On Some Special Substructures of Neutrosophic Rings and Their Properties

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Abstract

In this paper we introduce the notions of AH-ideal and AHS-ideal as new kinds of neutrosophic substructures defined in a neutrosophic ring. We investigate the properties of these substructures and some related concepts as AH-weak principal ideal, AH-weak prime ideal and AH-weak maximal ideal.

Keywords: Neutrosophic ring, AH-ideal, AHS-ideal, Neutrosophic substructure, AHS-homomorphism.

1. Introduction

Neutrosophy as a branch of philosophy introduced by Smarandache has many applications in both real world and mathematical concepts, especially, in algebra. The notion of neutrosophic groups and rings is defined by Kandasamy and Smarandache in [9], and studied widely in [3 , 4, 7]. Studies were carried out on neutrosophic rings, neutrosophic hyperring, and neutrosophic refined rings. See [1-2]. Neutrosophic rings have many interesting properties and substructures such as neutrosophic subrings and neutrosophic ideals. They are defined and studied widely. See [1 ,3 , 4]. In this work we focus on subsets with form P+QI where P,Q are ideals in the ring R. Two new kinds of neutrosophic substructures which we call AH-ideals and AHS-ideals can be defined by the previous aspect. We prove many theorems which describe their essential properties. Also, we introduce some related concepts such as AH-weak principal ideal, AH-weak prime ideal and AH-weak maximal ideal which have many interesting properties similar to the properties of the classical principal, maximal and prime ideals defined in classical rings.

For our purpose we introduce the concept of AHS-homomorhism and AHS-isomorphism.

Motivation

Since the neutrosophic ring under addition and multiplication (+ and $\times$) R(I)={a+bI ; $a, b \in R , R\ is\ a\ ring$} can be represented as R+RI [4], we are interested in studying the subsets with form P+QI; where P, Q are ideals in R, in addition to investigating their properties.

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2. Preliminaries

In this section we introduce a short revision of some theorems and definitions about ideals and neutrosophic ideals.

Definition 2.1:[8]

Let (R,+,×) be a ring and P be an ideal of R

(a) P is called prime if $a \times b \in P$ implies $a \in P \; or \; b \in P \; for \; a, b \in R$.

(b)P is called maximal if there is no proper ideal J is containing P.

(c) P is called principal if P=<a> for some a$\in R$.

(d) The set M = $\{x \in R \; ; \; \exists n \in Z \; such \; x^n \in P\}$ is called the root ideal of P and we denote it by $\sqrt{P}$.

Theorem 2.2:[8]

Let R, T be two commutative rings and $f: R \to T$ be a ring homomorphism; let P be an ideal in R and J an ideal in T such $J \neq T \; and \; kerf \leq P \neq R$, then

(a) P is prime in R if and only if f(P) is prime in T.

(b) P is maximal in R if and only if f(P) is maximal in T.

(c) J is prime in T if and only if $f^{-1}(J)$ is prime in R.

(d) J is maximal in T if and only if $f^{-1}(J)$ is maximal in R.

Definition 2.3:[9]

Let (R,+,×) be a ring, then R(I)=$\{a + bI \; ; \; a, b \in R\}$ is called the neutrosophic ring; where I is a neutrosophic indeterminate element with the condition $I^2 = I$.

Definition 2.4:[9]

Let R(I) be a neutrosophic ring, a non-empty subset P of R(I) is called a neutrosophic ideal if :

(a) P is a neutrosophic subring of R(I).

(b) for every $p \in P \; and \; r \in R(I)$, we have $r \times p \, , p \times r \in P$.

Theorem 2.5: [8]

Let P, Q be two ideals in the ring R, then $P \cap Q \, , P + Q \, , P \times Q$ are ideals in R.

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For definitions of P+Q, P×Q, [see 8, pp. 49-53].

3. Main concepts and discussion

Definition 3.1:

Let R be a ring and R(I) be the related neutrosophic ring and $P = P_0 + P_1 I = \{a_0 + a_1 I\ ;\ a_0 \in P_0\ , a_1 \in P_1\}$; P_0, P_1 are two subsets of R.

(a)We say that P is an AH-ideal if P_0, P_1 are ideals in the ring R.

