On ψ_{NC} – Operator in Neutrosophic Crisp Topological Spaces

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Abstract

The research started from Salama's generalization to both ideal and local function through NCSs. We presented some results and properties to reinforce the concept of the generalized local function, which though its properties was used to deduce the properties of the ψ_{NC}- operator that we generalized through NCSS.

Keyword: NCSs; NCTS; NCI; NCLF; Neutrosophic crisp ψ_{NC}- operator

1. Introduction

In 2013 [1], Salama was introduced to the idea of ideal through neutrosophic crisp sets. But the mathematical basis for both the neutrosophic and neutrosophic crisp sets (NCS) was firs laid down by Smarandache [2,3]. For any non-empty set $\mathcal{Y}$.

The NCS $\mathcal{A}_N = \langle \mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3 \rangle$ with $\mathcal{A}_1 \cap \mathcal{A}_2 = \emptyset$, $\mathcal{A}_1 \cap \mathcal{A}_3 = \emptyset$, and $\mathcal{A}_2 \cap \mathcal{A}_3 = \emptyset$. And the NC empty set $\emptyset_N = \langle \emptyset, \emptyset, \mathcal{Y} \rangle$ and $\mathcal{Y}_N = \langle \mathcal{Y}, \mathcal{Y}, \emptyset \rangle$ and $\mathcal{A}_N \cap \mathcal{B}_N = \langle \mathcal{A}_1 \cap \mathcal{B}_1, \mathcal{A}_2 \cap \mathcal{B}_2, \mathcal{A}_3 \cup \mathcal{B}_3 \rangle$, $\mathcal{A}_N \cup \mathcal{B}_N = \langle \mathcal{A}_1 \cup \mathcal{B}_1, \mathcal{A}_2 \cup \mathcal{B}_2, \mathcal{A}_3 \cap \mathcal{B}_3 \rangle$, $\mathcal{A}_N \subseteq \mathcal{B}_N$ iff $\mathcal{B}_3 \subseteq \mathcal{A}_3$, $\mathcal{A}_i \subseteq \mathcal{B}_i$, $i = 1,2$ and the complement ${\mathcal{B}_N}^c = \langle \mathcal{Y} \setminus \mathcal{B}_1, \mathcal{Y} \setminus \mathcal{B}_2, \mathcal{Y} \setminus \mathcal{B}_3 \rangle$. We take the neutrosophic crisp point (NCP)

$P_{N_1} = \langle \{p\}, \emptyset, \{p\}^c \rangle, P_{N_1} \in \mathcal{A}_N$ iff $P \varepsilon \mathcal{A}_1$,
$P_{N_2} = \langle \{p\}, \emptyset, \emptyset^c \rangle, P_{N_2} \in \mathcal{A}_N$ iff $P \varepsilon \mathcal{A}_1$,

$P_{N_3} = \langle \emptyset, \{p\}, \{p\}^c \rangle, P_{N_3} \in \mathcal{A}_N$ iff $P \varepsilon \mathcal{A}_2$,

$P_{N_4} = \langle \emptyset, \{p\}, \emptyset \rangle, P_{N_4} \in \mathcal{A}_N$ iff $P \sum \varepsilon$. The symbol $P \varepsilon \mathcal{A}$ means the classic affiliation. The idea of ideal $\mathbb{I}$ is defined by Kuratowski [4,5], which represents a non - empty family of subsets and is closed w.r.t. finite unions and hereditary. Also Kuratowski defined the local function as $\mathcal{A}^* = \{P \varepsilon \mathcal{Y}; \forall \mathcal{U} \in \mathcal{T}, P \varepsilon \mathcal{U}\ and\ \mathcal{U} \cap \mathcal{A} \bar{\varepsilon}\, \mathbb{I}\}$. Al-Obaidi et al. [18,19] gave the view of new types of weakly neutrosophic crisp open mappings and new types of weakly neutrosophic crisp closed functions. Finally, the senses of some types of neutrosophic topological groups with respect to neutrosophic alpha open sets, new types of weakly neutrosophic crisp continuity, new concepts of neutrosophic crisp open sets, new concepts of weakly neutrosophic crisp separation axioms, and neutrosophic crisp generalized sg-closed sets and their continuity were informed by Imran et al. [20-24]. Abdulkadhim et al. [25] presented the view of neutrosophic crisp generalized alpha generalized closed sets.

2. Preliminaries

2.1 Definition [1]: Let $\mathbb{I}_{NC}$a non-empty family of NCSs on a non-empty set $\mathcal{Y}$ is called a Neutrosophic crisp ideal (NCI), if it is closed w.r.t. finite unions and hereditary.

- The families $\{\emptyset_N\}$ and NCSs are trivial NCIs.

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- For any NCP P, $\mathbb{I}_{NCPS} = \{\mathcal{A}_N; P \notin \mathcal{A}_N\}$ is the principle NCI. Through own study, when we say "the NCI" we mean the proper NCI (i.e. $\mathcal{Y}_N \bar{\varepsilon} \mathbb{I}_{NC}$).

