Type-I extension Diophantine neutrosophic interval valued soft set in real life applications for a decision making

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Abstract

We describe certain operations and present the theory of the Type-I extension Diophantine neutrosophic interval valued soft set. Additionally, we go over an algorithm that uses the Type-I soft set model to address the decision-making problem. We present a similarity measure between two Type-I extension Diophantine neutrosophic interval valued soft sets and talk about how it might be used in practical applications. A few exemplary cases are provided to demonstrate their practical application in solving uncertain problems.

Keywords: Type-I extension Diophantine neutrosophic interval valued soft set, soft set model, decision making problem.

1 Introduction

Choosing the best course of action becomes more difficult for decision makers as real-world systems get more complicated. Even though condensing can be challenging, reaching a single objective is not insurmountable. Establishing objectives, rewards, and boundaries for their points of view has proven difficult for many firms. Every decision-maker is unable to arrive at the perfect solution by using flexible criteria on every real-world issue. As a result, decision-makers work to create more dependable and practical techniques for reaching the best choices. It is not always possible to address circumstances involving decision-making with ambiguity and uncertainty by applying strict, analytical methods. Many uncertain theories have been proposed,such as fuzzy set (FS),[1] intuitionistic FS (IFS),[2] vague set (VS),[3] Pythagorean FS (PFS)[4] and spherical FS (SFS).[5] An FS is a set of elements with levels of membership in a given set ranging from 0 to 1; these grades are referred to as an element membership value (MV) in the set. Later, Atanassov proposed the concept of an IFS that is divided into categories based on the non-membership value (NMV), which cannot exceed one.[2] When the combined grade value for MV and NMV is greater than one, we occasionally convey a single

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issue to the decision-making process. Yager[4] developed a new concept of PFS, a generalization of IFS, which is characterized by the square sum of its MV and NMV with a value of no more than one. These concepts are unable to capture a neutral state (neither favour nor disfavor). The concept of picture FS was developed by Cuong et al.[6] and it employs three pointers positive MV, neutral MV and negative MV with the sum of these three values not exceeding one. Florentin Smarandache[7] introduced the concept of neosophic logic. With this logic, the TD, ID, and FD of each proposition are evaluated. The NS consists of elements where the TD, ID and FD are ranked in $[0, 1]$. Neutosophy means neutral thought, and this neutrality separates fuzzy logic from intuitionistic fuzzy logic. The Pythagorean neutrosophic interval-valued set (PNIVS) was introduced by Florentin Smarandache et al.[8] as a set of interval values. There are several applications of single values of the NS, including medical diagnosis[9] and context analysis.[10] Recently, Palanikumar et al. discussed many algebraic structures with applications by[11]-.[14]

Molodtsov[15] proposed the theory of soft sets. In comparison with other uncertain theories, soft sets more accurately reflect the objectivity and complexity of decision making during actual situations. Moreover, the combination of soft sets with other mathematical models is also a critical research area. Maji proposed by the concept of fuzzy soft set[16] and intuitionistic fuzzy soft set.[17] These two theories are applied to solve various decision making problems. Yong Yang was discussed by picture fuzzy soft set.[18] In recent years, Peng[19] has extended fuzzy soft set to Pythagorean fuzzy soft set. This model solved a class of multi attribute decision making consists sum of the degree of membership and non membership value is exceeding unity but the sum of the squares is equal or not exceeding unity. Pinaki Majumdara discussed generalized fuzzy soft sets.[20] Recently many researchers discussed new aggregation operators with applications[21]-.[27] The goal of this study was to use the Type-I extension Diophantine neutrosophic interval valued soft set using soft set model. The introduction is found in section 1. A brief overview of the soft set, interval valued soft set concepts is presented in Section 2. A description of Type-I extension Diophantine neutrosophic interval valued soft set is presented in section 3. There is a discussion in section 4 about similarity measure based on this soft set model. This section explains applications, algorithm and numerical example are discussed in the section 5. Section 6 concludes the discussion with concluding remarks.

2 Preliminaries

Definition 2.1. Let $\mathcal{X}$ be a universal. The PyFS A in $\mathcal{X}$ is $A = \left\{\flat, \langle \zeta^t_A(\flat), \zeta^f_A(\flat)\rangle | \flat \in \mathcal{X}\right\}$, where $\zeta^t_A, \zeta^f_A : \mathcal{X} \to [0,1]$ denotes MG and NMG of $\flat \in \mathcal{X}$ to A, respectively with $0 \leq (\zeta^t_A(\flat))^2 + (\zeta^f_A(\flat))^2 \leq 1$. For $A = \langle \zeta^t_A, \zeta^f_A\rangle$ is called a Pythagorean fuzzy number (PyFN).

Definition 2.2. The PyIVFS A in $\mathcal{X}$ is $A = \left\{\flat, \left\langle \zeta^t_A(\flat), \zeta^f_A(\flat)\right\rangle \Big| \flat \in \mathcal{X}\right\}$, where $\zeta^t_A, \zeta^f_A : \mathcal{X} \to Int([0,1])$ denotes MG and NMG of $\flat \in \mathcal{X}$ to A, respectively with $0 \leq (\zeta^{t+}_A(\flat))^2 + (\zeta^{f+}_A(\flat))^2 \leq 1$. For $A = \left\langle \left[\zeta^{t-}_A, \zeta^{t+}_A\right], \left[\zeta^{f-}_A, \zeta^{f+}_A\right]\right\rangle$ is called a Pythagorean interval-valued FN (PyIVFN).

Definition 2.3. The NSS A in $\mathcal{X}$ is $A = \left\{\flat, \langle \zeta^t_A(\flat), \zeta^m_A(\flat), \zeta^f_A(\flat)\rangle | \flat \in \mathcal{X}\right\}$, where $\zeta^t_A, \zeta^m_A, \zeta^f_A : \mathcal{X} \to [0,1]$ represents TG, IG and FG of $\flat \in \mathcal{X}$ to A, respectively with $0 \leq \zeta^t_A(\flat) + \zeta^m_A(\flat) + \zeta^f_A(\flat) \leq 3$. For $A = \langle \zeta^t_A, \zeta^m_A, \zeta^f_A\rangle$ is called a neutrosophic number (NSN).

Definition 2.4. Let $\mathcal{X}$ be a non-empty set of the universe, Pythagorean neutrosophic interval valued set (PNIVS) A in $\mathcal{X}$ is an object having the following form : $\widehat{A} = \{\flat, \widehat{\zeta}_A(\flat), \widehat{\hbar}_A(\flat), \widehat{\ell}_A(\flat) | \flat \in \mathcal{X}\}$, where $\widehat{\zeta}_A(\flat) = [\zeta^-_A(\flat), \zeta^+_A(\flat)]$ and $\widehat{\hbar}_A(\flat) = [\hbar^-_A(\flat), \hbar^+_A(\flat)]$ and $\widehat{\ell}_A(\flat) = [\ell^-_A(\flat), \ell^+_A(\flat)]$ represent the degree of positive membership, degree of neutral membership and degree of negative membership of A respectively. Consider the mapping $\widehat{\zeta}_A : \mathcal{X} \to D[0,1], \widehat{\hbar}_A : \mathcal{X} \to D[0,1], \widehat{\ell}_A : \mathcal{X} \to D[0,1]$ and $0 \leq (\widehat{\zeta}_A(\flat))^2 + (\widehat{\hbar}_A(\flat))^2 + (\widehat{\ell}_A(\flat))^2 \leq 1$ means $0 \leq (\zeta^+_A(\flat))^2 + (\hbar^+_A(\flat))^2 + (\ell^+_A(\flat))^2 \leq 2$. Here $\widehat{A} = \left\langle [\zeta^-_A, \zeta^+_A], [\hbar^-_A, \hbar^+_A], [\ell^-_A, \ell^+_A]\right\rangle$ is called a Pythagorean neutrosophic interval valued number(PNIVN).

