Selection process based on new type neutrosophic interval-valued set applied to logarithm operator

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Abstract

We introduce the new type neutrosophic interval-valued set (NIVS) problems relevant to multiple attribute decision making (MADM). Pythagorean interval-valued fuzzy set (PIVFS) and neutrosophic set (NS) can be extended into new type neutrosophic interval-valued set. We discusses new type neutrosophic interval-valued weighted averaging (new type NIVWA), new type neutrosophic interval-valued weighted geometric (new type NIVWG), generalized new type neutrosophic interval-valued weighted averaging (new type GNIVWA) and generalized new type neutrosophic interval-valued weighted geometric (new type GNIVWG). A number of algebraic properties of new type NIVSs have been established such as associativity, distributivity and idempotency. Using expert judgments and criteria, we will be able to decide which options are the most appropriate. Several of the proposed and current models are also compared in order to demonstrate the reliability and usefulness of the models under study. Additionally, the findings of the study are fascinating and intriguing.

Keywords: MADM; new type NIVWA; new type NIVWG; new type GNIVWA; new type GNIVWG.

1 Introduction

Different techniques have been used by many authors to contribute to this field of study. As a result of the uncertainties, fuzzy set (FS),[1] intuitionistic FS (IFS),[2] interval valued FS (IVFS),[3] vague set (VS),[4] Pythagorean FS (PFS),[5] IVPFS,[6] spherical FS (SFS)[7] neutrosophic set (NS).[8] A membership degree (MD) in a set represents degrees of belongingness ranging from 0 to 1. IFSs are classified by the number of MDs with a nonmembership degree (NMD) fewer than one, which Atanassov introduced. In some cases, the MDs and NMDs combined are more than one, which causes the DM approach to create a single problem. As a generalization of IFS, PFS ensures that the square total of MD and NMD is not greater than one. It is impossible to demonstrate

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neutrality (neither favor nor disfavor). As part of the picture FS proposed by Cuong,[9] there are three pointers; a positive, neutral, and negative one. Palanikumar et al. discussed the new aggregating operator[10]-.[15] It was discussed by Liu et al.[16] that aggregation operators (AO) were introduced with generalized PFS. Challenges using DM approach with greater than one truth membership value (TMD), false membership value (FMD), and indeterminacy membership value (IMD). The SFS proposed by[7] provides a squared sum of TMD, IMD and FMD that is less than one. A TOPSIS-based conceptualization of the SFS can be found in.[17] Recently, Palanikumar et al. discussed the new aggregating operators[18]-.[20] Recently many researchers discussed the new applications such as neutrosophic and its extension[24]-.[27]

The concept of neutrosophic sets was introduced by Smarandache et al.[21] In FS and IFS, one of the biggest differences is the knowledge of neutral thought. Neutrosophy is the knowledge of neutral thought. This logic assigns a value to each proposition based on TMD, IMD, and FD. A universe with a value between 0 and 1 in which every element has a value between 0 and 1. Based on AOs,[22] presented a method for MCDM under interval NS. In many applications and AOs and its algebraic structures are explained in Palanikumar et al.[23] The section 1 contains an introduction. Information about PNS and FS can be found in section 2. In section 3, we define and describe some of the operations of new type NIVNs. In section 4, we provide AOs for a new type of NIVN. In Section 6, we provide the conclusion. There are two main outcomes of the paper: 1) New types of NIVS have been established with algebraic properties; they are associative, distributive, and idempotent. This is followed by a discussion of the new type NIVWA, new type NIVWG, new type GNIVWA and new type GNIVWA.

2 Preliminaries

In this section, we will review the concepts of PFS and PIVFS.

Definition 2.1. [5] Let $\mathbb{U}$ be the universal set. The PFS $\aleph = \Big\{\ell, \langle \wp^T_\aleph(\ell), \wp^F_\aleph(\ell)\rangle \big| \ell \in \mathbb{U}\Big\}$, $\wp^T_\aleph : \mathbb{U} \to [0,1]$ and $\wp^F_\aleph : \mathbb{U} \to [0,1]$ denotes MD and NMD of $\ell \in \mathbb{U}$ to $\aleph$, respectively and $0 \preceq (\wp^T_\aleph(\ell))^2 + (\wp^F_\aleph(\ell))^2 \preceq 1$. For convenience, $\aleph = \langle \wp^T_\aleph, \wp^F_\aleph \rangle$ is called the Pythagorean fuzzy number(PFN).

Definition 2.2. [6] The Pythagorean IVFS (PIVFS) $\aleph = \Big\{\ell, \Big\langle \widetilde{\wp^T_\aleph}(\ell), \widetilde{\wp^F_\aleph}(\ell)\Big\rangle \Big| \ell \in \mathbb{U}\Big\}$, where $\widetilde{\wp^T_\aleph} : \mathbb{U} \to Int([0,1])$ and $\widetilde{\wp^F_\aleph} : \mathbb{U} \to Int([0,1])$ denotes MD and NMD of $\ell \in \mathbb{U}$ to $\aleph$, respectively, and $0 \preceq (\wp^{T+}_\aleph(\ell))^2 + (\wp^{F+}_\aleph(\ell))^2 \preceq 1$. For convenience, $\aleph = \Big\langle \Big[\wp^{T-}_\aleph, \wp^{T+}_\aleph\Big], \Big[\wp^{F-}_\aleph, \wp^{F+}_\aleph\Big] \Big\rangle$ is called the PIVFN.

Definition 2.3. The Pythagorean neutrosophic set $\aleph = \Big\{\ell, \langle \wp^T_\aleph(\ell), \wp^I_\aleph(\ell), \wp^F_\aleph(\ell)\rangle \big| \ell \in \mathbb{U}\Big\}$, where $\wp^T_\aleph : \mathbb{U} \to [0,1]$, $\wp^I_\aleph : \mathbb{U} \to [0,1]$ and $\wp^F_\aleph : \mathbb{U} \to [0,1]$ denotes TMD, IMD and FMD of $\ell \in \mathbb{U}$ to $\aleph$, respectively and $0 \preceq (\wp^T_\aleph(\ell))^2 + (\wp^I_\aleph(\ell))^2 + (\wp^F_\aleph(\ell))^2 \preceq 2$. For convenience, $\aleph = \langle \wp^T_\aleph, \wp^I_\aleph, \wp^F_\aleph \rangle$ is called the Pythagorean neutrosophic number.

3 New type NIVN and its fundamental operations

There are several intriguing fundamental operations associated with the new type NIVN. This section L denotes log operations.

Definition 3.1. The new type NIVS $\aleph = \Big\{\ell, \Big\langle \Big[LT^-_\aleph(\ell), L(T^+_\aleph(\ell))\Big], \Big[LI^-_\aleph(\ell), LI^+_\aleph(\ell)\Big], \Big[LF^-_\aleph(\ell), L(F^+_\aleph(\ell))\Big] \Big\rangle \Big| \ell \in \mathbb{U}\Big\}$, $\widetilde{\wp^T_\aleph} : \mathbb{U} \to Int([0,1])$, $\widetilde{\wp^I_\aleph} : \mathbb{U} \to Int([0,1])$ and $\widetilde{\wp^F_\aleph} : \mathbb{U} \to Int([0,1])$ denotes TMD, IMD and FMD of $\ell \in \mathbb{U}$ to $\aleph$, respectively and $0 \preceq (L_{\Im_i} T^+_\aleph(\ell))^{\flat_1} + (L_{\Im_i} I^-_\aleph(\ell))^{\flat_2} + (L_{\Im_i} F^+_\aleph(\ell))^{\flat_3} \preceq 2$, where $\flat_1, \flat_2, \flat_3$ are positive integers and $\Im = \circledast[T^-_\aleph, T^+_\aleph], [I^+_\aleph, I^-_\aleph], [F^-_\aleph, F^+_\aleph]$. For convenience, $\aleph = \Big\langle \Big[LT^-_\aleph, L(T^+_\aleph)\Big], \Big[LI^-_\aleph, LI^+_\aleph\Big], \Big[LF^-_\aleph, L(F^+_\aleph)\Big] \Big\rangle$ is called the new type NIVN.

Definition 3.2. Let $\aleph = \Big\langle \Big[LT^-_\aleph, L(T^+_\aleph)\Big], \Big[LI^-_\aleph, LI^+_\aleph\Big], \Big[LF^-_\aleph, L(F^+_\aleph)\Big] \Big\rangle$ be the new type NIVN, the scorollarye function of $\aleph$ is defined as $\mathbb{S}(\aleph) = \frac{\mathbb{S}_1(\aleph)+\mathbb{S}_2(\aleph)}{2}$, $-1 \preceq \mathbb{S}(\aleph) \preceq 1$. where

$$\mathbb{S}_1(\aleph) = \left(\frac{\mathcal{P}}{2} + 1 - \frac{\mathcal{Q}}{2} + 1 - \frac{\mathcal{R}}{2}\right), \mathbb{S}_2(\aleph) = \left(\frac{\mathcal{P}}{2} + 1 - \frac{\mathcal{Q}}{2} + 1 - \frac{\mathcal{R}}{2}\right),$$

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The accuracy function of $\aleph$ is $\mathbb{A}(\aleph) = \frac{\mathbb{A}_1(\aleph)+\mathbb{A}_2(\aleph)}{2}$, where $0 \preceq \mathbb{A}(\aleph) \preceq 1$.

$$\mathbb{A}_1(\aleph) = \left(\frac{\mathcal{P}}{2} + 1 + \frac{\mathcal{Q}}{2} + 1 + \frac{\mathcal{R}}{2}\right), \mathbb{A}_2(\aleph) = \left(\frac{\mathcal{P}}{2} + 1 + \frac{\mathcal{Q}}{2} + 1 + \frac{\mathcal{R}}{2}\right),$$

where $\mathcal{P} = (L_{\Im_i} T^-_\aleph)^2 + (L_{\Im_i}(T^+_\aleph))^2, \mathcal{Q} = (L_{\Im_i} I^-_\aleph)^2 + (L_{\Im_i} I^+_\aleph)^2, \mathcal{R} = (L_{\Im_i} F^-_\aleph)^2 + (L_{\Im_i}(F^+_\aleph))^2$

Definition 3.3. Let $\aleph = \left\langle [LT^-_\aleph, L(T^+_\aleph)], [LI^-_\aleph, LI^+_\aleph], [LF^-_\aleph, L(F^+_\aleph)] \right\rangle$,
$\aleph_1 = \left\langle [LT^-_{\aleph\,1}, L(T^+_{\aleph\,1})], [LI^-_{\aleph\,1}, LI^+_{\aleph\,1}], [LF^-_{\aleph\,1}, L(F^+_{\aleph\,1})] \right\rangle$
and $\aleph_2 = \left\langle [LT^-_{\aleph\,2}, L(T^+_{\aleph\,2})], [LI^-_{\aleph\,2}, LI^+_{\aleph\,2}], [LF^-_{\aleph\,2}, L(F^+_{\aleph\,2})] \right\rangle$ be any three new type NIVNs and $\Im = \circledast[T_{\aleph_i}, T^+_{\aleph_i}], [I_{\aleph_i}, I_{\aleph_i}], [F_{\aleph_i}, F^+_{\aleph_i}]$. Their following operations are defined as follows:

1. $\aleph_1 \bigvee \aleph_2 =$
$$\left[\begin{array}{c} \left[\begin{array}{c} \sqrt[\flat_1]{(L_{\Im_i}T^-_{\aleph\,1})^{\flat_1} + (L_{\Im_i}T^-_{\aleph\,2})^{\flat_1} - (L_{\Im_i}T^-_{\aleph\,1})^{\flat_1} \cdot (L_{\Im_i}T^-_{\aleph\,2})^{\flat_1}}, \\ \sqrt[\flat_1]{(L_{\Im_i}(T^+_{\aleph\,1}))^{\flat_1} + (L_{\Im_i}(T^+_{\aleph\,2}))^{\flat_1} - (L_{\Im_i}(T^+_{\aleph\,1}))^{\flat_1} \cdot (L_{\Im_i}(T^+_{\aleph\,2}))^{\flat_1}} \end{array}\right], \\ \left[\begin{array}{c} \sqrt[\flat_2]{(L_{\Im_i}I^-_{\aleph\,1})^{\flat_2} + (L_{\Im_i}I^+_{\aleph\,2})^{\flat_2} - (L_{\Im_i}I^-_{\aleph\,1})^{\flat_2} \cdot (L_{\Im_i}I^+_{\aleph\,2})^{\flat_2}}, \\ \sqrt[\flat_2]{(L_{\Im_i}I^-_{\aleph\,1})^{\flat_2} + (L_{\Im_i}I^+_{\aleph\,2})^{\flat_2} - (L_{\Im_i}I^-_{\aleph\,1})^{\flat_2} \cdot (L_{\Im_i}I^+_{\aleph\,2})^{\flat_2}} \end{array}\right], \\ \left[L_{\Im_i}(F^-_{\aleph\,1})^{\flat_3} \cdot L_{\Im_i}(F^-_{\aleph\,2})^{\flat_3}, L_{\Im_i}(F^+_{\aleph\,1})^{\flat_3} \cdot L_{\Im_i}(F^+_{\aleph\,2})^{\flat_3}\right] \end{array}\right],$$

