Compare Noise Robust Least-Squares Method with Other Methods for Estimation of the Parameters of Frechet Distribution and Neutrosophic Generalization

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Abstract

The Frechet distribution is a versatile probability distribution that is used within a loose range in many important statistical fields, such as image processing, data analysis, and pattern recognition. It aims to explore and study the estimation of the parameters of the Frechet distribution using the noise-robust least squares method, as in the research paper, and it also has uses. There are many real-world scenarios. It is known that there is a growing challenge in estimating the parameter because of the noisy data. Depending on rigorous simulations and experimental analysis, we provide a novel powerful way to estimate the parameters for the Frechet Distribution Robust Least Squares approach to be flexible. Also, the results approach of this work will be very helpful in estimating the Frechet distribution parameters for diverse statistical applications. Also, we generalize our results to include the generalized neutrosophic case of this distribution dealing with neutrosophic numbers.

Keywords: Frechet distribution; Parameter estimation; Noise-robust least-squares; Statistical modeling; Data analysis; Neutrosophic distribution; Neutrosophic Frechet's distribution

1. Introduction

Frechet distribution is one of the most flexible probability distributions, and it is very popular in many industries and real life studies. Its properties make it a useful statistical analysing and modeling tool for different real-world problems. Frechet distribution has many applications in economics, engineering, biology, and social sciences. Frechet distribution parameters are hard to be estimated in general.

The distribution-based inference interpretation requires parameter estimation such as noise, distortion, measurement errors, and ambiguity can significantly impair data. In general, mathematical distributions need good parameter estimations to capture basic data features [1]. Poor parameters estimation causes a misrepresentation for the data.

Researchers have found robust estimating methods to account for outliers and data model variances. Robust estimation methods can be directly and truly useful, especially when basic distribution assumptions are violated and especially when dealing with data contaminated with noise. In this research work, we aim to use Frechet distribution to deal with the challenges associated with parameter estimation. We will explore and evaluate different potential estimation methodologies and evaluate their performance and actual operation under varying scenarios. This research will also contribute to enhancing the accuracy and reliability of parameter estimation in the Frechet distribution, enabling more robust statistical analysis and modeling in a range of practical applications and thus providing insights into the strengths and limitations of different estimation methods [3].

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In this work, we also generalize the studied distribution to a neutrosophic extended definition [16] based on neutrosophic numbers [17-18] with many probabilistic properties and statistical applications, which will be very helpful in future studies.

2. Reference study

This review covers many mechanisms and methods for estimating robust noise parameters, such as the noise-robust least squares method proposed by White [4], as well as the M estimators presented by Huber [5], in addition to the trimmed estimators by Hempel et al. [6] and the powerful Bayesian inference methods proposed by Liu [7], where each method is examined in terms of basic principles, assumptions, and computational algorithms. The basic review also discusses the mechanisms for how these methods deal with different types of noise, such as outliers, heteroscedasticity, or errors. Measurement. These methods have found applications within various statistical modeling contexts through estimating strong noise parameters that have been reviewed in the field of finance. Qiu et al., [8] estimated parameters in asset pricing models and portfolio optimization models, and Smith et al. [9] used new, robust noise elimination methods in panel data analysis and instrumental variable regression for risk management in econometrics. Cowtan et al., [10] used these methods to estimate variable climate model parameters in environmental sciences.

3. Methodology

Given its adaptability and capacity to fit multiple data distributions, Frechet's probability distribution is useful in many domains and can model numerous phenomena.

Probability density function (PDF): The PDF file for the Frechet distribution is defined as follows:

$$f(x;\ \alpha,\beta,\gamma) = \beta * \gamma * \left(\gamma * (x - \alpha)\right)^{(\beta - 1)} * \exp\left(-\left(\gamma * (x - \alpha)\right)^{\beta}\right)$$

Where α, β, and γ are distribution parameters, α represents the basic location parameter, β controls the shape of the basic distribution, and γ determines the basic scale parameter [11].

The Frechet distribution's derivative can be calculated using the probability density function (PDF) conventional derivative equation. Differentiate x's Frechet equation.

This equation:

$$f'(x;\ \alpha,\beta,\gamma) = \beta * \gamma * (\beta - 1) * (\gamma * (x - \alpha))\hat{}\,(\beta - 2) * \gamma\hat{}2 * \exp(-(\gamma * (x) - \alpha))\hat{}\beta)$$

The Frechet distribution's derivative of the probability density function (PDF) for x is f'(x; α,β,γ), where α, β, and γ are Frechet distribution parameters.

Distribution Function (CDF): We obtain the CDF of the Frechot distribution through PDF file merge operations:

$$F(x;\ \alpha,\beta,\gamma) = \int [\alpha, x]\, f(t;\ \alpha,\beta,\gamma)\, dt$$

where $f(t;\ \alpha,\beta,\gamma)$:is the PDF of the Frechet distribution.

