Efficient Algorithms for Fuzzy Centrality Measures in Large-Scale Social Networks

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Abstract

Numerous criteria are in place for social network applications. They require identification of network's core nodes. Traditional centrality measurements focus on specific node's direct connections or reachability. Often this disregards inherent ambiguity and complexity in real-world social networks. To address these constraints, we have introduced new method called Node Pack Fuzzy Information Centrality based on Pythagorean Neutrosophic Fuzzy Theory. Three essential values truth, falsity and indeterminacy have been added to this approach. This new approach provides a thorough depiction of social networks and it also offers a more sophisticated comprehension of connections between nodes. Complex and ambiguous interactions between entities can be effectively expressed using Pythagorean Neutrosophic values. Unlike traditional values, Pythagorean Neutrosophic values consider several uncertainty dimensions; this is a major improvement over traditional fuzzy value. Our approach handles relational complexity well and it includes self-weight for every node too. It represents each node's unique value, significance, or impact on the network. The network assessment is now more precise and contextual so we can assess centrality with greater precision. We applied this approach to a small academic network called university faculty/researchers. The application of Node Pack Fuzzy Information Centrality yielded promising results. It can enhance various activities associated with social network analysis. It can also offer valuable insights into the network architecture.

Keywords: Centrality measures; Influential nodes; Node pack fuzzy information centrality; Pythagorean neutrosophic fuzzy graph; Social Networks

1. Introduction

Social media platforms often get used for news delivery. They are also used for advertising and business. These platforms consist of individuals and entities. The central nodes help with the spread of information. Identifying these core nodes needs several criteria. This makes it a challenging task. Fuzzy graph models are often picked, because of the uncertainty. Graphs offer a visual representation of relationships. Zadeh [1] is the one who came up with fuzzy sets. These sets help with managing real-world uncertainties. Kauffman [2] introduced fuzzy graphs. Atanassov [3] suggested IFS: intuitionistic fuzzy sets, which are used in decision-making. They merge membership and non-membership degrees. Yager [4] introduced Pythagorean fuzzy sets later these sets refined precision. Smarandache [5] introduced neutrosophic sets and these sets grasp three values: truth, falsehood and indeterminacy. The concepts were elaborated on by these sets. Pythagorean neutrosophic sets can accommodate complex uncertainty. Bavelas [6][7] was first to estimate graph centrality. It was followed by Shimbel[8] who used shortest path approach. Katz [9] measured node influence by using Katz Centrality. Nieminen[10] introduced degree centrality where direct links are emphasized. Nonetheless, lack of activity in links can restrict it. Freeman[11] improved centrality. It now includes both Closeness and Betweenness Centrality. The shortest paths through a node are looked at by these measures. Estrada et al. [12] introduced subgraph centrality. Bonacich[13] introduced Eigenvector Centrality for complex networks. For the purposes of ranking nodes of influence Bae alongside Kim[14] developed Coreness Centrality. J. Wang et al. [15] focused on the development of Weighted

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Neighborhood Centrality. Zhang et al. [16] researched centrality within directionally connected fuzzy social networks. Q. Wang et al. [17] utilized fuzzy hypergraph theory in the exploration of structural centrality.

Crisp graphs treat all edges in an identical manner; fuzzy graphs on the other hand consider uncertainty. This quality suits them better for dynamic social networks in the real world. Fuzzy systems can enhance the accuracy of centrality assessments. They do this by accounting for uncertainty. S. Samanta et al. [18] created neutrosophic graphs based on neutrosophic sets. Ajay et al. [19] introduced Pythagorean neutrosophic fuzzy graphs. To identify social network leaders, T. Zhou et al. [20] used delicious.com's Leader Rank to compare rankings to fan counts. Ling et al. [21 proposed gravity centrality and discovered significant spreaders using Newton's gravity formula. Lu et al. [22] evaluated the relationship between member actions and action network topology, which is critical for targeted advertising. P. Wang et al. [23] investigated noteworthy nodes in biological networks. J. Sheng et al. [24] provided a technique that leverages both local and global structures to discover important nodes. X. Wang et al. [25] developed a semi-local metric for influential nodes. L. Panfeng et al. [26] proposed a voting-based technique for social networks. Venkata Rao Songa et al.[27] introduced a local measure for influential nodes in directed weighted networks using Pythagorean fuzzy sets.

The 2024 AD Scientific Index is an index that ranks academic institutions and researchers worldwide. It obtains its data from Google Scholar. This data consists of citation, h-index and i10-index. The index analyses the most recent output of 18,528 universities. These universities are spread across 219 nations. Google Scholar has a shortcoming: it contains papers, which are incorrectly attributed to scholars with the same names. This can potentially result in inaccurate data. Inflated publication numbers are another possible outcome. Pythagorean neutrosophic fuzzy graphs provide a more accurate representation of this data, especially when self-publication and collaborative research are taken into account. In this article, a specific measure called node pack fuzzy information centrality is used. It is used to pinpoint key nodes in these graphs. Nodes that bear influence will be detected. The process will be based on a suggested method only.

This paper is structured as follows: First, critical definitions of existing centrality methods are reviewed. Next, the concept of influential nodes in Pythagorean neutrosophic graphs is introduced, followed by an algorithm for modifying node pack fuzzy information centrality, along with relevant theorems. The method proposed is then utilized to identify influential nodes in a given network, which are then analysed. Finally, the results are discussed, and the study concludes.