2. $\aleph_1 \bigwedge \aleph_2 =$
$$\left[\begin{array}{c} \left[L_{\Im_i}(T^-_{\aleph\,1})^{\flat_1} \cdot L_{\Im_i}(T^-_{\aleph\,2})^{\flat_1}, L_{\Im_i}(T^+_{\aleph\,1})^{\flat_1} \cdot L_{\Im_i}(T^+_{\aleph\,2})^{\flat_1}\right], \\ \left[\begin{array}{c} \sqrt[\flat_2]{(L_{\Im_i}I^-_{\aleph\,1})^{\flat_2} + (L_{\Im_i}I^+_{\aleph\,2})^{\flat_2} - (L_{\Im_i}I^-_{\aleph\,1})^{\flat_2} \cdot (L_{\Im_i}I^+_{\aleph\,2})^{\flat_2}}, \\ \sqrt[\flat_2]{(L_{\Im_i}I^-_{\aleph\,1})^{\flat_2} + (L_{\Im_i}I^+_{\aleph\,2})^{\flat_2} - (L_{\Im_i}I^-_{\aleph\,1})^{\flat_2} \cdot (L_{\Im_i}I^+_{\aleph\,2})^{\flat_2}} \end{array}\right], \\ \left[\begin{array}{c} \sqrt[\flat_3]{(L_{\Im_i}F^-_{\aleph\,1})^{\flat_3} + (L_{\Im_i}F^-_{\aleph\,2})^{\flat_3} - (L_{\Im_i}F^-_{\aleph\,1})^{\flat_3} \cdot (L_{\Im_i}F^-_{\aleph\,2})^{\flat_3}}, \\ \sqrt[\flat_3]{(L_{\Im_i}(F^+_{\aleph\,1}))^{\flat_3} + (L_{\Im_i}(F^+_{\aleph\,2}))^{\flat_3} - (L_{\Im_i}(F^+_{\aleph\,1}))^{\flat_3} \cdot (L_{\Im_i}(F^+_{\aleph\,2}))^{\flat_3}} \end{array}\right] \end{array}\right],$$

3. $$\Psi \cdot \aleph = \left[\begin{array}{c} \left[\sqrt[\flat_1]{1 - \left(1 - (L_{\Im_i}T^-_\aleph)^{\flat_1}\right)^\Psi}, \sqrt[\flat_1]{1 - \left(1 - (L_{\Im_i}(T^+_\aleph))^{\flat_1}\right)^\Psi}\right], \\ \left[\sqrt[\flat_2]{1 - \left(1 - (L_{\Im_i}I^-_\aleph)^{\flat_2}\right)^\Psi}, \sqrt[\flat_2]{1 - \left(1 - (L_{\Im_i}I^+_\aleph)^{\flat_2}\right)^\Psi}\right], \\ \left[(L_{\Im_i}F^-_\aleph)^{\flat_3\Psi}, (L_{\Im_i}(F^+_\aleph))^{\flat_3\Psi}\right] \end{array}\right],$$

4. $$\aleph^\Psi = \left[\begin{array}{c} \left[(L_{\Im_i}T^-_\aleph)^{\flat_1\Psi}, (L_{\Im_i}(T^+_\aleph))^{\flat_1\Psi}\right], \\ \left[\sqrt[\flat_2]{1 - \left(1 - (L_{\Im_i}I^-_\aleph)^{\flat_2}\right)^\Psi}, \sqrt[\flat_2]{1 - \left(1 - (L_{\Im_i}I^+_\aleph)^{\flat_2}\right)^\Psi}\right], \\ \left[\sqrt[\flat_3]{1 - \left(1 - (L_{\Im_i}F^-_\aleph)^{\flat_3}\right)^\Psi}, \sqrt[\flat_3]{1 - \left(1 - (L_{\Im_i}(F^+_\aleph))^{\flat_3}\right)^\Psi}\right] \end{array}\right].$$

4 AOs based on new type NIVS approach

Based on the operational rules of new type NIVNs, the weighed averaging operators for new type NIVN are presented.

4.1 New type NIV weighted averaging(new type NIVWA) operator

Definition 4.1. Let $\aleph_i = \left\langle [LT^-_{\aleph\,i}, LT^+_{\aleph\,i}], [LI^{-+}_{\aleph\,i}, LI^{-+}_{\aleph\,i}], [LF^-_{\aleph\,i}, LF^+_{\aleph\,i}] \right\rangle$ be the family of new type NIVNs, $\zeta = (\zeta_1, \zeta_2, ..., \zeta_n)$ be the weight of $\aleph_i$, $\zeta_i \geq 0$ and $\bigwedge_{i=1}^n \zeta_i = 1$ and $\Im = \circledast[T^-_{\aleph_i}, T^+_{\aleph_i}], [I_{\aleph_i}, I_{\aleph_i}], [F_{\aleph_i}, F^+_{\aleph_i}]$. Then new type NIVWA operator is new type NIVWA $(\aleph_1, \aleph_2, ..., \aleph_n) = \bigwedge_{i=1}^n \zeta_i \aleph_i$ for $i = 1, 2, ..., n$.

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Theorem 4.2. *Let* $\aleph_i = \left\langle [LT^{-}_{\aleph\ i}, LT^{+}_{\aleph\ i}], [LI^{-+}_{\aleph\ i}, LI^{-+}_{\aleph\ i}], [LF^{-}_{\aleph\ i}, LF^{+}_{\aleph\ i}] \right\rangle$ *be the family of new type NIVNs. Then new type NIVWA* $(\aleph_1, \aleph_2, ..., \aleph_n) =$ *(associativity property).*

$$\left[\begin{array}{c} \left[\sqrt[\flat_1]{1 - \circledast_{i=1}^{n}\left(1 - (L_{\Im_i} T^{-}_{\aleph\ i})^{\flat_1}\right)^{\zeta_i}}, \sqrt[\flat_1]{1 - \circledast_{i=1}^{n}\left(1 - (L_{\Im_i}(T^{+}_{\aleph\ i}))^{\flat_1}\right)^{\zeta_i}}\right], \\ \left[\sqrt[\flat_2]{1 - \circledast_{i=1}^{n}\left(1 - (L_{\Im_i} I^{-}_{\aleph\ i})^{\flat_2}\right)^{\zeta_i}}, \sqrt[\flat_2]{1 - \circledast_{i=1}^{n}\left(1 - (L_{\Im_i} I^{+}_{\aleph\ i})^{\flat_2}\right)^{\zeta_i}}\right], \\ \left[\circledast_{i=1}^{n} (L_{\Im_i} F^{-}_{\aleph\ i})^{\flat_3\zeta_i}, \circledast_{i=1}^{n}(L_{\Im_i}(F^{+}_{\aleph\ i}))^{\flat_3\zeta_i}\right] \end{array}\right].$$

Proof. If $n = 2$, then new type NIVWA $(\aleph_1, \aleph_2) = \zeta_1\aleph_1 \bigvee \zeta_2\aleph_2$, where

$$\zeta_1\aleph_1 = \left[\begin{array}{c} \left[\sqrt[\flat_1]{1 - \left(1 - (L_{\Im_i} T^{-}_{\aleph\ 1})^{\flat_1}\right)^{\zeta_1}}, \sqrt[\flat_1]{1 - \left(1 - (L_{\Im_i}(T^{+}_{\aleph\ 1}))^{\flat_1}\right)^{\zeta_1}}\right], \\ \left[\sqrt[\flat_2]{1 - \left(1 - (L_{\Im_i} I^{-}_{\aleph\ 1})^{\flat_2}\right)^{\zeta_1}}, \sqrt[\flat_2]{1 - \left(1 - (L_{\Im_i} I^{+}_{\aleph\ 1})^{\flat_2}\right)^{\zeta_1}}\right], \\ \left[(L_{\Im_i} F^{-}_{\aleph\ 1})^{\flat_3\zeta_1}, (L_{\Im_i}(F^{+}_{\aleph\ 1}))^{\flat_3\zeta_1}\right] \end{array}\right]$$

and

$$\zeta_2\aleph_2 = \left[\begin{array}{c} \left[\sqrt[\flat_1]{1 - \left(1 - (L_{\Im_i} T^{-}_{\aleph\ 2})^{\flat_1}\right)^{\zeta_2}}, \sqrt[\flat_1]{1 - \left(1 - (L_{\Im_i}(T^{+}_{\aleph\ 2}))^{\flat_1}\right)^{\zeta_2}}\right], \\ \left[\sqrt[\flat_2]{1 - \left(1 - (L_{\Im_i} I^{-}_{\aleph\ 2})^{\flat_2}\right)^{\zeta_2}}, \sqrt[\flat_2]{1 - \left(1 - (L_{\Im_i} I^{+}_{\aleph\ 2})^{\flat_2}\right)^{\zeta_2}}\right], \\ \left[(L_{\Im_i} F^{-}_{\aleph\ 2})^{\flat_3\zeta_2}, (L_{\Im_i}(F^{+}_{\aleph\ 2}))^{\flat_3\zeta_2}\right] \end{array}\right].$$

Hence,

$$\zeta_1\aleph_1 \bigvee \zeta_2\aleph_2 = \left[\begin{array}{c} \left[\begin{array}{c} \sqrt[\flat_1]{\begin{array}{c}\left(1 - \left(1 - (L_{\Im_i} T^{-}_{\aleph\ 1})^{\flat_1}\right)^{\zeta_1}\right) + \left(1 - \left(1 - (L_{\Im_i} T^{-}_{\aleph\ 2})^{\flat_1}\right)^{\zeta_2}\right) \\ -\left(1 - \left(1 - (L_{\Im_i} T^{-}_{\aleph\ 1})^{\flat_1}\right)^{\zeta_1}\right) \cdot \left(1 - \left(1 - (L_{\Im_i} T^{-}_{\aleph\ 2})^{\flat_1}\right)^{\zeta_2}\right)\end{array}}, \\ \sqrt[\flat_1]{\begin{array}{c}\left(1 - \left(1 - (L_{\Im_i}(T^{+}_{\aleph\ 1}))^{\flat_1}\right)^{\zeta_1}\right) + \left(1 - \left(1 - (L_{\Im_i}(T^{+}_{\aleph\ 2}))^{\flat_1}\right)^{\zeta_2}\right) \\ -\left(1 - \left(1 - (L_{\Im_i}(T^{+}_{\aleph\ 1}))^{\flat_1}\right)^{\zeta_1}\right) \cdot \left(1 - \left(1 - (L_{\Im_i}(T^{+}_{\aleph\ 2}))^{\flat_1}\right)^{\zeta_2}\right)\end{array}} \end{array}\right], \\ \left[\begin{array}{c} \sqrt[\flat_2]{\begin{array}{c}\left(1 - \left(1 - (L_{\Im_i} I^{-}_{\aleph\ 1})^{\flat_2}\right)^{\zeta_1}\right) + \left(1 - \left(1 - (L_{\Im_i} I^{+}_{\aleph\ 2})^{\flat_2}\right)^{\zeta_2}\right) \\ -\left(1 - \left(1 - (L_{\Im_i} I^{-}_{\aleph\ 1})^{\flat_2}\right)^{\zeta_1}\right) \cdot \left(1 - \left(1 - (L_{\Im_i} I^{+}_{\aleph\ 2})^{\flat_2}\right)^{\zeta_2}\right)\end{array}}, \\ \sqrt[\flat_2]{\begin{array}{c}\left(1 - \left(1 - (L_{\Im_i} I^{-}_{\aleph\ 1})^{\flat_2}\right)^{\zeta_1}\right) + \left(1 - \left(1 - (L_{\Im_i} I^{+}_{\aleph\ 2})^{\flat_2}\right)^{\zeta_2}\right) \\ -\left(1 - \left(1 - (L_{\Im_i} I^{-}_{\aleph\ 1})^{\flat_2}\right)^{\zeta_1}\right) \cdot \left(1 - \left(1 - (L_{\Im_i} I^{+}_{\aleph\ 2})^{\flat_2}\right)^{\zeta_2}\right)\end{array}} \end{array}\right], \\ \left[(L_{\Im_i} F^{-}_{\aleph\ 1})^{\flat_3\zeta_1} \cdot (L_{\Im_i} F^{-}_{\aleph\ 2})^{\flat_3\zeta_2}, (L_{\Im_i}(F^{+}_{\aleph\ 1}))^{\flat_3\zeta_1} \cdot (L_{\Im_i}(F^{+}_{\aleph\ 2}))^{\flat_3\zeta_2}\right] \end{array}\right]$$

$$= \left[\begin{array}{c} \left[\begin{array}{c} \sqrt[\flat_1]{1 - \left(1 - (L_{\Im_i} T^{-}_{\aleph\ 1})^{\flat_1}\right)^{\zeta_1} \cdot \left(1 - (L_{\Im_i} T^{-}_{\aleph\ 2})^{\flat_1}\right)^{\zeta_2}}, \\ \sqrt[\flat_1]{1 - \left(1 - (L_{\Im_i}(T^{+}_{\aleph\ 1}))^{\flat_1}\right)^{\zeta_1} \cdot \left(1 - (L_{\Im_i}(T^{+}_{\aleph\ 2}))^{\flat_1}\right)^{\zeta_2}} \end{array}\right], \\ \left[\begin{array}{c} \sqrt[\flat_2]{1 - \left(1 - (L_{\Im_i} I^{-}_{\aleph\ 1})^{\flat_2}\right)^{\zeta_1} \cdot \left(1 - (L_{\Im_i} I^{+}_{\aleph\ 2})^{\flat_2}\right)^{\zeta_2}}, \\ \sqrt[\flat_2]{1 - \left(1 - (L_{\Im_i} I^{-}_{\aleph\ 1})^{\flat_2}\right)^{\zeta_1} \cdot \left(1 - (L_{\Im_i} I^{+}_{\aleph\ 2})^{\flat_2}\right)^{\zeta_2}} \end{array}\right], \\ \left[(L_{\Im_i} F^{-}_{\aleph\ 1})^{\flat_3\zeta_1} \cdot (L_{\Im_i} F^{-}_{\aleph\ 2})^{\flat_3\zeta_2}, (L_{\Im_i}(F^{+}_{\aleph\ 1}))^{\flat_3\zeta_1} \cdot (L_{\Im_i}(F^{+}_{\aleph\ 2}))^{\flat_3\zeta_2}\right] \end{array}\right].$$