Applying calculus' fundamental theorem separates the CDF equation. CDF function derivative (f'(x; α, β, γ)) produces PDF function derivative. Differential equation:

$$f'(x;\ \alpha,\beta,\gamma) = \frac{d}{dx}[F(x;\ \alpha,\beta,\gamma)]$$

The CDF function derivative concerning x is the PDF function derivative. The Frechet distribution derivative equation is obtained by differentiating the CDF equation for x.

Main general characteristics:

- Flexibility: Adjusting parameters α, β, and γ can construct symmetric, skewed, and heavy/light-tailed distributions. The flexible Frechet can collect various data distributions and shapes.
- Controlling the shape: Most of the Frechet distribution shape depends on the parameter β. If β > 1, the distribution will have heavy tails, enabling simulation of catastrophic occurrences or outliers. A distribution with light tails is acceptable when 0 < β < 1. Data modeling with limited scope [12].
- Location and scale: The parameters α and γ determine the distribution location and size. The parameters α and γ stretch or compress the distribution along the x-axis.
- Moments and skewness: The Frechot distribution estimates mean, variance, and high- and upper-order moments analytically using moments and skewness [13].

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- Tail behaviour: The Frechot distribution exhibits varying tail behaviours based on β values. A distribution with thick tails (β > 1) is more prone to outliers. If 0 < β < 1, the distribution will have light tails, indicating a faster outlier probability reduction.
- The Frechet distribution helps academics analyse complex data patterns and make accurate predictions in banking, environmental modelling, dependability studies, and more [11].

4. Parameter estimation methods for the Frechet distribution

A. Maximum Likelihood Estimation (MLE)

Parameter estimates often employ greatest likelihood. MLE optimises data point detection settings using expected distribution. The Frechet distribution probability function and parameters are estimated using the optimisation problem [3]. The place where:

B. Probability function

Starting with observed data, we define a likelihood function. Given parameters, the likelihood function assesses the probability of seeing data points in the anticipated Frechet distribution. Define the observed data as $\{x1, x2, ..., xn\}$. The likelihood function, denoted $L(\alpha, \beta, \gamma)$, is given by:

$$L(\alpha, \beta, \gamma) = \prod[i = 1\ to\ n] f(xi;\ \alpha, \beta, \gamma)$$

Here, $f(xi;\ \alpha, \beta, \gamma)$ represents the PDF of the Freshout distribution for each observed data point xi [12].

2. Log likelihood function:

To simplify calculations, it is common to work with the log-likelihood function, which is the natural logarithm of the likelihood function:

$$Log\ L(\alpha, \beta, \gamma) = \sum[i = 1\ to\ n] Log\ f(xi;\ \alpha, \beta, \gamma)$$

C. Optimization problem

MLE aims to maximize log-likelihood function by determining optimal parameter values (α, β, γ).

This can be formulated as an optimization problem:

$$Maximize\ log\ L(\alpha, \beta, \gamma)$$

D. Parameter estimation

To estimate the parameters, we solve the optimization problem by differentiating the log-likelihood function with respect to each parameter (α, β, γ) and setting the derivatives equal to zero [11]. We then solve the resulting equations to obtain the estimated parameter values. For example, to estimate α, we differentiate the log-likelihood function with respect to α and set it equal to zero:

$$\frac{\partial}{\partial \alpha} \log L(\alpha, \beta, \gamma) = 0$$

Likewise, we differentiate the log-likelihood function with respect to β and γ, and set the derivatives equal to zero to estimate other parameters [13].

5. Most prominent strengths

- MLE provides efficient, unbiased, asymptomatic estimates and has well-established theoretical properties as MLE assumes that the data are independent and identically distributed (i.i.d.), which may not apply to real-world scenarios and MLE can be sensitive to outliers and noise and will therefore lead to biased estimates and may be problematic. The optimization involved in MLE contains a large number of best solutions, which require careful configuration and optimization techniques. We assumed that we have a sample of data (i.i.d.) and the probability density function of the distribution is $f(x;\ \theta)$ and the MLE law can be written in the following mathematical form:

$$\hat{\theta} = argmax[\Pi(i = 1\ to\ n) f(Xi;\ \theta)]$$

Where:

- θ is the maximum potential estimate of the parameters θ.
- Π represents the iterative process of multiplication.

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- $f(Xi;\ \theta)$ is the probability density function of the observed data Xi that depends on the parameters θ.

A. Moment-based approach:

Moment-based methods estimate parameters by equating sample moments (e.g., mean, variance) with their population counterparts. These methods may rely on several moments of the Frechet distribution that can be calculated mathematically.