2. Preliminaries

A. Definitions

D.Ajay et al. [28] defined the Pythagorean neutrosophic fuzzy set as follows:

Definition 1: A Pythagorean Neutrosophic Fuzzy Set (PNFS) on a universe of discourse U is represented as P= $\{\langle x, T_P(x), I_P(x), F_P(x)\rangle \mid x \in U\}$, where: $T_P(x)$: U → [0,1] denotes the membership degree, $I_P(x)$: U → [0,1] represents the degree of indeterminacy, and $F_P(x)$: U → [0,1] signifies the non-membership degree of each element x∈U to the set P. These components are subject to the constraints: $0 \le (T_P(x))^2 + (F_P(x))^2 \le 1$ and $0 \le (I_P(x))^2 \le 1$ then

$$0 \le (T_P(x))^2 + (I_P(x))^2 + (F_P(x))^2 \le 2 \quad (1)$$

Here, $T_P(x)$ and $F_P(x)$ are dependent components and $I_p(x)$ is independent component.

Definition 2: A Pythagorean Neutrosophic Fuzzy Graph, Ġ = (V, σ, μ) consists with a non-empty set V = $\{v_1, v_2, ..., v_n\}$, together with a pair of functions σ = ($\sigma_T, \sigma_I, \sigma_F$): V → [0,1] and μ = ($\mu_T, \mu_I, \mu_F$): V × V → [0,1] such that for all $v_1, v_2 \in V$, and must satisfy:

$$\mu_T(v_1, v_2) \le \min\{\sigma_T(v_1), \sigma_T(v_2)\},$$
$$\mu_I(v_1, v_2) \le \min\{\sigma_I(v_1), \sigma_I(v_2),$$
$$\mu_F(v_1, v_2) \le \max\{\sigma_F(v_1), \sigma_F(v_2)\}. \quad (2)$$

Where $\mu_T(v_1, v_2)$, $\mu_I(v_1, v_2)$, and $\mu_F(v_1, v_2)$ represent the Truth, Indeterminacy and Falsity membership degree values of an edge (v_1,v_2) in the graph Ġ respectively. Here, $\sigma_T(v_1)$, $\sigma_I(v_1)$, and $\sigma_F(v_1)$ represents the Truth, Indeterminacy and Falsity membership values of the vertex v_1 in Ġ respectively.

Definition 3: In a Pythagorean Neutrosophic Fuzzy graph Ġ = (V, σ, μ), the strength of an edge (v_1, v_2) is denoted by $(S_T(v_1, v_2), S_I(v_1, v_2), S_F(v_1, v_2))$ and defined as

$$S_{T(v_1,v_2)} = \mu_T(v_1, v_2) / \min\{\sigma_T(v_1), \sigma_T(v_2)\},$$
$$S_{I(v_1,v_2)} = \mu_I(v_1, v_2) / \min\{\sigma_I(v_1), \sigma_I(v_2)\},$$
$$S_{F(v_1,v_2)} = \mu_F(v_1, v_2) / \max\{\sigma_F(v_1), \sigma_F(v_2)\}. \quad (3)$$

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Where $\mu_T(v_1, v_2)$, $\mu_I(v_1, v_2)$, and $\mu_F(v_1, v_2)$ represent the Truth, Indeterminacy and Falsity membership degree values of an edge (v_1, v_2) in the graph Ġ, respectively. $\sigma_T(v_1)$, $\sigma_I(v_1)$, and $\sigma_F(v_1)$ represents the Truth, Indeterminacy and Falsity membership values of vertex in Ġ, respectively.

B. Graph Centrality Metrics

Identifying core nodes within a complex network is an essential task. Several centrality measurement methods can aid in this process.

Degree centrality: Shaw [29] introduced degree centrality, which represents the count of direct connections that a node possesses. A node v_i , degree centrality is defined as:

$C_{v_i} = d_{v_i}$ (4)

where the degree of node v_i is d_{v_i}. The normalized degree centrality is:

$C'_{v_i} = \frac{d_{v_i}}{n-1}$ (5)

Here, *n* represents the total number of nodes. This method measures only direct connections, but nodes connected indirectly can sometimes share more information than direct ones. Thus, considering indirect influence is crucial for identifying the central node.

Node Pack Fuzzy Information Centrality: Venkata Rao Songa et al. [30] introduced the concept of entropy measure with the inbound and outbound significance of a node in a directed weighted network.

$$C_{v_i} = -\sum_{i=1}^{m}(\mu^2(v_i)\log\mu^2(v_i) + \vartheta^2(v_i)\log\vartheta^2(v_i) + \pi^2(v_i)\log\pi^2(v_i)), \quad \gamma = 1 \quad (6)$$

(or)

$$C_{v_i} = \sum_{i=1}^{m}\frac{1}{\gamma-1}\left(1-\left(\left(\mu^2(v_i)\right)^{\gamma} + \left(\vartheta^2(v_i)\right)^{\gamma}\right) + \left(\pi^2(v_i)\right)^{\gamma}\right), \quad \gamma \neq 1\ (\gamma > 0) \quad (7)$$

Here, m be the total number of nodes connected to node v_i in the network. The normalized Node Pack Fuzzy Information Centrality of v_i in a Pythagorean Neutrosophic Fuzzy Set, considering the dependency between $\mu(v_i)$ and $\vartheta(v_i)$, and the independence of $\pi(v_i)$, is defined for the equation (6) as:

$$C'_{v_i} = -\frac{2}{(n-1)\log(n)}\sum_{i=1}^{m}x^2(v_i)\log x^2(v_i) \quad (8)$$

Here, x is the membership value of node v_i, and $(n-1)\log(n)$ serves as the normalization factor based on the maximum possible entropy, where n represents the total count of nodes within the network.

