}''(\text{ş},\alpha)\right]=\frac{1}{\upsilon^2}\mathcal{K}[\underline{\mathrm{ų}}(\text{ş},\alpha)]-\frac{1}{\upsilon}\underline{\mathrm{ų}}(0,\alpha)-\underline{\mathrm{ų}}'(0,\alpha)$,

$\mathcal{K}[\overline{\mathrm{ų}}''(\text{ş},\alpha)]=\frac{1}{\upsilon^2}\mathcal{K}[\overline{\mathrm{ų}}(\text{ş},\alpha)]-\frac{1}{\upsilon}\overline{\mathrm{ų}}(0,\alpha)-\overline{\mathrm{ų}}'(0,\alpha)$ and $\mathrm{ų}(\text{ş})$ and $\mathrm{ų}'(\text{ş})$ are i-d using theorem 2.2 .

$$\underline{\mathrm{ų}''}(\text{ş},\alpha)=\underline{\mathrm{ų}}''(\text{ş},\alpha),\overline{\mathrm{ų}''}(\text{ş},\alpha)=\overline{\mathrm{ų}}''(\text{ş},\alpha)$$

Since $\mathrm{ų}(\text{ş})$ is i-d using theorem 2.1

$$\underline{\mathrm{ų}'}(0,\alpha)=\underline{\mathrm{ų}}'(0,\alpha) and\ \overline{\mathrm{ų}'}(0,\alpha)=\overline{\mathrm{ų}}'(0,\alpha)$$

$\mathcal{K}\left[\underline{\mathrm{ų}''}(\text{ş},\alpha)\right]=\frac{1}{\upsilon^2}\mathcal{K}[\underline{\mathrm{ų}}(\text{ş},\alpha)]-\frac{1}{\upsilon}\underline{\mathrm{ų}}(0,\alpha)-\underline{\mathrm{ų}}'(0,\alpha)$,

$\mathcal{K}[\overline{\mathrm{ų}''}(\text{ş},\alpha)]=\frac{1}{\upsilon^2}\mathcal{K}[\overline{\mathrm{ų}}(\text{ş},\alpha)]-\frac{1}{\upsilon}\overline{\mathrm{ų}}(0,\alpha)-\overline{\mathrm{ų}'}(0,\alpha)$

$$\left\{\frac{1}{\upsilon^2}\widehat{\mathcal{K}}[\mathrm{ų}(\text{ş})]\ominus\frac{1}{\upsilon}\mathrm{ų}(0)\right\}\ominus\mathrm{ų}'(0)=\left(\mathcal{K}\left[\underline{\mathrm{ų}''}(\text{ş},\alpha)\right],\mathcal{K}[\overline{\mathrm{ų}''}(\text{ş},\alpha)]\right)=\widehat{\mathcal{K}}[\mathrm{ų}''(\text{ş})]$$

$$\widehat{\mathcal{K}}[\mathrm{ų}''(\text{ş})]=\left\{\frac{1}{\upsilon^2}\widehat{\mathcal{K}}[\mathrm{ų}(\text{ş})]\ominus\frac{1}{\upsilon}\mathrm{ų}(0)\right\}\ominus\mathrm{ų}'(0)$$

b. when $\mathrm{ų}(\text{ş})$ is i-d and $\mathrm{ų}'(\text{ş})$ is ii-d

$$\frac{-1}{\upsilon}\mathrm{ų}(0)\ominus\left\{\frac{-1}{\upsilon^2}\widehat{\mathcal{K}}[\mathrm{ų}(\text{ş})]\right\}\ominus\mathrm{ų}'(0)=\frac{-1}{\upsilon}\overline{\mathrm{ų}}(0,\alpha)+\frac{-1}{\upsilon^2}\mathcal{K}[\overline{\mathrm{ų}}(\text{ş},\alpha)]-\overline{\mathrm{ų}'}(0,\alpha),$$
$$\frac{-1}{\upsilon}\underline{\mathrm{ų}}(0,\alpha)+\frac{-1}{\upsilon^2}\mathcal{K}\left[\underline{\mathrm{ų}}(\text{ş},\alpha)\right]-\underline{\mathrm{ų}}'(0,\alpha)\ .$$

Since $\mathcal{K}[\overline{\mathrm{ų}''}(\text{ş},\alpha)]=\frac{1}{\upsilon^2}\mathcal{K}[\overline{\mathrm{ų}}(\text{ş},\alpha)]-\frac{1}{\upsilon}\overline{\mathrm{ų}}(0,\alpha)-\overline{\mathrm{ų}'}(0,\alpha)$,