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Thus, new type NIVWA $(\aleph_1, \aleph_2)$

$$= \left[\begin{array}{c} \left[\sqrt[\flat_1]{1-\circledast_{i=1}^{n}\Big(1-(L_{\Im_i}T^-_{\aleph\ i})^{\flat_1}\Big)^{\zeta_i}},\ \sqrt[\flat_1]{1-\circledast_{i=1}^{n}\Big(1-(L_{\Im_i}(T^+_{\aleph\ i}))^{\flat_1}\Big)^{\zeta_i}}\right], \\ \left[\sqrt[\flat_2]{1-\circledast_{i=1}^{n}\Big(1-(L_{\Im_i}I^-_{\aleph\ i})^{\flat_2}\Big)^{\zeta_i}},\ \sqrt[\flat_2]{1-\circledast_{i=1}^{n}\Big(1-(L_{\Im_i}I^+_{\aleph\ i})^{\flat_2}\Big)^{\zeta_i}}\right], \\ \left[\circledast_{i=1}^{n}(L_{\Im_i}F^-_{\aleph\ i})^{\flat_3\zeta_i}, \circledast_{i=1}^{n}(L_{\Im_i}(F^+_{\aleph\ i}))^{\flat_3\zeta_i}\right] \end{array}\right].$$

It is valid for $n = l$ and $l \geq 3$.
Hence, new type NIVWA $(\aleph_1, \aleph_2, ..., \aleph_l)$

$$= \left[\begin{array}{c} \left[\sqrt[\flat_1]{1-\circledast_{i=1}^{l}\Big(1-(L_{\Im_i}T^-_{\aleph\ i})^{\flat_1}\Big)^{\zeta_i}},\ \sqrt[\flat_1]{1-\circledast_{i=1}^{l}\Big(1-(L_{\Im_i}(T^+_{\aleph\ i}))^{\flat_1}\Big)^{\zeta_i}}\right], \\ \left[\sqrt[\flat_2]{1-\circledast_{i=1}^{l}\Big(1-(L_{\Im_i}I^-_{\aleph\ i})^{\flat_2}\Big)^{\zeta_i}},\ \sqrt[\flat_2]{1-\circledast_{i=1}^{l}\Big(1-(L_{\Im_i}I^+_{\aleph\ i})^{\flat_2}\Big)^{\zeta_i}}\right], \\ \left[\circledast_{i=1}^{l}(L_{\Im_i}F^-_{\aleph\ i})^{\flat_3\zeta_i}, \circledast_{i=1}^{l}(L_{\Im_i}(F^+_{\aleph\ i}))^{\flat_3\zeta_i}\right] \end{array}\right].$$

If $n = l + 1$ and we apply, new type NIVWA $(\aleph_1, \aleph_2, ..., \aleph_l, \aleph_{l+1})$

$$= \left[\begin{array}{c} \left[\begin{array}{c} \sqrt[\flat_1]{\begin{array}{c}\bigwedge_{i=1}^{l}\Big(1-\Big(1-(L_{\Im_i}T^-_{\aleph\ i})^{\flat_1}\Big)^{\zeta_i}\Big)+\Big(1-\Big(1-(L_{\Im_i}T^-_{\aleph\ l+1})^{\flat_1}\Big)^{\zeta_{l+1}}\Big) \\ -\circledast_{i=1}^{l}\Big(1-\Big(1-(L_{\Im_i}T^-_{\aleph\ i})^{\flat_1}\Big)^{\zeta_i}\Big)\cdot\Big(1-\Big(1-(L_{\Im_i}T^-_{\aleph\ l+1})^{\flat_1}\Big)^{\zeta_{l+1}}\Big)\end{array}}, \\ \sqrt[\flat_1]{\begin{array}{c}\bigwedge_{i=1}^{l}\Big(1-\Big(1-(L_{\Im_i}(T^+_{\aleph\ i}))^{\flat_1}\Big)^{\zeta_i}\Big)+\Big(1-\Big(1-(L_{\Im_i}(T^+_{\aleph\ l+1}))^{\flat_1}\Big)^{\zeta_{l+1}}\Big) \\ -\circledast_{i=1}^{l}\Big(1-\Big(1-(L_{\Im_i}(T^+_{\aleph\ i}))^{\flat_1}\Big)^{\zeta_i}\Big)\cdot\Big(1-\Big(1-(L_{\Im_i}(T^+_{\aleph\ l+1}))^{\flat_1}\Big)^{\zeta_{l+1}}\Big)\end{array}} \end{array}\right], \\ \left[\begin{array}{c} \sqrt[\flat_2]{\begin{array}{c}\bigwedge_{i=1}^{l}\Big(1-\Big(1-(L_{\Im_i}I^-_{\aleph\ i})^{\flat_2}\Big)^{\zeta_i}\Big)+\Big(1-\Big(1-(L_{\Im_i}I^+_{\aleph\ l+1})^{\flat_2}\Big)^{\zeta_{l+1}}\Big) \\ -\circledast_{i=1}^{l}\Big(1-\Big(1-(L_{\Im_i}I^-_{\aleph\ i})^{\flat_2}\Big)^{\zeta_i}\Big)\cdot\Big(1-\Big(1-(L_{\Im_i}I^+_{\aleph\ l+1})^{\flat_2}\Big)^{\zeta_{l+1}}\Big)\end{array}}, \\ \sqrt[\flat_2]{\begin{array}{c}\bigwedge_{i=1}^{l}\Big(1-\Big(1-(L_{\Im_i}I^-_{\aleph\ i})^{\flat_2}\Big)^{\zeta_i}\Big)+\Big(1-\Big(1-(L_{\Im_i}I^+_{\aleph\ l+1})^{\flat_2}\Big)^{\zeta_{l+1}}\Big) \\ -\circledast_{i=1}^{l}\Big(1-\Big(1-(L_{\Im_i}I^-_{\aleph\ i})^{\flat_2}\Big)^{\zeta_i}\Big)\cdot\Big(1-\Big(1-(L_{\Im_i}I^+_{\aleph\ l+1})^{\flat_2}\Big)^{\zeta_{l+1}}\Big)\end{array}} \end{array}\right], \\ \left[\circledast_{i=1}^{l}(L_{\Im_i}F^-_{\aleph\ i})^{\flat_3\zeta_i}\cdot(L_{\Im_i}F^-_{\aleph\ l+1})^{\flat_3\zeta_{l+1}}, \circledast_{i=1}^{l}(L_{\Im_i}(F^+_{\aleph\ i}))^{\flat_3\zeta_i}\cdot(L_{\Im_i}(F^+_{\aleph\ l+1}))^{\flat_3\zeta_{l+1}}\right] \end{array}\right]$$

$$= \left[\begin{array}{c} \left[\sqrt[\flat_1]{1-\circledast_{i=1}^{l+1}\Big(1-(L_{\Im_i}T^-_{\aleph\ i})^{\flat_1}\Big)^{\zeta_i}},\ \sqrt[\flat_1]{1-\circledast_{i=1}^{l+1}\Big(1-(L_{\Im_i}(T^+_{\aleph\ i}))^{\flat_1}\Big)^{\zeta_i}}\right], \\ \left[\sqrt[\flat_2]{1-\circledast_{i=1}^{l+1}\Big(1-(L_{\Im_i}I^-_{\aleph\ i})^{\flat_2}\Big)^{\zeta_i}},\ \sqrt[\flat_2]{1-\circledast_{i=1}^{l+1}\Big(1-(L_{\Im_i}I^+_{\aleph\ i})^{\flat_2}\Big)^{\zeta_i}}\right], \\ \left[\circledast_{i=1}^{l+1}(L_{\Im_i}F^-_{\aleph\ i})^{\flat_3\zeta_i}, \circledast_{i=1}^{l+1}(L_{\Im_i}(F^+_{\aleph\ i}))^{\flat_3\zeta_i}\right] \end{array}\right].$$

Theorem 4.3. *(idempotency property) If all* $\aleph_i = \Big\langle [LT^-_{\aleph\ i}, LT^+_{\aleph\ i}], [LI^-_{\aleph\ i}, LI^+_{\aleph\ i}][LF^-_{\aleph\ i}, LF^+_{\aleph\ i}]\Big\rangle (i = 1, 2, ..., n)$ *are equal and* $\aleph_i = \aleph$*. Then new type NIVWA* $(\aleph_1, \aleph_2, ..., \aleph_n) = \aleph$*.*

Proof. Given that $[LT^-_{\aleph\ i}, LT^+_{\aleph\ i}] = [LT^-_{\aleph}, L(T^+_{\aleph})]$, $[LI^-_{\aleph\ i}, LI^+_{\aleph\ i}] = [LI^-_{\aleph}, LI^+_{\aleph}]$ and $[LF^-_{\aleph\ i}, LF^+_{\aleph\ i}] =$

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$[LF^{-}_{\aleph}, L(F^{+}_{\aleph})]$, for $i = 1, 2, ..., n$ and $\bigwedge_{i=1}^{n} \zeta_i = 1$. Now, new type NIVWA $(\aleph_1, \aleph_2, ..., \aleph_n)$

$$\begin{aligned}
&= \left[\begin{array}{c}\left[\sqrt[\flat_1]{1-\circledast_{i=1}^{n}\Big(1-(L_{\Im_i}T^{-}_{\aleph\ i})^{\flat_1}\Big)^{\zeta_i}},\ \sqrt[\flat_1]{1-\circledast_{i=1}^{n}\Big(1-(L_{\Im_i}(T^{+}_{\aleph\ i}))^{\flat_1}\Big)^{\zeta_i}}\right],\\ \left[\sqrt[\flat_2]{1-\circledast_{i=1}^{n}\Big(1-(L_{\Im_i}I^{-}_{\aleph\ i})^{\flat_2}\Big)^{\zeta_i}},\ \sqrt[\flat_2]{1-\circledast_{i=1}^{n}\Big(1-(L_{\Im_i}I^{+}_{\aleph\ i})^{\flat_2}\Big)^{\zeta_i}}\right],\\ \left[\circledast_{i=1}^{n}(L_{\Im_i}F^{-}_{\aleph\ i})^{\flat_3\zeta_i},\circledast_{i=1}^{n}(L_{\Im_i}(F^{+}_{\aleph\ i}))^{\flat_3\zeta_i}\right]\end{array}\right]\\
&= \left[\begin{array}{c}\left[\sqrt[\flat_1]{1-\Big(1-(L_{\Im_i}T^{-}_{\aleph})^{\flat_1}\Big)^{\bigwedge_{i=1}^{n}\zeta_i}},\ \sqrt[\flat_1]{1\Big(1-(L_{\Im_i}(T^{+}_{\aleph}))^{\flat_1}\Big)^{\bigwedge_{i=1}^{n}\zeta_i}}\right],\\ \left[\sqrt[\flat_2]{1-\Big(1-(L_{\Im_i}I^{-}_{\aleph})^{\flat_2}\Big)^{\bigwedge_{i=1}^{n}\zeta_i}},\ \sqrt[\flat_2]{1\Big(1-(L_{\Im_i}I^{+}_{\aleph})^{\flat_2}\Big)^{\bigwedge_{i=1}^{n}\zeta_i}}\right],\\ \left[(L_{\Im_i}F^{-}_{\aleph})^{\bigwedge_{i=1}^{n}\flat_3\zeta_i},(L_{\Im_i}(F^{+}_{\aleph}))^{\bigwedge_{i=1}^{n}\flat_3\zeta_i}\right]\end{array}\right]\\
&= \left[\begin{array}{c}\left[\sqrt[\flat_1]{1-\Big(1-(L_{\Im_i}T^{-}_{\aleph})^{\flat_1}\Big)},\ \sqrt[\flat_1]{1-\Big(1-(L_{\Im_i}(T^{+}_{\aleph}))^{\flat_1}\Big)}\right],\\ \left[\sqrt[\flat_2]{1-\Big(1-(L_{\Im_i}I^{-}_{\aleph})^{\flat_2}\Big)},\ \sqrt[\flat_2]{1-\Big(1-(L_{\Im_i}I^{+}_{\aleph})^{\flat_2}\Big)}\right],\\ \left[(L_{\Im_i}F^{-}_{\aleph})^{\flat_3},(L_{\Im_i}(F^{+}_{\aleph}))^{\flat_3}\right]\end{array}\right]\\
&= \aleph.
\end{aligned}$$

Theorem 4.4. *(boundedness property) Let* $\aleph_i = \Big\langle [LT^{-}_{\aleph\ ij}, L(T^{+}_{\aleph\ ij})], [LI^{-}_{\aleph\ ij}, LI^{+}_{\aleph\ ij}][LF^{-}_{\aleph\ ij}, L(F^{+}_{\aleph\ ij})]\Big\rangle (i = 1, 2, ..., n); (j = 1, 2, ..., i_j)$ *be the collection of new type NIVWA, where* $\underbrace{L_{\Im_i}T^{-}_{\aleph}} = \min L_{\Im_i}T^{-}_{\aleph\ ij}$, $\overbrace{L_{\Im_i}T^{-}_{\aleph}} = \max L_{\Im_i}T^{-}_{\aleph\ ij}$, $\underbrace{L_{\Im_i}(T^{+}_{\aleph})} = \min L_{\Im_i}(T^{+}_{\aleph\ ij})$, $\overbrace{L_{\Im_i}(T^{+}_{\aleph})} = \max L_{\Im_i}(T^{+}_{\aleph\ ij})$, $\underbrace{L_{\Im_i}I^{-}_{\aleph}} = \min L_{\Im_i}I^{-}_{\aleph\ ij}$, $\overbrace{L_{\Im_i}I^{+}_{\aleph}} = \max L_{\Im_i}I^{+}_{\aleph\ ij}$, $\underbrace{L_{\Im_i}I^{-}_{\aleph}} = \min L_{\Im_i}I^{-}_{\aleph\ ij}$, $\overbrace{L_{\Im_i}I^{+}_{\aleph}} = \max L_{\Im_i}I^{+}_{\aleph\ ij}$, $\underbrace{L_{\Im_i}F^{-}_{\aleph}} = \min L_{\Im_i}F^{-}_{\aleph\ ij}$, $\overbrace{L_{\Im_i}F^{-}_{\aleph}} = \max L_{\Im_i}F^{-}_{\aleph\ ij}$, $\underbrace{L_{\Im_i}(F^{+}_{\aleph})} = \min L_{\Im_i}(F^{+}_{\aleph\ ij})$, $\overbrace{L_{\Im_i}(F^{+}_{\aleph})} = \max L_{\Im_i}(F^{+}_{\aleph\ ij})$.