PageRank: L Page et al. [31] assesses the importance of web pages depends on their link structure, assuming that a page's value increases with the number and quality of links it receives from reputable sources. Let page X has citations from pages T1...Tn. Based on the damping factor d, and the number of outgoing links C(X), the rank of page X is calculated as:

Pr(X) = (1-d) + d (Pr(T1)/C(T1) + … + Pr (Tn)/C(Tn)) (9)

The normalized PageRank centrality is:

$Pr_i = (1-d) + d\sum_{i=1,i\neq j}^{N} I_{i,j}\frac{Pr_i}{n_i}$ (10)

Betweenness centrality: Freeman [32] proposed the betweenness centrality, which indicates how often a vertex appears on the shortest paths between other vertices. Betweenness centrality of a vertex v_i, is defined as:

$C_{v_i} = \sum_{j\neq k,i\neq j,k}\frac{h_{jk}(v_i)}{h_{jk}}$ (11)

Here, h_{jk} represent the shortest paths between vertices v_i and v_j , and $h_{jk}(v_i)$ is the count of those paths passing through vertex v_i. The normalized betweenness centrality is then calculated as follows.

$C'_{v_i} = \frac{\sum_{j\neq k,i\neq j,k}\frac{h_{jk}(v_i)}{h_{jk}}}{(n-1).(n-2)}$ (12)

Here, n represents the total count of vertices, and h_{jk} represents the shortest path between vertices v_i and v_j . $h_{jk}(v_i)$ indicates the number of these paths that pass through the vertex v_i.

Closeness centrality: Freeman [32] introduced closeness centrality, measuring how quickly a node can spread information across a network. It is defined as the reciprocal of the average distance from a node to all other nodes in the network; a higher closeness centrality indicates shorter average distances.

$$C_{v_i} = \frac{1}{\sum_{j=1}^{n} d(v_i, v_j)} \quad (13)$$

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Here, d (v_i, vj) denotes the distance between the two vertices v_i and v_j . The closeness centrality that has been standardized is

$$C'_{v_i} = \frac{n-1}{\sum_{j=1}^{n} d(v_i,v_j)} \quad (14)$$

Here, n is the total count of vertices and $d(v_i, v_j)$ is the distance between vertices v_i and v_j.

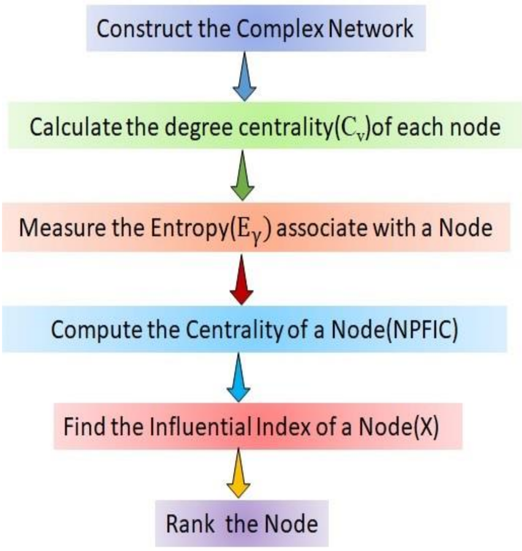

Figure 1. Steps of the Process

3. Methodology

The centrality computation establishes each node's relevance based on its connections. The influential index computation combines these centrality values to rank the nodes based on their network-wide influence.

A. Algorithm

Input:

- Acquire the real-world dataset and construct the complex network
- Required Parameters: α, β and γ.

Output:

- Centrality scores of researchers.

Steps:

Step 1. Calculate degree centrality of a node from its collaboration data.

Maximum number of collaborations represented by

max_collab=max (Collaborations)

Calculate the degree centrality of node i:

$C_v(i)$=Collaborations(i)/max_collab-1 (15)

Step 2. Calculate entropy of each node i from the equation (7):

$$E_\gamma(i) = \sum_{i=1}^{m} \frac{1}{\gamma - 1}\left(1 - T_i^\gamma - I_i^\gamma - F_i^\gamma\right) \quad (16)$$

Where T_i, I_i, and F_i are the truth, indeterminacy, and falsity degree values of node i.

Step 3. Compute Centrality Score for each node i

$$NPFIC(i) = \alpha\cdot C_v(i)+\beta\cdot (1-E_\gamma(i)) \quad (17)$$

This formula balances collaborative influence from (15) and uncertainty from (16)

Step 4. The importance index for node i is computed as

$$X_i = \frac{2 \times NPFIC_{T_i} \times \left(1 - NPFIC_{F_i}\right) + NPFIC_{I_i}}{3} \quad (18)$$

This formula combines the centrality of a node and other factors to give a single score that represents the influence or importance of X_i

Step 5. Repeat Step 1 to Step 4 to compute importance index of other vertices in the network.

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Step 6. Rank the node based on their importance index

$$\max \{X_i\} \ , i=1, 2, \ldots, n$$

End of Algorithm.