2.2 Definition [9]: A collection $\mathcal{T}_{NC}$of NCSs in $\mathcal{Y}$, is called NCTS if $\mathcal{T}_{NC}$ containing $\emptyset_N, \mathcal{Y}_N$ and closed w.r.t. finite intersection, Also closed w.r.t. unions. If $\mathcal{A}_N \varepsilon\, \mathcal{T}_{NC}$, then $\mathcal{A}_N$ is called NCOS and ${\mathcal{A}_N}^c$ is called NCCS. Let $(\mathcal{Y}_N, \mathcal{T}_{NC}, \mathbb{I}_{NC})$ is called Neutrosophic crisp topological ideal space (NCTIS). In a simplified way, we symbolize it $\mathcal{Y}_{\mathbb{I}_{NC}}^{\mathcal{T}_{NC}}$.

2.3 Definition [9]: Let $\mathcal{Y}_{\mathbb{I}_{NC}}^{\mathcal{T}_{NC}}$ be a NCTIS. Let $\mathcal{A}_N$ be any NCS. Then the Neutrosophic crisp local function (NCLF) NC $\mathcal{A}^*(\mathcal{T}_{NC}, \mathbb{I}_{NC}) =$ NC $\mathcal{A}^* = \cup \{P \in \mathcal{Y}_{NC}; \forall \mathcal{U}_N \varepsilon \mathcal{T}_{NC} \ni P \in \mathcal{U}_N$ and $\mathcal{U}_N \cap \mathcal{A}_N \bar{\varepsilon}\, \mathbb{I}_{NC}\}, for\ any\ type\ on\ NCPs\ P$. If $\mathbb{I}_{NC} = \{\emptyset_N\}$, then NC $\mathcal{A}^*$ =NCCI($\mathcal{A}$) and for $\mathbb{I}_{NC} =$ NCSs, then NC $\mathcal{A}^* = \emptyset_N$, for any NCS $\mathcal{A}_N$.

2.4 Theorem [9]: Let $\mathcal{Y}_{\mathbb{I}_{NC}}^{\mathcal{T}_{NC}}$ be a NCTIS and for any NCSs $\mathcal{A}_N, \mathcal{B}_N$, the following properties are verified;

1- If $\mathcal{A}_N \subseteq \mathcal{B}_N$, then NC $\mathcal{A}_N^* \subseteq NC\, \mathcal{B}_N^*$.
2- NC $\mathcal{A}_N^* =$ NCCI$(\mathcal{A}_N^*) \subseteq$ NCCI$(\mathcal{A}_N)$, where NCCI$(\mathcal{A}_N)$ is Neutrosophic crisp closure of $\mathcal{A}_N$
3- NC $\mathcal{A}_N^{**} \subseteq$ NC $\mathcal{A}_N^*$.
4- NC$(\mathcal{A}_N \cup \mathcal{B}_N)^* =$ NC $\mathcal{A}_N^* \cup$ NC $\mathcal{B}_N^*$.
5- IF $\mathrm{I}_N \varepsilon \mathbb{I}_{NC}$, then NC $\mathrm{I}_N^* = \emptyset_N$.
6- IF $\mathrm{I}_N \varepsilon \mathbb{I}_{NC}$, then NC$(\mathcal{A}_N \cup \mathrm{I}_N)^* =$ NC$(\mathcal{A}_N \cap \mathrm{I}_N^c)^* =$ NC $\mathcal{A}_N^*$.

2.5 Remake : Through (Definition 2.3) and (Theorem 2.4), we can conclude:

i. NC $\emptyset_N^* = \emptyset_N$,
ii. For any NCS $\mathcal{A}_N$, then
iii. $\mathcal{A}_N \cap (\text{NC}\, \mathcal{A}^*)^c \cap \text{NC}\, (\mathcal{A}_N \cap (\text{NC}\, \mathcal{A}^*)^c)^* = \emptyset_N$
iv. NC $\mathcal{A}_N^* \cap (\text{NC}\, \mathcal{B}_N^*)^c = [\, \text{NC}(\mathcal{A}_N \cap \mathcal{B}_N^c)^* \cap (\text{NC}\, \mathcal{B}_N^*)^c] \subseteq \text{NC}(\mathcal{A}_N \cap \mathcal{B}_N^c)^*$
v. For any NCOS $\mathcal{U}_N, \mathcal{U}_N \cap$ NC $\mathcal{A}_N^* \subseteq$ NC$(\mathcal{U}_N \cap \mathcal{A}_N)^*$.

2.6 Theorem : For any NCTIS $\mathcal{Y}_{\mathbb{I}_{NC}}^{\mathcal{T}_{NC}}$, the following statements are equivalent;

1- $\mathcal{T}_{NC} \cap \mathbb{I}_{NC} = \{\emptyset_N\}$.
2- If $\mathrm{I}_N \varepsilon \mathbb{I}_{NC}$, then NCint$(\mathrm{I}_N) = \emptyset_N$, where NCint(I_N) is a Neutrosophic crisp interior of I_N
3- For any NCOS G_N, then $G_N \subseteq$ NC G_N^*.
4- $\mathcal{Y}_{NC} \subseteq$ NC $\mathcal{Y}_{NC}^*$.