(b)We say that P is an AHS-ideal if $P_0 = P_1$.

(c) The AH-ideal P is called null if $P_0, P_1 \in \{R\ , O\ \}$.

Theorem 3.2:

Let R(I) be a neutrosophic ring and $P = P_0 + P_1 I$ be an AH-ideal, then P is not a neutrosophic ideal in general by the classical meaning.

Proof:

Since P_0, P_1 are ideals, they are subgroups of (R,+), thus $P = P_0 + P_1 I$ is a neutrosophic subgroup of (R(I),+). Now suppose that

$r = r_0 + r_1 I \in R(I), a = a_0 + a_1 I \in P.$

We have $a = \ r_0 a_0 + (r_1 a_1 + r_1 a_0 + r_0 a_1)I$, we remark that $r_1 a_1 + r_1 a_0 + r_0 a_1$does not nessecary belong to P_1 because a_0 $does\ not\ belong\ to\ P_1$ thus P is not supposed to be an ideal. See example 3.17.

It is easy to see that if $P_0 = P_1$, then $P = P_0 + P_1 I$ is a neutrosophic ideal in the classical meaning.

Remark 3.3:

We can define the right AH-ideal as P_0, P_1 are right ideals in R, and the left AH-ideal as P_0, P_1 are left ideals in R.

Definition 3.4:

Let R(I) be a neutrosophic ring and $P = P_0 + P_1 I\ , Q = Q_0 + Q_1 I$ be two AH-ideals. Then we define:

$P + Q = (P_0 + Q_0) + (P_1 + Q_1)I.$

$P \cap Q = (P_0 \cap Q_0) + (P_1 \cap Q_1)I.$

$P \times Q = \ P_0 Q_0 + (P_1 Q_0 + P_0 Q_1 + P_1 Q_1)I.$

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Theorem 3.5:

Let R(I) be a neutrosophic ring and $P = P_0 + P_1 I$, $Q = Q_0 + Q_1 I$ be two AH-ideals, then

P + Q and $P \cap Q$, P×Q are AH-ideals.

Proof:

$P_i \times Q_i$, $P_i + Q_i$, $P_i \cap Q_i$ for $i \in \{0,1\}$ are ideals in R as a result of Theorem 2.5, thus we get the proof. See example 3.17.

Definition 3.6:

Let R(I) be acommutativeneutrosophic ring and $P = P_0 + P_1 I$ be an AH-ideal then the AH-root of P can be defined as: $AH - Rad(P) = \sqrt{P_0} + \sqrt{P_1}\, I$.

Theorem 3.7:

Every AH-root of an AH-ideal is also AH-ideal.

Proof:

Since $\sqrt{P_i}$ is an ideal in R we get that $\sqrt{P_0} + \sqrt{P_1}\, I$ is an AH-ideal of the neutrosophic ring R(I).

It is easy to see that if P is an AHS-ideal then the AH-root of P is also an AHS-ideal because $\sqrt{P_0} = \sqrt{P_1}$.

Definition 3.8:

Let R(I) be a neutrosophic ring and $P = P_0 + P_1 I$ be an AH-ideal. Then we define the AH-factor as: $R(I)/P = R/P_0 + R/P_1\ I$.

Theorem 3.9:

Let R(I) be a neutrosophic ring and $P = P_0 + P_1 I$ be an AH-ideal then $R(I)/P$ is a ring with the following two binary operations

$[(x_0 + P_0) + (y_0 + P_1)I] + [(x_1 + P_0) + (y_1 + P_1)I] =$

$$[(x_0 + x_1 + P_0) + (y_0 + y_1 + P_1)I]$$

$[(x_0 + P_0) + (y_0 + P_1)I] \times [(x_1 + P_0) + (y_1 + P_1)I]$ =$[(x_0 \times x_1 + P_0) + (y_0 \times y_1 + P_1)I]$.

Proof:

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Since P_0, P_1 are ideals in R, then R/P_0 , R/P_1 are rings, so $R(I)/P$ under the previous operations is closed. It is obvious that $(R(I)/P\ ,+)$ is abelian neutrosophic group.

