On The Diagonalization Problem of Weak Fuzzy Complex Matrices Based On a Special Isomorphism

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Abstract

In this paper, we study the diagonalization problem of weak fuzzy complex matrices. To solve this problem we build a special algebraic isomorphism between the ring of weak fuzzy complex matrices and the direct product of the classical ring of real-entries matrices with itself, then we use it to solve the diagonalization problem by using the classical diagonalization problem for real matrices with the inverse isomorphism formula. Also, we illustrate many examples to explain the validity of our method.

Keywords: Weak fuzzy complex matrix; Weak fuzzy complex number; Diagonalization; Isomorphism

1. Introduction

The concept of weak fuzzy complex numbers was presented in [1] as a novel extension of real numbers in a similar way of building split-complex or dual numbers.

Many authors studied this class of generalized numbers, we can find weak fuzzy complex spaces [6], weak fuzzy complex numbers for computers [3], and even in number theory [4-5].

Weak fuzzy complex matrices were defined in [2], where they were handled by computing inverses and determinants.

In this work, we continue the previous efforts about studying weak fuzzy complex structures, where we study the diagonalization problem of weak fuzzy complex matrices. To solve this problem we build a special algebraic isomorphism between the ring of weak fuzzy complex matrices and the direct product of the classical ring of real-entries matrices with itself in a similar way of building the isomorphism between weak fuzzy complex numbers and the direct product of the real field with itself in [7], then we use it to solve the diagonalization problem by using the classical diagonalization problem for real matrices with the inverse isomorphism formula.

For the definition and the elementary properties of weak fuzzy complex matrices, see [2].

2. Main Discussion

Definition:

Let $A = \left(a_{ij} + Jb_{ij}\right)_{1\le i,j\le n}$ be a weak fuzzy complex square real matrix with $a_{ij}, b_{ij} \in \mathbb{R}$, then A is called diagonalizable if and only if there exists a diagonal weak fuzzy complex matrix D, and an invertible weak fuzzy complex matrix X such that:

$A = X^{-1}DX.$

Remark:

For any weak fuzzy complex matrix A, we can write it as $A = A_1 + A_2J$; A_1, A_2

are $n \times n$ square classical matrices.

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Definition:

Let W_μ be a ring of all square $n \times n$ weak fuzzy complex matrices, we define:

$f: W_\mu \to \mathbb{R}_{n\times n} \times \mathbb{R}_{n\times n}$ such that:

$f(A_1 + A_2 J) = (A_1 - \sqrt{t}A_2, A_1 + \sqrt{t}A_2)$

Theorem:

The mapping (*f*) is a ring isomorphism.

Proof:

Assume that $A = A_1 + A_2 J, B = B_1 + B_2 J$, then:

$A_1 = B_1, A_2 = B_2, A_1 - A_2\sqrt{t} = B_1 - B_2\sqrt{t}, A_1 + A_2\sqrt{t} = B_1 + B_2\sqrt{t}$, hence $f(A) = f(B)$, and (*f*) is well defined.

$A + B = (A_1 + B_1) + J(A_2 + B_2),$

$f(A + B) = \left(A_1 + B_1 - \sqrt{t}(A_2 + B_2), A_1 + B_1 + \sqrt{t}(A_2 + B_2)\right) = (A_1 - A_2\sqrt{t}, A_1 + A_2\sqrt{t}) + (B_1 - B_2\sqrt{t}, B_1 + B_2\sqrt{t}) = f(A) + f(B).$

$A \times B = (A_1 + A_2 J) \times (B_1 + B_2 J) = (A_1 B_1 + tA_2 B_2) + J(A_1 B_2 + A_2 B_1),$

$$f(A \times B) = (k_1, k_2): k_1 = A_1 B_1 + tA_2 B_2 - \sqrt{t}(A_1 B_2 + A_2 B_1) = \left(A_1 - \sqrt{t}A_2\right)\left(B_1 - \sqrt{t}B_2\right), k_2 = A_1 B_1 + tA_2 B_2 + \sqrt{t}(A_1 B_2 + A_2 B_1) = \left(A_1 + \sqrt{t}A_2\right)\left(B_1 + \sqrt{t}B_2\right),$$

Thus $f(A \times B) = f(A) \times f(B)$.

If $f(A) = (0,0)$, then:$\begin{cases} A_1 - \sqrt{t}A_2 \\ A_1 + \sqrt{t}A_2 \end{cases} \Longrightarrow \{A_1 = A_2 = 0$ and $k_{er}(f) = \{0\}$.