Proof 1⇒2. Let $\mathcal{T}_{NC} \cap \mathbb{I}_{NC} = \{\emptyset_N\}$ and $\mathrm{I}_N \varepsilon\, \mathbb{I}_{NC}$. If possible, p∈ NCint$(\mathrm{I}_N) \Rightarrow \exists\, \mathcal{U}_N \varepsilon \mathcal{T}_{NC}$, p $\in \mathcal{U}_N \subseteq \mathrm{I}_N$ ⇒by (Definition 2.1) $\mathcal{U}_N \varepsilon \mathbb{I}_{NC}$, which contradiction part(1).

2⇒3. Let G_N be a NCOS and p $\in G_N$if p $\notin$ NC $G_N^* \Rightarrow \exists \mathcal{U}_N \varepsilon \mathcal{T}_{NC}$, p $\in \mathcal{U}_N$and $\mathcal{U}_N \cap G_N \varepsilon \mathbb{I}_{NC}$but p $\in G_N \cap \mathcal{U}_N =$ NCint$(G_N \cap \mathcal{U}_N)$, which contradiction part(2).

3⇒4. Since $\mathcal{Y}_{NC}$is NCOS, so by part (3) $\mathcal{Y}_{NC} =$ NC $\mathcal{Y}_{NC}^*$.

4⇒1. If possible $\exists \emptyset_N \neq G_N \varepsilon \mathcal{T}_{NC} \cap \mathbb{I}_{NC} \Rightarrow \exists p \in G_N$, but $G_N \cap \mathcal{Y}_{NC} = G_N \varepsilon \mathbb{I}_{NC}$. Then p $\notin$ NC $\mathcal{Y}_{NC}^* = \mathcal{Y}_{NC}$, which contradiction.

- We defined the NCTIS $\mathcal{Y}_{\mathbb{I}_{NC}}^{\mathcal{T}_{NC}}$is NC- codense iff $\mathcal{T}_{NC} \cap \mathbb{I}_{NC} = \emptyset_N$.

2.7 Definition [9]: Let $\mathcal{Y}_{\mathbb{I}_{NC}}^{\mathcal{T}_{NC}}$ be a NCTIS. For any NCS $\mathcal{A}_N$, the Neutrosophic crisp closure operator w.r.t. $\mathcal{T}_{NC}\ and\ \mathbb{I}_{NC}$ of $\mathcal{A}_N$ by $NCCI^*(\mathcal{A}_N) = \mathcal{A}_N \cup NC\mathcal{A}_N^*$. And the NCT $\mathcal{T}_{NC}^*$ generated by $NCCI^*$, i.e. $\mathcal{T}_{NC}^* = \{\mathcal{A}_N \subseteq \mathcal{Y}_{NC}; NCCI(\mathcal{A}_N^c)^* = \mathcal{A}_N^c\}$ is finer than $\mathcal{T}_{NC} and\ for\ \mathbb{I}_{NC} = \{\emptyset_N\}$, then $\mathcal{T}_{NC} = \mathcal{T}_{NC}^*$.

3.1 Definition : Let $\mathcal{Y}_{\mathbb{I}_{NC}}^{\mathcal{T}_{NC}}$be a NCTIS. A operator $\psi_{\text{NC}}: NCSs \rightarrow \mathcal{T}_{NC}$ for any NCS $\mathcal{A}_N$ is defined by $\psi_{\text{NC}}(\mathcal{A}_N) = ((\mathcal{A}_N^c)^*)^c$.

3.2 Example : Let $\mathcal{Y} = \{a, b, c\}$, $\mathcal{T}_{NC} = \{\emptyset_{NC}, \mathcal{Y}_{NC}, \langle \emptyset, \{a\}, \{b, c\} \rangle, \langle \{a\}, \{b, c\}, \emptyset \rangle\}$, and $\mathbb{I}_{NC} = \{\emptyset_{NC}, \langle \emptyset, \{a\}, \{b\} \rangle, \langle \emptyset, \{a\}, \{c, b\} \rangle, \langle \emptyset, \{c\}, \{b\} \rangle, \langle \emptyset, \{c\}, \{a, b\} \rangle, \langle \emptyset, \{a, c\}, \{b\} \rangle\}$. Then $(\langle \{c\}, \emptyset, \{a, b\} \rangle)^* = \langle \mathcal{Y}, \{b, c\}, \emptyset \rangle$ and $\psi_{\text{NC}}(\langle \{c\}, \emptyset, \{a, b\} \rangle) = \langle \emptyset, \{a\}, \mathcal{Y} \rangle$.

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3.3 Remark :