Addition is well defined, suppose that $[(x_0 + P_0) + (y_0 + P_1)I] = [(x_1 + P_0) + (y_1 + P_1)I]$ so $(x_0 + P_0) = (x_1 + P_0)$ and $(y_0 + P_1) = (y_1 + P_1)$ thus $x_1 - x_0 \in P_0, y_1 - y_0 \in P_1$

$[(x_2 + P_0) + (y_2 + P_1)I] = [(x_3 + P_0) + (y_3 + P_1)I]$ so $(x_2 + P_0) = (x_3 + P_0)$ and $(y_2 + P_1) = (y_3 + P_1)$ thus $x_3 - x_2 \in P_0, y_3 - y_2 \in P_1$

$[(x_0 + P_0) + (y_0 + P_1)I] + [(x_2 + P_0) + (y_2 + P_1)I] = [(x_0 + x_2 + P_0) + (y_0 + y_2 + P_1)I]$ and

$$[(x_1 + P_0) + (y_1 + P_1)I] + [(x_3 + P_0) + (y_3 + P_1)I] = [(x_1 + x_3 + P_0) + (y_1 + y_3 + P_1)I]$$

We can see that $(x_1 + x_3) - (x_0 + x_2) = (x_1 - x_0) + (x_3 - x_2) \in P_0$ and $(y_1 + y_3) - (y_0 + y_2) = (y_1 - y_0) + (y_3 - y_2) \in P_1$ thus $[(x_1 + x_3 + P_0) + (y_1 + y_3 + P_1)I] = [(x_0 + x_2 + P_0) + (y_0 + y_2 + P_1)I]$.

Multiplication is well defined. Since $x_1 - x_0 \in P_0$ then $(x_1 - x_0) \times x_2 = x_1 \times x_2 - x_0 \times x_2 \in P_0$, by the same we find $x_1 \times (x_3 - x_2) = x_1 \times x_3 - x_1 \times x_2 \in P_0$ that implies $(x_1 \times x_2 - x_0 \times x_2) + (x_1 \times x_3 - x_1 \times x_2) = (x_1 \times x_3 - x_0 \times x_2) \in P_0$

By the same argument we find $(y_1 \times y_3 - y_0 \times y_2) \in P_1$ thus $(y_1 \times y_3 + P_1) = (y_0 \times y_2 + P_1)$ and $(x_1 \times x_3 + P_0) = (x_0 \times x_2 + P_0)$; thus addition and multiplication are well defined.

The multiplication is associative and distributive with respect to addition.

Let $x = (x_0 + P_0) + (y_0 + P_1)I$, $y = (x_1 + P_0) + (y_1 + P_1)I$, $z = (x_2 + P_0) + (y_2 + P_1)I$ be three elements in $R(I)/P$ we have:

$x \times (y + z) = [(x_0 + P_0) + (y_0 + P_1)I\ ] \times [(x_1 + x_2 + P_0) + (y_1 + y_2 + P_1)I] =$

$[x_0 \times (x_1 + x_2) + P_0] + [y_0 \times (y_1 + y_2) + P_1]I = [x_0 \times x_1 + x_0 \times x_2 + P_0] + [y_0 \times y_1 + y_0 \times y_2 + P_1]I =$

$[(x_0 + P_0) + (y_0 + P_1)I] \times [(x_1 + P_0) + (y_1 + P_1)I] + [(x_0 + P_0) + (y_0 + P_1)I] \times [(x_2 + P_0) + (y_2 + P_1)I] = x \times y + x \times z$.

Following the same argument, we can prove that $(y + z) \times x = y \times x + z \times x$.

Thus we get the proof.

Definiton 3.10:

Let R(I), T(J) be two neutrosophic rings and the map $f: R(I) \to T(J)$ we say that f is aneutrosophic AHS-homomrphism if

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The restriction of the map f on R is a ring homomorphism from R to T i.e.$f_R: R \to T$is homomorphism and

$f(a + b\mathrm{I}) = f_R(a) + f_R(b)J.$

We say that R(I), T(J) are AHS-isomomrphic neutrosophic rings if there is a neutrosophic AHS-homomorphism

$f: R(I) \to T(J)$which is a bijective map i.e (R $\cong T$), we say that f is a neutrosophic AHS-isomorphism.