For every $(C_1, C_2) \in R_{n\times n} \times R_{n\times n}$, there exists:

$A = \frac{1}{2}(C_1 + C_2) + \frac{1}{2\sqrt{t}} J(C_1 - C_2) \in W_\mu$ such that:

$f(A) = (C_1, C_2)$, thus (*f*) is a bijection, and $W_\mu \cong R_{n\times n} \times R_{n\times n}$.

Eigen values and vectors:

Let $X = X_1 + X_2 J$ be an eigenvector of $A = A_1 + A_2 J$ with $a = a_1 + a_2 J \in W_c$ as the corresponding eigen value, then: $A.X = a.X$.

By using the mapping (*f*), we can get:

$$\begin{cases} \left(A_1 - \sqrt{t}A_2\right)\left(X_1 - \sqrt{t}X_2\right) = \left(a_1 - \sqrt{t}a_2\right)\left(X_1 - \sqrt{t}X_2\right) \\ \left(A_1 + \sqrt{t}A_2\right)\left(X_1 + \sqrt{t}X_2\right) = \left(a_1 + \sqrt{t}a_2\right)\left(X_1 + \sqrt{t}X_2\right) \end{cases}$$

Thus:

$X_1 - \sqrt{t}X_2$ is an eigen vector of $A_1 - \sqrt{t}A_2$ with $a_1 - \sqrt{t}a_2$ as the corresponding eigen value.

$X_1 + \sqrt{t}X_2$ is an eigen vector of $A_1 + \sqrt{t}A_2$ with $a_1 + \sqrt{t}a_2$ as the corresponding eigen value.

Example:

For $J^2 = t = \frac{1}{4}$, let us try to compute the eigen values/ vectors of

$$A = \begin{pmatrix} 2 + 4J & 2J \\ 12 + 2J & -4 + 2J \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 1 & -4 \end{pmatrix} + J\begin{pmatrix} 4 & 2 \\ 2 & 2 \end{pmatrix}.$$

$$A_1 - \sqrt{t}A_2 = \begin{pmatrix} 2 & 0 \\ 1 & -4 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} 4 & 2 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 0 & -5 \end{pmatrix}.$$

$$\det\left[\left(A_1 - \sqrt{t}A_2 - cI\right)\right] = 0 \Longrightarrow \begin{vmatrix} -c & -1 \\ 0 & -5 - c \end{vmatrix} = 0 \Longrightarrow -c(-5 - c) = 0 \Longrightarrow c \in \{0, -5\}.$$

For the eigenvectors of $A_1 - \sqrt{+}A_2$:

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$\begin{pmatrix}0 & -1\\0 & -5\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}\Rightarrow y=0\Rightarrow V_1=(1,0)$

$\begin{pmatrix}5 & -1\\0 & 0\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}\Rightarrow y=5x\Rightarrow V_2=(1,5)$

$A_1+\sqrt{t}A_2=\begin{pmatrix}2 & 0\\1 & -4\end{pmatrix}+\frac{1}{2}\begin{pmatrix}4 & 2\\2 & 2\end{pmatrix}=\begin{pmatrix}4 & 1\\2 & -3\end{pmatrix}.$

$\begin{vmatrix}4-c & 1\\2 & -3-c\end{vmatrix}=0\Rightarrow -12-4c+3c+c^2-2=0\Rightarrow c^2-c-14=0\Rightarrow c\in\left\{\frac{1+\sqrt{57}}{2},\frac{1-\sqrt{57}}{2}\right\}.$

For the eigenvectors of $A_1+\sqrt{+}A_2$:

$$\begin{pmatrix}\frac{7-\sqrt{57}}{2} & 1\\ 2 & \frac{-7-\sqrt{57}}{2}\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}\Rightarrow$$

$$y+\left(\frac{7-\sqrt{57}}{2}\right)x=0\Rightarrow y=\frac{-7+\sqrt{57}}{2}x, V_1'=\left(1,\frac{-7+\sqrt{57}}{2}\right)$$

$$\begin{pmatrix}\frac{7+\sqrt{57}}{2} & 1\\ 2 & \frac{-7+\sqrt{57}}{2}\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}\Rightarrow$$

$\left(\frac{7+\sqrt{57}}{2}\right)x+y=0\Rightarrow V_2'=\left(1,\frac{-7-\sqrt{57}}{2}\right).$

Let $a=a_1+a_2J$ be an eigen value of A with $X=X_1+X_2J$ as the eigen vector, then:

$$\begin{cases}a_1-\sqrt{t}a_2=a_1-\frac{1}{2}a_2\in\{0,-5\}\\ a_1+\sqrt{t}a_2=a_1+\frac{1}{2}a_2\in\left\{\frac{1+\sqrt{57}}{2},\frac{1-\sqrt{57}}{2}\right\}\end{cases}$$