$\mathcal{K}\left[\underline{\mathrm{ų}''}(\text{ş},\alpha)\right]=\frac{1}{\upsilon^2}\mathcal{K}\left[\underline{\mathrm{ų}}(\text{ş},\alpha)\right]-\frac{1}{\upsilon}\underline{\mathrm{ų}}(0,\alpha)-\underline{\mathrm{ų}'}(0,\alpha)$ **,** $\mathrm{ų}(\text{ş})$ is i-d and $\mathrm{ų}'(\text{ş})$ is ii-d using theorem 2.2

$\overline{\mathrm{ų}''}(\text{ş},\alpha)=\underline{\mathrm{ų}}''(\text{ş},\alpha),\underline{\mathrm{ų}''}(\text{ş},\alpha)=\overline{\mathrm{ų}}''(\text{ş},\alpha)$,

Since $\mathrm{ų}(\text{ş})$ is i-d using theorem 2.1

$$\overline{\mathrm{ų}'}(0,\alpha)=\overline{\mathrm{ų}}'(0,\alpha),\underline{\mathrm{ų}'}(0,\alpha)=\underline{\mathrm{ų}}'(0,\alpha)$$

$$\mathcal{K}\left[\underline{\mathrm{ų}''}(\text{ş},\alpha)\right]=\frac{-1}{\upsilon}\overline{\mathrm{ų}}(0,\alpha)+\frac{1}{\upsilon^2}\mathcal{K}[\overline{\mathrm{ų}}(\text{ş},\alpha)]-\overline{\mathrm{ų}'}(0,\alpha)$$

$$\mathcal{K}[\overline{\mathrm{ų}''}(\text{ş},\alpha)]=\frac{-1}{\upsilon}\underline{\mathrm{ų}}(0,\alpha)+\frac{1}{\upsilon^2}\mathcal{K}\left[\underline{\mathrm{ų}}(\text{ş},\alpha)\right]-\underline{\mathrm{ų}'}(0,\alpha)$$

$$\frac{-1}{\upsilon}\mathrm{ų}(0)\ominus\left\{\frac{-1}{\upsilon^2}\widehat{\mathcal{K}}[\mathrm{ų}(\text{ş})]\right\}\ominus\mathrm{ų}'(0)=\left(\mathcal{K}\left[\underline{\mathrm{ų}''}(\text{ş},\alpha)\right],\mathcal{K}[\overline{\mathrm{ų}''}(\text{ş},\alpha)]\right)=\widehat{\mathcal{K}}[\mathrm{ų}''(\text{ş})]$$

$$\widehat{\mathcal{K}}[\mathrm{ų}''(\text{ş})]=\frac{-1}{\upsilon}\mathrm{ų}(0)\ominus\left\{\frac{-1}{\upsilon^2}\widehat{\mathcal{K}}[\mathrm{ų}(\text{ş})]\right\}\ominus\mathrm{ų}'(0)$$

c. $\mathrm{ų}'(\text{ş})$ is i-d and $\mathrm{ų}(\text{ş})$ is ii-d.

$$\frac{-1}{\upsilon}\mathrm{ų}(0)\ominus\left\{\frac{-1}{\upsilon^2}\widehat{\mathcal{K}}[\mathrm{ų}(\text{ş})]\right\}\ominus\mathrm{ų}'(0)=\frac{-1}{\upsilon}\overline{\mathrm{ų}}(0,\alpha)+\frac{1}{\upsilon^2}\mathcal{K}[\overline{\mathrm{ų}}(\text{ş})]-\underline{\mathrm{ų}'}(0,\alpha),$$
$$\frac{-1}{\upsilon}\underline{\mathrm{ų}}(0,\alpha)+\frac{1}{\upsilon^2}\mathcal{K}\left[\underline{\mathrm{ų}}(\text{ş})\right]-\overline{\mathrm{ų}'}(0,\alpha)$$

Since $\mathcal{K}[\overline{\mathrm{ų}}''(\text{ş},\alpha)]=\frac{1}{\upsilon^2}\mathcal{K}[\overline{\mathrm{ų}}(\text{ş},\alpha)]-\frac{1}{\upsilon}\overline{\mathrm{ų}}(0,\alpha)-\overline{\mathrm{ų}}'(0,\alpha)$