Example 3.11:

Suppose that R = $(Z_6, +, \times), T = (Z_{10}, +, \times)$ are two rings, we have $f: R(I) \to T(J); f(a + bI) = 5a + 5bJ$ is an AHS-homomorphism because $f_R: R \to T$; $f_R(a) = 5a$ is a homomorphism between R and T.

The previous example shows that AHS-homomorphism is not supposed to be a neutrosophic ring homomorphism defined in[3] because $f(I) = f(0 + 1.I) = f(0) + f(1)J = 0 + 5J = 5J \neq J$.

It is easy to see that if $f: R(I) \to T(J)$ is a neutrosophic AHS-homomorphism then $f\big(R(I)\big) = f_R(R) + f_R(R)J$.

The AH-kernel of $f: R(I) \to T(J)$ can be defined as $AH - Kerf = Kerf_R + Kerf_R J$

In the last example we have $Kerf_R = \{0,2,4\}$ thus $AH - kerf = Kerf_R + Kerf_R I$= {0, 2, 4, 2I, 4I, 2+4I, 2+2I, 4+2I, 4+4I }.

If Q = $Q_0 + Q_1 J$ is an AH-ideal of T(J),then the inverse image of Q is

$f^{-1}(Q) = f_T^{-1}(Q_0) + f_T^{-1}(Q_1)I.$

Theorem 3.12:

Let R(I), T(J) be two neutrosophic rings and f: R(I)→T(J) is a neutrosophic ring AHS-homomorphism, let $P = P_0 + P_1 I$ be an AH-ideal of R(I) and $Q = Q_0 + Q_1 J$ be an AH-ideal of T(J), then we have

(a) f(P) is an AH-ideal of f(R(I)).

(b) $f^{-1}(Q)$ is an AH-ideal of R(I).

(c) If P is AHS-ideal of R(I), then f(P) is an AHS-ideal of f(R(I)).

(d) $AH - kerf = kerf_R + kerf_R I$ is an AHS-ideal;f_R is the restriction of f on the ring R.

(e) The AH-factor $R(I)/kerf\ \ is\ AHS - isomorphic\ to\ f(R(I))$.

Proof:

(a) Since f can be restricted on R, by Definition 3.10, we can write

$f(P) = f_R(P_0) + f_R(P_1)J$. Since $f_R(P_i); i \in \{0,1\}$ is an ideal in f(R), thus f(P) is an AH-ideal in f(R(I)).

(b) Since $f^{-1}(Q) = f_T^{-1}(Q_0) + f_T^{-1}(Q_1)I$ and $f_T^{-1}(Q_i);\ i \in \{0,1\}$ is an ideal in R so $f^{-1}(Q)$ is an AH-ideal of T(J).

(c) We have $P_0 = P_1$, so $f_R(P_0) = f_R(P_1)$ and f(P) must be an AHS-ideal.

(d) Since $kerf_R$ is an ideal of R then $AH - kerf = kerf_R + kerf_R I$ is an AHS-ideal of R(I).

(e) Since f is a ring homomorphism, then $R/kerf_R \cong f(R)$ so we get:

$R(I)/kerf = R/kerf_R + R/kerf_R J \cong f_R(R) + f_R(R)J = f(R(I))$.

We mean by the symbol ≅ the concept of AHS-isomorphism introduced in Definition 3.10.

[For more clarity see Examples 3.17 and 3.18].

Definition 3.13:

Let R(I) be a neutrosophic commutative ring and $P = P_0 + P_1 I$ be an AH-ideal. Then we say that

(a) P is a weak prime AH-ideal if P_0, P_1 are prime ideals in R.

(b) P is a weak maximal AH-ideal if P_0, P_1 are maximal ideals in R.

(c) P is a weak principal AH-ideal if P_0, P_1 are principal ideals in R.

Definition 3.14:

Let R(I) be a commutative neutrosophicring,we call it a weak principal AH-ring if every AH-ideal is a weak AH-principal ideal.

Theorem 3.15:

Let R(I), T(J) be two commutative neutrosophic rings with a neutrosophicAHS-homomorphism f: R(I)→T(J) then

If P = $P_0 + P_1 I$ is an AHS- ideal of R(I) and AH-Ker f ≤ P≠ $R(I)$ then

(a) P is a weak prime AHS-ideal if and only if f(P) is a weak prime AHS-ideal in f(R(I)).