Remark 2.5. Given that $\widehat{A_1} = \langle \zeta_{\widehat{A_1}}, \hbar_{\widehat{A_1}}, \ell_{\widehat{A_1}}\rangle$, $\widehat{A_2} = \langle \zeta_{\widehat{A_2}}, \hbar_{\widehat{A_2}}, \ell_{\widehat{A_2}}\rangle$ and $\widehat{A_3} = \langle \zeta_{\widehat{A_3}}, \hbar_{\widehat{A_3}}, \ell_{\widehat{A_3}}\rangle$ are any three PNIVNs over $(\mathcal{X}, E)$, then the following properties are holds:
(i) $\widehat{A^c_1} = \langle \ell_{\widehat{A_1}}, \hbar_{\widehat{A_1}}, \zeta_{\widehat{A_1}}\rangle$
(ii) $\widehat{A_1} \uplus \widehat{A_2} = \left\langle \max(\zeta_{\widehat{A_1}}, \zeta_{\widehat{A_2}}), \min(\hbar_{\widehat{A_1}}, \hbar_{\widehat{A_2}}), \min(\ell_{\widehat{A_1}}, \ell_{\widehat{A_2}})\right\rangle$

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(iii) $\widehat{A_1} \bigotimes \widehat{A_2} = \Big\langle \min(\zeta_{\widehat{A_1}}, \zeta_{\widehat{A_2}}), \min(\hbar_{\widehat{A_1}}, \hbar_{\widehat{A_2}}), \max(\ell_{\widehat{A_1}}, \ell_{\widehat{A_2}}) \Big\rangle$

(iv) $\widehat{A_1} \leq \widehat{A_2}$ iff $\zeta_{\widehat{A_1}} \leq \zeta_{\widehat{A_2}}$ and $\hbar_{\widehat{A_1}} \leq \hbar_{\widehat{A_2}}$ and $\ell_{\widehat{A_1}} \geq \ell_{\widehat{A_2}}$.

Remark 2.6. Let $\mathcal{X} = \{\flat_1, \flat_2, ..., \flat_n\}$ be a non-empty set of the universe and $E = \{e_1, e_2, ..., e_m\}$ be a set of parameter. The pair $(\mathcal{X}, E)$ is a soft universe. Consider the mapping $\widehat{\mathcal{P}} : E \to D(I)^{\mathcal{X}}$ and $\wp$ be a fuzzy subset of E, ie. $\wp : E \to I = [0, 1]$, where $D(I)^{\mathcal{X}}$ is the collection of all interval valued fuzzy subsets of $\mathcal{X}$. Let $\widehat{\mathcal{P}}_\wp : E \to D(I)^{\mathcal{X}} \times I$ be a function defined as $\widehat{\mathcal{P}}_\wp(e) = \Big(\widehat{\mathcal{P}}(e)(\flat), \wp(e)\Big), \forall \flat \in \mathcal{X}$. Then $\widehat{\mathcal{P}}_\wp$ is called a Type-I generalized interval valued fuzzy soft set (Type-I GIVFSS) on $(\mathcal{X}, E)$. Here for each parameter e_i, $\widehat{\mathcal{P}}_\wp(e_i) = \Big(\widehat{\mathcal{P}}(e_i)(\flat), \wp(e_i)\Big), \forall \flat \in \mathcal{X}$ indicates not only the degree of belongingness of the elements of $\mathcal{X}$ in $\widehat{\mathcal{P}}(e_i)$ but also the degree of possibility fuzzy of such belongingness which is represented by $\wp(e_i)$. So we can write $\widehat{\mathcal{P}}_\wp(e_i)$ as follows: $\widehat{\mathcal{P}}_\wp(e_i) = \Big(\Big\{\frac{\flat_1}{\widehat{\mathcal{P}}(e_i)(\flat_1)}, \frac{\flat_2}{\widehat{\mathcal{P}}(e_i)(\flat_2)}, ..., \frac{\flat_n}{\widehat{\mathcal{P}}(e_i)(\flat_n)}\Big\}, \wp(e_i)\Big)$, where $\widehat{\mathcal{P}}(e_i)(\flat_1)$, $\widehat{\mathcal{P}}(e_i)(\flat_2)$,...,$\widehat{\mathcal{P}}(e_i)(\flat_n)$ are the degrees of belongingness and $\wp(e_i)$ is the degree of possibility fuzzy of such belongingness.

3 Type-I EDioNSIVSS

Here Type-I EDioNSIVSS stands for Type-I extension Diophantine neutrosophic interval valued fuzzy soft set.

Definition 3.1. Let $\mathcal{X} = \{\flat_1, \flat_2, ..., \flat_n\}$ be a non-empty set of the universe and $E = \{e_1, e_2, ..., e_m\}$ be a set of parameter. The pair $(\mathcal{X}, E)$ is called a soft universe. Suppose that $\widehat{\mathcal{P}} : E \to DioNS\widehat{\mathcal{P}}(\mathcal{X})$ and p is a neutrosophic subset of E. That is $p : E \to [0, 1]$, where $DioNS\widehat{\mathcal{P}}(\mathcal{X})$ denotes the collection of all Diophantine neutrosophic interval valued subsets of $\mathcal{X}$. If $\widehat{\mathcal{P}}_{\tau,p} : E \to DioNS\widehat{\mathcal{P}}(\mathcal{X}) \times [0, 1]$ is a function defined as $\widehat{\mathcal{P}}_{\tau,p}(e) = \Big(\widehat{\mathcal{P}}(e)(\flat), \tau(e)(\flat), p(e)\Big), x \in \mathcal{X}$, then $\widehat{\mathcal{P}}_{\tau,p}$ is a Type-I EDioNSIVSS on $(\mathcal{X}, E)$. For each parameter e, $\widehat{\mathcal{P}}_{\tau,p}(e_i) = \Bigg(\Big\{\frac{\flat_1}{(\zeta_{\widehat{\mathcal{P}}(e_i)}(\flat_1), \hbar_{\widehat{\mathcal{P}}(e_i)}(\flat_1), \ell_{\widehat{\mathcal{P}}(e_i)}(\flat_1)), (\alpha_1, \beta_1, \gamma_1)}, ..., \frac{\flat_n}{(\zeta_{\widehat{\mathcal{P}}(e_i)}(\flat_n), \hbar_{\widehat{\mathcal{P}}(e_i)}(\flat_n), \ell_{\widehat{\mathcal{P}}(e_i)}(\flat_n)), (\alpha_n, \beta_n, \gamma_n)}\Big\},$ $(p_1(e_i), p_2(e_i), p_3(e_i))\Bigg)$.

To demonstrate the above Definition, we provide a numerical example as follows:

Example 3.2. Let $\mathcal{X} = \{\flat_1, \flat_2, \flat_3\}$ and a set of parameter $E = \{e_1, e_2, e_3\}$. Suppose that $\widehat{\mathcal{P}}_{\tau_1,p} : E \to S\widehat{\mathcal{P}}(\mathcal{X}) \times [0, 1]$ is given by

$$\widehat{\mathcal{P}}_{\tau,p}(e_1) = \left(\left\{\begin{array}{c} \frac{\flat_1}{([0.55,0.60],[0.25,0.30],[0.65,0.70]),(0.5,0.35,0.15)} \\ \frac{\flat_2}{([0.70,0.85],[0.25,0.35],[0.60,0.65]),(0.4,0.3,0.2)} \\ \frac{\flat_3}{([0.50,0.55],[0.40,0.45],[0.60,0.80]),(0.55,0.25,0.1)} \end{array}\right\}, (0.60, 0.70, 0.45)\right);$$

$$\widehat{\mathcal{P}}_{\tau,p}(e_2) = \left(\left\{\begin{array}{c} \frac{\flat_1}{([0.55,0.60],[0.35,0.45],[0.75,0.85]),(0.25,0.45,0.3)} \\ \frac{\flat_2}{([0.50,0.60],[0.45,0.55],[0.60,0.70]),(0.4,0.5,0.1)} \\ \frac{\flat_3}{([0.50,0.65],[0.30,0.35],[0.65,0.75]),(0.3,0.3,0.4)} \end{array}\right\}, (0.65, 0.40, 0.50)\right);$$

$$\widehat{\mathcal{P}}_{\tau,p}(e_3) = \left(\left\{\begin{array}{c} \frac{\flat_1}{([0.45,0.50],[0.60,0.65],[0.75,0.90]),(0.5,0.2,0.1)} \\ \frac{\flat_2}{([0.55,0.60],[0.70,0.75],[0.60,0.70]),(0.45,0.25,0.2)} \\ \frac{\flat_3}{([0.35,0.40],[0.55,0.65],[0.65,0.75]),(0.35,0.25,0.2)} \end{array}\right\}, (0.55, 0.60, 0.75)\right)$$