$\mathcal{K}\left[\underline{\mathrm{ų}}''(\text{ş},\alpha)\right]=\frac{1}{\upsilon^2}\mathcal{K}\left[\underline{\mathrm{ų}}(\text{ş}),\alpha\right]-\frac{1}{\upsilon}\underline{\mathrm{ų}}(0,\alpha)-\underline{\mathrm{ų}}'(0,\alpha)$, $\mathrm{ų}(\text{ş})$ is ii-d and $\mathrm{ų}'(\text{ş})$ is i-d using theorem 2.2

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$$\overline{ц''}(ş,\alpha)=\underline{ц}''(ş,\alpha),\underline{ц''}(ş,\alpha)=\overline{ц}''(ş,\alpha)$$

Since ц(ş) is ii-d using theorem 2.1

$$\overline{ц'}(0,\alpha)=\underline{ц}'(0,\alpha),\underline{ц'}(0,\alpha)=\overline{ц}'(0,\alpha)$$

$$\mathcal{K}\left[\underline{ц''}(ş,\alpha)\right]=\frac{1}{\mathcal{v}^2}\mathcal{K}[\overline{ц}(ş,\alpha)]-\frac{1}{\mathcal{v}}\overline{ц}(0,\alpha)-\underline{ц'}(0,\alpha)$$

$$\mathcal{K}[\overline{ц''}(ş,\alpha)]=\frac{1}{\mathcal{v}^2}\mathcal{K}\left[\underline{ц}(ş,\alpha)\right]-\frac{1}{\mathcal{v}}\underline{ц}(0,\alpha)-\overline{ц'}(0,\alpha)$$

$$\frac{-1}{\mathcal{v}}ц(0)\ominus\left\{\frac{-1}{\mathcal{v}^2}\hat{\mathcal{K}}[ц(ş)]\right\}\ominus ц'(0)=\left(\mathcal{K}\left[\overline{ц''}(ş,\alpha)\right],\mathcal{K}\left[\underline{ц''}(ş,\alpha)\right]\right)=\hat{\mathcal{K}}[ц''(ş)]$$

$$\hat{\mathcal{K}}[ц''(ş)]=\frac{-1}{\mathcal{v}}ц(0)\ominus\left\{\frac{-1}{\mathcal{v}^2}\hat{\mathcal{K}}[ц(ş)]\right\}\ominus ц'(0)$$

d. when ц(ş) and ц′(ş) are ii-d

$$\frac{1}{\mathcal{v}^2}\hat{\mathcal{K}}[ц(ş)]\ominus\frac{1}{\mathcal{v}}ц(0)\ominus ц'(0)=\begin{pmatrix}\frac{1}{\mathcal{v}^2}\underline{ц}[ц(ş,\alpha)]-\frac{1}{\mathcal{v}}\underline{ц}(0,\alpha)-\overline{ц'}(0,\alpha)\\ ,\frac{1}{\mathcal{v}^2}\overline{ц}[ц(ş,\alpha)]-\frac{1}{\mathcal{v}}\overline{ц}(0,\alpha)-\underline{ц'}(0,\alpha)\end{pmatrix}$$

Since

$$\mathcal{K}\left[\underline{ц''}(ş,\alpha)\right]=\frac{1}{\mathcal{v}^2}\mathcal{K}\left[\underline{ц}(ş,\alpha)\right]-\frac{1}{\mathcal{v}}\underline{ц}(0,\alpha)-\underline{ц'}(0,\alpha)$$

$\mathcal{K}[\overline{ц}''(ş,\alpha)]=,\frac{1}{\mathcal{v}^2}\mathcal{K}[\overline{ц}(ş,\alpha)]-\frac{1}{\mathcal{v}}\overline{ц}(0,\alpha)-\overline{ц}'(0,\alpha)$, (ş) and ц′(ş) are ii-d using theorem 2.2

$$\underline{ц''}(ş,\alpha)=\underline{ц}''(ş,\alpha),\overline{ц''}(ş,\alpha)=\overline{ц}''(ş,\alpha)$$