(b) P is a weak maximal AHS-ideal if and only if f(P) is a weak maximal AHS-ideal in f(R(I)).

(c) If Q = $Q_0 + Q_1 J$ is an AH-ideal of T(J) then it is a weak prime AH-ideal if and only if $f^{-1}(Q)$ is a weak prime in R(I).

(d) If Q = $Q_0 + Q_1 J$is an AHS-ideal of T(J) then it is a weak maximal AHS-ideal if and only if $f^{-1}(Q)$ is a weak maximal in R(I).

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Proof:

(a) We have AH-Ker f $\leq$ P so $kerf_R \leq P_0$, $kerf_R \leq P_1$.We can find

$f_R(P_0) = f_R(P_1)$and both of them are ideals in f(R); thus $f(P) = f_R(P_0) + f_R(P_1)J$ is a weak prime AHS-ideal in f(R(I)) if and only if P is a weak prime AH-ideal in R(I) as a result of theorem 2.2.

(b) Following the same argument, we can get the proof.

(c) We have that $f^{-1}(Q) = {f_T}^{-1}(Q_0) + {f_T}^{-1}(Q_1)I$, and ${f_T}^{-1}(Q_0), {f_T}^{-1}(Q_1)$ are prime in R if and only if Q_0, Q_1 are prime in T, then the proof holds.

(d) We have $f^{-1}(Q) = {f_T}^{-1}(Q_0) + {f_T}^{-1}(Q_1)I$, and ${f_T}^{-1}(Q_0), {f_T}^{-1}(Q_1)$ are maximal in R if and only if Q_0, Q_1 are maximal in T by theorem (2.2), thus the proof holds.

Remark 3.16:

It is easy to see that if P is an AH-ideal in R(I) then (a) and (b) are still true.

(c), (d) are still true if Q is an AHS-ideal.

Example 3.17:

In this example we clarify some of introduced concepts.

Let R(I)=$Z_6(I)$, $P_0 = \{0,2,4\}, P_1 = \{0,3\}$ are two ideals in Z_6 then we have

(a) P=$P_0 + P_1 I$ = {0,2,4 ,2+3I,4+3I, 3I} is an AH-ideal.

(b) Q=$P_1 + P_1 I = \{0,3,3+3I,3I\}$ is an AHS-ideal because $P_1 = P_1$.

(c) We have: R/P_0={P_0 , 1+P_0 } and $R/P_1 = \{P_1 , 1+P_1 , 2+P_1\}$; thus the AH-factor

$R(I)/P$ = { $P_0 + P_1 I,\ P_0 + (1+P_1)I, P_0 + (2+P_1)I$, (1+$P_0$) + $P_1 I$, (1+P_0) + $(1+P_1)I$, (1+P_0) + $(2+P_1)I$ }

We shoud remark that $P_0 = P_0 + 0.I$ and 0 = 0 + 0.I.

(d)We can clarify the addition on the AH-factor $R(I)/P$ as:

$[P_0 + (1+P_1)I] + [(1+P_0) + (2+P_1)I]$= [(0+1)+$P_0$] + [(1+2)+$P_1$] I = (1+$P_0$) + (3+$P_1$) I = (1+$P_0$) + $P_1 I$.

We can clarify the multiplication on the AH-factor $R(I)/P$ as:

$[P_0 + (1+P_1)I] \times [(1+P_0) + (2+P_1)I\,] = [(0\times 1) + P_0] + [(1\times 2) + P_1]I = P_0 + (2+P_1)I$.

(e) We can see that $P \cap Q = (P_0 \cap P_1) + (P_1 \cap P_1)I = \{0\} + P_1 I = \{0,3I\}$ which it is an AH-ideal.

(f) P+Q = $(P_0 + P_1) + (P_1 + P_1)I = R + P_1 I = \{0,1,2,3,4,5,3I, 1 + 3I, 2 + 3I, \ldots\ldots 5 + 3I\}$.