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Definition 3.3. Let $\mathcal{X}$ be a non-empty set of the universe and E be a set of parameter. Suppose that $\widehat{\mathcal{P}}_{\tau_1,p}$ and $\widehat{\mathcal{Q}}_{\tau_2,q}$ are two Type-I EDioNSIVSSs on $(\mathcal{X}, E)$. Now $\widehat{\mathcal{P}}_{\tau_1,p}$ is a Type-I EDioNSIVSS subset of $\widehat{\mathcal{Q}}_{\tau_2,q}$ (denoted by $\widehat{\mathcal{P}}_{\tau_1,p} \sqsubseteq \widehat{\mathcal{Q}}_{\tau_2,q}$) if and only if
(i) $\widehat{\mathcal{P}}(e)(\flat) \sqsubseteq \widehat{\mathcal{Q}}(e)(\flat)$ if $\zeta_{\widehat{\mathcal{P}}(e)}(\flat) \leq \zeta_{\widehat{\mathcal{Q}}(e)}(\flat),\ \hbar_{\widehat{\mathcal{P}}(e)}(\flat) \leq \hbar_{\widehat{\mathcal{Q}}(e)}(\flat),\ \ \ell_{\widehat{\mathcal{P}}(e)}(\flat) \geq \ell_{\widehat{\mathcal{Q}}(e)}(\flat)$,
(ii) $\tau_1(\flat)(e) \leq \tau_1(\flat)(e)$,
(iii) $p(e) \leq q(e), \forall\, e \in E$ and $\forall\, \flat \in \mathcal{X}$.

4 Find similarity measure

Let $\mathcal{X} = \{\flat_1, \flat_2, ..., \flat_m\}$ be a non-empty set of the universe and $E = \{e_1, e_2, ..., e_n\}$ be a set of parameters. Suppose that $\widehat{\mathcal{P}}_{\tau_1,p}$ and $\widehat{\mathcal{Q}}_{\tau_2,q}$ are two Type-I EDioNSIVSSs on $(\mathcal{X}, E)$. The similarity measure between two Type-I EDioNSIVSSs $\widehat{\mathcal{P}}_{\tau_1,p}$ and $\widehat{\mathcal{Q}}_{\tau_2,q}$ is denoted by $Sim(\widehat{\mathcal{P}}_{\tau_1,p}, \widehat{\mathcal{Q}}_{\tau_2,q})$ and is defined as $Sim(\widehat{\mathcal{P}}_{\tau_1,p}, \widehat{\mathcal{Q}}_{\tau_2,q}) = \varphi(\widehat{\mathcal{P}}, \widehat{\mathcal{Q}}) \cdot \psi(p, q)$.

where $$\varphi(\widehat{\mathcal{P}}, \widehat{\mathcal{Q}}) = \frac{1}{m}\sum_{j=1}^{m} \begin{bmatrix} \min\left\{T_1^-\left(\widehat{\mathcal{P}}(e)(\flat_j), \widehat{\mathcal{Q}}(e)(\flat_j)\right), T_2^-\left(\widehat{\mathcal{P}}(e)(\flat_j), \widehat{\mathcal{Q}}(e)(\flat_j)\right), S^-\left(\widehat{\mathcal{P}}(e)(\flat_j), \widehat{\mathcal{Q}}(e)(\flat_j)\right)\right\}, \\ \max\left\{T_1^+\left(\widehat{\mathcal{P}}(e)(\flat_j), \widehat{\mathcal{Q}}(e)(\flat_j)\right), T_2^+\left(\widehat{\mathcal{P}}(e)(\flat_j), \widehat{\mathcal{Q}}(e)(\flat_j)\right), S^+\left(\widehat{\mathcal{P}}(e)(\flat_j), \widehat{\mathcal{Q}}(e)(\flat_j)\right)\right\} \end{bmatrix}$$

and $\psi(p, q) = 1 - \frac{\sum_{i=1}^{n}\left|p(e_i) - q(e_i)\right|}{\sum_{i=1}^{n}\left|p(e_i) + q(e_i)\right|}$.

Where, $\widehat{T_1}\left(\widehat{\mathcal{P}}(e)(\flat_j), \widehat{\mathcal{Q}}(e)(\flat_j)\right) =$

$$\left[\frac{\sum_{i=1}^{n}\left(\alpha_i\zeta^-_{\mathcal{P}(e_i)}(\flat_j)\cdot\alpha'_i\zeta^-_{\mathcal{Q}(e_i)}(\flat_j)\right)}{\sum_{i=1}^{n}\left(1-\sqrt{\left(1-{\alpha_i}^2\zeta^{2-}_{\mathcal{P}(e_i)}(\flat_j)\right)\cdot\left(1-{\alpha'_i}^2\zeta^{2-}_{\mathcal{Q}(e_i)}(\flat_j)\right)}\right)}, \frac{\sum_{i=1}^{n}\left(\alpha_i\zeta^+_{\mathcal{P}(e_i)}(\flat_j)\cdot\alpha'_i\zeta^+_{\mathcal{Q}(e_i)}(\flat_j)\right)}{\sum_{i=1}^{n}\left(1-\sqrt{\left(1-{\alpha_i}^2\zeta^{2+}_{\mathcal{P}(e_i)}(\flat_j)\right)\cdot\left(1-{\alpha'_i}^2\zeta^{2+}_{\mathcal{Q}(e_i)}(\flat_j)\right)}\right)}\right]$$

$\widehat{T_2}\left(\widehat{\mathcal{P}}(e)(\flat_j), \widehat{\mathcal{Q}}(e)(\flat_j)\right) =$

$$\left[\frac{\sum_{i=1}^{n}\left(\beta_i^2\hbar^{2-}_{\mathcal{P}(e_i)}(\flat_j)\cdot{\beta'_i}^2\hbar^{2-}_{\mathcal{Q}(e_i)}(\flat_j)\right)}{\sum_{i=1}^{n}\left(1-\sqrt{\left(1-\beta_i^4\hbar^{4-}_{\mathcal{P}(e_i)}(\flat_j)\right)\cdot\left(1-{\beta'_i}^4\hbar^{4-}_{\mathcal{Q}(e_i)}(\flat_j)\right)}\right)}, \frac{\sum_{i=1}^{n}\left(\beta_i^2\hbar^{2+}_{\mathcal{P}(e_i)}(\flat_j)\cdot{\beta'_i}^2\hbar^{2+}_{\mathcal{Q}(e_i)}(\flat_j)\right)}{\sum_{i=1}^{n}\left(1-\sqrt{\left(1-\beta_i^4\hbar^{4+}_{\mathcal{P}(e_i)}(\flat_j)\right)\cdot\left(1-{\beta'_i}^4\hbar^{4+}_{\mathcal{Q}(e_i)}(\flat_j)\right)}\right)}\right]$$