Since ц(ş) is ii-d using theorem 1.2

$$\overline{ц'}(0,\alpha)=\underline{ц}'(0,\alpha),\underline{ц'}(0,\alpha)=\overline{ц}'(0,\alpha)$$

$$\mathcal{K}\left[\underline{ц''}(ş,\alpha)\right]=\frac{1}{\mathcal{v}^2}\mathcal{K}\left[\underline{ц}(ş,\alpha)\right]-\frac{1}{\mathcal{v}}\underline{ц}(0,\alpha)-\overline{ц'}(0,\alpha)$$

$$\mathcal{K}[\overline{ц''}(ş,\alpha)]=\frac{1}{\mathcal{v}^2}\mathcal{K}[\overline{ц}(ş,\alpha)]-\frac{1}{\mathcal{v}}\overline{ц}(0,\alpha)-\underline{ц'}(0,\alpha)$$

$$\frac{1}{\mathcal{v}^2}\hat{\mathcal{K}}[ц(ş)]\ominus\frac{1}{\mathcal{v}}ц(0)\ominus ц'(0)=\left(\mathcal{K}\left[\underline{ц''}(ş,\alpha)\right],\mathcal{K}\left[\overline{ц''}(ş,\alpha)\right]\right)=\hat{\mathcal{K}}[ц''(ş)]$$

$$\hat{\mathcal{K}}[ц''(ş)]=\frac{1}{\mathcal{v}^2}\hat{\mathcal{K}}[ц(ş)]\ominus\frac{1}{\mathcal{v}}ц(0)\ominus ц'(0)$$

Theorem 2.4 [14-17] Given a continuous valued function at Neutrosophic $ц: R\to H(R)$ it may be represented as:

$ц_T(ş)=\left[\underline{ц}_{T\alpha}(ş),\overline{ц}_{T\alpha}(ş)\right]$ **,** for each $\alpha\in[0,1]$

$ц_I(ş)=\left[\underline{ц}_{I\beta}(ş),\overline{ц}_{I\beta}(ş)\right]$, for each $\beta\in[0,1]$

$ц_F(ş)=[\underline{ц}_{F\gamma}(ş),\overline{ц}_{F\gamma}(ş)]$,for each $\gamma\in[0,1]$

Then

1. $\underline{ц}_{T\alpha}$ and $\overline{ц}_{T\alpha}$ are differentiable functions and $ц'(ş)=\left[\underline{ц}'_{T\alpha}(ş),\overline{ц}'_{T\alpha}(ş)\right]$,if $ц_T$ is i-d.
2. $\underline{ц}_{T\alpha}$ and $\overline{ц}_{T\alpha}$ are differentiable functions and $ц'(ş)=\left[\underline{ц}'_{T\alpha}(ş),\overline{ц}'_{T\alpha}(ş)\right]$,if $ц_T$ is ii-d.
3. $\underline{ц}_{I\beta}$ and $\overline{ц}_{I\beta}$ are differentiable functions and $ц'(ş)=\left[\underline{ц}'_{I\beta}(ş),\overline{ц}'_{I\beta}(ş)\right]$,if $ц_I$ is i-d.
4. $\underline{ц}_{I\beta}$ and $\overline{ц}_{I\beta}$ are differentiable functions and $ц'(ş)=\left[\underline{ц}'_{I\beta}(ş),\overline{ц}'_{I\beta}(ş)\right]$,if $ц_I$ is ii-d.

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5. $\underline{ų}_{F\gamma}(ş)$ and $\overline{ų}_{F\gamma}(ş)$ are differentiable functions and $ų'(ş) = \left[\underline{ų}'_{T\gamma}(ş), \overline{ų}'_{T\gamma}(ş)\right]$if $ų_F$ is i-d.

6. $\underline{ų}_{F\gamma}(ş)$ and $\overline{ų}_{F\gamma}(ş)$ are differentiable functions and $ų'(ş) = \left[\underline{ų}'_{T\gamma}(ş), \overline{ų}'_{T\gamma}(ş)\right]$,if $ų_F$ is ii-d.

We omit the proof since it is similar to the theorem 2.2 proof.

3. Environment Neutrosophic for 2nd ORD

Consider the general 2nd ORD provided as follows:

$$r''(ş) = ų(ş, r(ş), r'(ş)) \tag{3.1}$$

Withe initial conditions $r(ş_0) = r_0, r'(ş_0) = ɯ_0$, where $ų: [ş_0, \Gamma] \times \mathbb{R} \to \mathbb{R}$.