Then, $\Big\langle [\underbrace{L_{\Im_i}T^{-}_{\aleph}}, \underbrace{L_{\Im_i}(T^{+}_{\aleph})}], [\underbrace{L_{\Im_i}I^{-}_{\aleph}}, \underbrace{L_{\Im_i}I^{+}_{\aleph}}], [\overbrace{L_{\Im_i}F^{-}_{\aleph}}, \overbrace{L_{\Im_i}(F^{+}_{\aleph})}]\Big\rangle$

$$\begin{aligned}
&\preceq \ new\ type\ NIVWA(\aleph_1, \aleph_2, ..., \aleph_n)\\
&\preceq \ \Big\langle [\overbrace{L_{\Im_i}T^{-}_{\aleph}}, \overbrace{L_{\Im_i}(T^{+}_{\aleph})}], [\overbrace{L_{\Im_i}I^{-}_{\aleph}}, \overbrace{L_{\Im_i}I^{+}_{\aleph}}], [\underbrace{L_{\Im_i}F^{-}_{\aleph}}, \underbrace{L_{\Im_i}(F^{+}_{\aleph})}]\Big\rangle.
\end{aligned}$$

where $1 \preceq i \preceq n,\ j = 1, 2, ..., i_j$.

Proof. Since, $\underbrace{L_{\Im_i}T^{-}_{\aleph}} = \min L_{\Im_i}T^{-}_{\aleph\ ij}$, $\overbrace{L_{\Im_i}T^{-}_{\aleph}} = \max L_{\Im_i}T^{-}_{\aleph\ ij}$

$\underbrace{L_{\Im_i}(T^{+}_{\aleph})} = \min L_{\Im_i}(T^{+}_{\aleph\ ij})$, $\overbrace{L_{\Im_i}(T^{+}_{\aleph})} = \max L_{\Im_i}(T^{+}_{\aleph\ ij})$ and $\underbrace{L_{\Im_i}T^{-}_{\aleph}} \preceq L_{\Im_i}T^{-}_{\aleph\ ij} \preceq \overbrace{L_{\Im_i}T^{-}_{\aleph}}$ and $\underbrace{L_{\Im_i}(T^{+}_{\aleph})} \preceq L_{\Im_i}(T^{+}_{\aleph})_{ij} \preceq \overbrace{L_{\Im_i}(T^{+}_{\aleph})}$.

Now, $\underbrace{L_{\Im_i}T^{-}_{\aleph}} + \underbrace{L_{\Im_i}(T^{+}_{\aleph})}$

$$\begin{aligned}
&= \sqrt[\flat_1]{1-\circledast_{i=1}^{n}\Big(1-(\underbrace{L_{\Im_i}T^{-}_{\aleph}})^{\flat_1}\Big)^{\zeta_i}} + \sqrt[\flat_1]{1-\circledast_{i=1}^{n}\Big(1-(\underbrace{L_{\Im_i}(T^{+}_{\aleph})})^{\flat_1}\Big)^{\zeta_i}}\\
&\preceq \sqrt[\flat_1]{1-\circledast_{i=1}^{n}\Big(1-(L_{\Im_i}T^{-}_{\aleph\ ij})^{\flat_1}\Big)^{\zeta_i}} + \sqrt[\flat_1]{1-\circledast_{i=1}^{n}\Big(1-(L_{\Im_i}(T^{+}_{\aleph\ ij}))^{\flat_1}\Big)^{\zeta_i}}\\
&\preceq \sqrt[\flat_1]{1-\circledast_{i=1}^{n}\Big(1-(\overbrace{L_{\Im_i}T^{-}_{\aleph}})^{\flat_1}\Big)^{\zeta_i}} + \sqrt[\flat_1]{1-\circledast_{i=1}^{n}\Big(1-(\overbrace{L_{\Im_i}(T^{+}_{\aleph})})^{\flat_1}\Big)^{\zeta_i}}\\
&= \overbrace{L_{\Im_i}T^{-}_{\aleph}} + \overbrace{L_{\Im_i}(T^{+}_{\aleph})}.
\end{aligned}$$

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Since, $\underbrace{L_{\Im_i} I^-_{\aleph}} = \min L_{\Im_i} I^-_{\aleph\, ij}$, $\overbrace{L_{\Im_i} I^+_{\aleph}} = \max L_{\Im_i} I^+_{\aleph\, ij}$, $\underbrace{L_{\Im_i} I^-_{\aleph}} = \min L_{\Im_i} I^-_{\aleph\, ij}$, $\overbrace{L_{\Im_i} I^+_{\aleph}} = \max L_{\Im_i} I^+_{\aleph\, ij}$ and $\underbrace{L_{\Im_i} I^-_{\aleph}} \preceq L_{\Im_i} I^-_{\aleph\, ij} \preceq \overbrace{L_{\Im_i} I^-_{\aleph}}$ and $\underbrace{L_{\Im_i} I^+_{\aleph}} \preceq L_{\Im_i} I^+_{\aleph\, ij} \preceq \overbrace{L_{\Im_i} I^+_{\aleph}}$. Now,

$$\begin{aligned}
\underbrace{L_{\Im_i} I^-_{\aleph}} + \underbrace{L_{\Im_i} I^+_{\aleph}} &= \sqrt[\flat_2]{1 - \circledast_{i=1}^{n}\Big(1 - (\underbrace{L_{\Im_i} I^-_{\aleph}})^{\flat_2}\Big)^{\zeta_i}} + \sqrt[\flat_2]{1 - \circledast_{i=1}^{n}\Big(1 - (\underbrace{L_{\Im_i} I^+_{\aleph}})^{\flat_2}\Big)^{\zeta_i}} \\
&\preceq \sqrt[\flat_2]{1 - \circledast_{i=1}^{n}\Big(1 - (L_{\Im_i} I^-_{\aleph\, ij})^{\flat_2}\Big)^{\zeta_i}} + \sqrt[\flat_2]{1 - \circledast_{i=1}^{n}\Big(1 - (L_{\Im_i} I^+_{\aleph\, ij})^{\flat_2}\Big)^{\zeta_i}} \\
&\preceq \sqrt[\flat_2]{1 - \circledast_{i=1}^{n}\Big(1 - (\overbrace{L_{\Im_i} I^-_{\aleph}})^{\flat_2}\Big)^{\zeta_i}} + \sqrt[\flat_2]{1 - \circledast_{i=1}^{n}\Big(1 - (\overbrace{L_{\Im_i} I^+_{\aleph}})^{\flat_2}\Big)^{\zeta_i}} \\
&= \overbrace{L_{\Im_i} I^-_{\aleph}} + \overbrace{L_{\Im_i} I^+_{\aleph}}.
\end{aligned}$$

Since, $\underbrace{L_{\Im_i} F^-_{\aleph}} = \min L_{\Im_i} F^-_{\aleph\, ij}$, $\overbrace{L_{\Im_i} F^-_{\aleph}} = \max L_{\Im_i} F^-_{\aleph\, ij}$ $\underbrace{L_{\Im_i}(F^+_{\aleph})} = \min L_{\Im_i}(F^+_{\aleph\, ij})$, $\overbrace{L_{\Im_i}(F^+_{\aleph})} = \max L_{\Im_i}(F^+_{\aleph\, ij})$ and $\underbrace{L_{\Im_i} F^-_{\aleph}} \preceq L_{\Im_i} F^-_{\aleph\, ij} \preceq \overbrace{L_{\Im_i} F^-_{\aleph}}$ and $\underbrace{L_{\Im_i}(F^+_{\aleph})} \preceq L_{\Im_i}(F^+_{\aleph\, ij}) \preceq \overbrace{L_{\Im_i}(F^+_{\aleph})}$. Now,

$$\begin{aligned}
\underbrace{L_{\Im_i} F^-_{\aleph}} + \underbrace{L_{\Im_i}(F^+_{\aleph})} &= \circledast_{i=1}^{n}(\underbrace{L_{\Im_i} F^-_{\aleph}})^{\flat_3 \zeta_i} + \circledast_{i=1}^{n}(\underbrace{L_{\Im_i}(F^+_{\aleph})})^{\flat_3 \zeta_i} \\
&\preceq \circledast_{i=1}^{n}(L_{\Im_i} F^-_{\aleph\, ij})^{\flat_3 \zeta_i} + \circledast_{i=1}^{n}(L_{\Im_i}(F^+_{\aleph\, ij}))^{\flat_3 \zeta_i} \\
&\preceq \circledast_{i=1}^{n}(\overbrace{L_{\Im_i} F^-_{\aleph}})^{\flat_3 \zeta_i} + \circledast_{i=1}^{n}(\overbrace{L_{\Im_i}(F^+_{\aleph})})^{\flat_3 \zeta_i} \\
&= \overbrace{L_{\Im_i} F^-_{\aleph}} + \overbrace{L_{\Im_i}(F^+_{\aleph})}.
\end{aligned}$$

Therefore,

$$\left[\frac{\left(\sqrt[\flat_1]{1 - \circledast_{i=1}^{n}\Big(1 - (\underbrace{L_{\Im_i} T^-_{\aleph}})^{\flat_1}\Big)^{\zeta_i}}\right)^2 + \left(\sqrt[\flat_1]{1 - \circledast_{i=1}^{n}\Big(1 - (\underbrace{L_{\Im_i}(T^+_{\aleph})})^{\flat_1}\Big)^{\zeta_i}}\right)^2}{2} + 1 - \frac{\left(\sqrt[\flat_2]{1 - \circledast_{i=1}^{n}\Big(1 - (\underbrace{L_{\Im_i} I^-_{\aleph}})^{\flat_2}\Big)^{\zeta_i}}\right)^2 + \left(\sqrt[\flat_2]{1 - \circledast_{i=1}^{n}\Big(1 - (\underbrace{L_{\Im_i} I^+_{\aleph}})^{\flat_2}\Big)^{\zeta_i}}\right)^2}{2} + 1 - \frac{\left(\circledast_{i=1}^{n}(\overbrace{L_{\Im_i} F^-_{\aleph}})^{\flat_3 \zeta_i}\right)^2 + \left(\circledast_{i=1}^{n}(\overbrace{L_{\Im_i}(F^+_{\aleph})})^{\flat_3 \zeta_i}\right)^2}{2}\right]$$

$$= \left[\frac{\left(\sqrt[\flat_1]{1 - \circledast_{i=1}^{n}\Big(1 - (L_{\Im_i} T^-_{\aleph\, ij})^{\flat_1}\Big)^{\zeta_i}}\right)^2 + \left(\sqrt[\flat_1]{1 - \circledast_{i=1}^{n}\Big(1 - (L_{\Im_i}(T^+_{\aleph\, ij}))^{\flat_1}\Big)^{\zeta_i}}\right)^2}{2} + 1 - \frac{\left(\sqrt[\flat_2]{1 - \circledast_{i=1}^{n}\Big(1 - (L_{\Im_i} I^-_{\aleph\, ij})^{\flat_2}\Big)^{\zeta_i}}\right)^2 + \left(\sqrt[\flat_2]{1 - \circledast_{i=1}^{n}\Big(1 - (L_{\Im_i} I^+_{\aleph\, ij})^{\flat_2}\Big)^{\zeta_i}}\right)^2}{2} + 1 - \frac{\left(\circledast_{i=1}^{n}(L_{\Im_i} F^-_{\aleph\, ij})^{\flat_3 \zeta_i}\right)^2 + \left(\circledast_{i=1}^{n}(L_{\Im_i}(F^+_{\aleph\, ij}))^{\flat_3 \zeta_i}\right)^2}{2}\right]$$

$$= \left[\frac{\left(\sqrt[\flat_1]{1 - \circledast_{i=1}^{n}\Big(1 - (\overbrace{L_{\Im_i} T^-_{\aleph}})^{\flat_1}\Big)^{\zeta_i}}\right)^2 + \left(\sqrt[\flat_1]{1 - \circledast_{i=1}^{n}\Big(1 - (\overbrace{L_{\Im_i}(T^+_{\aleph})})^{\flat_1}\Big)^{\zeta_i}}\right)^2}{2} + 1 - \frac{\left(\sqrt[\flat_2]{1 - \circledast_{i=1}^{n}\Big(1 - (\overbrace{L_{\Im_i} I^-_{\aleph}})^{\flat_2}\Big)^{\zeta_i}}\right)^2 + \left(\sqrt[\flat_2]{1 - \circledast_{i=1}^{n}\Big(1 - (\overbrace{L_{\Im_i} I^+_{\aleph}})^{\flat_2}\Big)^{\zeta_i}}\right)^2}{2} + 1 - \frac{\left(\circledast_{i=1}^{n}(\underbrace{L_{\Im_i} F^-_{\aleph}})^{\flat_3 \zeta_i}\right)^2 + \left(\circledast_{i=1}^{n}(\underbrace{L_{\Im_i}(F^+_{\aleph})})^{\flat_3 \zeta_i}\right)^2}{2}\right].$$

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Hence, $\left\langle [\underbrace{LT^{-}_{\aleph}}, \underbrace{L(T^{+}_{\aleph})}], [\underbrace{LI^{-}_{\aleph}}, \underbrace{LI^{+}_{\aleph}}], [\overbrace{LF^{-}_{\aleph}}, \overbrace{L(F^{+}_{\aleph})}] \right\rangle$

$$\begin{aligned} &\preceq \quad new\ type\ NIVWA(\aleph_1, \aleph_2, ..., \aleph_n) \\ &\preceq \quad \left\langle [\overbrace{LT^{-}_{\aleph}}, \overbrace{L(T^{+}_{\aleph})}], [\overbrace{LI^{-}_{\aleph}}, \overbrace{LI^{+}_{\aleph}}], [\underbrace{LF^{-}_{\aleph}}, \underbrace{L(F^{+}_{\aleph})}] \right\rangle. \end{aligned}$$