$\widehat{S}\left(\widehat{\mathcal{P}}(e)(\flat_j), \widehat{\mathcal{Q}}(e)(\flat_j)\right) =$

$$\left[1-\sqrt{\left|\left[\frac{\sum_{i=1}^{n}\left({\gamma_i}^2\ell^{2-}_{\mathcal{P}(e_i)}(\flat_j)-{\gamma'_i}^2\ell^{2+}_{\mathcal{Q}(e_i)}(\flat_j)\right)}{\sum_{i=1}^{n}1+\left({\gamma_i}^2\ell^{2+}_{\mathcal{P}(e_i)}(\flat_j)\cdot{\gamma'_i}^2\ell^{2+}_{\mathcal{Q}(e_i)}(\flat_j)\right)}, \frac{\sum_{i=1}^{n}\left({\gamma_i}^2\ell^{2+}_{\mathcal{P}(e_i)}(\flat_j)-{\gamma'_i}^2\ell^{2-}_{\mathcal{Q}(e_i)}(\flat_j)\right)}{\sum_{i=1}^{n}1+\left({\gamma_i}^2\ell^{2-}_{\mathcal{P}(e_i)}(\flat_j)\cdot{\gamma'_i}^2\ell^{2-}_{\mathcal{Q}(e_i)}(\flat_j)\right)}\right]\right|}\right],$$

for $j = 1, 2, ..., m$.

Theorem 4.1. *Let $\widehat{\mathcal{P}}_{\tau_1,p}$, $\widehat{\mathcal{Q}}_{\tau_2,q}$ and $\widehat{\mathcal{R}}_{\tau_3,r}$ be the any three Type-I EDioNSIVSSs over $(\mathcal{X}, E)$. Then $\widehat{\mathcal{P}}_{\tau_1,p} \sqsubseteq \widehat{\mathcal{Q}}_{\tau_2,q} \sqsubseteq \widehat{\mathcal{R}}_{\tau_3,r}$ implies $Sim(\widehat{\mathcal{P}}_{\tau_1,p}, \widehat{\mathcal{R}}_{\tau_3,r}) \leq Sim(\widehat{\mathcal{Q}}_{\tau_2,q}, \widehat{\mathcal{R}}_{\tau_3,r})$.*

Proof. For $j = 1, 2, ..., m$

$$\left\{\begin{array}{l} \widehat{\mathcal{P}}_{\tau_1,p} \sqsubseteq \widehat{\mathcal{Q}}_{\tau_2,q} \implies \left\{\begin{array}{l} \left[\alpha_i\zeta^-_{\mathcal{P}(e_i)}(\flat_j), \alpha_i\zeta^+_{\mathcal{P}(e_i)}(\flat_j)\right] \leq \left[\alpha'_i\zeta^-_{\mathcal{Q}(e_i)}(\flat_j), \alpha'_i\zeta^+_{\mathcal{Q}(e_i)}(\flat_j)\right] \\ \left[\beta_i\hbar^-_{\mathcal{P}(e_i)}(\flat_j), \beta_i\hbar^+_{\mathcal{P}(e_i)}(\flat_j)\right] \leq \left[\beta'_i\hbar^-_{\mathcal{Q}(e_i)}(\flat_j), \beta'_i\hbar^+_{\mathcal{Q}(e_i)}(\flat_j)\right] \\ \left[\gamma_i\ell^-_{\mathcal{P}(e_i)}(\flat_j), \gamma_i\ell^+_{\mathcal{P}(e_i)}(\flat_j)\right] \geq \left[\gamma'_i\ell^-_{\mathcal{Q}(e_i)}(\flat_j), \gamma'_i\ell^+_{\mathcal{Q}(e_i)}(\flat_j)\right] \\ \left[p^-(e_i), p^+(e_i)\right] \leq \left[q^-(e_i), q^+(e_i)\right] \end{array}\right\} \\ \widehat{\mathcal{P}}_{\tau_1,p} \sqsubseteq \widehat{\mathcal{R}}_{\tau_3,r} \implies \left\{\begin{array}{l} \left[\alpha_i\zeta^-_{\mathcal{P}(e_i)}(\flat_j), \alpha_i\zeta^+_{\mathcal{P}(e_i)}(\flat_j)\right] \leq \left[\alpha''_i\zeta^-_{\mathcal{R}(e_i)}(\flat_j), \alpha''_i\zeta^+_{\mathcal{R}(e_i)}(\flat_j)\right] \\ \left[\beta_i\hbar^-_{\mathcal{P}(e_i)}(\flat_j), \beta_i\hbar^+_{\mathcal{P}(e_i)}(\flat_j)\right] \leq \left[\beta''_i\hbar^-_{\mathcal{R}(e_i)}(\flat_j), \beta''_i\hbar^+_{\mathcal{R}(e_i)}(\flat_j)\right] \\ \left[\gamma_i\ell^-_{\mathcal{P}(e_i)}(\flat_j), \gamma_i\ell^+_{\mathcal{P}(e_i)}(\flat_j)\right] \geq \left[\gamma''_i\ell^-_{\mathcal{R}(e_i)}(\flat_j), \gamma''_i\ell^+_{\mathcal{R}(e_i)}(\flat_j)\right] \\ \left[p^-(e_i), p^+(e_i)\right] \leq \left[r^-(e_i), r^+(e_i)\right] \end{array}\right\} \\ \widehat{\mathcal{Q}}_{\tau_2,q} \sqsubseteq \widehat{\mathcal{R}}_{\tau_3,r} \implies \left\{\begin{array}{l} \left[\alpha'_i\zeta^-_{\mathcal{Q}(e_i)}(\flat_j), \alpha'_i\zeta^+_{\mathcal{Q}(e_i)}(\flat_j)\right] \leq \left[\alpha''_i\zeta^-_{\mathcal{R}(e_i)}(\flat_j), \alpha''_i\zeta^+_{\mathcal{R}(e_i)}(\flat_j)\right] \\ \left[\beta'_i\hbar^-_{\mathcal{Q}(e_i)}(\flat_j), \beta'_i\hbar^+_{\mathcal{Q}(e_i)}(\flat_j)\right] \leq \left[\beta''_i\hbar^-_{\mathcal{R}(e_i)}(\flat_j), \beta''_i\hbar^+_{\mathcal{R}(e_i)}(\flat_j)\right] \\ \left[\gamma'_i\ell^-_{\mathcal{Q}(e_i)}(\flat_j), \gamma'_i\ell^+_{\mathcal{Q}(e_i)}(\flat_j)\right] \geq \left[\gamma''_i\ell^-_{\mathcal{R}(e_i)}(\flat_j), \gamma''_i\ell^+_{\mathcal{R}(e_i)}(\flat_j)\right] \\ \left[q^-(e_i), q^+(e_i)\right] \leq \left[r^-(e_i), r^+(e_i)\right] \end{array}\right\} \end{array}\right\} \ldots\ldots(*)$$

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Clearly, $\left[\left(\alpha_i\zeta^{-}_{\mathcal{P}(e_i)}(\flat_j)\cdot\alpha_i^{''}\zeta^{-}_{\mathcal{R}(e_i)}(\flat_j)\right),\left(\alpha_i\zeta^{+}_{\mathcal{P}(e_i)}(\flat_j)\cdot\alpha_i^{''}\zeta^{+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right]\leq$