Suppose that the initial values r_0 and $ɯ_0$,which are known as Neutrosophic numbers, are characterized by uncertain and the lower and upper bound of truth, indeterminacy and falsity. The fuzzy differential equations for initial values that follow are therefore derived from equation 3.1:

$r''(ş) = r(ş, r(ş), r'(ş)),\ \ 0 \le ş \le \Gamma$ such that ,

$$\left.\begin{aligned} r_T(ş_0) = r_0 = \left[\underline{r}'_{T\alpha}(0), \overline{r}'_{T\alpha}(0)\right], \quad 0 \le \alpha \le 1 \\ r'_T(ş_0) = ɯ_0 = \left[\underline{ɯ}'_{T\alpha}(0), \overline{ɯ}'_{T\alpha}(0)\right], \quad 0 \le \alpha \le 1 \end{aligned}\right\} \quad 3.2$$

$$\left.\begin{aligned} r_I(ş_0) = r_0 = \left[\underline{r}'_{I\beta}(0), \overline{r}'_{I\beta}(0)\right], \quad 0 \le \beta \le 1 \\ r'_I(ş_0) = ɯ_0 = \left[\underline{ɯ}'_{I\beta}(0), \overline{ɯ}'_{I\beta}(0)\right], \quad 0 \le \beta \le 1 \end{aligned}\right\} \quad 3.3$$

$$\left.\begin{aligned} r_F(ş_0) = r_0 = \left[\underline{r}'_{F\gamma}(0), \overline{r}'_{F\gamma}(0)\right], \quad 0 \le \gamma \le 1 \\ r'_F(ş_0) = ɯ_0 = \left[\underline{ɯ}'_{F\gamma}(0), \overline{ɯ}'_{F\gamma}(0)\right], \quad 0 \le \gamma \le 1 \end{aligned}\right\} \quad 3.4$$

When the given second order differential equation is transformed using the Neutrosophic Kamaml Transform, we obtain

$$\mathcal{K}[r''(ş)] = \mathcal{K}[ų(ş, r(ş), r'(ş))]$$

Case 1: Theorem 2.4 [18-20] gives us the following if $r(ş)$ and $r'(ş)$ are first form differentiable functions or if $r(ş)$ and $r'(ş)$ are ii-d functions.

$$r''(ş) = [\underline{r}''(ş), \overline{r}''(ş)].$$

The differential equation yields the following result:

$$\underline{r''}_{T\alpha}(ş) = \underline{r}_{T\alpha}(ş, r(ş), r'(ş)), \underline{r}_{T\alpha}(ş_0) = \underline{r}_{T\alpha}(0), \underline{r}'_{T\alpha}(ş_0) = \underline{ɯ}_{T\alpha}(ş_0).$$

$$\overline{r''}_{T\alpha}(ş) = \overline{r}_{T\alpha}(ş, r(ş), r'(ş)), \overline{r}_{T\alpha}(ş_0) = \overline{r}_{T\alpha}(0), \overline{r'}_{T\alpha}(ş_0) = \overline{ɯ}_{T\alpha}(ş_0).$$

$$\underline{r''}_{I\beta}(ş) = \underline{r}_{I\beta}(ş, r(ş), r'(ş)), \underline{r}_{I\beta}(ş_0) = \underline{r}_{I\beta}(0), \underline{r}'_{I\beta}(ş_0) = \underline{ɯ}_{I\beta}(ş_0).$$

$$\overline{r''}_{I\beta}(ş) = \overline{r}_{I\beta}(ş, r(ş), r'(ş)), \overline{r}_{I\beta}(ş_0) = \overline{r}_{I\beta}(0), \overline{r'}_{I\beta}(ş_0) = \overline{ɯ}_{I\beta}(ş_0).$$

$$\underline{r''}_{F\gamma}(ş) = \underline{r}_{F\gamma}(ş, r(ş), r'(ş)), \underline{r}_{F\gamma}(ş_0) = \underline{r}_{F\gamma}(0), \underline{r}'_{F\gamma}(ş_0) = \underline{ɯ}_{F\gamma}(ş_0).$$

$$\overline{r''}_{F\gamma}(ş) = \overline{r}_{F\gamma}(ş, r(ş), r'(ş)), \overline{r}_{F\gamma}(ş_0) = \overline{r}_{F\gamma}(0), \overline{r'}_{F\gamma}(ş_0) = \overline{ɯ}_{F\gamma}(ş_0).$$

by Neutrosophic Kamal transform ,we get

$\hat{\mathcal{K}}[ų''(ş)] = \left\{\frac{1}{v^2}\hat{\mathcal{K}}[ų(ş)] \ominus \frac{1}{v}ų(0)\right\} \ominus ų'(0)$