B. Example

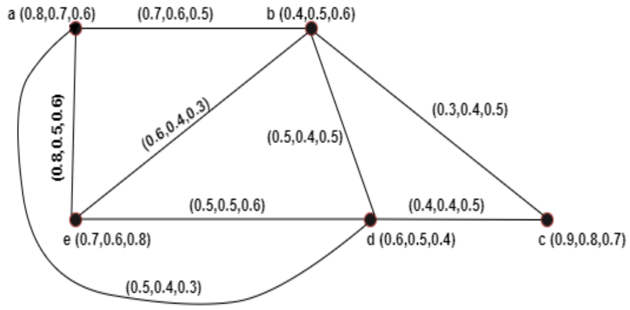

Figure 2. A Pythagorean Neutrosophic Fuzzy Graph

Figure 2 illustrates a Pythagorean Neutrosophic Fuzzy graph, G = (V, σ, μ) with the vertex set V= {a, b, c, d, e}. Table 1 lists the truth(T), indeterminacy(I), and falsity(F) values for each vertex, while Table 2 provides these values for all edges.

Table 1: Vertex membership values in Figure 2

σ	**a**	**b**	**c**	**d**	**e**
σ_T	0.8	0.4	0.9	0.6	0.7
σ_I	0.4	0.5	0.6	0.3	0.6
σ_F	0.6	0.6	0.7	0.4	0.4

Table 2: Edge membership values in Figure 2

μ	**a, b**	**a, e**	**a, d**	**b, c**	**b, d**	**b, e**	**c, d**	**d, e**
μ_T	0.7	0.8	0.5	0.3	0.5	0.6	0.4	0.5
μ_I	0.6	0.5	0.4	0.4	0.4	0.4	0.4	0.5
μ_F	0.5	0.6	0.3	0.5	0.5	0.3	0.5	0.6

For each node from Figure 2 calculate degree centrality. Let $\gamma = 2$ and calculate Havrda-Charvát Entropy for node a is $E_\gamma(a) = (0.36, 0.51, 0.64)$. Using the same procedure, the HC entropy for vertices b, c, d, and e is calculated, and all results are summarized in Table 3.

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Table 3: Entropy values in Figure 2

Vertices	E_T	E_I	E_F
a	0.36	0.51	0.64
b	0.84	0.75	0.64
c	0.19	0.36	0.51
d	0.64	0.75	0.84
e	0.51	0.64	0.36

Let us consider, the value of α = 0.7 and β = 0.3 in this specific case and calculate node pack fuzzy information centrality of a vertex a is $\mathrm{NPFIC}(a) = (0.72, 0.67, 0.63)$.
The centrality values for vertices b, c, d, and e are computed in a similar manner, with all the results summarized in Table 4.

Table 4: Centrality Scores of each vertex in Figure 2

Vertices	$NPFIC_T$	$NPFIC_I$	$NPFIC_F$
a	0.72	0.67	0.63
b	0.75	0.78	0.81
c	0.59	0.54	0.50
d	0.81	0.78	0.75
e	0.67	0.63	0.72

The importance index of vertex a is determined as follows:

$$X_a = \frac{2 \times (0.72 \times (1 - 0.63) + 0.67)}{3} = 0.401$$

Similarly, the importance index for vertices b, c, d, and e is calculated using the same approach, with the results summarized in Table 5.

Table 5: Importance Index of each vertex in Figure 2

Vertex	Influential Index
X_a	0.401
X_b	0.355
X_c	0.377
X_d	0.395
X_e	0.335

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The most influential node in this Pythagorean Neutrosophic environment is the one with the highest importance index, i.e., max $\{X_a, X_b, X_c, X_d, X_e = X_a$. Therefore, the most influential vertex from Figure 2 is vertex a.
$\therefore\ \mathbf{X_a > X_d > X_c > X_b > X_e}$

Axiom 1: Let $\dot{G}$=(V,σ,μ) represent, Pythagorean Neutrosophic Fuzzy Graph, where |V| = n, and let the centrality of a vertex v_i within this graph is denoted as C_{V_i} = (C_{T_i}, C_{I_i}, C_{F_i}) then $0 \leq (C_{T_i})^2 + (C_{I_i})^2 + (C_{F_i})^2 \leq 2$.
Proof: In a Pythagorean Neutrosophic Fuzzy Set (PNFS), the vertex v_i centrality is indicated by C_{v_i}= (C_{T_i}, C_{I_i}, C_{F_i}) where C_{T_i} (truth) and C_{F_i} (falsity) are dependent, and C_{I_i} (indeterminacy) is independent. To prove that $0 \leq (C_{T_i})^2 + (C_{I_i})^2 + (C_{F_i})^2 \leq 2$:
As C_{T_i} and C_{F_i} are dependent components, then their squared sum satisfies $(C_{T_i})^2 + (C_{F_i})^2 \leq 1$ and C_{I_i} is the independent component so that $(C_{I_i})^2$ can take any value up to 1. The maximum possible value of $(C_{T_i})^2 + (C_{I_i})^2 + (C_{F_i})^2$ occurs when both the truth-falsity sum and the indeterminacy value are maximized, yielding: $(C_{T_i})^2 + (C_{I_i})^2 + (C_{F_i})^2 \leq 1+1=2$
Thus, the inequality $(C_{T_i})^2 + (C_{I_i})^2 + (C_{F_i})^2 \leq 2$ holds.