$\left[\left(\alpha_i^{'}\zeta^{-}_{\mathcal{Q}(e_i)}(\flat_j)\cdot\alpha_i^{''}\zeta^{-}_{\mathcal{R}(e_i)}(\flat_j)\right),\left(\alpha_i^{'}\zeta^{+}_{\mathcal{Q}(e_i)}(\flat_j)\cdot\alpha_i^{''}\zeta^{+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right]$ implies that

$$\left[\sum_{i=1}^{n}\left(\alpha_i\zeta^{-}_{\mathcal{P}(e_i)}(\flat_j)\cdot\alpha_i^{''}\zeta^{-}_{\mathcal{R}(e_i)}(\flat_j)\right),\sum_{i=1}^{n}\left(\alpha_i\zeta^{+}_{\mathcal{P}(e_i)}(\flat_j)\cdot\alpha_i^{''}\zeta^{+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right]\leq$$

$$\left[\sum_{i=1}^{n}\left(\alpha_i^{'}\zeta^{-}_{\mathcal{Q}(e_i)}(\flat_j)\cdot\alpha_i^{''}\zeta^{-}_{\mathcal{R}(e_i)}(\flat_j)\right),\sum_{i=1}^{n}\left(\alpha_i^{'}\zeta^{+}_{\mathcal{Q}(e_i)}(\flat_j)\cdot\alpha_i^{''}\zeta^{+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right] \quad ....(1)$$

for $j=1,2,...,m$

Clearly, $\left[{\alpha_i}^2\zeta^{2-}_{\mathcal{P}(e_i)}(\flat_j),\ {\alpha_i}^2\zeta^{2+}_{\mathcal{P}(e_i)}(\flat_j)\right]\leq\left[{\alpha_i^{'}}^2\zeta^{2-}_{\mathcal{Q}(e_i)}(\flat_j),\ {\alpha_i^{'}}^2\zeta^{2+}_{\mathcal{Q}(e_i)}(\flat_j)\right]\leq\left[{\alpha_i^{''}}^2\zeta^{2-}_{\mathcal{R}(e_i)}(\flat_j),\ {\alpha_i^{''}}^2\zeta^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right]$

implies that

$$\left[-{\alpha_i}^2\zeta^{2+}_{\mathcal{P}(e_i)}(\flat_j),\ -{\alpha_i}^2\zeta^{2-}_{\mathcal{P}(e_i)}(\flat_j)\right]\geq\left[-{\alpha_i^{'}}^2\zeta^{2+}_{\mathcal{Q}(e_i)}(\flat_j),\ -{\alpha_i^{'}}^2\zeta^{2-}_{\mathcal{Q}(e_i)}(\flat_j)\right]\geq\left[-{\alpha_i^{''}}^2\zeta^{2+}_{\mathcal{R}(e_i)}(\flat_j),\ -{\alpha_i^{''}}^2\zeta^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right]$$

and

$$\left[1-{\alpha_i}^2\zeta^{2-}_{\mathcal{P}(e_i)}(\flat_j),\ 1-{\alpha_i}^2\zeta^{2+}_{\mathcal{P}(e_i)}(\flat_j)\right]\geq\left[1-{\alpha_i^{'}}^2\zeta^{2-}_{\mathcal{Q}(e_i)}(\flat_j),\ 1-{\alpha_i^{'}}^2\zeta^{2+}_{\mathcal{Q}(e_i)}(\flat_j)\right]\geq\left[1-{\alpha_i^{''}}^2\zeta^{2-}_{\mathcal{R}(e_i)}(\flat_j),\ 1-{\alpha_i^{''}}^2\zeta^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right]$$

and

$$\left[\left(\left(1-{\alpha_i}^2\zeta^{2-}_{\mathcal{P}(e_i)}(\flat_j)\right)\cdot\left(1-{\alpha_i^{''}}^2\zeta^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right)\right),\ \left(\left(1-{\alpha_i}^2\zeta^{2+}_{\mathcal{P}(e_i)}(\flat_j)\right)\cdot\left(1-{\alpha_i^{''}}^2\zeta^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)\right]\geq$$

$$\left[\left(\left(1-{\alpha_i^{'}}^2\zeta^{2-}_{\mathcal{Q}(e_i)}(\flat_j)\right)\cdot\left(1-{\alpha_i^{''}}^2\zeta^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right)\right),\ \left(\left(1-{\alpha_i^{'}}^2\zeta^{2+}_{\mathcal{Q}(e_i)}(\flat_j)\right)\cdot\left(1-{\alpha_i^{''}}^2\zeta^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)\right]$$

and

$$\left[\sqrt{\left(\left(1-{\alpha_i}^2\zeta^{2-}_{\mathcal{P}(e_i)}(\flat_j)\right)\cdot\left(1-{\alpha_i^{''}}^2\zeta^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)},\ \sqrt{\left(\left(1-{\alpha_i}^2\zeta^{2+}_{\mathcal{P}(e_i)}(\flat_j)\right)\cdot\left(1-{\alpha_i^{''}}^2\zeta^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)}\right]\geq$$

$$\left[\sqrt{\left(\left(1-{\alpha_i^{'}}^2\zeta^{2-}_{\mathcal{Q}(e_i)}(\flat_j)\right)\cdot\left(1-{\alpha_i^{''}}^2\zeta^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)},\ \sqrt{\left(\left(1-{\alpha_i^{'}}^2\zeta^{2+}_{\mathcal{Q}(e_i)}(\flat_j)\right)\cdot\left(1-{\alpha_i^{''}}^2\zeta^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)}\right]$$

and

$$1-\left[\sqrt{\left(\left(1-{\alpha_i}^2\zeta^{2-}_{\mathcal{P}(e_i)}(\flat_j)\right)\cdot\left(1-{\alpha_i^{''}}^2\zeta^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)},\ \sqrt{\left(\left(1-{\alpha_i}^2\zeta^{2+}_{\mathcal{P}(e_i)}(\flat_j)\right)\cdot\left(1-{\alpha_i^{''}}^2\zeta^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)}\right]\leq$$

$$1-\left[\sqrt{\left(\left(1-{\alpha_i^{'}}^2\zeta^{2-}_{\mathcal{Q}(e_i)}(\flat_j)\right)\cdot\left(1-{\alpha_i^{''}}^2\zeta^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)},\ \sqrt{\left(\left(1-{\alpha_i^{'}}^2\zeta^{2+}_{\mathcal{Q}(e_i)}(\flat_j)\right)\cdot\left(1-{\alpha_i^{''}}^2\zeta^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)}\right]$$

and

$$\left[1-\sqrt{\left(\left(1-{\alpha_i}^2\zeta^{2+}_{\mathcal{P}(e_i)}(\flat_j)\right)\cdot\left(1-{\alpha_i^{''}}^2\zeta^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)},\ 1-\sqrt{\left(\left(1-{\alpha_i}^2\zeta^{2-}_{\mathcal{P}(e_i)}(\flat_j)\right)\cdot\left(1-{\alpha_i^{''}}^2\zeta^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)}\right]\leq$$

$$\left[1-\sqrt{\left(\left(1-{\alpha_i^{'}}^2\zeta^{2+}_{\mathcal{Q}(e_i)}(\flat_j)\right)\cdot\left(1-{\alpha_i^{''}}^2\zeta^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)},\ 1-\sqrt{\left(\left(1-{\alpha_i^{'}}^2\zeta^{2-}_{\mathcal{Q}(e_i)}(\flat_j)\right)\cdot\left(1-{\alpha_i^{''}}^2\zeta^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)}\right]$$

and

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$$\left\{\begin{array}{l}\left[\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-{\alpha_i}^2\zeta^{2+}_{\mathcal{P}(e_i)}(\flat_j)\right)\cdot\left(1-{\alpha''_i}^2\zeta^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)}\right),\ \sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-{\alpha_i}^2\zeta^{2-}_{\mathcal{P}(e_i)}(\flat_j)\right)\cdot\left(1-{\alpha''_i}^2\zeta^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)}\right)\right]\leq\\ \left[\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-{\alpha'_i}^2\zeta^{2+}_{\mathcal{Q}(e_i)}(\flat_j)\right)\cdot\left(1-{\alpha''_i}^2\zeta^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)}\right),\ \sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-{\alpha'_i}^2\zeta^{2-}_{\mathcal{Q}(e_i)}(\flat_j)\right)\cdot\left(1-{\alpha''_i}^2\zeta^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)}\right)\right]\end{array}\right\}..(2)$$