Theorem 4.5. *(monotonicity property) Let* $\aleph_i = \left\langle [LT^{-}_{\aleph\ t_{ij}}, L(T^{+}_{\aleph\ t_{ij}})], [LI^{-}_{\aleph\ t_{ij}}, LI^{+}_{\aleph\ t_{ij}}], [LF^{-}_{\aleph\ t_{ij}}, L(F^{+}_{\aleph\ t_{ij}})] \right\rangle$ *and* $\zeta_i = \left\langle [LT^{-}_{\aleph\ h_{ij}}, L(T^{+}_{\aleph\ h_{ij}})], [LI^{-}_{\aleph\ h_{ij}}, LI^{+}_{\aleph\ h_{ij}}], [LF^{-}_{\aleph\ h_{ij}}, L(F^{+}_{\aleph\ h_{ij}})] \right\rangle (i = 1, 2, ..., n); (j = 1, 2, ..., i_j)$ *be the families of new type NIVWAs. For any i, if there is* $\left(L_{\Im_i} T^{-}_{\aleph\ t_{ij}}\right)^2 + \left(L_{\Im_i}(T^{+}_{\aleph\ t_{ij}})\right)^2 \preceq \left(L_{\Im_i} T^{-}_{\aleph\ h_{ij}}\right)^2 + \left(L_{\Im_i}(T^{+}_{\aleph\ h_{ij}})\right)^2$ *and* $\left(L_{\Im_i} I^{-}_{\aleph\ t_{ij}}\right)^2 + \left(L_{\Im_i} I^{+}_{\aleph\ t_{ij}}\right)^2 \geq \left(L_{\Im_i} I^{-}_{\aleph\ h_{ij}}\right)^2 + \left(L_{\Im_i} I^{+}_{\aleph\ h_{ij}}\right)^2$ *and* $\left(L_{\Im_i} F^{-}_{\aleph\ t_{ij}}\right)^2 + \left(L_{\Im_i}(F^{+}_{\aleph})_{t_{ij}}\right)^2 \geq \left(L_{\Im_i} F^{-}_{\aleph\ h_{ij}}\right)^2 + \left(L_{\Im_i}(F^{+}_{\aleph\ h_{ij})}\right)^2$ *or* $\aleph_i \preceq \mathscr{W}_i$. *Then new type NIVWA* $(\aleph_1, \aleph_2, ..., \aleph_n) \preceq$ *new type NIVWA* $(\mathscr{W}_1, \mathscr{W}_2, ..., \mathscr{W}_n)$.

Proof. For any i, $\left(L_{\Im_i} T^{-}_{\aleph\ t_{ij}}\right)^{\flat_1} + \left(L_{\Im_i}(T^{+}_{\aleph\ t_{ij}})\right)^{\flat_1} \preceq \left(L_{\Im_i} T^{-}_{\aleph\ h_{ij}}\right)^{\flat_1} + \left(L_{\Im_i}(T^{+}_{\aleph\ h_{ij}})\right)^{\flat_1}$.
Therefore, $1 - \left(L_{\Im_i} T^{-}_{\aleph\ t_i}\right)^{\flat_1} + 1 - \left(L_{\Im_i}(T^{+}_{\aleph\ t_i})\right)^{\flat_1} \geq 1 - \left(L_{\Im_i} T^{-}_{\aleph\ h_i}\right)^{\flat_1} + 1 - \left(L_{\Im_i}(T^{+}_{\aleph\ h_i})\right)^{\flat_1}$.
Hence, $\circledast_{i=1}^{n} \left(1 - \left(L_{\Im_i} T^{-}_{\aleph\ t_i}\right)^{\flat_1}\right)^{\zeta_i} + \circledast_{i=1}^{n} \left(1 - \left(L_{\Im_i}(T^{+}_{\aleph\ t_i})\right)^{\flat_1}\right)^{\zeta_i} \geq$
$\circledast_{i=1}^{n} \left(1 - \left(L_{\Im_i} T^{-}_{\aleph\ h_i}\right)^{\flat_1}\right)^{\zeta_i} + \circledast_{i=1}^{n} \left(1 - \left(L_{\Im_i}(T^{+}_{\aleph\ h_i})\right)^{\flat_1}\right)^{\zeta_i}$
and $\sqrt[\flat_1]{1 - \circledast_{i=1}^{n} \left(1 - \left(L_{\Im_i} T^{-}_{\aleph\ t_i}\right)^{\flat_1}\right)^{\zeta_i}} + \sqrt[\flat_1]{1 - \circledast_{i=1}^{n} \left(1 - \left(L_{\Im_i}(T^{+}_{\aleph\ t_i)}\right)^{\flat_1}\right)^{\zeta_i}}$
$\preceq \sqrt[\flat_1]{1 - \circledast_{i=1}^{n} \left(1 - \left(L_{\Im_i} T^{-}_{\aleph\ h_i}\right)^{\flat_1}\right)^{\zeta_i}} + \sqrt[\flat_1]{1 - \circledast_{i=1}^{n} \left(1 - \left(L_{\Im_i}(T^{+}_{\aleph\ h_i)}\right)^{\flat_1}\right)^{\zeta_i}}$.

For any i, $\left(L_{\Im_i} I^{-}_{\aleph\ t_{ij}}\right)^{\flat_2} + \left(L_{\Im_i} I^{+}_{\aleph\ t_{ij}}\right)^{\flat_2} \geq \left(L_{\Im_i} I^{-}_{\aleph\ h_{ij}}\right)^{\flat_2} + \left(L_{\Im_i} I^{+}_{\aleph\ h_{ij}}\right)^{\flat_2}$.
Therefore, $1 - \left(L_{\Im_i} I^{-}_{\aleph\ t_i}\right)^{\flat_2} + 1 - \left(L_{\Im_i} I^{+}_{\aleph\ t_i}\right)^{\flat_2} \preceq 1 - \left(L_{\Im_i} I^{-}_{\aleph\ h_i}\right)^{\flat_2} + 1 - \left(L_{\Im_i} I^{+}_{\aleph\ h_i}\right)^{\flat_2}$.
Hence, $\circledast_{i=1}^{n} \left(1 - \left(L_{\Im_i} I^{-}_{\aleph\ t_i}\right)^{\flat_2}\right)^{\zeta_i} + \circledast_{i=1}^{n} \left(1 - \left(L_{\Im_i} I^{+}_{\aleph\ t_i}\right)^{\flat_2}\right)^{\zeta_i} \preceq$
$\circledast_{i=1}^{n} \left(1 - \left(L_{\Im_i} I^{-}_{\aleph\ h_i}\right)^{\flat_2}\right)^{\zeta_i} + \circledast_{i=1}^{n} \left(1 - \left(L_{\Im_i} I^{+}_{\aleph\ h_i}\right)^{\flat_2}\right)^{\zeta_i}$
implies that $\sqrt[\flat_2]{1 - \circledast_{i=1}^{n} \left(1 - \left(L_{\Im_i} I^{-}_{\aleph\ t_i}\right)^{\flat_2}\right)^{\zeta_i}} + \sqrt[\flat_2]{1 - \circledast_{i=1}^{n} \left(1 - \left(L_{\Im_i} I^{+}_{\aleph\ t_i}\right)^{\flat_2}\right)^{\zeta_i}}$

$$\geq \sqrt[\flat_2]{1 - \circledast_{i=1}^{n} \left(1 - \left(L_{\Im_i} I^{-}_{\aleph\ h_i}\right)^{\flat_2}\right)^{\zeta_i}} + \sqrt[\flat_2]{1 - \circledast_{i=1}^{n} \left(1 - \left(L_{\Im_i} I^{+}_{\aleph\ h_i}\right)^{\flat_2}\right)^{\zeta_i}}.$$

Hence, $1 - \sqrt[\flat_2]{1 - \circledast_{i=1}^{n} \left(1 - \left(L_{\Im_i} I^{-}_{\aleph\ t_i}\right)^{\flat_2}\right)^{\zeta_i}} + \sqrt[\flat_2]{1 - \circledast_{i=1}^{n} \left(1 - \left(L_{\Im_i} I^{+}_{\aleph\ t_i}\right)^{\flat_2}\right)^{\zeta_i}}$
$\preceq 1 - \sqrt[\flat_2]{1 - \circledast_{i=1}^{n} \left(1 - \left(L_{\Im_i} I^{-}_{\aleph\ h_i}\right)^{\flat_2}\right)^{\zeta_i}} + \sqrt[\flat_2]{1 - \circledast_{i=1}^{n} \left(1 - \left(L_{\Im_i} I^{+}_{\aleph\ h_i}\right)^{\flat_2}\right)^{\zeta_i}}$.

For any i, $\left(L_{\Im_i} F^{-}_{\aleph\ t_{ij}}\right)^{\flat_3} + \left(L_{\Im_i}(F^{+}_{\aleph\ t_{ij})}\right)^{\flat_3} \geq \left(L_{\Im_i} F^{-}_{\aleph\ h_{ij}}\right)^{\flat_3} + \left(L_{\Im_i}(F^{+}_{\aleph\ h_{ij})}\right)^{\flat_3}$.
Therefore, $1 - \frac{\left(\circledast_{i=1}^{n} L_{\Im_i} F^{-}_{\aleph\ t_{ij}}\right)^{\flat_3} + \left(\circledast_{i=1}^{n} L_{\Im_i}(F^{+}_{\aleph\ t_{ij})}\right)^{\flat_3}}{2} \preceq 1 - \frac{\left(\circledast_{i=1}^{n} L_{\Im_i} F^{-}_{\aleph\ h_{ij}}\right)^{\flat_3} + \left(\circledast_{i=1}^{n} L_{\Im_i}(F^{+}_{\aleph\ h_{ij})}\right)^{\flat_3}}{2}$.

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$$= \left[\frac{\left(\sqrt[\flat_2]{1 - \circledast_{i=1}^{n}\left(1-(L_{\Im_i}T^{-}_{\aleph\ ti})^{\flat_2}\right)^{\zeta_i}}\right)^2 + \left(\sqrt[\flat_2]{1 - \circledast_{i=1}^{n}\left(1-(L_{\Im_i}(T^{+}_{\aleph\ ti}))^{\flat_2}\right)^{\zeta_i}}\right)^2}{2} \atop +1 - \frac{\left(\sqrt[\flat_2]{1 - \circledast_{i=1}^{n}\left(1-(L_{\Im_i}I^{-}_{\aleph\ ti})^{\flat_2}\right)^{\zeta_i}}\right)^2 + \left(\sqrt[\flat_2]{1 - \circledast_{i=1}^{n}\left(1-(L_{\Im_i}I^{+}_{\aleph\ ti})^{\flat_2}\right)^{\zeta_i}}\right)^2}{2 \atop +1 - \frac{\left(\circledast_{i=1}^{n}(L_{\Im_i}F^{-}_{\aleph\ tij})\right)^2 + \left(\circledast_{i=1}^{n}(L_{\Im_i}(F^{+}_{\aleph\ tij}))\right)^2}{2}} \right]$$

$$= \left[\frac{\left(\sqrt[\flat_2]{1 - \circledast_{i=1}^{n}\left(1-(L_{\Im_i}T^{-}_{\aleph\ hi})^{\flat_2}\right)^{\zeta_i}}\right)^2 + \left(\sqrt[\flat_2]{1 - \circledast_{i=1}^{n}\left(1-(L_{\Im_i}(T^{+}_{\aleph\ hi}))^{\flat_2}\right)^{\zeta_i}}\right)^2}{2} \atop +1 - \frac{\left(\sqrt[\flat_2]{1 - \circledast_{i=1}^{n}\left(1-(L_{\Im_i}I^{-}_{\aleph\ hi})^{\flat_2}\right)^{\zeta_i}}\right)^2 + \left(\sqrt[\flat_2]{1 - \circledast_{i=1}^{n}\left(1-(L_{\Im_i}I^{+}_{\aleph\ hi})^{\flat_2}\right)^{\zeta_i}}\right)^2}{2 \atop +1 - \frac{\left(\circledast_{i=1}^{n}(L_{\Im_i}F^{-}_{\aleph\ hij})\right)^2 + \left(\circledast_{i=1}^{n}(L_{\Im_i}(F^{+}_{\aleph\ hij}))\right)^2}{2}} \right].$$

Hence, new type NIVWA $(\aleph_1, \aleph_2, ..., \aleph_n) \preceq$ new type NIVWA $(\mathscr{W}_1, \mathscr{W}_2, ..., W_n)$.

4.2 new type NIV weighted geometric(new type NIVWG) operator

Definition 4.6. Let $\aleph_i = \left\langle [LT^{-}_{\aleph\ i}, LT^{+}_{\aleph\ i}], [LI^{-+}_{\aleph\ i}, LI^{-+}_{\aleph\ i}], [LF^{-}_{\aleph\ i},\ LF^{+}_{\aleph\ i}] \right\rangle$ be the family of new type NIVNs. Then new type NIVWG operator is new type NIVWG $(\aleph_1, \aleph_2, ..., \aleph_n) = \circledast_{i=1}^{n} \aleph_i^{\zeta_i}$ $(i = 1, 2, ..., n)$.