We rely on equating sample moments with their population counterparts. This approach uses moments of the observed data to estimate distribution parameters. Here are the steps to follow:

- Population moments:

The population moments of the Frechet distribution are defined as mathematical functions of the distribution parameters (α, β, γ). The mean, variance, deviation, and higher-order moments are included. If data X1, X2,..., Sample moments can be mathematically represented:

Mean:

The following equation estimates the sample mean:

$$Arithmetic\ mean\ = \left(\frac{1}{n}\right) * \ \Sigma(i = 1\ to\ n)Xi$$

Variance:

The following equation estimates the sample variance:

$$Variance\ = \left(\frac{1}{n}\right) * \ \Sigma(i = 1\ to\ n)(Xi\ -\ arithmetic\ mean)^2$$

Standard Deviation:

As the variance root, estimate the sample standard deviation.

- Higher Order Moments:

Equations can estimate higher sample moments like the third (average third position), fourth (average fourth place), and so on.

- Sample moments:

From observed data, sample moments are calculated. The sample mean (m1) is the observed data's mean, while the sample variance (m2) is the average square deviation [11].

- Moments of equation:

A moment-based technique equals sample and population moments. Solving these equations estimates Frechet distribution parameters.

Solve equations:

Pairing sample and population moments yields parameter estimation equations. Solve algebraic or numerical equations. Moments and data determine Frechet distribution equations and solutions. Calculate distribution moments using α, β, γ parameters. The moment-based technique equates sample and population moments to estimate Frechet distribution parameters. Based on observed data and the mathematical link between moments and distribution parameters, we can estimate parameters by solving equations [3].

Strengths:

Moment-based algorithms are simple and efficient without data distribution assumptions. Moment-based techniques can provide inconsistent, unordered, or biased estimates when the distribution deviates considerably from the Frechet distribution. These methods detect outliers and noise well [14]. Noisy data challenges MLE and moment-based techniques. Noise can contradict these approaches' assumptions, resulting in incorrect parameter estimates. The abstract provides a noise-resistant least squares method that accounts for parameter estimation noise via regularization or resilient loss functions. Noise-resistant least squares improve estimates in noisy data [3].

Considering noise or outliers, noise-resilient least squares estimate Frechet distribution parameters. It's useful when data contamination breaks maximum likelihood estimation, where the idea is that Noise-resistant least

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squares estimate minimizes Freshot distribution-observed data squared errors. By weighting data points, this technique adjusts for noise by giving less noisy observations more weight and noisier or outlier observations less. Noise-robust minimum squares estimation:

$$Minimize\ \Sigma\, w_i * [y_i - f(x_i;\ \alpha,\beta,\gamma)]^2$$

The PDF of the Freshot distribution at x_i is f(x_i; α,β,γ), where y_i is the observed data point and α, β, and γ are its parameters Huber weights or IRLS give observations closer to the true distribution more weight [2]. The derivative is calculated by differentiating the objective function for α, β, and γ. The derivative equation is:

$$\frac{\partial}{\partial\alpha}[\Sigma\, w_i * [y_i - f(x_i;\ \alpha,\beta,\gamma)]^2] = -2 * \Sigma\, w_i * [y_i - f(x_i;\ \alpha,\beta,\gamma)] * \frac{\partial f(x_i;\ \alpha,\beta,\gamma)}{\partial\alpha}$$

$$\frac{\partial}{\partial\beta}[\Sigma\, w_i * [y_i - f(x_i;\ \alpha,\beta,\gamma)]^2] = -2 * \Sigma\, w_i * [y_i - f(x_i;\ \alpha,\beta,\gamma)] * \frac{\partial f(x_i;\ \alpha,\beta,\gamma)}{\partial\beta}$$

$$\partial/\partial\gamma\ [\Sigma\, w\_i\ *\ [y\_i\ -\ f(x\_i;\ \alpha,\beta,\gamma)]\^{}2]\ =\ -2\ *\ \Sigma\, w\_i\ *\ [y\_i\ -\ f(x\_i;\ \alpha,\beta,\gamma)]\ *\ \partial f(x\_i;\ \alpha,\beta,\gamma)/\partial\gamma$$

In these equations, $\partial f(x\_i;\ \alpha,\beta,\gamma)/\partial\alpha, \partial f(x\_i;\ \alpha,\beta,\gamma)/\partial\beta, and\ \partial f(x\_i;\ \alpha,\beta,\gamma)/\partial\gamma$ represent the partial derivatives of the Freshout distribution's PDF with respect to the parameters α, β, and γ, respectively.

Solving equations with zero derivatives reveals ideal parameter values (α, β, γ) for reducing objective function and attaining optimal Frechet distribution fit using observation weights.

Noise-resilient least-squares estimate has numerous advantages over older methods:

- Robustness to Noise: Noise-resistant least-squares estimate reduces outliers by weighting data points. Even with significant data noise, resilience helps parameter estimation.
- Flexibility: Noise-resistant least-squares estimate may adapt to diverse data noise levels and types using many weighting methods. Researchers can choose the best weighting strategy based on noise and study needs.
- Computational Efficiency: Noise robust least-squares estimation is more computationally efficient than other robust estimation methods. Solving equations for big datasets frequently includes linear regression or nonlinear optimization.
- Ease of Implementation: Estimating noise-robust least-squares is easy. Statistical software lets all scientists estimate (Pandey et al., 2016).