Axiom 2: Let Ġ = (V, σ, μ) represent, Pythagorean Neutrosophic Fuzzy Graph with |V| = n. The centrality of a vertex v_i is denoted as (C_{T_i}, C_{I_i}, C_{F_i}), so the weighted centrality of the vertex v_i is (W_{T_i}, W_{I_i}, W_{F_i}), then $0 \leq (W_{T_i})^2 + (W_{I_i})^2 + (W_{F_i})^2 \leq 2$.
Proof: In Pythagorean Neutrosophic Fuzzy Graph with |V| = n, then the centrality of a vertex v_i is represented by the triplet (C_{T_i}, C_{I_i}, C_{F_i}), so the weighted centralities (W_{T_i}, W_{I_i}, W_{F_i}) of a vertex are derived in a way that respects the Pythagorean neutrosophic principles as well. Since each measure in (C_{T_i}, C_{I_i}, C_{F_i}) is related to (W_{T_i}, W_{I_i}, W_{F_i}) linearly or through scaling, we can infer: $(W_{T_i})^2 + (W_{I_i})^2 + (W_{F_i})^2$ can be at most a scaled version of $(C_{T_i})^2 + (C_{I_i})^2 + (C_{F_i})^2$. For ($W_{T_i}$, W_{I_i}, W_{F_i}) their squared values are non-negative, and in the worst case: $(W_{T_i})^2$=1, $(W_{I_i})^2$=1, $(W_{F_i})^2$=0. Thus, the maximum value of $(W_{T_i})^2 + (W_{I_i})^2 + (W_{F_i})^2$ is 2. Therefore, the inequality $0 \leq (W_{T_i})^2 + (W_{I_i})^2 + (W_{F_i})^2 \leq 2$ holds true, confirming the bounds for weighted centralities in a Pythagorean Neutrosophic Fuzzy Graph.

Axiom 3: Let Ġ = (V, σ, μ) be a Pythagorean Neutrosophic Fuzzy Graph, where the weighted centrality of a vertex v_i is represented as (W_{Ti}, W_{Ii}, W_{Fi}), and the influence index of vertex v_i, represented by X_{V_i}, is equal to 0 when its weighted centrality is either (0, n, 0) or (n,1, 0), where n lies within the interval [0,1].
Proof: Consider a Pythagorean Neutrosophic Fuzzy Graph, denoted as Ġ = (V, σ, μ), where V represents the set of vertices, and the pair of membership functions of vertices $\sigma = (\sigma_{T_i}, \sigma_{I_i}, \sigma_{F_i})$ and their edges membership functions $\mu = (\mu_{T_i}, \mu_{I_i}, \mu_{F_i})$ denote the degree of collaboration (truth), the level of indecision (indeterminacy), and the degree of non-collaboration (falsity) for each vertex $v_i \in V$ and weighted centrality of vertex v_i be represented by (W_{T_i}, W_{I_i}, W_{F_i}). The importance index of vertex v_i is given by:

$$X_{v_1} = \frac{2 \times W_{T_i} \times (1 - W_{F_i}) + W_{I_i}}{3}$$

Given that the weighted centrality values of W_{T_i}, W_{I_i}, and W_{F_i} are constrained by $(W_{T_i})^2 + (W_{F_i})^2 \leq 1$ and $0 \leq (W_{I_i})^2 \leq 1$, the formula ensures that higher truth membership (W_{T_i}) and lower falsity membership (W_{F_i}) will maximize the X_{v_1}, while indecision W_{I_i} moderates the effect. The influential index X_{V_i}=0 is possible when W_{T_i}= 0 and W_{I_i}= 0, or when W_{F_i}= 0 and W_{I_i} = 0. Thus, $X_{v_1} = 0$ when the weighted centrality is either (0, n, 0) or (n,1,0), where n ∈ [0,1].

Axiom 4: Consider a Pythagorean Neutrosophic fuzzy collaboration network represented as Ġ = (V, σ, μ), where the Collaboration Stability (CS) of the network is defined by:

$$CS(\dot{G}) = \frac{1}{|E|}\sum_{(V_i, V_j) \in E}\left(1 - |\mu_{T_i} - \mu_{I_i}|\right) \qquad (19)$$

where |E| represents the total number of collaboration edges, μ_{T_i} denotes the truth degree, and μ_{I_i} indicates the indeterminacy degree in the collaboration.

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Proof: A comprehensive collaboration network displays dependability and efficacy. The formula establishes the network's overall stability by minimizing the difference between the truth and indeterminacy values for all cooperative relationships. When the difference is modest and positive, the network is considered more stable.

4. Implementation

"AD Scientific Index" has recently released 2024 rankings. They ranked global scientists and universities. Google Scholar data was used to assess certain metrics. Metrics such as h-index, i10 index and citation counts. But Google Scholar data can sometimes have inaccuracies. Inaccuracies like fake articles (falsity) and missing or conflicting information (indeterminacy). The index focuses on publications and citations. It unfortunately overlooks essential factor like research collaboration. Research institutions could benefit from publishing faculty rankings. These rankings consider additional factors like research collaboration and self-publication. They also consider citation metrics. Such actions will surely enhance both research quality and institutional reputation. To validate proposed ranking method a small network of researchers is analyzed. In this network, each individual is represented as a node. Edges indicate collaboration forming a complete graph. Score determine rank of node. Score depends on academic collaboration i.e., centrality. It also takes into account self-publishing and citation metrics. Weights are self-weights. There are twelve individuals in the network from Figure 3, denoted as A to L. In Figure 4 a graph is depicted and it shows a group of connected nodes A to I based on collaboration. Nodes J, K, and L have no collaborations. They are not displayed in the graph of Figure 4. The data from Google Scholar includes metrics like total publications recent publications, and citations. It also includes h-index, i10-index and Journal quality as metrics. These metrics are used to calculate node's membership values. The values are truth (T) indeterminacy (I) and falsity (F). These values assess the reliability of a researcher’s profile. The errors and inconsistencies in the data are accounted for. Now let us discuss researchers J, K and L. They have not engaged in any collaboration with others at this institution. We treat the vertices J, K and L as distinct nodes for the purpose of centrality calculation.