Theorem 4.7. *Let $\aleph_i = \left\langle [LT^{-}_{\aleph\ i}, LT^{+}_{\aleph\ i}], [LF^{-}_{\aleph\ i}, LF^{+}_{\aleph\ i}] \right\rangle$ be the family of new type NIVNs. Then new type NIVWG $(\aleph_1, \aleph_2, ..., \aleph_n)$*

$$= \left[\begin{array}{c} \left[\circledast_{i=1}^{n}(L_{\Im_i}T^{-}_{\aleph\ i})^{\flat_1\zeta_i}, \circledast_{i=1}^{n}(L_{\Im_i}(T^{+}_{\aleph\ i}))^{\flat_1\zeta_i}\right], \\ \left[\sqrt[\flat_2]{1 - \circledast_{i=1}^{n}\left(1-(L_{\Im_i}I^{-}_{\aleph\ i})^{\flat_2}\right)^{\zeta_i}}, \sqrt[\flat_2]{1 - \circledast_{i=1}^{n}\left(1-(L_{\Im_i}I^{+}_{\aleph\ i})^{\flat_2}\right)^{\zeta_i}}\right], \\ \left[\sqrt[\flat_3]{1 - \circledast_{i=1}^{n}\left(1-(L_{\Im_i}F^{-}_{\aleph\ i})^{\flat_3}\right)^{\zeta_i}}, \sqrt[\flat_3]{1 - \circledast_{i=1}^{n}\left(1-(L_{\Im_i}(F^{+}_{\aleph\ i}))^{\flat_3}\right)^{\zeta_i}}\right] \end{array} \right].$$

Proof. This proof based on Theorem 4.2.

Theorem 4.8. *If all $\aleph_i = \left\langle [LT^{-}_{\aleph\ i}, LT^{+}_{\aleph\ i}], [LI^{-}_{\aleph\ i}, LI^{+}_{\aleph\ i}][LF^{-}_{\aleph\ i},\ LF^{+}_{\aleph\ i}] \right\rangle$ are equal and $\aleph_i = \aleph$, for $i = 1, 2, ..., n$. Then new type NIVWG $(\aleph_1, \aleph_2, ..., \aleph_n) = \aleph$.*

Proof. This proof based on Theorem 4.3.

Corollary 4.9. *The new type NIVWG operator is used to satisfy the boundedness and monotonicity properties.*

Proof. This proof based on Theorem 4.4 and Theorem 4.5.

4.3 Generalized new type NIVWA (new type GNIVWA) operator

Definition 4.10. Let $\aleph_i = \left\langle [LT^{-}_{\aleph\ i}, LT^{+}_{\aleph\ i}], [LI^{-}_{\aleph\ i}, LI^{+}_{\aleph\ i}], [LF^{-}_{\aleph\ i},\ LF^{+}_{\aleph\ i}] \right\rangle$ be the family of new type NIVN. Then new type GNIVWA $(\aleph_1, \aleph_2, ..., \aleph_n) = \left(\bigwedge_{i=1}^{n} \zeta_i \aleph_i^{\Psi}\right)^{1/\Psi}$ is called the new type GNIVWA operator.

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Theorem 4.11. *Let* $\aleph_i = \left\langle [LT^-_{\aleph\ i}, LT^+_{\aleph\ i}], [LI^-_{\aleph\ i}, LI^+_{\aleph\ i}], [LF^-_{\aleph\ i}, LF^+_{\aleph\ i}] \right\rangle$ *be the family of new type NIVNs. Then new type GNIVWA* $(\aleph_1, \aleph_2, ..., \aleph_n)$

$$= \left[\begin{array}{c} \left[\left(\sqrt[\flat_1]{1-\circledast_{i=1}^n\left(1-\left((L_{\Im_i}T^-_{\aleph\ i})^{\flat_1}\right)^{\flat_1}\right)^{\zeta_i}}\right)^{1/\Psi}, \left(\sqrt[\flat_1]{1-\circledast_{i=1}^n\left(1-\left((L_{\Im_i}(T^+_{\aleph\ i}))^{\flat_1}\right)^{\flat_1}\right)^{\zeta_i}}\right)^{1/\Psi}\right], \\ \left[\left(\sqrt[\flat_2]{1-\circledast_{i=1}^n\left(1-\left((L_{\Im_i}I^-_{\aleph\ i})^{\flat_2}\right)^{\flat_2}\right)^{\zeta_i}}\right)^{1/\Psi}, \left(\sqrt[\flat_2]{1-\circledast_{i=1}^n\left(1-\left((L_{\Im_i}I^+_{\aleph\ i})^{\flat_2}\right)^{\flat_2}\right)^{\zeta_i}}\right)^{1/\Psi}\right], \\ \left[\begin{array}{c} \sqrt[\flat_3]{1-\left(1-\left(\circledast_{i=1}^n\left(\sqrt[\flat_3]{1-\left(1-(L_{\Im_i}F^-_{\aleph\ i})^{\flat_3}\right)^{\flat_3}}\right)^{\zeta_i}\right)^{\flat_3}\right)^{1/\Psi}}, \\ \sqrt[\flat_3]{1-\left(1-\left(\circledast_{i=1}^n\left(\sqrt[\flat_3]{1-\left(1-(L_{\Im_i}(F^+_{\aleph\ i}))^{\flat_3}\right)^{\flat_3}}\right)^{\zeta_i}\right)^{\flat_3}\right)^{1/\Psi}} \end{array}\right] \end{array}\right].$$

Proof. We have, $\bigwedge_{i=1}^n \zeta_i \aleph_i^{\Psi}$

$$= \left[\begin{array}{c} \left[\sqrt[\flat_1]{1-\circledast_{i=1}^n\left(1-\left((L_{\Im_i}T^-_{\aleph\ i})^{\flat_1}\right)^{\flat_1}\right)^{\zeta_i}}, \sqrt[\flat_1]{1-\circledast_{i=1}^n\left(1-\left((L_{\Im_i}(T^+_{\aleph\ i}))^{\flat_1}\right)^{\flat_1}\right)^{\zeta_i}}\right], \\ \left[\sqrt[\flat_2]{1-\circledast_{i=1}^n\left(1-\left((L_{\Im_i}I^-_{\aleph\ i})^{\flat_2}\right)^{\flat_2}\right)^{\zeta_i}}, \sqrt[\flat_2]{1-\circledast_{i=1}^n\left(1-\left((L_{\Im_i}I^+_{\aleph\ i})^{\flat_2}\right)^{\flat_2}\right)^{\zeta_i}}\right], \\ \left[\circledast_{i=1}^n\left(\sqrt[\flat_3]{1-\left(1-(L_{\Im_i}F^-_{\aleph\ i})^{\flat_3}\right)^{\flat_3}}\right)^{\zeta_i}, \circledast_{i=1}^n\left(\sqrt[\flat_3]{1-\left(1-(L_{\Im_i}(F^+_{\aleph\ i}))^{\flat_3}\right)^{\flat_3}}\right)^{\zeta_i}\right] \end{array}\right].$$

If $n = 2$, then $\zeta_1\aleph_1 \bigvee \zeta_2\aleph_2$

$$= \left[\begin{array}{c} \left[\begin{array}{c} \sqrt[\flat_1]{\begin{array}{c}\left(\sqrt[\flat_1]{1-\left(1-\left((L_{\Im_i}T^-_{\aleph\ 1})^{\flat_1}\right)^{\flat_1}\right)^{\zeta_1}}\right)^{\flat_1} + \left(\sqrt[\flat_1]{1-\left(1-\left((L_{\Im_i}T^-_{\aleph\ 2})^{\flat_1}\right)^{\flat_1}\right)^{\zeta_1}}\right)^{\flat_1} \\ -\left(\sqrt[\flat_1]{1-\left(1-\left((L_{\Im_i}T^-_{\aleph\ 1})^{\flat_1}\right)^{\flat_1}\right)^{\zeta_1}}\right)^{\flat_1}\cdot\left(\sqrt[\flat_1]{1-\left(1-\left((L_{\Im_i}T^-_{\aleph\ 2})^{\flat_1}\right)^{\flat_1}\right)^{\zeta_1}}\right)^{\flat_1}\end{array}}, \\ \sqrt[\flat_1]{\begin{array}{c}\left(\sqrt[\flat_1]{1-\left(1-\left((L_{\Im_i}(T^+_{\aleph\ 1}))^{\flat_1}\right)^{\flat_1}\right)^{\zeta_1}}\right)^{\flat_1} + \left(\sqrt[\flat_1]{1-\left(1-\left((L_{\Im_i}(T^+_{\aleph\ 2}))^{\flat_1}\right)^{\flat_1}\right)^{\zeta_1}}\right)^{\flat_1} \\ -\left(\sqrt[\flat_1]{1-\left(1-\left((L_{\Im_i}(T^+_{\aleph\ 1}))^{\flat_1}\right)^{\flat_1}\right)^{\zeta_1}}\right)^{\flat_1}\cdot\left(\sqrt[\flat_1]{1-\left(1-\left((L_{\Im_i}(T^+_{\aleph\ 2}))^{\flat_1}\right)^{\flat_1}\right)^{\zeta_1}}\right)^{\flat_1}\end{array}} \end{array}\right], \\ \left[\begin{array}{c} \sqrt[\flat_2]{\begin{array}{c}\left(\sqrt[\flat_2]{1-\left(1-\left((L_{\Im_i}I^-_{\aleph\ 1})^{\flat_2}\right)^{\flat_2}\right)^{\zeta_1}}\right)^{\flat_2} + \left(\sqrt[\flat_2]{1-\left(1-\left((L_{\Im_i}I^-_{\aleph\ 2})^{\flat_2}\right)^{\flat_2}\right)^{\zeta_1}}\right)^{\flat_2} \\ -\left(\sqrt[\flat_2]{1-\left(1-\left((L_{\Im_i}I^-_{\aleph\ 1})^{\flat_2}\right)^{\flat_2}\right)^{\zeta_1}}\right)^{\flat_2}\cdot\left(\sqrt[\flat_2]{1-\left(1-\left((L_{\Im_i}I^-_{\aleph\ 2})^{\flat_2}\right)^{\flat_2}\right)^{\zeta_1}}\right)^{\flat_2}\end{array}}, \\ \sqrt[\flat_2]{\begin{array}{c}\left(\sqrt[\flat_2]{1-\left(1-\left((L_{\Im_i}I^+_{\aleph\ 1})^{\flat_2}\right)^{\flat_2}\right)^{\zeta_1}}\right)^{\flat_2} + \left(\sqrt[\flat_2]{1-\left(1-\left((L_{\Im_i}I^+_{\aleph\ 2})^{\flat_2}\right)^{\flat_2}\right)^{\zeta_1}}\right)^{\flat_2} \\ -\left(\sqrt[\flat_2]{1-\left(1-\left((L_{\Im_i}I^+_{\aleph\ 1})^{\flat_2}\right)^{\flat_2}\right)^{\zeta_1}}\right)^{\flat_2}\cdot\left(\sqrt[\flat_2]{1-\left(1-\left((L_{\Im_i}I^+_{\aleph\ 2})^{\flat_2}\right)^{\flat_2}\right)^{\zeta_1}}\right)^{\flat_2}\end{array}} \end{array}\right], \\ \left[\begin{array}{c} \left(\sqrt[\flat_3]{1-\left(1-(L_{\Im_i}F^-_{\aleph\ 1})^{\flat_3}\right)^{\flat_3}}\right)^{\zeta_1}\cdot\left(\sqrt[\flat_3]{1-\left(1-(L_{\Im_i}F^-_{\aleph\ 2})^{\flat_3}\right)^{\flat_3}}\right)^{\zeta_1}, \\ \left(\sqrt[\flat_3]{1-\left(1-(L_{\Im_i}(F^+_{\aleph\ 1}))^{\flat_3}\right)^{\flat_3}}\right)^{\zeta_1}\cdot\left(\sqrt[\flat_3]{1-\left(1-(L_{\Im_i}(F^+_{\aleph\ 2}))^{\flat_3}\right)^{\flat_3}}\right)^{\zeta_1} \end{array}\right] \end{array}\right]$$

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$$= \left[\begin{array}{c} \left[\sqrt[\flat_1]{1-\circledast_{i=1}^{\flat_1}\left(1-\left((L_{\Im_i}T^-_{\aleph\ 1})^{\flat_1}\right)^{\flat_1}\right)^{\zeta_i}},\ \sqrt[\flat_1]{1-\circledast_{i=1}^{\flat_1}\left(1-\left((L_{\Im_i}(T^+_{\aleph\ 1}))^{\flat_1}\right)^{\flat_1}\right)^{\zeta_i}}\right], \\ \left[\sqrt[\flat_2]{1-\circledast_{i=1}^{\flat_2}\left(1-\left((L_{\Im_i}I^-_{\aleph\ 1})^{\flat_2}\right)^{\flat_2}\right)^{\zeta_i}},\ \sqrt[\flat_2]{1-\circledast_{i=1}^{\flat_2}\left(1-\left((L_{\Im_i}I^+_{\aleph\ 1})^{\flat_2}\right)^{\flat_2}\right)^{\zeta_i}}\right], \\ \left[\circledast_{i=1}^{\flat_3}\left(\sqrt[\flat_3]{1-\left(1-(L_{\Im_i}F^-_{\aleph\ i})^{\flat_3}\right)^{\flat_3}}\right)^{\zeta_i},\circledast_{i=1}^{\flat_3}\left(\sqrt[\flat_3]{1-\left(1-(L_{\Im_i}(F^+_{\aleph\ i}))^{\flat_3}\right)^{\flat_3}}\right)^{\zeta_i}\right] \end{array}\right].$$

It is valid for $n=l$ and $l\geq 3$.
Hence, $\bigwedge_{i=1}^{l}\zeta_i\aleph_i^{\Psi}=$

$$\left[\begin{array}{c} \left[\sqrt[\flat_1]{1-\circledast_{i=1}^{l}\left(1-\left((L_{\Im_i}T^-_{\aleph\ 1})^{\flat_1}\right)^{\flat_1}\right)^{\zeta_i}},\ \sqrt[\flat_1]{1-\circledast_{i=1}^{l}\left(1-\left((L_{\Im_i}(T^+_{\aleph\ 1}))^{\flat_1}\right)^{\flat_1}\right)^{\zeta_i}}\right], \\ \left[\sqrt[\flat_2]{1-\circledast_{i=1}^{l}\left(1-\left((L_{\Im_i}I^-_{\aleph\ 1})^{\flat_2}\right)^{\flat_2}\right)^{\zeta_i}},\ \sqrt[\flat_2]{1-\circledast_{i=1}^{l}\left(1-\left((L_{\Im_i}I^+_{\aleph\ 1})^{\flat_2}\right)^{\flat_2}\right)^{\zeta_i}}\right], \\ \left[\circledast_{i=1}^{l}\left(\sqrt[\flat_3]{1-\left(1-(L_{\Im_i}F^-_{\aleph\ i})^{\flat_3}\right)^{\flat_3}}\right)^{\zeta_i},\circledast_{i=1}^{l}\left(\sqrt[\flat_3]{1-\left(1-(L_{\Im_i}F^-_{\aleph\ i})^{\flat_3}\right)^{\flat_3}}\right)^{\zeta_i}\right] \end{array}\right].$$