By employing the lower and upper functions, to obtain

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$$\mathcal{K}\left[\underline{ɥ}_{T\alpha}(ş, r(ş), r'(ş))\right] = \frac{1}{\upsilon^2}\mathcal{K}[\underline{r}_{T\alpha}(ş)] - \frac{1}{\upsilon}\underline{r}_{T\alpha}(0) - \underline{r}'_{T\alpha}(0)$$

$$\mathcal{K}[\overline{ɥ}_{T\alpha}(ş, r(ş), r'(ş))] = \frac{1}{\upsilon^2}\mathcal{K}[\bar{r}_{T\alpha}(ş)] - \frac{1}{\upsilon}\bar{r}_{T\alpha}(0) - \bar{r}'_{T\alpha}(0)$$

$$\mathcal{K}\left[\underline{ɥ}_{I\beta}(ş, r(ş), r'(ş))\right] = \frac{1}{\upsilon^2}\mathcal{K}[\underline{r}_{I\beta}(ş)] - \frac{1}{\upsilon}\underline{r}_{I\beta}(0) - \underline{r}'_{I\beta}(0)$$

$$\mathcal{K}\left[\overline{ɥ}_{I\beta}(ş, r(ş), r'(ş))\right] = \frac{1}{\upsilon^2}\mathcal{K}[\bar{r}_{I\beta}(ş)] - \frac{1}{\upsilon}\bar{r}_{I\beta}(0) - \bar{r}'_{I\beta}(0)$$

$$\mathcal{K}\left[\underline{ɥ}_{F\gamma}(ş, r(ş), r'(ş))\right] = \frac{1}{\upsilon^2}\mathcal{K}[\underline{r}_{F\gamma}(ş)] - \frac{1}{\upsilon}\underline{r}_{F\gamma}(0) - \underline{r}'_{F\gamma}(0)$$

$$\mathcal{K}\left[\overline{ɥ}_{F\gamma}(ş, r(ş), r'(ş))\right] = \frac{1}{\upsilon^2}\mathcal{K}[\bar{r}_{F\gamma}(ş)] - \frac{1}{\upsilon}\bar{r}_{F\gamma}(0) - \bar{r}'_{F\gamma}(0)$$

The inverse Neutrosophic Kamal Transform is used to solve this and yield the following:

$$\underline{r}_{T\alpha}(ş), \bar{r}_{T\alpha}(ş), \underline{r}_{I\beta}(ş), \bar{r}_{I\beta}(ş), \underline{r}_{F\gamma}(ş), \bar{r}_{F\gamma}(ş)$$

Case2: [20-24] We may obtain the following from theorem 2.4: if $r(ş)$ is a i-d and $r'(ş)$ is a ii-d or if $r(ş)$ is a ii-d and $r'(ş)$ is a i-d functions,

$$r''(ş) = [\underline{r}''(ş), \bar{r}''(ş)].$$

The following is the result of the differential equation:

$$\underline{r''}_{T\alpha}(ş) = \underline{r}_{T\alpha}(ş, r(ş), r'(ş)), \underline{r}_{T\alpha}(ş_0) = \underline{r}_{T\alpha}(0), \underline{r}'_{T\alpha}(ş_0) = \underline{ɯ}_{T\alpha}(ş_0).$$

$$\overline{r''}_{T\alpha}(ş) = \bar{r}_{T\alpha}(ş, r(ş), r'(ş)), \bar{r}_{T\alpha}(ş_0) = \bar{r}_{T\alpha}(0), \bar{r}'_{T\alpha}(ş_0) = \overline{ɯ}_{T\alpha}(ş_0).$$

$$\underline{r''}_{I\beta}(ş) = \underline{r}_{I\beta}(ş, r(ş), r'(ş)), \underline{r}_{I\beta}(ş_0) = \underline{r}_{I\beta}(0), \underline{r}'_{I\beta}(ş_0) = \underline{ɯ}_{I\beta}(ş_0).$$

$$\overline{r''}_{I\beta}(ş) = \bar{r}_{I\beta}(ş, r(ş), r'(ş)), \bar{r}_{I\beta}(ş_0) = \bar{r}_{I\beta}(0), \bar{r}'_{I\beta}(ş_0) = \overline{ɯ}_{I\beta}(ş_0).$$

$$\underline{r''}_{F\gamma}(ş) = \underline{r}_{F\gamma}(ş, r(ş), r'(ş)), \underline{r}_{F\gamma}(ş_0) = \underline{r}_{F\gamma}(0), \underline{r}'_{F\gamma}(ş_0) = \underline{ɯ}_{F\gamma}(ş_0).$$

$$\overline{r''}_{F\gamma}(ş) = \bar{r}_{F\gamma}(ş, r(ş), r'(ş)), \bar{r}_{F\gamma}(ş_0) = \bar{r}_{F\gamma}(0), \bar{r}'_{F\gamma}(ş_0) = \overline{ɯ}_{F\gamma}(ş_0).$$

by Neutrosophic Kamal transform ,we get

$$\widehat{\mathcal{K}}[ɥ''(ş)] = \frac{-1}{\upsilon}ɥ(0) \ominus \left\{\frac{-1}{\upsilon^2}\widehat{\mathcal{K}}[ɥ(ş)]\right\} \ominus ɥ'(0)$$