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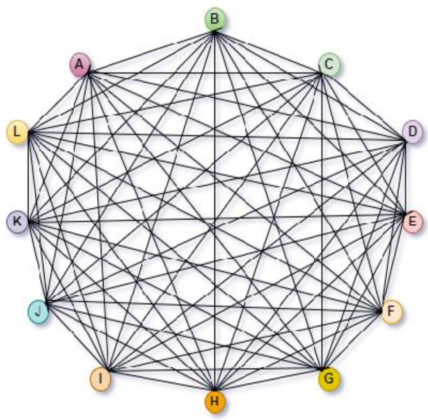

Figure 3. Researchers Collaboration Network in an Institution

Therefore, the centrality of these vertices is (0.0, 0.0, 0.0) for J, (0.0, 0.0, 0.0) for K, and (0.0, 0.0, 0.0) for L. The membership values for truth (T), indeterminacy (I), and falsity (F) for vertices J, K, and L are as follows: Vertex J has values of (0.8, 0.01, 0.3), vertex K has (0.6, 0.0, 0.3), and vertex L has (0.3, 0.0, 0.0). The T, I, and F values for edges are based on research collaboration, derived from parameters related to publications between two researchers. It includes any of the following parameters:

(i) Total publications
(ii) Publications in the last 6 years
(iii) Total citations
(iv) Citations in the last 6 years
(v) Quality of the journal (SCI/SCIE/Scopus/UGC Care)
(vi) Impact Factor (IF) of the journal
(vii) Journal quartiles (Q1, Q2, Q3, Q4).

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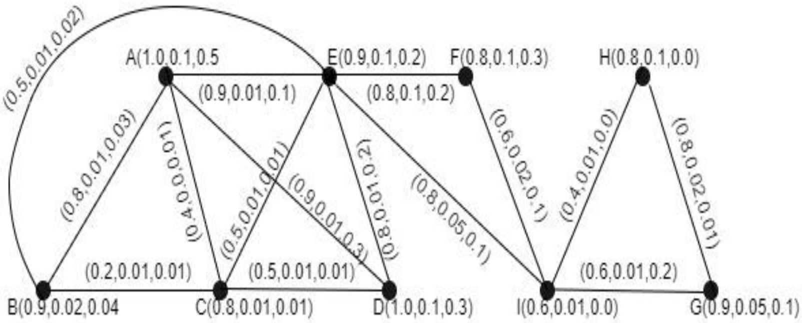

Figure 4. Pythagorean Neutrosophic Fuzzy Graph (From Figure 3)

For each researcher from Figure 4 calculate degree centrality. Let $\gamma = 2$ and calculate Havrda-Charvát entropy for all researchers and compute centrality scores by combine degree centrality and Havrda-Charvát entropy. The influential index for all researchers is calculated, and are presented in Table 6.

Table 6: Influential Index of Researcher in Figure 4

Vertex	Influential Index
X_A	0.406
X_B	0.391
X_C	0.401
X_D	0.411
X_E	0.438
X_F	0.289
X_G	0.335
X_H	0.300
X_I	0.358
X_J	0.166
X_K	0.092
X_L	0.024

The most influential researcher is determined by finding the researcher with the highest influential index, specifically,
$\max \{X_A, X_B, X_C, X_D, X_E, X_F, X_G, X_H, X_I, X_J, X_K, X_L\} = X_E$.
Therefore, the most influential researcher of this institution is E. In the same way, we can identify the second most influential researcher from this institution and create a rank list based on the influential index. This ranking is displayed in Table 7. High Centrality with low entropy ranks highest, moderate values rank in the middle and low centrality with heigh entropy ranks lowest. PNFS-based centrality accounts for the inherent fuzziness of relationships between academics, which can arise from informal collaborations, ambiguous co-authorship roles, or variable academic output. This method is particularly effective when relationships are imprecise.

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Table 7: Rank the Researcher in Figure 4

Rank	Vertex	Influential Index
1	E	0.438
2	D	0.411
3	A	0.406
4	C	0.401
5	B	0.391
6	I	0.358
7	G	0.335
8	H	0.300
9	F	0.289
10	J	0.166
11	K	0.092
12	L	0.024

5. Results Analysis

Node influence measurement hinges on weights and nodal connectivity. In this research, the most influential node was pinpointed. This was done through three factors: collaboration, self-publishing and citations. To measure collaboration in research, centrality measures are used. In contrast, node self-weights are used to evaluate citation and self-publication. Therefore, assessment of node centrality in a Pythagorean Neutrosophic Fuzzy Graph gives realistic results. However, proposed method has limitations. It is quite challenging to compute truth, falsity and indeterminacy. This computation is based on original data. Currently there are no available methods for gathering such type of information. Figure 5 and 6 compare indices of influence. In addition, they compare researcher rankings from divergent centrality measures. These figures show an interesting statistic. 83% of researchers achieved highest influential indices. These indices are higher than those of any other are centralities.