If $n=l+1$ and we apply, then $\bigwedge_{i=1}^{l}\zeta_i\aleph_i^{\Psi}+\zeta_{l+1}\aleph_{l+1}^{\Psi}=\bigwedge_{i=1}^{l+1}\zeta_i\aleph_i^{\Psi}$.
Now,$\bigwedge_{i=1}^{l}\zeta_i\aleph_i^{\Psi}+\zeta_{l+1}\aleph_{l+1}^{\Psi}=\zeta_1\aleph_1^{\Psi}\bigvee\zeta_2\aleph_2^{\Psi}\bigvee...\bigvee\zeta_l\aleph_l^{\Psi}\bigvee\zeta_{l+1}\aleph_{l+1}^{\Psi}$

$$= \left[\begin{array}{c} \left[\begin{array}{c} \sqrt[\flat_1]{\begin{array}{c}\left(\sqrt[\flat_1]{1-\circledast_{i=1}^{l}\left(1-\left((L_{\Im_i}T^-_{\aleph\ i})^{\flat_1}\right)^{\flat_1}\right)^{\zeta_i}}\right)^{\flat_1}+\left(\sqrt[\flat_1]{1-\left(1-\left((L_{\Im_i}T^-_{\aleph\ l+1})^{\flat_1}\right)^{\flat_1}\right)^{\zeta_1}}\right)^{\flat_1}, \\ -\left(\sqrt[\flat_1]{1-\circledast_{i=1}^{l}\left(1-\left((L_{\Im_i}T^-_{\aleph\ i})^{\flat_1}\right)^{\flat_1}\right)^{\zeta_i}}\right)^{\flat_1}\cdot\left(\sqrt[\flat_1]{1-\left(1-\left((L_{\Im_i}T^-_{\aleph\ l+1})^{\flat_1}\right)^{\flat_1}\right)^{\zeta_1}}\right)^{\flat_1}\end{array}}, \\ \sqrt[\flat_1]{\begin{array}{c}\left(\sqrt[\flat_1]{1-\circledast_{i=1}^{l}\left(1-\left((L_{\Im_i}(T^+_{\aleph\ i}))^{\flat_1}\right)^{\flat_1}\right)^{\zeta_i}}\right)^{\flat_1}+\left(\sqrt[\flat_1]{1-\left(1-\left((L_{\Im_i}(T^+_{\aleph\ l+1}))^{\flat_1}\right)^{\flat_1}\right)^{\zeta_1}}\right)^{\flat_1} \\ -\left(\sqrt[\flat_1]{1-\circledast_{i=1}^{l}\left(1-\left((L_{\Im_i}(T^+_{\aleph\ i}))^{\flat_1}\right)^{\flat_1}\right)^{\zeta_i}}\right)^{\flat_1}\cdot\left(\sqrt[\flat_1]{1-\left(1-\left((L_{\Im_i}(T^+_{\aleph\ l+1}))^{\flat_1}\right)^{\flat_1}\right)^{\zeta_1}}\right)^{\flat_1}\end{array}} \end{array}\right], \\ \left[\begin{array}{c} \sqrt[\flat_2]{\begin{array}{c}\left(\sqrt[\flat_2]{1-\circledast_{i=1}^{l}\left(1-\left((L_{\Im_i}I^-_{\aleph\ i})^{\flat_2}\right)^{\flat_2}\right)^{\zeta_i}}\right)^{\flat_2}+\left(\sqrt[\flat_2]{1-\left(1-\left((L_{\Im_i}I^-_{\aleph\ l+1})^{\flat_2}\right)^{\flat_2}\right)^{\zeta_1}}\right)^{\flat_2}, \\ -\left(\sqrt[\flat_2]{1-\circledast_{i=1}^{l}\left(1-\left((L_{\Im_i}I^-_{\aleph\ i})^{\flat_2}\right)^{\flat_2}\right)^{\zeta_i}}\right)^{\flat_2}\cdot\left(\sqrt[\flat_2]{1-\left(1-\left((L_{\Im_i}I^-_{\aleph\ l+1})^{\flat_2}\right)^{\flat_2}\right)^{\zeta_1}}\right)^{\flat_2}\end{array}} \\ \sqrt[\flat_2]{\begin{array}{c}\left(\sqrt[\flat_2]{1-\circledast_{i=1}^{l}\left(1-\left((L_{\Im_i}I^+_{\aleph\ i})^{\flat_2}\right)^{\flat_2}\right)^{\zeta_i}}\right)^{\flat_2}+\left(\sqrt[\flat_2]{1-\left(1-\left((L_{\Im_i}I^+_{\aleph\ l+1})^{\flat_2}\right)^{\flat_2}\right)^{\zeta_1}}\right)^{\flat_2} \\ -\left(\sqrt[\flat_2]{1-\circledast_{i=1}^{l}\left(1-\left((L_{\Im_i}I^+_{\aleph\ i})^{\flat_2}\right)^{\flat_2}\right)^{\zeta_i}}\right)^{\flat_2}\cdot\left(\sqrt[\flat_2]{1-\left(1-\left((L_{\Im_i}I^+_{\aleph\ l+1})^{\flat_2}\right)^{\flat_2}\right)^{\zeta_1}}\right)^{\flat_2}\end{array}} \end{array}\right], \\ \left[\begin{array}{c} \circledast_{i=1}^{l}\left(\sqrt[\flat_3]{1-\left(1-(L_{\Im_i}F^-_{\aleph\ i})^{\flat_3}\right)^{\flat_3}}\right)^{\zeta_i}\cdot\left(\sqrt[\flat_3]{1-\left(1-(L_{\Im_i}F^-_{\aleph\ l+1})^{\flat_3}\right)^{\flat_3}}\right)^{\zeta_1}, \\ \circledast_{i=1}^{l}\left(\sqrt[\flat_3]{1-\left(1-(L_{\Im_i}(F^+_{\aleph\ i}))^{\flat_3}\right)^{\flat_3}}\right)^{\zeta_i}\cdot\left(\sqrt[\flat_3]{1-\left(1-(L_{\Im_i}(F^+_{\aleph\ l+1}))^{\flat_3}\right)^{\flat_3}}\right)^{\zeta_1} \end{array}\right] \end{array}\right].$$

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Thus,

$$\bigwedge_{i=1}^{l+1}\zeta_i\aleph_i^{\Psi}=\left[\begin{array}{c}\left[\sqrt[\flat_1]{1-\circledast_{i=1}^{l+1}\left(1-\left((L_{\Im_i}T^-_{\aleph\ 1})^{\flat_1}\right)^{\flat_1}\right)^{\zeta_i}},\sqrt[\flat_1]{1-\circledast_{i=1}^{l+1}\left(1-\left((L_{\Im_i}(T^+_{\aleph\ 1}))^{\flat_1}\right)^{\flat_1}\right)^{\zeta_i}}\right],\\ \left[\sqrt[\flat_2]{1-\circledast_{i=1}^{l+1}\left(1-\left((L_{\Im_i}I^-_{\aleph\ 1})^{\flat_2}\right)^{\flat_2}\right)^{\zeta_i}},\sqrt[\flat_2]{1-\circledast_{i=1}^{l+1}\left(1-\left((L_{\Im_i}I^+_{\aleph\ 1})^{\flat_2}\right)^{\flat_2}\right)^{\zeta_i}}\right],\\ \left[\circledast_{i=1}^{l+1}\left(\sqrt[\flat_3]{1-\left(1-(L_{\Im_i}F^-_{\aleph\ i})^{\flat_3}\right)^{\flat_3}}\right)^{\zeta_i},\circledast_{i=1}^{l+1}\left(\sqrt[\flat_3]{1-\left(1-(L_{\Im_i}F^-_{\aleph\ i})^{\flat_3}\right)^{\flat_3}}\right)^{\zeta_i}\right]\end{array}\right].$$

Hence,

$$\left(\bigwedge_{i=1}^{l+1}\zeta_i\aleph_i^{\Psi}\right)^{1/\Psi}$$

$$=\left[\begin{array}{c}\left[\left(\sqrt[\flat_1]{1-\circledast_{i=1}^{l+1}\left(1-\left((L_{\Im_i}T^-_{\aleph\ i})^{\flat_1}\right)^{\flat_1}\right)^{\zeta_i}}\right)^{1/\Psi},\left(\sqrt[\flat_1]{1-\circledast_{i=1}^{l+1}\left(1-\left((L_{\Im_i}(T^+_{\aleph\ i}))^{\flat_1}\right)^{\flat_1}\right)^{\zeta_i}}\right)^{1/\Psi}\right],\\ \left[\left(\sqrt[\flat_2]{1-\circledast_{i=1}^{l+1}\left(1-\left((L_{\Im_i}I^-_{\aleph\ i})^{\flat_2}\right)^{\flat_2}\right)^{\zeta_i}}\right)^{1/\Psi},\left(\sqrt[\flat_2]{1-\circledast_{i=1}^{l+1}\left(1-\left((L_{\Im_i}I^+_{\aleph\ i})^{\flat_2}\right)^{\flat_2}\right)^{\zeta_i}}\right)^{1/\Psi}\right],\\ \left[\begin{array}{c}\sqrt[\flat_3]{1-\left(1-\left(\circledast_{i=1}^{l+1}\left(\sqrt[\flat_3]{1-\left(1-(L_{\Im_i}F^-_{\aleph\ i})^{\flat_3}\right)^{\flat_3}}\right)^{\zeta_i}\right)^{\flat_3}\right)^{1/\Psi}},\\ \sqrt[\flat_3]{1-\left(1-\left(\circledast_{i=1}^{l+1}\left(\sqrt[\flat_3]{1-\left(1-(L_{\Im_i}(F^+_{\aleph\ i}))^{\flat_3}\right)^{\flat_3}}\right)^{\zeta_i}\right)^{\flat_3}\right)^{1/\Psi}}\end{array}\right]\end{array}\right].$$

It is valid for $l\geq 1$.

Remark 4.12. If $\zeta_i=1$, then new type GNIVWA operator is modified to the new type NIVWA operator.

Theorem 4.13. *If all* $\aleph_i=\left\langle[LT^-_{\aleph\ i},LT^+_{\aleph\ i}],[LI^-_{\aleph\ i},LI^+_{\aleph\ i}][LF^-_{\aleph\ i},\ LF^+_{\aleph\ i}]\right\rangle$ *are equal and* $\aleph_i=\aleph$*. Then new type GNIVWA* $(\aleph_1,\aleph_2,...,\aleph_n)=\aleph$*.*

Proof. This proof based on Theorem 4.3.

Remark 4.14. We use the new type GNIVWA operator to satisfy boundedness and monotonicity conditions.

Proof. Based on Theorems 4.4 and 4.5.

4.4 Generalized new type NIVWG (new type GNIVWG) operator

Definition 4.15. Let $\aleph_i=\left\langle[LT^-_{\aleph\ i},LT^+_{\aleph\ i}],[LI^-_{\aleph\ i},LI^+_{\aleph\ i}],[LF^-_{\aleph\ i},LF^+_{\aleph\ i}]\right\rangle$ be the family of new type NIVNs. Then new type GNIVWG $(\aleph_1,\aleph_2,...,\aleph_n)=\frac{1}{\Psi}\left(\circledast_{i=1}^{n}(\Psi\aleph_i)^{\zeta_i}\right)$ is called the new type GNIVWG operator.

Theorem 4.16. *Let* $\aleph_i=\left\langle[LT^-_{\aleph\ i},LT^+_{\aleph\ i}],[LI^-_{\aleph\ i},LI^+_{\aleph\ i}],[LF^-_{\aleph\ i},\ LF^+_{\aleph\ i}]\right\rangle$ *be the family of new type NIVNs. Then new type GNIVWG*$(\aleph_1,\aleph_2,...,\aleph_n)$

$$=\left[\begin{array}{c}\left[\begin{array}{c}\sqrt[\flat_1]{1-\left(1-\left(\circledast_{i=1}^{n}\left(\sqrt[\flat_1]{1-\left(1-(L_{\Im_i}T^-_{\aleph\ i})^{\flat_1}\right)^{\flat_1}}\right)^{\zeta_i}\right)^{\flat_1}\right)^{1/\Psi}},\\ \sqrt[\flat_1]{1-\left(1-\left(\circledast_{i=1}^{n}\left(\sqrt[\flat_1]{1-\left(1-(L_{\Im_i}(T^+_{\aleph\ i}))^{\flat_1}\right)^{\flat_1}}\right)^{\zeta_i}\right)^{\flat_1}\right)^{1/\Psi}}\end{array}\right],\\ \left[\left(\sqrt[\flat_2]{1-\circledast_{i=1}^{n}\left(1-\left((L_{\Im_i}I^-_{\aleph\ i})^{\flat_2}\right)^{\flat_2}\right)^{\zeta_i}}\right)^{1/\Psi},\left(\sqrt[\flat_2]{1-\circledast_{i=1}^{n}\left(1-\left((L_{\Im_i}I^+_{\aleph\ i})^{\flat_2}\right)^{\flat_2}\right)^{\zeta_i}}\right)^{1/\Psi}\right],\\ \left[\left(\sqrt[\flat_3]{1-\circledast_{i=1}^{n}\left(1-\left((L_{\Im_i}F^-_{\aleph\ i})^{\flat_3}\right)^{\flat_3}\right)^{\zeta_i}}\right)^{1/\Psi},\left(\sqrt[\flat_3]{1-\circledast_{i=1}^{n}\left(1-\left((L_{\Im_i}(F^+_{\aleph\ i}))^{\flat_3}\right)^{\flat_3}\right)^{\zeta_i}}\right)^{1/\Psi}\right]\end{array}\right].$$

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Proof. This proof based on the Theorem 4.11.