Utilizing the lower and upper functions ,to have

$$\mathcal{K}\left[\underline{ɥ}_{T\alpha}(ş, r(ş), r'(ş))\right] = -\frac{1}{\upsilon}\bar{r}_{T\alpha}(0) + \frac{1}{\upsilon^2}\mathcal{K}[\bar{r}_{T\alpha}(ş)] - \bar{r}'_{T\alpha}(0)$$

$$\mathcal{K}[\overline{ɥ}_{T\alpha}(ş, r(ş), r'(ş))] = -\frac{1}{\upsilon}\underline{r}_{T\alpha}(0) + \frac{1}{\upsilon^2}\mathcal{K}[\underline{r}_{T\alpha}(ş)] - \underline{r}'_{T\alpha}(0)$$

$$\mathcal{K}\left[\underline{ɥ}_{I\beta}(ş, r(ş), r'(ş))\right] = -\frac{1}{\upsilon}\bar{r}_{I\beta}(0) + \frac{1}{\upsilon^2}\mathcal{K}[\bar{r}_{I\beta}(ş)] - \bar{r}'_{I\beta}(0)$$

$$\mathcal{K}\left[\overline{ɥ}_{I\beta}(ş, r(ş), r'(ş))\right] = -\frac{1}{\upsilon}\underline{r}_{I\beta}(0) + \frac{1}{\upsilon^2}\mathcal{K}[\underline{r}_{I\beta}(ş)] - \underline{r}'_{I\beta}(0)$$

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$$\mathcal{K}\left[\underline{u}_{F\gamma}(ş,r(ş),r'(ş))\right] = -\frac{1}{\vartheta}\bar{r}_{F\gamma}(0) + \frac{1}{\vartheta^2}\mathcal{K}[\bar{r}_{F\gamma}(ş)] - \bar{r}'_{F\gamma}(0)$$

$$\mathcal{K}\left[\overline{u}_{F\gamma}(ş,r(ş),r'(ş))\right] = -\frac{1}{\vartheta}\underline{r}_{F\gamma}(0) + \frac{1}{\vartheta^2}\mathcal{K}[\underline{r}_{F\gamma}(ş)] - \underline{r}'_{F\gamma}(0)$$

We employ the inverse Neutrosophic Kamal Transform to solve this and obtain the following:

$\underline{r}_{T\alpha}(ş), \bar{r}_{T\alpha}(ş), \underline{r}_{I\beta}(ş), \bar{r}_{I\beta}(ş), \underline{r}_{F\gamma}(ş), \bar{r}_{F\gamma}(ş)$.

4. Example Illustrative

Think about the initial value problem with Neutrosophic:

$$r''(ş) + r(ş) = 0$$

$$r_T(0) = (\alpha - 0.1, 0.1 - \alpha), r'_T(0) = (\alpha - 0.1, 0.1 - \alpha)$$

$$r_I(0) = (\beta - 1, 1 - \beta), r'_\beta(0) = (\beta - 1, 1 - \beta)$$

$$r_F(0) = (\gamma - 0.5, 0.5 - \gamma), r'_\gamma(0) = (\gamma - 0.5, 0.5 - \gamma)$$

Using the provided second order differential equation and the Neutrosophic Kamal transform:

$$\hat{\mathcal{K}}[r''(ş)] + \hat{\mathcal{K}}[r(ş)] = 0$$

Case1: $\hat{\mathcal{K}}[r''(ş)] = \left\{\frac{1}{\vartheta^2}\hat{\mathcal{K}}[r(ş)] \ominus \frac{1}{\vartheta}r(0)\right\} \ominus r'(0)$

For $\alpha -$ cut

$$\frac{1}{\vartheta^2}\mathcal{K}[\underline{r}_T(ş,\alpha)] - \frac{1}{\vartheta}\underline{r}_T(0,\alpha) - \underline{r}'_T(0,\alpha) + \mathcal{K}[\underline{r}_T(ş,\alpha)] = 0$$