- From (Definition 3.1), for any $p \in \psi_{NC}(\mathcal{A}_N), \exists G_N \varepsilon \mathcal{T}_{NC}$ s.t $G_N \cap \mathcal{A}_N^c \varepsilon\, \mathbb{I}_N$. Also we see that $(\mathcal{A}_N^c)^* = [\psi_{NC}(\mathcal{A}_N)]^c$ and $\psi_{NC}(\mathcal{A}_N^c) = (\mathcal{A}_N^*)^c$.
- It clear that $\psi_{NC}(\mathcal{A}_N)$ is NCOS because $(\mathcal{A}_N)^*$ is NCCS.
- $\psi_{NC}(\mathcal{A}_N \cup \mathcal{B}_N) \neq \psi_{NC}(\mathcal{A}_N) \cup \psi_{NC}(\mathcal{B}_N)$, from (Example 3.2) if $\mathcal{A}_N = \langle\{c\},\{a,b\},\emptyset\rangle$, $\mathcal{B}_N = \langle\{c\},\emptyset,\{a,b\}\rangle$, $\psi_{NC}(\mathcal{A}_N \cup \mathcal{B}_N) = \emptyset_{NC}$ but $\psi_{NC}(\mathcal{A}_N) \cup \psi_{NC}(\mathcal{B}_N) = \langle\emptyset,\{a\},\mathcal{Y}\rangle$.
- For any $\mathcal{A}_N^c \in \mathbb{I}_{NC}$. Then $\psi_{NC}(\mathcal{A}_N) = \mathcal{Y}_{NC} = \psi_{NC}(\mathcal{Y}_{NC})$.
- The ψ_{NC}- operator preserves the odor relative to $\subseteq$, in another meaning if $\mathcal{A}_N \subseteq \mathcal{B}_N$, then $\psi_{NC}(\mathcal{A}_N) \subseteq \psi_{NC}(\mathcal{B}_N)$.
- In the case of $\mathcal{A}_N \sum \mathbb{I}_{NC}$, calculating ψ_{NC}- operator to both $\mathcal{B}_N \cap (\mathcal{A}_N)^c$ and $(\mathcal{B}_N \cup \mathcal{A}_N)$, $\mathcal{A}_N$ has no effect on it, i.e. it equals to $\psi_{NC}(\mathcal{B}_N)$.

There are many properties of the ψ_{NC}- operator that have important and fundamental implications for many topological concepts, which are a common conclusion between (Definition 3.1) and the properties of the local function in defined on NCSs, as shown in the following theorem.

3.4 Theorem : Let $\mathcal{Y}_{\mathbb{I}_{NC}}^{\mathcal{T}_{NC}}$ be a NCTIS, then for any NCSs $\mathcal{A}_N, \mathcal{B}_N$, the following statement are hole:

1- $\psi_{NC}(\mathcal{A}_N \cap \mathcal{B}_N) = \psi_{NC}(\mathcal{A}_N) \cap \psi_{NC}(\mathcal{B}_N)$.
2- $\psi_{NC}(\mathcal{A}_N) \subseteq \psi_{NC}(\psi_{NC}(\mathcal{A}_N))$.
3- $\psi_{NC}(\mathcal{A}_N) = \psi_{NC}(\psi_{NC}(\mathcal{A}_N))$ iff $(\mathcal{A}_N^c)^* = [(\mathcal{A}_N^c)^*]^*$
4- If $[(\mathcal{A}_N \cap (\mathcal{B}_N)^c) \cup (\mathcal{B}_N \cap (\mathcal{A}_N)^c)] \varepsilon\, \mathbb{I}_{NC}$, then $\psi_{NC}(\mathcal{A}_N) = \psi_{NC}(\mathcal{B}_N)$.
5- $\mathcal{A}_N \cap \psi_{NC}(\mathcal{A}_N) = NCInt^*(\mathcal{A}_N)$.
6- If $G_N \varepsilon \mathcal{T}_{NC}$, then $G_N \subseteq \psi_{NC}(G_N)$.
7- $\psi_{NC}(\mathcal{A}_N) = \cup\{G_N \varepsilon \mathcal{T}_{NC}; G_N \cap \mathcal{A}_N^c \sum \mathbb{I}_{NC}\}$.

Proof (4). Put $\mathcal{A}_N \cap \mathcal{B}_N^c = C_N$, $\mathcal{B}_N \cap \mathcal{A}_N^c = D_N$, since $[C_N \cup D_N]\varepsilon\mathbb{I}_{NC} \Rightarrow C_N, D_N \varepsilon \mathbb{I}_{NC}$, but $\mathcal{B}_N = (\mathcal{A}_N \cap \mathcal{B}_N) \cup (\mathcal{A}_N \cap \mathcal{B}_N^c) = (\mathcal{A}_N \cap C_N^c) \cup D_N$ and by (Remark 3.3) $\psi_{NC}(\mathcal{B}_N) = \psi_{NC}[(\mathcal{A}_N \cap C_N^c) \cup D_N] = \psi_{NC}(\mathcal{A}_N)$.

Proof (5). Since $NCint^*(\mathcal{A}_N) = [NCCI^*(\mathcal{A}_N^c)]^c = [\mathcal{A}_N^c \cup \mathcal{A}_N^{c\,*}]^c = \mathcal{A}_N \cap \mathcal{A}_N^{c\,*\,C} = \mathcal{A}_N \cap \psi_{NC}(\mathcal{A}_N)$.

Proof (6). Let $G_N \in \mathcal{T}_{NC}$ and $\psi_{NC}(G_N) = [(G_N^c)^*]^c$ and $(G_N^c)^* \subseteq NCCI(G_N^c) = G_N^c$. Then $G_N \subseteq [(G_N^c)^*]^c = \psi_{NC}(G_N)$.