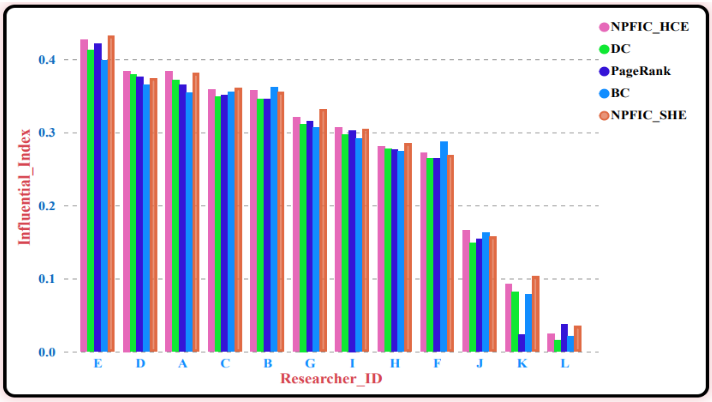

Figure 5. Influential Indices of various Centralities

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Figure 7 and 8 shows that NPFIC has the strongest association with BC, surpassing other centralities despite varying influence indices. The accuracy and efficiency of NPFIC in identifying central nodes is clearly outperforming than other centralities and these results offer valuable insights into social network structures. Researchers who may not rank highly using traditional methods can appear more central due to their roles in bridging uncertainty or fostering indirect collaborations, which are critical for knowledge dissemination. This highlights the importance of considering both strong and weak ties in academic networks. Top-ranked researchers tend to be those who frequently collaborate across departments and research areas, enhancing their role as key information disseminators. Researchers with moderate centrality may have fewer direct collaborations, but their unique position between specialized groups allows them to facilitate interdisciplinary research and making them more influential when evaluated using Pythagorean Neutrosophic Fuzzy System-based methods.

Figure 6. Different Centralities provided Ranks

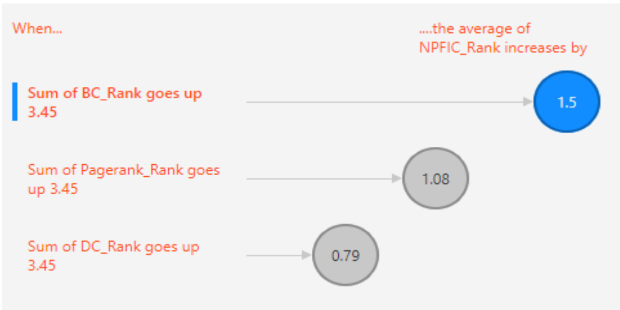

Figure 7. Impact of Key Influencers

Low-ranked researchers might have fewer localized collaborations. They could have less influence overall. This is particularly true when we factor in fuzzy ties in the network. Pythagorean Neutrosophic Fuzzy Sets are now a part of centrality calculations. These are used for academic collaboration networks. This includes our proposed method. The method presents potent approach to rank researchers. This method allows for deeper comprehension of influence dynamics within academic communities. It counts for uncertainties. It considers indirect relationships and this leads to better research management. It also improves collaboration strategies. By *Axiom 4*, we compute the Collaboration Stability (CS) for the network in Figure 1 and the value is 0.8875. This network is considered more stable and it is because the difference is smaller and positive. This is evident from equation 19.

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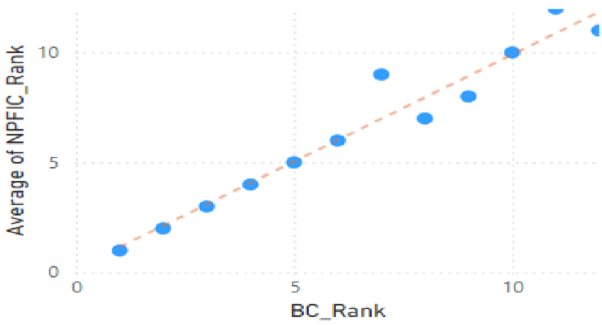

Figure 8. BC and NPFIC Rank Correlation

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6. Conclusion

This study advances Pythagorean neutrosophic fuzzy graphs in social networks by refining degree centrality, which often overestimates influence for inactive nodes with many connections. We introduce a modified measure, Node Pack Fuzzy Information Centrality (NPFIC), incorporating node (researcher) weight and research collaboration to better identify influential researchers. It integrates each node's self-weight reflecting individual merit, influence or importance yielding a more precise and context-sensitive measure of centrality and also demonstrated its effectiveness in identifying influential nodes and improving traditional centrality metrics by mitigating issues like the overestimation of inactive, highly connected nodes. By leveraging Pythagorean Neutrosophic values, the proposed approach captures the complexity of relationships (in-strength and out-strength impact of a researcher) more accurately than traditional fuzzy methods, considering uncertainties by Havrda-Charvát entropy and self-weight by calculating centrality. It is clearly expressed that the proposed centrality measure identifying influential nodes in more nuanced way by incorporating Pythagorean neutrosophic fuzzy values in large-scale social networks like academic collaboration networks. The network assessment is now more precise and contextual so we able to assess centrality with greater precision. This framework also sets the foundation for further exploration of Pythagorean Neutrosophic Fuzzy Graph operations and bipolar Pythagorean Neutrosophic Fuzzy Graphs in real-world applications.

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References

[1] L. A. Zadeh, "Fuzzy sets," Information and Control, vol. 8, pp. 338–353, 1965.

[2] Kauffman, Introduction à la Théorie des Sous-ensembles Flous, Masson et Cie Editeurs, Paris, 1973.

[3] K. Atanassov, "Intuitionistic fuzzy sets," Fuzzy Sets and Systems, vol. 20, pp. 87–96, 1986.

[4] R. R. Yager, "Pythagorean fuzzy subsets," in Proc. Joint IFSA World Congress and NAFIPS Annual Meeting, Edmonton, Canada, pp. 57–61, 2013.