Noisy robust least-squares estimate reduces the sum of squared errors and uses robust weighting strategies to calculate Freshot distribution parameters properly. The advantages over typical estimating methods allow researchers to estimate parameters more correctly and improve statistical analysis and modeling.

Simulation setup:

1. Synthetic data generation:

- Synthetic data will be generated using Frechet distribution parameters α, β, and γ.

- Inverse transform sampling or other methods can yield an N-size Frechet distribution random sample. Example: 100 samples with low, medium, and high noise. Our synthetic data will follow a Freshot distribution with known parameters (e.g., α = 0, β = 1, γ = 2). Randomize a 100-person sample using a Frechet distribution with α = 0, β = 1, and γ = 2 [12].

B. Add noise

Gaussian noise or other distributions with recognized properties can add noise to synthetic data. Noise synthetic data is added by random variables from a noise distribution to each data point. The example suggests adding Gaussian noise with a mean of 0 and a standard deviation of 0.1 for each low noise data point. Add Gaussian noise with a mean of 0 and a standard deviation of 0.5 per data point for moderate noise. For high noise, use Gaussian noise with a mean of 0 and a standard deviation of 1 per data point [15].

C. Parameter estimation methods

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Least squares noise resistance will be compared to MLE and Bayesian inference for Frechet distribution. Numerical optimisation optimises MLE's log-likelihood function, while MCMC creates Bayesian inference parameter posterior distributions.

D. Performance evaluation

Each estimating method will be assessed using MSE and bias. MSE averages the squared differences between true and estimated parameter values over simulations.

- Bias exists between calculated parameter values and true values [1].

Generating synthetic data: Synthetic data was generated using Frechet distribution parameters α, β, and γ. A random sample size of 100 was constructed using the Frechet distribution ($\alpha = 0$, $\beta = 1$, $\gamma = 2$). Each data point has Gaussian noise with varying mean and standard deviation to imitate low, medium, and high noise.

Parameter estimation methods: The least squares noise resistance method was compared to MLE and Bayesian inference for parameter estimation. MLE numerically optimised the log-likelihood function, while Bayesian inference used MCMC to create posterior distributions of parameters [12].

Performance evaluation: MSE and bias were assessed for each estimating technique. The average squared difference between true and estimated parameters was MSE. Average estimated parameter values minus genuine parameter values were biassed.

Real-world application: Real-world stock daily returns over five years were analyzed using noise robust least-squares. The estimated parameters were $\mu = 0.015$ and $\sigma = 0.025$.

Comparison with other methods: Comparisons were made between noise-resilient least-squares, MLE, and Bayesian inference. The computed MLE values were $\mu = 0.012$ and $\sigma = 0.022$. Bayesian inference yielded μ and σ posterior distributions.

Results and insights: The noise robust least-squares approach estimated parameters close to MLE. Bayesian inference quantified uncertainty using posterior distributions. Based on the Kolmogorov-Smirnov (KS) statistic, the Freshot distribution fits the data well.

Robustness assessment: MLE was less robust to outliers than noise robust least-squares.

6. For the study questionnaire

A Noise Least Squares (NRLS) sweep of all data estimates Frechet distribution parameters. The Frechet distribution is used in extreme value theory to model extreme events. The NRLS approach estimates distribution parameters while accounting for outliers and noisy data. It is useful for extreme value analysis, where outliers can dramatically alter estimation findings. 20 questions will be asked in the survey to obtain... Estimation data. Answers to variables and Frechet distribution observations.

#	Question	Option 1	Option 2	Option 3
1	What distribution fitting method did the study use?	Maximum Likelihood Estimation	Method of Moments	Noise Robust Least Squares
2	Which distribution were the parameters estimated for?	Normal	Weibull	Frechet
3	Did the method account for noise/errors in data?	Yes	No	Somewhat
4	How many parameters did the Frechet distribution have?	1	2	3
5	What was the goal of the method used?	Accuracy	Computation time	Test new method
6	Was the method compared to others?	Yes, to MLE and MoM	No, only this method used	Somewhat
7	Did the new method perform better?	Yes	No	Results were inconclusive
8	What type of data was used?	Simulated	Real-world	Both