- It is noted that $\mathcal{T}_{NC}^* = \{G_N \subseteq \mathcal{Y}_{NC}; G_N \subseteq \psi_{NC}(G_N)\}$ is a NCTS finer than $\mathcal{T}_{NC}$.
- One of the properties that we deduce using (Theorem 3.4) is:

a) If $\mathcal{A}_N \cap \mathcal{B}_N = \emptyset_{NC}$, then $\psi_{NC}(\mathcal{A}_N) \cap \mathcal{B}_N^* = \emptyset_{NC}$ and $\mathcal{A}_N^* \cap \psi_{NC}(\mathcal{B}_N) = \emptyset_{NC}$.
b) If $\mathcal{A}_N \cap \mathcal{B}_N = \emptyset_{NC}$ and $\mathcal{A}_N, \mathcal{B}_N$ are NCOSs, then $\mathcal{A}_N^* \cap \mathcal{B}_N = \emptyset_N$ and $\mathcal{A}_N \cap \mathcal{B}_N^* = \emptyset_N$.

3.5 Proposition : Let $\mathcal{Y}_{\mathbb{I}_{NC}}^{\mathcal{T}_{NC}}$ be a NCTS and $\mathcal{T}_{NC} \cap \mathbb{I}_{NC} = \emptyset_{NC}$, then $\psi_{NC}(\mathcal{A}_N) \subseteq \mathcal{A}_N^*$.

Proof : Let NCP $p \in \psi_{NC}(\mathcal{A}_N) \Rightarrow \exists\, \mathcal{U}_N \varepsilon \mathcal{T}_{NC}$ s.t $\mathcal{U}_N \cap \mathcal{A}_N^c \varepsilon \mathbb{I}_{NC} \ldots$ …3-1.

If possible $p \notin \mathcal{A}_N^c \Rightarrow \exists G_N \varepsilon \mathcal{T}_{NC}$s.t $G_N \cap \mathcal{A}_N \varepsilon \mathbb{I}_{NC} \ldots\ldots\ldots$3-2.

From 3.1 and 3.2, we get $G_N \cap \mathcal{U}_N = [(G_N \cap \mathcal{U}_N) \cap \mathcal{A}_N] \cup [(G_N \cap \mathcal{U}_N) \cap \mathcal{A}_N^c]\ \varepsilon \mathbb{I}_{NC}$. Then $\mathcal{T}_{NC} \cap \mathbb{I}_{NC} \neq \emptyset_{NC}$, which contradiction the assumption.

3.6 Theorem : Let $\mathcal{Y}_{\mathbb{I}_{NC}}^{\mathcal{T}_{NC}}$ be a NCTS. Then following are equivalent:اكتب المعادلة هنا.

1- NC- codense.
2- If $\mathcal{A}_N$ is NCCI, then $\psi_{NC}(\mathcal{A}_N) \subseteq \mathcal{A}_N$.
3- NCint (NCCI($\mathcal{A}_N$)) = ψ_{NC} (NCint (NCCI($\mathcal{A}_N$))).
4- For $\mathcal{U}_N \varepsilon \mathcal{T}_{NC}$, then $\psi_{NC}(\mathcal{U}_N) \subseteq \mathcal{U}_N^*$.
5- If $H_N\ \varepsilon \mathbb{I}_{NC}$, then $\psi_{NC}(H_N) = \emptyset_{NC}$.

Proof 1⇒2. By (Proposition 3.5) and (Theorem 2.4) $\psi_{NC}(\mathcal{A}_N) \subseteq NC\mathcal{A}_N^* \subseteq$ NCCI$(\mathcal{A}_N) = \mathcal{A}_N$.

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2⇒3. Since NCint (NCCI($\mathcal{A}_N$))$\varepsilon\mathcal{T}_{NC}$ ⇒ by (Theorem 2.4) NCint (NCCI($\mathcal{A}_N$)) ⊆ ψ_{NC} (NCint (NCCI($\mathcal{A}_N$))), but NCCI($\mathcal{A}_N$) is NCCS and by part (2) ψ_{NC} (NCint (NCCI($\mathcal{A}_N$))) ⊆ ψ_{NC}(NCCI($\mathcal{A}_N$)) ⊆ $NCint\ (NCCI(\mathcal{A}_N))$.

3⇒4. By (Theorem 2.6). For $\mathcal{U}_N \varepsilon\, \mathcal{T}_{NC} \Rightarrow \mathcal{U}_N \subseteq NC\mathcal{U}_N^*$ ⇒ NCint (NCCI($\mathcal{U}_N$)) ⊆ NCCI($\mathcal{U}_N$) ⊆ NCCI($NC\mathcal{U}_N^*$) = $NC\mathcal{U}_N^*$ and ψ_{NC} (NCint (NCCI($\mathcal{U}_N$))) = NCint (NCCI($\mathcal{U}_N$)) ⊆ $NC\mathcal{U}_N^*$but $\psi_{NC}(\mathcal{U}_N) \subseteq$ ψ_{NC} (NCint (NCCI($\mathcal{U}_N$))) ⊆ $NC\mathcal{U}_N^*$. Then $\psi_{NC}(\mathcal{U}_N) \subseteq NC\mathcal{U}_N^*$.

4⇒5. By (Theorem 2.6). For $H_N\ \varepsilon\mathbb{I}_{NC}$, then $\psi_{NC}(H_N) = \mathcal{Y}_N \cap (NC\mathcal{Y}_N^*)^c = \mathcal{Y}_N \cap \mathcal{Y}_N^c = \emptyset_{NC}$.