Equation (1) is divided by (2),

$$\frac{\left[\sum_{i=1}^{n}\left(\alpha_i\zeta^{-}_{\mathcal{P}(e_i)}(\flat_j)\cdot\alpha''_i\zeta^{-}_{\mathcal{R}(e_i)}(\flat_j)\right),\ \sum_{i=1}^{n}\left(\alpha_i\zeta^{+}_{\mathcal{P}(e_i)}(\flat_j)\cdot\alpha''_i\zeta^{+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-{\alpha_i}^2\zeta^{2+}_{\mathcal{P}(e_i)}(\flat_j)\right)\cdot\left(1-{\alpha''_i}^2\zeta^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)}\right),\ \sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-{\alpha_i}^2\zeta^{2-}_{\mathcal{P}(e_i)}(\flat_j)\right)\cdot\left(1-{\alpha''_i}^2\zeta^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)}\right)\right]}\leq$$
$$\frac{\left[\sum_{i=1}^{n}\left(\alpha'_i\zeta^{-}_{\mathcal{Q}(e_i)}(\flat_j)\cdot\alpha''_i\zeta^{-}_{\mathcal{R}(e_i)}(\flat_j)\right),\ \sum_{i=1}^{n}\left(\alpha'_i\zeta^{+}_{\mathcal{Q}(e_i)}(\flat_j)\cdot\alpha''_i\zeta^{+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-{\alpha'_i}^2\zeta^{2+}_{\mathcal{Q}(e_i)}(\flat_j)\right)\cdot\left(1-{\alpha''_i}^2\zeta^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)}\right),\ \sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-{\alpha'_i}^2\zeta^{2-}_{\mathcal{Q}(e_i)}(\flat_j)\right)\cdot\left(1-{\alpha''_i}^2\zeta^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)}\right)\right]}$$

implies that

$$\left\{\begin{array}{l}\left[\frac{\left[\sum_{i=1}^{n}\left(\alpha_i\zeta^{-}_{\mathcal{P}(e_i)}(\flat_j)\cdot\alpha''_i\zeta^{-}_{\mathcal{R}(e_i)}(\flat_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-{\alpha_i}^2\zeta^{2-}_{\mathcal{P}(e_i)}(\flat_j)\right)\cdot\left(1-{\alpha''_i}^2\zeta^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)}\right)\right]},\ \frac{\left[\sum_{i=1}^{n}\left(\alpha_i\zeta^{+}_{\mathcal{P}(e_i)}(\flat_j)\cdot\alpha''_i\zeta^{+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-{\alpha_i}^2\zeta^{2+}_{\mathcal{P}(e_i)}(\flat_j)\right)\cdot\left(1-{\alpha''_i}^2\zeta^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)}\right)\right]}\right]\leq\\ \left[\frac{\left[\sum_{i=1}^{n}\left(\alpha'_i\zeta^{-}_{\mathcal{Q}(e_i)}(\flat_j)\cdot\alpha''_i\zeta^{-}_{\mathcal{R}(e_i)}(\flat_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-{\alpha'_i}^2\zeta^{2-}_{\mathcal{Q}(e_i)}(\flat_j)\right)\cdot\left(1-{\alpha''_i}^2\zeta^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)}\right)\right]},\ \frac{\left[\sum_{i=1}^{n}\left(\alpha'_i\zeta^{+}_{\mathcal{Q}(e_i)}(\flat_j)\cdot\alpha''_i\zeta^{+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-{\alpha'_i}^2\zeta^{2+}_{\mathcal{Q}(e_i)}(\flat_j)\right)\cdot\left(1-{\alpha''_i}^2\zeta^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)}\right)\right]}\right]\end{array}\right\}$$

Therefore $\widehat{T_1}\left(\widehat{\mathcal{P}}(e)(\flat_j),\widehat{\mathcal{R}}(e)(\flat_j)\right)\leq\widehat{T_1}\left(\widehat{\mathcal{Q}}(e)(\flat_j),\widehat{\mathcal{R}}(e)(\flat_j)\right)$ ....(3)

Clearly, $\left[\left({\beta_i}^2\hbar^{2-}_{\mathcal{P}(e_i)}(\flat_j)\cdot{\beta''_i}^2\hbar^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right),\left({\beta_i}^2\hbar^{2+}_{\mathcal{P}(e_i)}(\flat_j)\cdot{\beta''_i}^2\hbar^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right]\leq$
$\left[\left({\beta'_i}^2\hbar^{2-}_{\mathcal{Q}(e_i)}(\flat_j)\cdot{\beta''_i}^2\hbar^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right),\left({\beta'_i}^2\hbar^{2+}_{\mathcal{Q}(e_i)}(\flat_j)\cdot{\beta''_i}^2\hbar^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right]$

implies that

$$\left[\sum_{i=1}^{n}\left({\beta_i}^2\hbar^{2-}_{\mathcal{P}(e_i)}(\flat_j)\cdot{\beta''_i}^2\hbar^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right),\sum_{i=1}^{n}\left({\beta_i}^2\hbar^{2+}_{\mathcal{P}(e_i)}(\flat_j)\cdot{\beta''_i}^2\hbar^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right]\leq$$
$$\left[\sum_{i=1}^{n}\left({\beta'_i}^2\hbar^{2-}_{\mathcal{Q}(e_i)}(\flat_j)\cdot{\beta''_i}^2\hbar^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right),\sum_{i=1}^{n}\left({\beta'_i}^2\hbar^{2+}_{\mathcal{Q}(e_i)}(\flat_j)\cdot{\beta''_i}^2\hbar^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right] \quad ....(4)$$

for $j=1,2,...,m$

Clearly, $\left[{\beta_i}^4\hbar^{4-}_{\mathcal{P}(e_i)}(\flat_j),\ {\beta_i}^4\hbar^{4+}_{\mathcal{P}(e_i)}(\flat_j)\right]\leq\left[{\beta'_i}^4\hbar^{4-}_{\mathcal{Q}(e_i)}(\flat_j),\ {\beta'_i}^4\hbar^{4+}_{\mathcal{Q}(e_i)}(\flat_j)\right]\leq\left[{\beta''_i}^4\hbar^{4-}_{\mathcal{R}(e_i)}(\flat_j),\ {\beta''_i}^4\hbar^{4+}_{\mathcal{R}(e_i)}(\flat_j)\right]$.

implies that

$$\left[-{\beta_i}^4\hbar^{4+}_{\mathcal{P}(e_i)}(\flat_j),\ -{\beta_i}^4\hbar^{4-}_{\mathcal{P}(e_i)}(\flat_j)\right]\geq\left[-{\beta'_i}^4\hbar^{4+}_{\mathcal{Q}(e_i)}(\flat_j),\ -{\beta'_i}^4\hbar^{4-}_{\mathcal{Q}(e_i)}(\flat_j)\right]\geq\left[-{\beta''_i}^4\hbar^{4+}_{\mathcal{R}(e_i)}(\flat_j),\ -{\beta''_i}^4\hbar^{4-}_{\mathcal{R}(e_i)}(\flat_j)\right]$$

and

$$\left[1-{\beta_i}^4\hbar^{4-}_{\mathcal{P}(e_i)}(\flat_j),\ 1-{\beta_i}^4\hbar^{4+}_{\mathcal{P}(e_i)}(\flat_j)\right]\geq\left[1-{\beta'_i}^4\hbar^{4-}_{\mathcal{Q}(e_i)}(\flat_j),\ 1-{\beta'_i}^4\hbar^{4+}_{\mathcal{Q}(e_i)}(\flat_j)\right]\geq\left[1-{\beta''_i}^4\hbar^{4-}_{\mathcal{R}(e_i)}(\flat_j),\ 1-{\beta''_i}^4\hbar^{4+}_{\mathcal{R}(e_i)}(\flat_j)\right]$$

and

$$\left[\left(\left(1-{\beta_i}^4\hbar^{4-}_{\mathcal{P}(e_i)}(\flat_j)\right)\cdot\left(1-{\beta''_i}^4\hbar^{4-}_{\mathcal{R}(e_i)}(\flat_j)\right)\right),\ \left(\left(1-{\beta_i}^4\hbar^{4+}_{\mathcal{P}(e_i)}(\flat_j)\right)\cdot\left(1-{\beta''_i}^4\hbar^{4+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)\right]\geq$$