Remark 4.17. If $\zeta_i = 1$, then new type GNIVWG operator is converted to the new type NIVWG operator.

Remark 4.18. New type GNIVWG operators satisfy boundedness and monotonicity properties.

Proof. This proof based on Theorem 4.4 and Theorem 4.5.

Corollary 4.19. *If all* $\aleph_i = \left\langle [LT^-_{\aleph\ i}, LT^+_{\aleph\ i}], [LI^-_{\aleph\ i}, LI^+_{\aleph\ i}][LF^-_{\aleph\ i}, LF^+_{\aleph\ i}] \right\rangle$ *are equal and* $\aleph_i = \aleph$, *for* $i = 1, 2, ..., n$. *Then new type GNIVWG*$(\aleph_1, \aleph_2, ..., \aleph_n) = \aleph$.

Proof. This proof based on Theorem 4.3.

5 MADM using new type NIVS data

Let $\aleph = \{\aleph_a, \aleph_b, ..., \aleph_n\}$ be the alternatives, $C = \{C_1, C_2, ..., C_m\}$ be the attributes, $w = \{\vartheta_1, \vartheta_2, ..., \vartheta_m\}$ be the weights of attributes, $\aleph_{ij} = \left\langle LT_{\aleph ij}, LI_{\aleph ij}, LF_{\aleph ij} \right\rangle$ is denote new type NIVS of $\aleph_i$ in C_j. Here, $\wp^T_{\aleph} : \mathbb{U} \to Int([0,1])$, $\wp^I_{\aleph} : \mathbb{U} \to Int([0,1])$ and $\wp^F_{\aleph} : \mathbb{U} \to Int([0,1])$ denotes TMD, IMD and FMD of $\ell \in \mathbb{U}$ to $\aleph$, respectively and $0 \preceq (L_{\Im_i} TU_{\aleph}(\ell))^{\flat_1} + (L_{\Im_i} IU_{\aleph}(\ell))^{\flat_2} + (L_{\Im_i} FU_{\aleph}(\ell))^{\flat_3} \preceq 2$, where $\flat_1, \flat_2, \flat_3$ are positive integers and $\Im = (T_{\aleph} \times I_{\aleph} \times F_{\aleph})$, where $\times$ denote usual multiplications.

5.1 Algorithm

Step-1: In order to make the new type NIVS value.
Step-2: A choice is made regarding which values to use for the weighted averaging (geometric) process. Let $\mathscr{D} = (\aleph_{ij})_{n\times m}$ is into $\mathscr{D} = (\aleph_{ij})_{n\times m}$.
Step-3: Calculate the positive and negative ideal values: $\aleph^P = \left\langle [1,1], [1,1], [0,0] \right\rangle$ and $\aleph^N = \left\langle [0,0], [0,0], [1,1] \right\rangle$.
Step-4: It is important to find the difference between the ideal values of each option in order to find the ED:

$$\mathscr{D}^P_i = \mathscr{D}_E\left(\aleph_i, \aleph^P\right); \ \mathscr{D}^N_i = \mathscr{D}_E\left(\aleph_i, \aleph^N\right).$$

Step-5: A distance between two points can be calculated using the following formula:

$$\mathscr{D}^\star_i = \frac{\mathscr{D}^N_i}{\mathscr{D}^P_i + \mathscr{D}^N_i}.$$

Step-6: In this case, the best output will be obtained using $\sup \mathscr{D}^\star_i$. Choosing the right option in order to solve a problem is referred to as a decision. A diagram of the proposed methods can be seen in Figure-1.

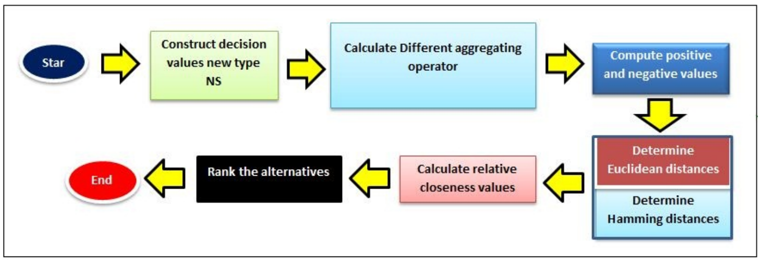

Figure-1. algorithm for proposed models.

5.2 Real life example

People can use personal computers at home, in college, or in their businesses. People can view favorable colors on computers by using a monitor, which is similar to a screen. In addition, the resolution rate is higher, so people can see more clearly. It is possible to add printers, larger speakers, desktop scanners, and hard drives that are more energy efficient to a personal computer. Laptops are lightweight and portable computers that are easy to transport, making them ideal for people to take on business trips, vacations, or anywhere else. In order to choose the right laptop (alternative), a customer has to decide between five types of laptops such as $\infty = \{\infty_a, \infty_b, \infty_c, \infty_d, \infty_e\}$. . Battery life performance is taken into account (C_1), storage capacity is taken into consideration (C_2), the operating system version is taken into consideration (C_3) and the overall cost is taken into consideration (C_4), with their corollaryresponding weights $w = \{0.4, 0.3, 0.2, 0.1\}$. We aim to select the best alternative from a large number of alternatives based on expert assessments. Laptops can be purchased for a variety of reasons. In order to purchase a product or service, a consumer goes through several stages. Table-1 shows that decision making information's are

Table 1: Decision making informations

	C_1	C_2	C_3	C_4
∞_a	$\langle[0.7, 0.75], [0.6, 0.65], [0.5, 0.55]\rangle$	$\langle[0.8, 0.85], [0.5, 0.55], [0.5, 0.65]\rangle$	$\langle[0.45, 0.8], [0.65, 0.7], [0.7, 0.75]\rangle$	$\langle[0.65, 0.8], [0.65, 0.7], [0.45, 0.55]\rangle$
∞_b	$\langle[0.75, 0.8], [0.55, 0.6], [0.65, 0.7]\rangle$	$\langle[0.6, 0.65], [0.55, 0.6], [0.75, 0.8]\rangle$	$\langle[0.55, 0.7], [0.6, 0.65], [0.8, 0.85]\rangle$	$\langle[0.6, 0.8], [0.55, 0.6], [0.3, 0.4]\rangle$
∞_c	$\langle[0.6, 0.65], [0.7, 0.75], [0.6, 0.65]\rangle$	$\langle[0.7, 0.75], [0.6, 0.65], [0.8, 0.85]\rangle$	$\langle[0.7, 0.75], [0.7, 0.75], [0.45, 0.75]\rangle$	$\langle[0.55, 0.65], [0.75, 0.8], [0.55, 0.6]\rangle$
∞_d	$\langle[0.55, 0.6], [0.75, 0.8], [0.7, 0.75]\rangle$	$\langle[0.55, 0.8], [0.35, 0.45], [0.35, 0.6]\rangle$	$\langle[0.75, 0.8], [0.35, 0.45], [0.7, 0.75]\rangle$	$\langle[0.6, 0.85], [0.8, 0.85], [0.6, 0.8]\rangle$
∞_e	$\langle[0.65, 0.7], [0.65, 0.7], [0.65, 0.75]\rangle$	$\langle[0.7, 0.75], [0.5, 0.65], [0.55, 0.7]\rangle$	$\langle[0.8, 0.85], [0.55, 0.65], [0.75, 0.85]\rangle$	$\langle[0.75, 0.8], [0.8, 0.9], [0.55, 0.75]\rangle$

Using the new type NIVWA operator, you can obtain the following aggregate data. New type NIVWA is shown in table-2.

Table 2: new type NIVWA operator

	$new\ type\ NIVWA\ operator\ (\flat_1, \flat_2, \flat_3) = (1, 1, 1)$
∞_a	$\langle[0.2374, 0.2597], [0.264, 0.2654], [0.2422, 0.2446]\rangle$
∞_b	$\langle[0.233, 0.2568], [0.252, 0.2522], [0.1728, 0.1754]\rangle$
∞_c	$\langle[0.2444, 0.2517], [0.2638, 0.2661], [0.2078, 0.2168]\rangle$
∞_d	$\langle[0.2667, 0.3096], [0.2686, 0.2695], [0.2248, 0.2545]\rangle$
∞_e	$\langle[0.2741, 0.279], [0.2724, 0.2816], [0.2367, 0.2501]\rangle$

The optimum value of each alternative should be determined by analyzing its positive and negative values

$\infty^P = \langle[1, 1], [1, 1], [0, 0]\rangle$ and $\infty^N = \langle[0, 0], [0, 0], [1, 1]\rangle$.
ED between each alternatives with both ideal values are $\mathscr{D}_1^P$=0.0653, $\mathscr{D}_2^P$=0.0886, $\mathscr{D}_3^P$=0.0746, $\mathscr{D}_4^P$=0.0854, $\mathscr{D}_5^P$=0.077 and $\mathscr{D}_1^N$=0.5471, $\mathscr{D}_2^N$=0.5238, $\mathscr{D}_3^N$=0.5378, $\mathscr{D}_4^N$=0.5269, $\mathscr{D}_5^N$=0.5354.
The following formula is used to calculate relative closeness: $\mathscr{D}_1^\star = 0.8933$, $\mathscr{D}_2^\star = 0.8553$, $\mathscr{D}_3^\star = 0.8782$, $\mathscr{D}_4^\star = 0.8605$, $\mathscr{D}_5^\star = 0.8743$.
The following are the alternatives ranked in order of preference: $\infty_a > \infty_c > \infty_e > \infty_d > \infty_b$. As a result, ∞_a is the best option for the purchasing.

5.3 Compared to existing models and proposed models

An interaction MADM approach for Pythagorean neutrosophic normal interval-valued aggregation operators was recently introduced by Palanikumar et al.[10] Several distance techniques have recently been applied to MADM challenges for spherical vague sets.[11] We proposed ED is based on new type NIVWA, new type NIVWG, new type GNIVWA and new type GNIVWG, respectively. There are several proposed methods shown in Table-3.

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Table 3: Proposed methods

$(\flat_1, \flat_2, \flat_3)$	WA	WG	GWA	GWG
ED $(1,1,1)$ $(proposed)$	$\infty_a > \infty_c > \infty_e$ $\infty_d > \infty_b$	$\infty_a > \infty_b > \infty_c$ $\infty_d > \infty_e$	$\infty_a > \infty_c > \infty_e$ $\infty_d > \infty_b$	$\infty_a > \infty_b > \infty_c$ $\infty_d > \infty_e$
ED $(1,1,2)$ $(proposed)$	$\infty_a > \infty_e > \infty_c$ $\infty_d > \infty_b$	$\infty_a > \infty_b > \infty_c$ $\infty_d > \infty_e$	$\infty_a > \infty_e > \infty_c$ $\infty_d > \infty_b$	$\infty_a > \infty_b > \infty_c$ $\infty_d > \infty_e$

Table-4 shows that existing methods.

Table 4: Existing methods

	WA	WG	GWA	GWG
ED [10]	$\infty_a > \infty_b > \infty_d$ $\infty_e > \infty_c$	$\infty_a > \infty_b > \infty_e$ $\infty_d > \infty_c$	$\infty_a > \infty_b > \infty_d$ $\infty_e > \infty_c$	$\infty_a > \infty_b > \infty_e$ $\infty_d > \infty_c$
ED [11]	$\infty_a > \infty_b > \infty_d$ $\infty_e > \infty_c$	$\infty_a > \infty_b > \infty_e$ $\infty_d > \infty_c$	$\infty_a > \infty_b > \infty_d$ $\infty_e > \infty_c$	$\infty_a > \infty_b > \infty_e$ $\infty_d > \infty_c$

5.4 Data Analysis

Accorollaryding to the WA operator, we present the relative closeness values and their order in Table-5:

Table 5: Relative closeness values

$q - values$	$\mathscr{D}_1^\star$	$\mathscr{D}_2^\star$	$\mathscr{D}_3^\star$	$\mathscr{D}_4^\star$	$\mathscr{D}_5^\star$	$Order$
$(1,1,1)$	0.8933	0.8553	0.8782	0.8605	0.8743	$\infty_a > \infty_c > \infty_e > \infty_d > \infty_b$
$(1,1,2)$	0.7462	0.6923	0.7218	0.7155	0.7257	$\infty_a > \infty_e > \infty_c > \infty_d > \infty_b$
$(2,2,2)$	0.7162	0.6919	0.7107	0.7004	0.7085	$\infty_a > \infty_c > \infty_e > \infty_d > \infty_b$
$(2,2,3)$	0.6572	0.6221	0.6446	0.6354	0.6469	$\infty_a > \infty_e > \infty_c > \infty_d > \infty_b$
$(2,3,2)$	0.6715	0.6621	0.6743	0.6716	0.6689	$\infty_c > \infty_d > \infty_e > \infty_a > \infty_b$

As a result, ∞_c instead of ∞_a. Similarly, we can apply to new type NIVWG, new type GNIVWA and new type GNIVWG operators.

6 Conclusion

A number of methods have been developed for dealing with uncertain information such as FSs, IFSs, PFSs and NSSs. AO rules have been proposed for new type NIVWA, new type NIVWG, new type GNIVWA, and new type GNIVWG. We discussed the scorollarye function based on new type of NIVS. Future discussions will cover the following topics:

1. An investigation of the new type NIVS of soft sets and expert sets.
2. Investigating Pythagorean cubic FSs and spherical cubic FSs.
3. The problem of MADM can be solved using other decision-making methodologies based on square root cubic fuzzy sets.

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