Applying the intial conditions,we get

$$\left(\frac{1}{\vartheta^2} + 1\right)\mathcal{K}[\underline{r}_T(ş,\alpha)] = (\alpha - 0.1)\frac{1}{\vartheta} + (\alpha - 0.1)$$

We solve this equation of $\mathcal{K}[\underline{r}_T(ş,\alpha)]$ to get

$$\mathcal{K}[\underline{r}_T(ş,\alpha)] = \frac{\vartheta^2}{\vartheta^2 + 1}(\alpha - 0.1)\frac{1}{\vartheta} + \frac{\vartheta^2}{\vartheta^2 + 1}(\alpha - 0.1)$$

$$\underline{r}_T(ş,\alpha) = (\alpha - 0.1)\frac{\vartheta}{\vartheta^2 + 1} + (\alpha - 0.1)\frac{\vartheta^2}{\vartheta^2 + 1}$$

$$\underline{r}_T(ş,\alpha) = (\alpha - 0.1)\cos ş + (\alpha - 0.1)\sin ş$$

$$\frac{1}{\vartheta^2}\mathcal{K}[\bar{r}_T(ş,\alpha)] - \frac{1}{\vartheta}\bar{r}'_T(0,\alpha) - \bar{r}_T(0,\alpha) + \mathcal{K}[\bar{r}_T(ş,\alpha)] = 0$$

Applying the intial condition and solve this equation of $\mathcal{K}[\bar{r}_T(ş,\alpha)]$, we get

$$\bar{r}_T(ş,\alpha) = (0.1 - \alpha)\cos ş + (0.1 - \alpha)şin\ ş$$

For $\beta -$ cut

$$\underline{r}_I(ş,\beta) = (\beta - 1)\cos ş + (\beta - 1)\sin ş$$

$$\bar{r}_I(ş,\beta) = (1 - \beta)\cos ş + (1 - \beta)şin\ ş$$

For$\gamma -$ cut

$$\underline{r}_F(ş,\gamma) = (\gamma - 0.5)\cos ş + (\gamma - 0.5)\sin ş$$

$$\bar{r}_F(ş,\gamma) = (0.5 - \gamma)\cos ş + (0.5 - \gamma)şin\ ş$$

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Case 2:Since $\hat{\mathcal{K}}[r''(ş)] = \frac{-1}{\upsilon} r(0) \ominus \left\{\frac{-1}{\upsilon^2} \hat{\mathcal{K}}[r(ş)]\right\} \ominus r'(0)$

For $\alpha -$ cut

$$-\frac{1}{\upsilon}\bar{r}_T(0,\alpha) + \frac{1}{\upsilon^2}\mathcal{K}[\bar{r}_T(ş,\alpha)] - \underline{r}'_T(0,\alpha) + \mathcal{K}[\underline{r}_T(ş,\alpha)] = 0$$

$$-\frac{1}{\upsilon}\underline{r}_T(0,\alpha) + \frac{1}{\upsilon^2}\mathcal{K}[\underline{r}_T(ş,\alpha)] - \bar{r}'_T(0,\alpha) + \mathcal{K}[\bar{r}_T(ş,\alpha)] = 0$$

Applying the intial conditionsand solve these equation of $\mathcal{K}[\underline{r}_T(ş,\alpha)]$ and $\mathcal{K}[\bar{r}_T(ş,\alpha)]$,we get

$$\underline{r}_T(ş,\alpha) = (\alpha - 0.1)\cosh ş + (\alpha - 0.1)\sinh ş$$

$$\bar{r}_T(ş,\alpha) = (0.1 - \alpha)\cosh ş + (0.1 - \alpha) ş inr\ ş$$

For $\beta -$ cut

$$\underline{r}_I(ş,\beta) = (\beta - 1)\cosh ş + (\beta - 1)\sinh ş$$

$$\bar{r}_I(ş,\beta) = (1 - \beta)\cosh ş + (1 - \beta) ş inr\ ş$$

For$\gamma -$ cut

$$\underline{r}_F(ş,\gamma) = (\gamma - 0.5)\cosh ş + (\gamma - 0.5)\sinh ş$$

$$\bar{r}_F(ş,\gamma) = (0.5 - \gamma)\cosh ş + (0.5 - \gamma) ş inr\ ş$$

5. Conclusion

The results highlight the potential of this technique to offer a robust solution framework, contributing significantly to the field of differential equations and opening avenues for future research in handling uncertainties in mathematical models**.**

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