Example 3.18:

Suppose that R = $(Z_6, +, \times), T = (Z_{10}, +, \times)$ are two commutative rings, we have $f: R(I) \to T(J); f(a + bI) = 5a + 5bJ$.

f is a neutrosophicAHS-homomrphism because$f_R: R \to T$; $f_R(a) = 5a$ is a homomorphism.

We have: $P = Kerf_R = \{0,2,4\}, f_R(R) = \{0,5\}, R/P \cong f_R(R) = \{0,5\}, R/P = \{ P , (1 + P)\}$.

The AH-factor $R(I)/Kerf = R/P + R/P\ J$={ (P + P.J), (P + [1+P]J), ([1+P]+P J), ([1+P]+[1+P]J) }

Which is AH-isomomrphic to f(R(I))= $f_R(R) + f_R(R)J = \{0,5\} + (\{0,5\})J = \{ 0, 5,5J, 5 + 5J\}$.

Q=$P_1 + P_1 I = \{0,3,3 + 3I, 3I\}$is an AHS-ideal defined in Example 3.17, we have $f(Q) = \{0,5,5J, 5 + 5J\}$, which is an AHS-ideal of T(J).

$S_0 = \{0,2,4,6,8\}$is a ideal of T thus S= $S_0 + S_0 J = \{0,2,4,6,8,2J, 4J, 6J, 8J, 2 + 2J, 2 + 4J, 2 + 6J, 2 + 8J, 4 + 2J, 4 + 4J, 4 + 8J, 4 + 6J, 8 + 2J$, 8+4J, 8+6J,8+8J} is an AHS-ideal of T(J).

$f_T^{-1}(S_0) = \{0,2,4\}$ so $f^{-1}(S) = f_T^{-1}(S_0) + f_T^{-1}(S_0)I = \{0,2,4,2I, 4I, 2 + 2I, 2 + 4I, 4 + 2I, 4 + 4I\}$is an AHS-ideal of R(I).

Example 3.19:

In the ring R $(Z_6, +, \times)$ we have two maximal ideals P={0,3}, Q={0,2,4}thus P+QI and Q+PI are two weak maximal AH-ideals of R(I).

Example 3.20:

(a) In the ring R=$(Z_8, +, \times)$ we have only one maximal ideal P={0,2,4,6} so P+PI is a weak maximal AHS-ideal of R(I).

(b) We have Q ={0,4 } is an ideal in R,$\sqrt{Q}$= {0,2,4,6}= P thus the AH-root of Q+QI is equal to P+PI.

Example 3.21:

In the ring (Z,+, ×) each ideal P is principal thus each AH-ideal S=P+QI is weak principal AH-ideal so Z(I) is a weak principal AH-ring.

Example 3.22:

(a) In the ring (Z,+, ×), P= <3>, Q=<2> are two prime and maximal ideals so P+QI ={3n+2mI; n, m∈ Z } is weak prime AH-ideal and weak maximal AH-ideal.

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(b)The map $f_Z: Z \to Z_6; f(a) = a \; mod \; 6$ is a homomorphism so the related AH-homomorphism is

$f: Z(I) \to Z_6(J); f(a + bI) = [a \; mod \; 6] + \; [b \; mod6]J$ and AH-kerf = 6Z + 6ZI is contained in P+QI.

(c) $f(P + QI) = f(P) + f(Q)J = \{0,3\} + \{0,,2,,4\}J$ which is a weak maximal / prime AH-ideal in $Z_6(I)$ since

{0,3 } , {0,2,4} are maximal and prime in Z_6.

(d) Since Q= {0,2,4} is maximal in Z_6, P=Q+QJ is a weak maximal / prime AH-ideal of $Z_6(J)$ and we find

$f^{-1}(P) = f_{Z_6}^{-1}(Q) + f_{Z_6}^{-1}(Q)I$ = < 2 > + < 2 >I which is a weak maximal/ prime AHS-ideal in Z(I).

Conclusion

In this article we introduced the concepts of AH-ideals and AHS-ideals in a neutrosophic ring. Some related concepts as weak principal ideal, AH-weak prime ideal and AH-weak maximal ideal are presented with some useful tools as AHS-homomorphism/isomorphism. We investigated the essential properties of these concepts and proved many related theorems concerning these properties.

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