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$$\left[\left(\left(1-{\beta_i'}^4\hbar^{4-}_{\mathcal{Q}(e_i)}(\flat_j)\right)\cdot\left(1-{\beta_i''}^4\hbar^{4-}_{\mathcal{R}(e_i)}(\flat_j)\right)\right),\ \left(\left(1-{\beta_i'}^4\hbar^{4+}_{\mathcal{Q}(e_i)}(\flat_j)\right)\cdot\left(1-{\beta_i''}^4\hbar^{4+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)\right]$$

and

$$\left[\sqrt{\left(\left(1-{\beta_i}^4\hbar^{4-}_{\mathcal{P}(e_i)}(\flat_j)\right)\cdot\left(1-{\beta_i''}^4\hbar^{4-}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)},\ \sqrt{\left(\left(1-{\beta_i}^4\hbar^{4+}_{\mathcal{P}(e_i)}(\flat_j)\right)\cdot\left(1-{\beta_i''}^4\hbar^{4+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)}\right]\geq$$
$$\left[\sqrt{\left(\left(1-{\beta_i'}^4\hbar^{4-}_{\mathcal{Q}(e_i)}(\flat_j)\right)\cdot\left(1-{\beta_i''}^4\hbar^{4-}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)},\ \sqrt{\left(\left(1-{\beta_i'}^4\hbar^{4+}_{\mathcal{Q}(e_i)}(\flat_j)\right)\cdot\left(1-{\beta_i''}^4\hbar^{4+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)}\right]$$

and

$$1-\left[\sqrt{\left(\left(1-{\beta_i}^4\hbar^{4-}_{\mathcal{P}(e_i)}(\flat_j)\right)\cdot\left(1-{\beta_i''}^4\hbar^{4-}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)},\ \sqrt{\left(\left(1-{\beta_i}^4\hbar^{4+}_{\mathcal{P}(e_i)}(\flat_j)\right)\cdot\left(1-{\beta_i''}^4\hbar^{4+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)}\right]\leq$$
$$1-\left[\sqrt{\left(\left(1-{\beta_i'}^4\hbar^{4-}_{\mathcal{Q}(e_i)}(\flat_j)\right)\cdot\left(1-{\beta_i''}^4\hbar^{4-}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)},\ \sqrt{\left(\left(1-{\beta_i'}^4\hbar^{4+}_{\mathcal{Q}(e_i)}(\flat_j)\right)\cdot\left(1-{\beta_i''}^4\hbar^{4+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)}\right]$$

and

$$\left[1-\sqrt{\left(\left(1-{\beta_i}^4\hbar^{4+}_{\mathcal{P}(e_i)}(\flat_j)\right)\cdot\left(1-{\beta_i''}^4\hbar^{4+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)},\ 1-\sqrt{\left(\left(1-{\beta_i}^4\hbar^{4-}_{\mathcal{P}(e_i)}(\flat_j)\right)\cdot\left(1-{\beta_i''}^4\hbar^{4-}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)}\right]\leq$$
$$\left[1-\sqrt{\left(\left(1-{\beta_i'}^4\hbar^{4+}_{\mathcal{Q}(e_i)}(\flat_j)\right)\cdot\left(1-{\beta_i''}^4\hbar^{4+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)},\ 1-\sqrt{\left(\left(1-{\beta_i'}^4\hbar^{4-}_{\mathcal{Q}(e_i)}(\flat_j)\right)\cdot\left(1-{\beta_i''}^4\hbar^{4-}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)}\right]$$

and

$$\left\{\begin{array}{l}\left[\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-{\beta_i}^4\hbar^{4+}_{\mathcal{P}(e_i)}(\flat_j)\right)\cdot\left(1-{\beta_i''}^4\hbar^{4+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)}\right),\ \sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-{\beta_i}^4\hbar^{4-}_{\mathcal{P}(e_i)}(\flat_j)\right)\cdot\left(1-{\beta_i''}^4\hbar^{4-}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)}\right)\right]\leq\\ \left[\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-{\beta_i'}^4\hbar^{4+}_{\mathcal{Q}(e_i)}(\flat_j)\right)\cdot\left(1-{\beta_i''}^4\hbar^{4+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)}\right),\ \sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-{\beta_i'}^4\hbar^{4-}_{\mathcal{Q}(e_i)}(\flat_j)\right)\cdot\left(1-{\beta_i''}^4\hbar^{4-}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)}\right)\right]\end{array}\right\}..(5)$$

Equation (4) is divided by (5),

$$\frac{\left[\sum_{i=1}^{n}\left({\beta_i}^2\hbar^{2-}_{\mathcal{P}(e_i)}(\flat_j)\cdot{\beta_i''}^2\hbar^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right),\ \sum_{i=1}^{n}\left({\beta_i}^2\hbar^{2+}_{\mathcal{P}(e_i)}(\flat_j)\cdot{\beta_i''}^2\hbar^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-{\beta_i}^4\hbar^{4+}_{\mathcal{P}(e_i)}(\flat_j)\right)\cdot\left(1-{\beta_i''}^4\hbar^{4+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)}\right),\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-{\beta_i}^4\hbar^{4-}_{\mathcal{P}(e_i)}(\flat_j)\right)\cdot\left(1-{\beta_i''}^4\hbar^{4-}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)}\right)\right]}\leq$$
$$\frac{\left[\sum_{i=1}^{n}\left({\beta_i'}^2\hbar^{2-}_{\mathcal{Q}(e_i)}(\flat_j)\cdot{\beta_i''}^2\hbar^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right),\ \sum_{i=1}^{n}\left({\beta_i'}^2\hbar^{2+}_{\mathcal{Q}(e_i)}(\flat_j)\cdot{\beta_i''}^2\hbar^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-{\beta_i'}^4\hbar^{4+}_{\mathcal{Q}(e_i)}(\flat_j)\right)\cdot\left(1-{\beta_i''}^4\hbar^{4+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)}\right),\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-{\beta_i'}^4\hbar^{4-}_{\mathcal{Q}(e_i)}(\flat_j)\right)\cdot\left(1-{\beta_i''}^4\hbar^{4-}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)}\right)\right]}$$

implies that

$$\left\{\begin{array}{l}\left[\frac{\left[\sum_{i=1}^{n}\left({\beta_i}^2\hbar^{2-}_{\mathcal{P}(e_i)}(\flat_j)\cdot{\beta_i''}^2\hbar^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-\beta_i{}^4\hbar^{4-}_{\mathcal{P}(e_i)}(\flat_j)\right)\cdot\left(1-{\beta_i''}^4\hbar^{4-}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)}\right)\right]},\ \frac{\left[\sum_{i=1}^{n}\left({\beta_i}^2\hbar^{2+}_{\mathcal{P}(e_i)}(\flat_j)\cdot{\beta_i''}^2\hbar^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-\beta_i{}^4\hbar^{4+}_{\mathcal{P}(e_i)}(\flat_j)\right)\cdot\left(1-{\beta_i''}^4\hbar^{4+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)}\right)\right]}\right]\leq\\ \left[\frac{\left[\sum_{i=1}^{n}\left({\beta_i'}^2\hbar^{2-}_{\mathcal{Q}(e_i)}(\flat_j)\cdot{\beta_i''}^2\hbar^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-{\beta_i'}^4\hbar^{4-}_{\mathcal{Q}(e_i)}(\flat_j)\right)\cdot\left(1-{\beta_i''}^4\hbar^{4-}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)}\right)\right]},\ \frac{\left[\sum_{i=1}^{n}\left({\beta_i'}^2\hbar^{2+}_{\mathcal{Q}(e_i)}(\flat_j)\cdot{\beta_i''}^2\hbar^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-{\beta_i'}^4\hbar^{4+}_{\mathcal{Q}(e_i)}(\flat_j)\right)\cdot\left(1-{\beta_i''}^4\hbar^{4+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right)}\right)\right]}\right]\end{array}\right\}$$