[5] F. Smarandache, Neutrosophy neutrosophic probability, set, and logic, Amer. Res. Press, Rehoboth, USA, p. 105, 1998.

[6] Bavelas, "A mathematical model for group structures," Appl. Anthropol., vol. 7, pp. 16–30, 1948.

[7] Bavelas, "Communication patterns in task-oriented groups," J. Acoust. Soc. Am., vol. 22, pp. 725–730, 1950.

[8] Shimbel, "Structural parameters of communication networks," Bull. Math. Biophys., vol. 15, no. 4, pp. 501–507, 1953.

[9] L. Katz, "A new status index derived from sociometric analysis," Psychometrika, vol. 18, no. 1, pp. 39–43, 1953.

[10] J. Nieminen, "On the centrality in a graph," Scand. J. Psychol., vol. 15, pp. 322–336, 1974.

[11] L. C. Freeman, "Centrality in social networks: conceptual clarification," Soc. Netw., vol. 1, pp. 215–239, 1978.

[12] E. Estrada and J. A. Rodriguez-Velazquez, "Subgraph centrality in complex networks," Phys. Rev., vol. 71, p. 05610, 2005.

[13] P. Bonacich, "Some unique properties of eigenvector centrality," Soc. Netw., vol. 29, pp. 555–564, 2007.

[14] J. Bae and S. Kim, "Identifying and ranking influential spreaders in complex networks by neighborhood coreness," Physica A, vol. 395, pp. 549–559, 2014.

[15] J. Wang et al., "A novel weight neighborhood centrality algorithm for identifying influential spreaders in complex networks," Physica A, vol. 395, pp. 30121–30128, 2017.

DOI: https://doi.org/10.54216/JCIM.160103
Received: October 12, 2024 Revised: January 01, 2025 Accepted: January 29, 2025

[16] Y.-G. Zhang, R.-J. Hu, Q. Li, and W.-C. Ma, "Centrality measures in directed fuzzy social networks," Fuzzy Inf. Eng., vol. 7, no. 1, pp. 115–128, 2015.

[17] Q. Wang and Z. T. Gong, "Structural centrality in fuzzy social networks based on fuzzy hypergraph theory," Comput. Math. Organ. Theory, vol. 26, pp. 236–254, 2020.

[18] S. Samanta, V. K. Dubey, and B. Sarkar, "Measure of influences in social networks," Appl. Soft Comput., p. 106858, 2020, doi: 10.1016/j.asoc.2020.106858.

[19] Ajay et al., "Identifying influential nodes in directed weighted networks using Pythagorean fuzzy sets," J. Theor. Appl. Inf. Technol., vol. 101, no. 1, pp. 374–385, 2024.

[20] T. Zhou, L. Lu, Y. C. Zhang, and C. H. Yeung, "Leaders in social networks: the delicious case," PLoS ONE, vol. 6, no. 6, e21202, 2011, doi: 10.1371/journal.pone.0021202.

[21] M. Ling, M. Chuang, H. Zhang, and B. Wang, "Identifying influential spreaders in complex networks based on gravity formula," Physica A, vol. 451, pp. 205–212, 2016, doi: 10.1016/j.physa.2015.12.162.

[22] Q. Lu, Q. Li, and S. S. Liao, "A graph-based action network framework to identify prestigious members through member's prestige evolution," Decis. Support Syst., vol. 53, no. 1, pp. 44–54, 2012.

[23] P. Wang, J. Lu, and X. Yu, "Identification of important nodes in directed biological networks: a network motif approach," PLoS ONE, vol. 9, no. 8, e106132, 2014.

[24] J. Sheng, J. Dai, B. Wang, G. Duan, J. Long, J. Zhang, K. Guan, S. Hu, L. Chen, and W. Guan, "Identifying influential nodes in complex networks based on global and local structure," Physica A, vol. 541, p. 123262, 2020.

[25] X. Wang, W. Slamu, W. Guo, S. Wang, and Y. Ren, "A novel semi-local measure of identifying influential nodes in complex networks," Chaos Solitons Fractals, vol. 158, p. 112037, 2022.

[26] L. Panfeng, L. Li, F. Shiyu, and Y. Yukai, "Identifying influential nodes in social networks: a voting approach," Chaos Solitons Fractals, vol. 52, p. 111309, 2021.

[27] V. R. Songa and P. Bodapati, "Identifying influential nodes in directed weighted networks using Pythagorean fuzzy sets," J. Theor. Appl. Inf. Technol., vol. 101, no. 1, pp. 374–385, 2024.

[28] Ajay and P. Chellamani, "Pythagorean neutrosophic fuzzy graphs and their application in complex networks," Int. J. Comput. Intell. Appl., vol. 22, no. 4, pp. 278–289, 2023.

[29] M. E. Shaw, "Group structure and the behavior of individuals in small groups," J. Psychol., vol. 38, pp. 139–149, 1954.

[30] V. R. Songa and P. Bodapati, "Identifying influential nodes in directed weighted networks using Pythagorean fuzzy sets," J. Theor. Appl. Inf. Technol., vol. 101, no. 1, pp. 374–385, 2024.

[31] S. Brin and L. Page, "The anatomy of a large-scale hypertextual web search engine," Comput. Networks ISDN Syst., vol. 30, pp. 107–117, 1997.

[32] L. C. Freeman, "Centrality in social networks: conceptual clarification," Soc. Netw., vol. 1, pp. 215–239, 1978.

DOI: https://doi.org/10.54216/JCIM.160103
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