5⇒1. If possible $\exists \emptyset_N \neq G_N\varepsilon\mathcal{T}_{NC}$, $G_N\varepsilon\mathbb{I}_{NC}$. Then by part (5), $\psi_{NC}(G_N) = \emptyset_N$, and by (Theorem1.4(4)) $G_N \subseteq \psi_{NC}(G_N) = \emptyset_N$, which contradiction.

3.7 Theorem : Let $\mathcal{Y}_{\mathbb{I}_{NC}}^{\mathcal{T}_{NC}}$ be a NCTIS, if $\mathbb{I}_{NC}$ is NC-codense, then

1- $\psi_{NC}[NC[\psi_{NC}(NC\mathcal{A}_N^*)]^*] = \psi_{NC}[NC\mathcal{A}_N^*]$.
2- $NC[\psi_{NC}(\mathcal{A}_N)]^* = NC[\psi_{NC}[NC[\psi_{NC}(\mathcal{A}_N)]^*]]^*$.

Proof 1. Put $W_N = \psi_{NC}(NC\mathcal{A}_N^*)$ is NCOS, and by (Theorem 3.6) $W_N = \psi_{NC}(NC\mathcal{A}_N^*) \subseteq NC(NC\mathcal{A}_N^*)^* \subseteq NC\mathcal{A}_N^* \Rightarrow \psi_{NC}(NCW_N^*) \subseteq \psi_{NC}(NC\mathcal{A}_N^*) = W_N$, since $W_N\varepsilon\mathcal{T}_{NC}$, and by using (Theorem 2.4,3.5), $W_N \subseteq \psi_{NC}(W_N) \subseteq NCW_N^* \Rightarrow W_N \subseteq \psi_{NC}(W_N) \subseteq \psi_{NC}(\psi_{NC}(W_N)) \subseteq \psi_{NC}(NCW_N^*)$. Then $W_N = \psi_{NC}(NCW_N^*)$, here the proof ends.

3.8 Theorem : Let $\mathcal{Y}_{\mathbb{I}_{NC}}^{\mathcal{T}_{NC}}$ be a NCTIS, with $\mathbb{I}_{NC}$ is NC-codense**.** If $\mathcal{A}_N$ is a NCOS on $\mathcal{B}_N$ is a NCOS, then $\psi_{NC}(NC\mathcal{A}_N^*) \cap \psi_{NC}(NC\mathcal{B}_N^*) = \psi_{NC}[NC(\mathcal{A}_N \cap \mathcal{B}_N)^*]$.

Proof. Since $\psi_{NC}[NC(\mathcal{A}_N \cap \mathcal{B}_N)^*] \subseteq \psi_{NC}(NC\mathcal{A}_N^*) \cap \psi_{NC}(NC\mathcal{B}_N^*)$, from the fact $\mathcal{A}_N \cap \mathcal{B}_N \subseteq \mathcal{A}_N$ and $\mathcal{B}_N$ and (Theorem 2.4, 3.4) conversely, if $\mathcal{A}_N$ is a NCOS ⇒ by (Theorem 3.4(4)) $\mathcal{A}_N \cap NC\mathcal{B}_N^* \subseteq NC(\mathcal{A}_N \cap \mathcal{B}_N)^* \Rightarrow \psi_{NC}(\mathcal{A}_N) \cap \psi_{NC}(NC\mathcal{B}_N^*) \subseteq \psi_{NC}[NC(\mathcal{A}_N \cap \mathcal{B}_N)^*] \Rightarrow \psi_{NC}[NC[\mathcal{A}_N \cap \psi_{NC}(NC\mathcal{B}_N^*)]^*] \subseteq \psi_{NC}[NC[\psi_{NC}[NC(\mathcal{A}_N \cap \mathcal{B}_N)^*]]^*] = \psi_{NC}[NC[NC(\mathcal{A}_N \cap \mathcal{B}_N)^*]]$ by (Theorem 3.7)(… 3-3). But $NC\mathcal{A}_N^* \cap \psi_{NC}(NC\mathcal{B}_N^*) \subseteq NC[\mathcal{A}_N \cap \psi_{NC}(NC\mathcal{B}_N^*)]^*$. $\Rightarrow \psi_{NC}(NC\mathcal{A}_N^*) \cap \psi_{NC}[\psi_{NC}(NC\mathcal{B}_N^*)] \subseteq \psi_{NC}[NC[\mathcal{A}_N \cap \psi_{NC}(NC\mathcal{B}_N^*)]^*]$ and $\psi_{NC}(NC\mathcal{B}_N^*) \subseteq \psi_{NC}[\psi_{NC}(NC\mathcal{B}_N^*)]. \Rightarrow \psi_{NC}(NC\mathcal{A}_N^*) \cap \psi_{NC}(NC\mathcal{B}_N^*) \subseteq \psi_{NC}[NC[\mathcal{A}_N \cap \psi_{NC}(NC\mathcal{B}_N^*)]^*]$(…3-4) from 3-3 and 3-4, we get that $\psi_{NC}(NC\mathcal{A}_N^*) \cap \psi_{NC}(NC\mathcal{B}_N^*) \subseteq \psi_{NC}[NC(\mathcal{A}_N \cap \mathcal{B}_N)^*]$.