If $\begin{cases}a_1-\frac{a_2}{2}=0\\ a_1+\frac{a_2}{2}=\frac{1+\sqrt{57}}{2}\end{cases}\Rightarrow\begin{cases}a_1=\frac{1+\sqrt{57}}{4}\\ a_2=\frac{1+\sqrt{57}}{2}\end{cases}\Rightarrow a=\frac{1+\sqrt{57}}{4}(1+2J).$

If $\begin{cases}a_1-\frac{a_2}{2}=0\\ a_1+\frac{a_2}{2}=\frac{1-\sqrt{57}}{2}\end{cases}\Rightarrow\begin{cases}a_1=\frac{1-\sqrt{57}}{4}\\ a_2=\frac{1-\sqrt{57}}{2}\end{cases}\Rightarrow a=\frac{1-\sqrt{57}}{4}(1+2J).$

If $\begin{cases}a_1-\frac{a_2}{2}=-5\\ a_1+\frac{a_2}{2}=\frac{1+\sqrt{57}}{2}\end{cases}\Rightarrow\begin{cases}a_1=\frac{-9+\sqrt{57}}{4}\\ a_2=\frac{11+\sqrt{57}}{2}\end{cases}\Rightarrow a=\frac{-9+\sqrt{57}}{4}+\frac{11+\sqrt{57}}{2}J.$

If $\begin{cases}a_1-\frac{a_2}{2}=-5\\ a_1+\frac{a_2}{2}=\frac{1-\sqrt{57}}{2}\end{cases}\Rightarrow\begin{cases}a_1=\frac{-9-\sqrt{57}}{4}\\ a_2=\frac{11-\sqrt{57}}{2}\end{cases}\Rightarrow a=\frac{-9-\sqrt{57}}{4}+\frac{11-\sqrt{57}}{2}J.$

For the eigen vectors:

$$\begin{cases}X_1-\frac{1}{2}X_2\in\{V_1=(1,0),V_2=(1,5)\}\\ X_1+\frac{1}{2}X_2\in\left\{V_1'=\left(1,\frac{-7+\sqrt{57}}{2}\right),V_2'=\left(1,\frac{-7-\sqrt{57}}{2}\right)\right\}\end{cases}$$

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If $\begin{cases} X_1 - \frac{1}{2}X_2 = (1,0) \\ X_1 + \frac{1}{2}X_2 = \left(1, \frac{-7+\sqrt{57}}{2}\right) \end{cases} \Rightarrow \begin{cases} X_1 = \left(1, \frac{-7+\sqrt{57}}{4}\right) \\ X_2 = \left(0, \frac{-7+\sqrt{57}}{2}\right) \end{cases} \Rightarrow X = \left(1, \frac{-7+\sqrt{57}}{4} + \frac{-7+\sqrt{57}}{2}J\right).$

If $\begin{cases} X_1 - \frac{1}{2}X_2 = (1,0) \\ X_1 + \frac{1}{2}X_2 = \left(1, \frac{-7-\sqrt{57}}{2}\right) \end{cases} \Rightarrow \begin{cases} X_1 = \left(1, \frac{-7-\sqrt{57}}{4}\right) \\ X_2 = \left(0, \frac{-7-\sqrt{57}}{2}\right) \end{cases} \Rightarrow X = \left(1, \frac{-7-\sqrt{57}}{4} + \frac{-7-\sqrt{57}}{2}J\right).$

If $\begin{cases} X_1 - \frac{1}{2}X_2 = (1,5) \\ X_1 + \frac{1}{2}X_2 = \left(1, \frac{-7+\sqrt{57}}{2}\right) \end{cases} \Rightarrow \begin{cases} X_1 = \left(1, \frac{3+\sqrt{57}}{4}\right) \\ X_2 = \left(0, \frac{-17+\sqrt{57}}{2}\right) \end{cases} \Rightarrow X = \left(1, \frac{3+\sqrt{57}}{4} + \frac{-17+\sqrt{57}}{2}J\right).$

If $\begin{cases} X_1 - \frac{1}{2}X_2 = (1,5) \\ X_1 + \frac{1}{2}X_2 = \left(1, \frac{-7-\sqrt{57}}{2}\right) \end{cases} \Rightarrow \begin{cases} X_1 = \left(1, \frac{3-\sqrt{57}}{4}\right) \\ X_2 = \left(0, \frac{-17-\sqrt{57}}{2}\right) \end{cases} \Rightarrow X = \left(1, \frac{3-\sqrt{57}}{4} + \frac{-17-\sqrt{57}}{2}J\right).$

3. Diagonal zing Weak Fuzzy Complex Matrices

According to the definition of mapping (*f*), we can obtain the following result:

A weak fuzzy complex matrix $A = A_1 + A_2J$ is diagonalizable if and only if: $\begin{cases} A_1 - \sqrt{t}A_2 \\ A_1 + \sqrt{t}A_2 \end{cases}$ are diagonalizable.