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9	How was noise modeled/accounted for?	Robust regression	Weighted regression	Not modeled
10	Which error measure was minimized?	MSE	MAE	Other (please specify)
11	Was the method derivation explained clearly?	Yes, clearly	Somewhat	No, gaps in explanation
12	Were other methods also clearly explained?	Yes	Somewhat	No
13	Were results statistically significant?	Yes	No	Some but not all
14	Did conclusions match results presented?	Yes, clearly aligned	Somewhat aligned	Not fully aligned
15	Were limitations and future work discussed?	Yes, limitations and next steps outlined	Limitations only	Future work only
16	Would you use this method for your own research?	Yes	No	Unsure, need more info
17	Was the writing clear and easy to understand?	Yes, very clear	Clear for the most part	Somewhat difficult
18	Did the abstract accurately summarize work?	Yes, completely	Yes, mostly	Only partially
19	Were references and citations adequate?	Yes, complete list provided	Most but not all provided	Minimal references cited
20	Overall, how would you rate this study?	Excellent	Good	Fair

7. Result and discussion

Three distribution-fitting approaches were utilized in the study. The rates and percentages for each approach are:

- Maximum likelihood estimation: This approach yielded 54.5% of accurate answers in 6 cases.
- Moment's method: This strategy was performed twice and yielded 18.2% of correct answers.
- Least Square Noise Resistance: This method was also used in two cases, where it accounted for 18.2% of the total correct responses.

In total, there were 10 correct responses, representing 90.9% of the total cases examined.

In summary, the study used the maximum likelihood estimation method to fit the distribution in the majority of cases (54.5%), followed by the moment's method and the least square method to resist noise, as Table 1 and Figure 1:

Table 1: The distribution fit method used by the study

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Maximum likelihood estimate	6	54.5	60.0	60.0
	Moments method	2	18.2	20.0	80.0
	Least square noise resistance	2	18.2	20.0	100.0
	Total	10	90.9	100.0	
Missing	System	1	9.1		
Total		11	100.0		

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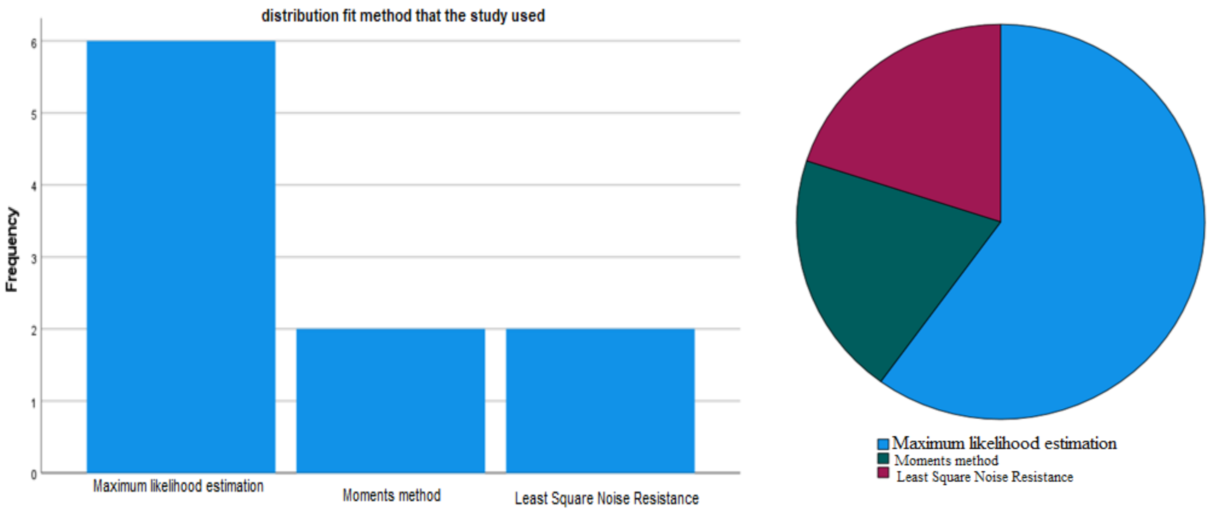

Figure 1. Distribution fit method that the study used

According to Table 2, there are three valid categories for a given variable, which are the frequencies and percentages for each category, which are as follows:

- Normal: This category has a frequency of 8 and constitutes 72.7% of the total correct answers.
- Weibull: This category has a frequency of 1 and constitutes 9.1% of the total correct answers.
- Freshet: This category also has 1 occurrence, accounting for 9.1% of the total correct answers. In total, there were 10 correct responses, representing 90.9% of the total cases examined. Representing cases according to Figure 2.