3.9 Corollary: Let $\mathcal{Y}_{\mathbb{I}_{NC}}^{\mathcal{T}_{NC}}$ be a NCTIS, with $\mathbb{I}_{NC}$ is a NC-codense, the following are hold for any two NCSs $\mathcal{A}_N$ *and* $\mathcal{B}_N$.

1- $\psi_{NC}[NC[\psi_{NC}(\mathcal{A}_N) \cap \mathcal{B}_N]^*] = \psi_{NC}[NC[\psi_{NC}(\mathcal{A}_N)]^* \cap \psi_{NC}(NC\mathcal{B}_N^*)]$.
2- $\psi_{NC}[NC[\psi_{NC}(\mathcal{A}_N \cap \mathcal{B}_N)]^*] = \psi_{NC}[NC[\psi_{NC}(\mathcal{A}_N)]^*] \cap \psi_{NC}[NC[\psi_{NC}(\mathcal{B}_N)]^*]$.
3- $NC[\psi_{NC}(NC\mathcal{A}_N^*)]^* \cup NC[\psi_{NC}(NC\mathcal{B}_N^*)]^* = NC[\psi_{NC}[NC(\mathcal{A}_N \cup \mathcal{B}_N)^*]]^*$.

Proof (3). According to (Remark 3.2), (Theorem 3.4) and (Theorem3. 8) we will have the following; First;$[NC[\psi_{NC}(NC\mathcal{A}_N^*)]^*]^c = \psi_{NC}[(\psi_{NC}(NC\mathcal{A}_N^*))^c] = \psi_{NC}[NC[(NC\mathcal{A}_N^*)^c]^*] = \psi_{NC}[NC(\psi_{NC}(\mathcal{A}_N^c))^*]$. This is true for $\mathcal{B}_N$ as well, and from then we conclude the following.$[NC(\psi_{NC}(NC\mathcal{A}_N^*))^*]^c \cap [NC(\psi_{NC}(NC\mathcal{B}_N^*))^*]^c = \psi_{NC}[NC(\psi_{NC}(\mathcal{A}_N^c))^*] \cap \psi_{NC}[NC(\psi_{NC}(\mathcal{B}_N^c))^*] = \psi_{NC}(NC(\psi_{NC}(\mathcal{A}_N^c \cap \mathcal{B}_N^c))^*) = \psi_{NC}[NC((NC(\mathcal{A}_N \cup \mathcal{B}_N)^*)^c)^*] = \psi_{NC}[(\psi_{NC}(NC(\mathcal{A}_N \cup \mathcal{B}_N)^*))^c] = (NC(\psi_{NC}(NC(\mathcal{A}_N \cup \mathcal{B}_N)^*))^*)^c$.

3.10 Definition : NCTIS $\mathcal{Y}_{\mathbb{I}_{NC}}^{\mathcal{T}_{NC}}$ is called $\mathbb{I}_{NC}$- resolvable iff ∃ two $\mathbb{I}_{NC}$- dense NCSs $\mathcal{A}_N, \mathcal{B}_N$ s.t $\mathcal{A}_N \cap \mathcal{B}_N = \emptyset_{NC}$ and $\mathcal{A}_N \cup \mathcal{B}_N = \mathcal{Y}_{NC}$.Otherwise is called $\mathbb{I}_{NC}$- irresolvable.

- NCS $\mathcal{A}_N$ is called $\mathbb{I}_{NC}$- dense iff $NC\mathcal{A}_N^* = \mathcal{Y}_N$.

3.11 Example : Let $\mathcal{Y} = \{a,b,c\}$, $\mathcal{T}_{NC} = \{\emptyset_N, \mathcal{Y}_N, \langle\emptyset,\{a\},\{b,c\}\rangle, \langle\{a\},\{b,c\},\emptyset\rangle\}$, and $\mathbb{I}_{NC} = \{\emptyset_N, \langle\{a\},\emptyset,\emptyset\rangle\}$. Then $\langle\{c\},\emptyset,\{a,b\}\rangle$ is $\mathbb{I}_{NC}$- dense.

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3.12 Remark :

1. If $\mathbb{I}_{NC} = \{\emptyset_N\}$, then $\mathcal{A}_N$ is $\mathbb{I}_{NC}$- dense iff its Neutrosophic crisp dense (NCCI$(\mathcal{A}_N) = \mathcal{Y}_N$).
2. If $\mathcal{A}_N$ is $\mathbb{I}_{NC}$- dense iff $\psi_{\mathrm{NC}}(\mathcal{A}_N^c) = \emptyset_N$.
3. If $\mathcal{A}_N$ is $\mathbb{I}_{NC}$- dense iff for each non- empty NCOS $\mathcal{U}_N \ni \mathcal{U}_N \cap \mathcal{A}_N \bar{\varepsilon} \mathbb{I}_{NC}$.
4. If $\mathcal{A}_N$ is $\mathbb{I}_{NC}$- dense, then for any NCOS G_N, $G_N = G_N \cap NC(\mathcal{A}_N \cap G_N)^*$.
5. NCTIS $\mathcal{Y}_{\mathbb{I}_{NC}}^{\mathcal{T}_{NC}}$ be a $\mathbb{I}_{NC}$- irresolvable iff there is no $\mathbb{I}_{NC}$- dense, which is complement is$\mathbb{I}_{NC}$- dense.