To diagonalize $A = A_1 + A_2J$, we diagonalize $\begin{cases} A_1 - \sqrt{t}A_2 \\ A_1 + \sqrt{t}A_2 \end{cases}$ as follows:

$\begin{cases} A_1 - \sqrt{t}A_2 = T_1^{-1}D_1T_1 \text{ ; } T_1 \text{ is an ivertible matrix} \\ A_1 + \sqrt{t}A_2 = T_2^{-1}D_2T_2 \text{ ; } T_2 \text{ is an ivertible matrix} \end{cases}$

D_1, D_2 are diagonal matrices.

By using the isomorphism (*f*), we can write:

$$f(A) = \left(A_1 - \sqrt{+}A_2, A_1 + \sqrt{+}A_2\right) =$$
$$(T_1^{-1}D_1T_1, T_2^{-1}D_2T_2) =$$
$$(T_1^{-1}, T_2^{-1}) \times (D_1, D_2) \times (T_1, T_2)$$

there for:

$$A = f^{-1}(T_1^{-1}, T_2^{-1}) \times f^{-1}(D_1, D_2) \times f^{-1}(T_1, T_2) =$$
$$\left[\frac{1}{2}(T_1^{-1} + T_2^{-1}) + \frac{1}{2\sqrt{t}}J(T_2^{-1} - T_1^{-1})\right] \times \left[\frac{1}{2}(D_1 + D_2) + \frac{1}{2\sqrt{t}}J(D_2 - D_1)\right] \times \left[\frac{1}{2}(T_1 + T_2) + \frac{1}{2\sqrt{t}}J(T_2 - T_1)\right].$$

Example:

Let's diagonalize the matrix:

$$A = \begin{pmatrix} \frac{5}{2} - \frac{3}{2}J & 1 - 3J \\ \frac{1}{2} + \frac{3}{2}J & \frac{5}{2} + \frac{9}{2}J \end{pmatrix} = \begin{pmatrix} \frac{5}{2} & 1 \\ \frac{1}{2} & \frac{5}{2} \end{pmatrix} + J\begin{pmatrix} -\frac{3}{2} & -3 \\ \frac{3}{2} & \frac{9}{2} \end{pmatrix} = A_1 + A_2J \text{ ; } J^2 = t = \frac{1}{9}.$$

$$\begin{cases} A_1 - \sqrt{t}A_2 = A_1 - \frac{1}{3}A_2J = \begin{pmatrix} 3 & 2 \\ 0 & 1 \end{pmatrix} \\ A_1 + \sqrt{t}A_2 = A_1 + \frac{1}{3}A_2J = \begin{pmatrix} 2 & 0 \\ 1 & 4 \end{pmatrix} \end{cases}$$

The eigen values of $A_1 - \sqrt{+}A_2$ are: {3,1}.

$\begin{pmatrix} 0 & 2 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow y = 0 \Rightarrow V_3 = (1,0)$

$\begin{pmatrix} 2 & -2 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow x = y \Rightarrow V_1 = (1,1)$

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$\Rightarrow T_1 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, T_1^{-1} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$

The eigen values of $A_1 + \sqrt{t}A_2$ are: {2,4}.

$\begin{pmatrix} 0 & 0 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow x = -2y \Rightarrow V_2 = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$