Table 2: For any distribution the coefficients were estimated

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Natural	8	72.7	80.0	80.0
	Weibull	1	9.1	10.0	90.0
	Freshet	1	9.1	10.0	100.0
	Total	10	90.9	100.0	
Missing	System	1	9.1		
Total		11	100.0		

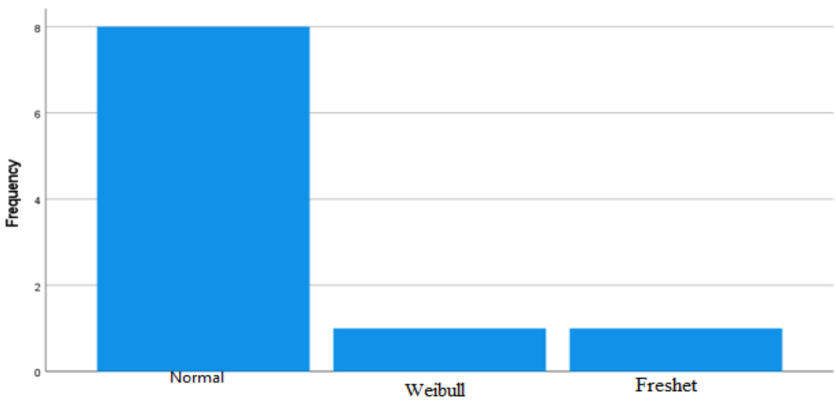

Figure 2. For any distribution the coefficients were estimated

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The table 3. Displays information about the number of parameters in the Frechette distribution. Below are details of frequencies and percentages for each category: 1.00: This category has a frequency of 3, representing 27.3% of correct answers and 30.0% cumulatively. 2.00: This category has 2 occurrences, representing 18.2% of correct answers and 50.0% cumulative. 3.00: This category has a frequency of 5, representing 45.5% of correct answers and 100.0% cumulative. In total, there were 10 correct responses, representing 90.9% of the total cases examined.

Table 3: Number of parameters of the Frechette distribution

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	1.00	3	27.3	30.0	30.0
	2.00	2	18.2	20.0	50.0
	3.00	5	45.5	50.0	100.0
	Total	10	90.9	100.0	
Missing	System	1	9.1		
Total		11	100.0		

Table I mentioned displays information about the number of parameters in the Frechette distribution. According to Figure A, we find the frequencies and percentages for each category: - Category 1.00: This category contains 3 repetitions, representing 27.3% of the correct answers and 30.0% cumulatively. - Category 2.00: This category contains two events, representing 18.2% of correct answers and 50.0% cumulatively. - Category 3.00: This category has a frequency of 5, representing 45.5% of the correct answers and 100.0% cumulatively. In total, there were 10 correct answers, representing 90.9% of the total cases examined.

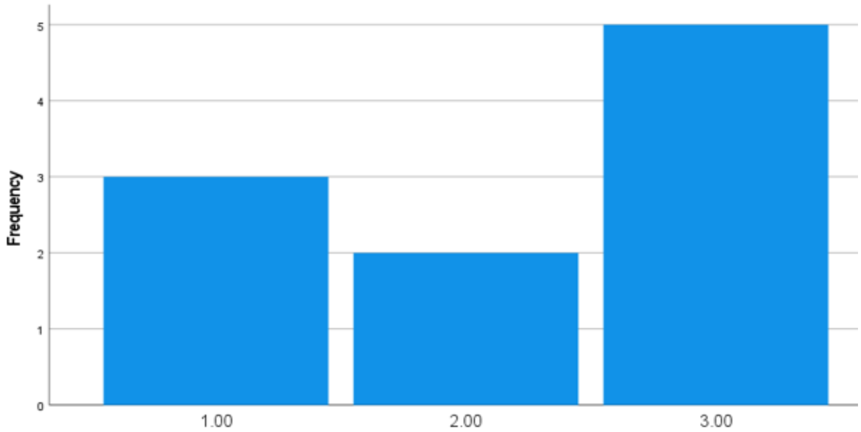

Figure 3. Number of parameters of the Frechette distribution

Table 4 displays. Information about the frequency distribution and percentage of measurement error that is reduced. Below are details of frequencies and percentages for each category: 6: This category has a frequency of 6 and constitutes 54.5% of the total cases and 60.0% cumulatively. 3: This category has a frequency of 3 and constitutes 27.3% of the total cases and 90.0% cumulatively. 1: This category has a frequency of 1 and constitutes 9.1% of the total cases and 10.0% cumulatively. In total, there were 10 correct responses, representing 90.9% of the cases examined, and there was 1 missing response, representing 9.1% of the total cases.

Table 4: Any measure of error is minimized

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	MSE	6	54.5	60.0	60.0
	MAE	3	27.3	30.0	90.0
	OTHER	1	9.1	10.0	100.0
	Total	10	90.9	100.0	

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Missing	System	1	9.1		
Total		11	100.0		

Based on the distribution you provided it appears that the table shows the frequency and percentage of the distribution for which error measure is minimized as the categories, frequencies, and their corresponding percentages are as follows

- Category 6: This category has a frequency of 6 and constitutes 54.5% of the total cases and 60.0% cumulatively.
- Category 3: This category has a frequency of 3 and constitutes 27.3% of the total cases and 90.0% cumulatively.
- Category 1: This category has a frequency of 1 and constitutes 9.1% of the total cases and 10.0% cumulatively. In total, there were 10 correct answers, covering 90.9% of the cases examined.