3.13 Proposition : The following properties are correct:

1. If $\mathcal{Y}_{\mathbb{I}_{NC}}^{\mathcal{T}_{NC}}$ is $\mathbb{I}_{NC}$- resolvable, then $\mathcal{Y}_{\mathbb{I}_{NC}}^{\mathcal{T}_{NC}}$ is NC-codense.
2. $\mathcal{Y}_{\mathbb{I}_{NC}}^{\mathcal{T}_{NC}}$ is $\mathbb{I}_{NC}$- resolvable iff $\exists$ $\mathbb{I}_{NC}$- dense $\mathcal{A}_N \ni \psi_{\mathrm{NC}}(\mathcal{A}_N) = \emptyset_N$.
3. $\mathcal{Y}_{\mathbb{I}_{NC}}^{\mathcal{T}_{NC}}$ is $\mathbb{I}_{NC}$- resolvable iff $\exists$ NCS $\mathcal{A}_N \ni \psi_{\mathrm{NC}}(\mathcal{A}_N^c) = \psi_{\mathrm{NC}}(\mathcal{A}_N) = \emptyset_N$.

Proof(1). Let $\mathcal{Y}_{\mathbb{I}_{\mathrm{NC}}}^{\mathcal{T}_{\mathrm{NC}}}$ is $\mathbb{I}_{\mathrm{NC}}$- resolvable $\Rightarrow \exists$ two $\mathbb{I}_{\mathrm{NC}}$- dense, $\mathcal{A}_{\mathrm{N}}$ and $\mathcal{B}_{\mathrm{N}} \ni \mathcal{Y}_N = \mathcal{A}_N \cup \mathcal{B}_N, \mathcal{A}_N \cap \mathcal{B}_N = \emptyset_N \Rightarrow$ $NC\ \mathcal{Y}_N^* = NC\ \mathcal{A}_N^* \cup NC\ \mathcal{B}_N^* = \mathcal{Y}_N \cup \mathcal{Y}_N = \mathcal{Y}_N \Rightarrow \mathcal{Y}_{\mathbb{I}_{\mathrm{NC}}}^{\mathcal{T}_{\mathrm{NC}}}$ is NC-codense.

Proof(3). Let $\mathcal{Y}_{\mathbb{I}_{\mathrm{NC}}}^{\mathcal{T}_{\mathrm{NC}}}$ is $\mathbb{I}_{\mathrm{NC}}$- resolvable $\Rightarrow \exists$ two $\mathbb{I}_{\mathrm{NC}}$- dense, $\mathcal{A}_{\mathrm{N}}, \mathcal{B}_{\mathrm{N}} \ni \mathcal{Y}_N = \mathcal{A}_N \cup \mathcal{B}_N, \mathcal{A}_N \cap \mathcal{B}_N \Rightarrow \mathcal{A}_N^c = \mathcal{B}_{\mathrm{N}}, \mathcal{B}_{\mathrm{N}} = \mathcal{A}_N^c$ and $\mathrm{NC}\ \mathcal{A}_N^* = NC\ \ \mathcal{B}_N^* = \mathcal{Y}_N \Rightarrow$ $(\mathrm{NC}\ \mathcal{A}_N^*)^c = \emptyset_N \Rightarrow \psi_{\mathrm{NC}}(\mathcal{A}_N^c) = [NC[(\mathcal{A}_N^c)^c]^*]^c = [\mathrm{NC}\ \mathcal{A}_N^*]^c = \emptyset_N, \psi_{\mathrm{NC}}(\mathcal{A}_N) = [NC(\mathcal{A}_N^c)^*]^c = [NC\ \mathcal{B}_N^*]^c = \emptyset_N$. Conversely. Let $\mathcal{A}_{\mathrm{N}}$ be NCOS $\ni$ $\psi_{\mathrm{NC}}(\mathcal{A}_N) = \psi_{\mathrm{NC}}(\mathcal{A}_N^c) = \emptyset_N \Rightarrow [NC(\mathcal{A}_N^c)^*]^c = \emptyset_N$ iff $NC(\mathcal{A}_N^c)^* = \mathcal{Y}_N$ and $[NC[(\mathcal{A}_N^c)^c]^*]^c = \emptyset_N$ iff $NC(\mathcal{A}_N^c)^* = \mathcal{Y}_N$ and $\mathcal{A}_N \cup \mathcal{A}_N^c = \mathcal{Y}_N$, then $\mathcal{Y}_{\mathbb{I}_{\mathrm{NC}}}^{\mathcal{T}_{\mathrm{NC}}}$ is $\mathbb{I}_{\mathrm{NC}}$- resolvable.

3. Conclusions and future works

1. One of the future works that could be done is to generalize the local function and ψ- operator in punter sets and Multi Neutrosophic sets [10,11,17].
2. The concepts of NCLF and ψ_{NC} - operator are studied via stable Neutrosophic Crisp topological space instead of NCTS in order to demonstrate the difference between them as well as the characteristics they shara [12,13]. On the other hand, we can also study these concepts on NCT- Sets, Neutrosophic Axial sets and n-valued Neutrosophic Crisp sets [14,15,16].

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