$\begin{pmatrix} -2 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow x = 0 \Rightarrow V_4 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\Rightarrow T_2 = \begin{pmatrix} 1 & 0 \\ -1 & -2 \end{pmatrix}, T_2^{-1} = \frac{-1}{2}\begin{pmatrix} -2 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{-1}{2} & \frac{-1}{2} \end{pmatrix}.$

$$T = \frac{1}{2}(T_1 + T_2) + \frac{1}{2\sqrt{t}}J(T_2 - T_1) = \frac{1}{2}\begin{pmatrix} 2 & 1 \\ -1 & -1 \end{pmatrix} + \frac{3}{2}J\begin{pmatrix} 0 & -1 \\ -1 & -3 \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2} - \frac{3}{2}J \\ \frac{-1}{2} - \frac{3}{2}J & \frac{-1}{2} - \frac{9}{2}J \end{pmatrix}$$

$$T^{-1} = \frac{1}{2}(T_1^{-1} + T_2^{-1}) + \frac{1}{2\sqrt{t}}J(T_2^{-1} - T_1^{-1}) = \frac{1}{2}\begin{pmatrix} 2 & -1 \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix} + \frac{3}{2}J\begin{pmatrix} 0 & 1 \\ -\frac{1}{2} & -\frac{3}{2} \end{pmatrix} = \begin{pmatrix} 1 & \frac{-1}{2} + \frac{3}{2}J \\ \frac{-1}{4} - \frac{3}{4}J & \frac{1}{4} - \frac{9}{4}J \end{pmatrix}$$

$$D = \frac{1}{2}(D_1 + D_2) + \frac{1}{2\sqrt{t}}J(D_2 - D_1) = \frac{1}{2}\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} + \frac{3}{2}J\begin{pmatrix} -1 & 0 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} \frac{5}{2} - \frac{3}{2}J & 0 \\ 0 & \frac{5}{2} + \frac{9}{2}J \end{pmatrix}.$$

$A = T^{-1} \times D \times T$.

Example:

For $J^2 = t = \frac{1}{4}$, take: $A = A_1 + A_2J = \begin{pmatrix} -1 & 1 & 1 \\ 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{pmatrix} + J\begin{pmatrix} -6 & 0 & 2 \\ 0 & -6 & -1 \\ 0 & 2 & -2 \end{pmatrix} =$

$$\begin{pmatrix} -1 - 6J & 1 & 1 + 2J \\ 0 & -6J & \frac{1}{2} - J \\ 0 & 1 + 2J & -2J \end{pmatrix}$$

We have:

$A_1 - \sqrt{t}A_2 = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 1 \end{pmatrix}, A_1 + \sqrt{t}A_2 =$

$\begin{pmatrix} -4 & 1 & 2 \\ 0 & -3 & 0 \\ 0 & 2 & -1 \end{pmatrix}$

The eigen values of $A_1 - \sqrt{t}A_2$ are:

$$\{2,3,1\}$$

The eigen vectors of $A_1 - \sqrt{t}A_2$ are:

$\{V_2 = (1,0,0), V_3 = (0,1,0), V_1 = (0,0,1)\}$ and:

$A_1 - \sqrt{t}A_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}(A_1 - \sqrt{t}A_2)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$

where $T_1 = U_{3\times3}, T^{-1} = U_{3\times3}, D_1 = A_1 - \sqrt{t}A_2$.

The eigen values of $A_1 + \sqrt{t}A_2$ are:

$$\{-4, -3, -1\}$$

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For the eigen vectors:

$$\begin{pmatrix}0&1&2\\0&1&0\\0&2&3\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}\Rightarrow\begin{cases}y+2z=0\\y=0\\2y+3z=0\end{cases}\Rightarrow y=z=0\Rightarrow V_{-4}=(1,0,0).$$

$$\begin{pmatrix}-1&1&2\\0&0&0\\0&2&2\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}\Rightarrow\begin{cases}-x+y+2z=0\\2y+2z=0\end{cases}\Rightarrow\begin{cases}z=-y\\x=-y\end{cases}\Rightarrow V_{-3}=(-1,1,-1).$$

$$\begin{pmatrix}-3&1&2\\0&-2&0\\0&2&0\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}\Rightarrow\begin{cases}-3x+y+2z=0\\-2y=0\end{cases}\Rightarrow\begin{cases}y=0\\z=\frac{3}{2}x\end{cases}\Rightarrow V_{-1}=\left(1,0,\frac{3}{2}\right).$$

Thus: $D_2=\begin{pmatrix}-4&0&0\\0&-3&0\\0&0&-1\end{pmatrix}, T_2=\begin{pmatrix}1&-1&1\\0&1&0\\0&-1&\frac{3}{2}\end{pmatrix}$.

Hence D, T are easy to be computed.

4. Conclusion

In this paper, we studied the diagonalization problem of weak fuzzy complex matrices. To solve this problem we built a special algebraic isomorphism between the ring of weak fuzzy complex matrices and the direct product of the classical ring of real-entries matrices with itself, then we used it to solve the diagonalization problem by using the classical diagonalization problem for real matrices with the inverse isomorphism formula. Also, we illustrated many examples to explain the validity of our method.

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