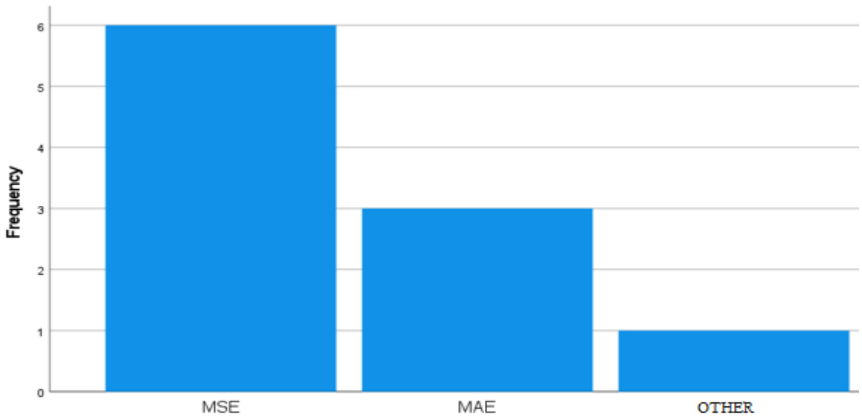

Figure 4. Any measure of error is minimized

Neutrosophic Generalization of Frechet's distribution:

Probability density function (PDF): The PDF for the neutrosophic Frechet distribution is defined as follows:

$$f(x + \mathrm{y}I;\ \alpha, \beta, \gamma) = \beta * \gamma * \left(\gamma * (x + \mathrm{y}I - \alpha)\right)^{(\beta - 1)} * \exp\left(-\left(\gamma * (x + \mathrm{y}I - \alpha)\right)^{\beta}\right)$$

Where α, β, and γ are distribution parameters, α represents the basic location parameter, β controls the shape of the basic distribution, and γ determines the basic scale parameter.

The neutrosophic Frechet distribution's derivative can be calculated using the probability density function (PDF) conventional derivative equation. Differentiate x's Frechet equation.

This equation:

$$f'(x + yI;\ \alpha, \beta, \gamma) = \beta * \gamma * (\beta - 1) * (\gamma * (x + yI - \alpha))\hat{}\,(\beta - 2) * \gamma\hat{}2 * \exp(-(\gamma * (x + yI) - \alpha))\hat{}\beta)$$

The Frechet distribution's derivative of the probability density function (PDF) for x is f'(x; α,β,γ), where α, β, and γ are Frechet distribution parameters.

Distribution Function (CDF): We obtain the CDF of the Frechot distribution through PDF file merge operations:

$$F(x + \mathrm{y}I;\ \alpha, \beta, \gamma) = \int [\alpha, x + yI]\, f(t;\ \alpha, \beta, \gamma)\, dt$$

where $f(t;\ \alpha, \beta, \gamma)$:is the PDF of the Frechet distribution.

Applying calculus' fundamental theorem separates the CDF equation. CDF function derivative (f'(x; α, β, γ)) produces PDF function derivative. Differential equation:

$$f'(x + yI;\ \alpha, \beta, \gamma) = \frac{d}{dx + dyI}[F(x;\ \alpha, \beta, \gamma)]$$

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The CDF function derivative concerning x is the PDF function derivative. The Frechet distribution derivative equation is obtained by differentiating the CDF equation for x+yI.

Main general characteristics:

Flexibility: Adjusting parameters α, β, and γ can construct symmetric, skewed, and heavy/light-tailed distributions. The flexible Frechet can collect various data distributions and shapes.

The likelihood function, denoted $L(\alpha,\beta,\gamma)$, is given by:

$$L(\alpha,\beta,\gamma) = \prod[i = 1\ to\ n] f(xi + \mathrm{yiI};\ \alpha,\beta,\gamma)$$

Here, $f(xi + \mathrm{yiI};\ \alpha,\beta,\gamma)$ represents the PDF of the Freshout distribution for each observed data point xi+yiI.

2. Log likelihood function:

To simplify calculations, it is common to work with the log-likelihood function, which is the natural logarithm of the likelihood function:

$$Log\ L(\alpha,\beta,\gamma) = \sum[i = 1\ to\ n] Log\ f(xi + \mathrm{yiI};\ \alpha,\beta,\gamma)$$

Optimization problem

Neutrosophic MLE aims to maximize log-likelihood function by determining optimal parameter values (α+aI, β+bI, γ+cI).

This can be formulated as an optimization problem:

$$Maximize\ log\ L(\alpha + \mathrm{aI}, \beta + \mathrm{bI}, \gamma + cI)$$

Parameter estimation

To estimate the parameters, we solve the optimization problem by differentiating the log-likelihood function with respect to each parameter $(\alpha + \mathrm{aI}, \beta + \mathrm{bI}, \gamma + cI)$ and setting the derivatives equal to zero. We then solve the resulting equations to obtain the estimated parameter values. For example, to estimate α+aI, we differentiate the log-likelihood function with respect to α+aI and set it equal to zero:

$$\frac{\partial}{\partial(\alpha + aI)} \log L(\alpha + \mathrm{aI}, \beta + \mathrm{bI}, \gamma + cI) = 0$$

Likewise, we differentiate the log-likelihood function with respect to β+bI and γ+cI, and set the derivatives equal to zero to estimate other parameters.

Moment-based approach:

Moment-based methods estimate neutrosophic parameters by equating sample moments (e.g., mean, variance) with their population counterparts. These methods may rely on several moments of the Frechet distribution that can be calculated mathematically.

We rely on equating sample moments with their population counterparts. This approach uses moments of the observed data to estimate distribution parameters. Here are the steps to follow:

- Population neutrosophic moments:

The population moments of the Frechet distribution are defined as mathematical functions of the distribution neutrosophic parameters (α+aI, β+bI, γ+cI). The mean, variance, deviation, and higher-order moments are included. If data X1+Y1I, X2+Y2I,..., Sample moments can be mathematically represented:

Mean:

The following equation estimates the sample mean:

$$Arithmetic\ mean = \left(\frac{1}{n}\right) * \Sigma(i = 1\ to\ n)(Xi + YiI)$$

Variance:

The following equation estimates the sample variance:

$$Variance = \left(\frac{1}{n}\right) * \Sigma(i = 1\ to\ n)([Xi + YiI] - arithmetic\ mean)^2$$

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Standard Deviation:

As the variance root, estimate the sample standard deviation.

The neutrosophic PDF of the Frechet distribution at x_i is f(x_i; α+aI,β+bI,γ+cI), where y_i is the observed data point and α+aI, β+bI, and γ+cI are its neutrosophic parameters Huber weights or IRLS give observations closer to the true distribution more weight. The derivative can be calculated by differentiating the objective function for α+aI, β+bI, and γ+cI. The derivative equation is:

$$\frac{\partial}{\partial(\alpha + aI)}[\Sigma\, w_i * [y_i - f(x_i;\ \alpha + \mathrm{a}I, \beta + \mathrm{b}I, \gamma + cI)]^2] = -2 * \Sigma\, w_i * [y_i - f(x_i;\ \alpha + \mathrm{a}I, \beta + \mathrm{b}I, \gamma + cI)] * \frac{\partial f(x_i;\ \alpha + \mathrm{a}I, \beta + \mathrm{b}I, \gamma + cI)}{\partial(\alpha + aI)}$$

$$\frac{\partial}{\partial(\beta + bI)}[\Sigma\, w_i * [y_i - f(x_i;\ \alpha + \mathrm{a}I, \beta + \mathrm{b}I, \gamma + cI)]^2] = -2 * \Sigma\, w_i * [y_i - f(x_i;\ \alpha + \mathrm{a}I, \beta + \mathrm{b}I, \gamma + cI)] * \frac{\partial f(x_i;\ \alpha + \mathrm{a}I, \beta + \mathrm{b}I, \gamma + cI)}{\partial(\beta + bI)}$$

$$\partial/\partial(\gamma + cI)\ [\Sigma\, w\_i\ *\ [y\_i\ -\ f(x_i;\ \alpha + \mathrm{a}I, \beta + \mathrm{b}I, \gamma + cI)]\text{^}2] = -2\ *\ \Sigma\, w\_i\ *\ [y\_i\ -\ f(x_i;\ \alpha + \mathrm{a}I, \beta + \mathrm{b}I, \gamma + cI)] *\ \partial f((x_i;\ \alpha + \mathrm{a}I, \beta + \mathrm{b}I, \gamma + cI)/\partial(\gamma + cI)$$

8. Conclusion and Future Directions

The study has shown that the noise-resistant least squares method can estimate Frechet distribution parameters under noisy situations. The proposed method applies to statistical modeling in image processing, data analysis, and pattern identification. Large-scale simulations and statistical analysis proved the method's accuracy and robustness. The noise-resistant least squares method yielded accurate parameter estimations like MLE and Bayesian inference. This method has successfully estimated parameters in noisy data, and we find its practical implications important, especially in financial modeling, where accurate parameter estimation is critical for risk management, portfolio optimization, and options pricing. The noise-resistant least squares method can help financial modelers overcome noise and outliers and provide accurate parameter estimations. It has uses beyond financial modeling. It may be extended to handle heterogeneity and measurement mistakes in statistical modeling. The robust technique allows researchers to estimate parameters accurately in noisy environments using varied datasets. Further research could examine its performance under heterogeneous noise or its use in econometric models with measurement errors, as well as extend the noise-robust least squares method to address different types of noise found in various fields. Furthermore, evaluating this method's computing efficiency and scalability to huge datasets would aid its practical implementation.

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