Therefore $\widehat{T_2}\left(\widehat{\mathcal{P}}(e)(\flat_j),\widehat{\mathcal{R}}(e)(\flat_j)\right)\leq\widehat{T_2}\left(\widehat{\mathcal{Q}}(e)(\flat_j),\widehat{\mathcal{R}}(e)(\flat_j)\right)$ ....(6)

Clearly, $\left[{\gamma_i}^2\ell^{2-}_{\mathcal{P}(e_i)}(\flat_j),{\gamma_i}^2\ell^{2+}_{\mathcal{P}(e_i)}(\flat_j)\right]\ \geq\ \left[{\gamma_i'}^2\ell^{2-}_{\mathcal{Q}(e_i)}(\flat_j),{\gamma_i'}^2\ell^{2+}_{\mathcal{Q}(e_i)}(\flat_j)\right]\ \geq\ \left[{\gamma_i''}^2\ell^{2-}_{\mathcal{R}(e_i)}(\flat_j),{\gamma_i''}^2\ell^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right]$

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$$\left[\gamma_i^2 \ell^{2-}_{\mathcal{P}(e_i)}(\flat_j), \gamma_i^2 \ell^{2+}_{\mathcal{P}(e_i)}(\flat_j)\right] - \left[\gamma_i^{''2} \ell^{2-}_{\mathcal{R}(e_i)}(\flat_j), \gamma_i^{''2} \ell^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right] \geq \left[\gamma_i^{'2} \ell^{2-}_{\mathcal{Q}(e_i)}(\flat_j), \gamma_i^{'2} \ell^{2+}_{\mathcal{Q}(e_i)}(\flat_j)\right]$$
$$-\left[\gamma_i^{''2} \ell^{2-}_{\mathcal{R}(e_i)}(\flat_j), \gamma_i^{''2} \ell^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right]$$

implies

$$\left|\left[\gamma_i^2 \ell^{2-}_{\mathcal{P}(e_i)}(\flat_j), \gamma_i^2 \ell^{2+}_{\mathcal{P}(e_i)}(\flat_j)\right] - \left[\gamma_i^{''2} \ell^{2-}_{\mathcal{R}(e_i)}(\flat_j), \gamma_i^{''2} \ell^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right]\right| \geq \left|\left[\gamma_i^{'2} \ell^{2-}_{\mathcal{Q}(e_i)}(\flat_j), \gamma_i^{'2} \ell^{2+}_{\mathcal{Q}(e_i)}(\flat_j)\right]\right.$$
$$\left.-\left[\gamma_i^{''2} \ell^{2-}_{\mathcal{R}(e_i)}(\flat_j), \gamma_i^{''2} \ell^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right]\right|$$

$$\left|\left[\gamma_i^2 \ell^{2-}_{\mathcal{P}(e_i)}(\flat_j) - \gamma_i^{''2} \ell^{2+}_{\mathcal{R}(e_i)}(\flat_j)\ ,\ \gamma_i^2 \ell^{2+}_{\mathcal{P}(e_i)}(\flat_j) - \gamma_i^{''2} \ell^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right]\right| \geq$$
$$\left|\left[\gamma_i^{'2} \ell^{2-}_{\mathcal{Q}(e_i)}(\flat_j) - \gamma_i^{''2} \ell^{2+}_{\mathcal{R}(e_i)}(\flat_j)\ ,\ \gamma_i^{'2} \ell^{2+}_{\mathcal{Q}(e_i)}(\flat_j) - \gamma_i^{''2} \ell^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right]\right|.$$

Hence $\left|\left[\sum_{i=1}^{n}\left(\gamma_i^2 \ell^{2-}_{\mathcal{P}(e_i)}(\flat_j) - \gamma_i^{''2} \ell^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)\ ,\ \sum_{i=1}^{n}\left(\gamma_i^2 \ell^{2+}_{\mathcal{P}(e_i)}(\flat_j) - \gamma_i^{''2} \ell^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right)\right]\right| \geq$

$\left|\left[\sum_{i=1}^{n}\left(\gamma_i^{'2} \ell^{2-}_{\mathcal{Q}(e_i)}(\flat_j) - \gamma_i^{''2} \ell^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)\ ,\ \sum_{i=1}^{n}\left(\gamma_i^{'2} \ell^{2+}_{\mathcal{Q}(e_i)}(\flat_j) - \gamma_i^{''2} \ell^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right)\right]\right|$ ....(7)

Also, $\left[\left(\gamma_i^2 \ell^{2-}_{\mathcal{P}(e_i)}(\flat_j) \cdot \gamma_i^{''2} \ell^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right), \left(\gamma_i^2 \ell^{2+}_{\mathcal{P}(e_i)}(\flat_j) \cdot \gamma_i^{''2} \ell^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right] \geq$

$\left[\left(\gamma_i^{'2} \ell^{2-}_{\mathcal{Q}(e_i)}(\flat_j) \cdot \gamma_i^{''2} \ell^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right), \left(\gamma_i^{'2} \ell^{2+}_{\mathcal{Q}(e_i)}(\flat_j) \cdot \gamma_i^{''2} \ell^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right]$

implies that

$$\left|\left[\left(\gamma_i^2 \ell^{2-}_{\mathcal{P}(e_i)}(\flat_j) \cdot \gamma_i^{''2} \ell^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right), \left(\gamma_i^2 \ell^{2+}_{\mathcal{P}(e_i)}(\flat_j) \cdot \gamma_i^{''2} \ell^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right]\right| \geq$$
$$\left|\left[\left(\gamma_i^{'2} \ell^{2-}_{\mathcal{Q}(e_i)}(\flat_j) \cdot \gamma_i^{''2} \ell^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right), \left(\gamma_i^{'2} \ell^{2+}_{\mathcal{Q}(e_i)}(\flat_j) \cdot \gamma_i^{''2} \ell^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right]\right|$$

$$\left|\left[\sum_{i=1}^{n} 1 + \left(\gamma_i^2 \ell^{2-}_{\mathcal{P}(e_i)}(\flat_j) \cdot \gamma_i^{''2} \ell^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right), \sum_{i=1}^{n} 1 + \left(\gamma_i^2 \ell^{2+}_{\mathcal{P}(e_i)}(\flat_j) \cdot \gamma_i^{''2} \ell^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right]\right| \geq$$

$\left|\left[\sum_{i=1}^{n} 1 + \left(\gamma_i^{'2} \ell^{2-}_{\mathcal{Q}(e_i)}(\flat_j) \cdot \gamma_i^{''2} \ell^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right), \sum_{i=1}^{n} 1 + \left(\gamma_i^{'2} \ell^{2+}_{\mathcal{Q}(e_i)}(\flat_j) \cdot \gamma_i^{''2} \ell^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right]\right|$, for $j = 1, 2, ..., m$........(8)

Equation (7) is divided by (8), we get

$$\frac{\left|\left[\sum_{i=1}^{n}\left(\gamma_i^2 \ell^{2-}_{\mathcal{P}(e_i)}(\flat_j) - \gamma_i^{''2} \ell^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)\ ,\ \sum_{i=1}^{n}\left(\gamma_i^2 \ell^{2+}_{\mathcal{P}(e_i)}(\flat_j) - \gamma_i^{''2} \ell^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right)\right]\right|}{\left|\left[\sum_{i=1}^{n} 1 + \left(\gamma_i^2 \ell^{2-}_{\mathcal{P}(e_i)}(\flat_j) \cdot \gamma_i^{''2} \ell^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right), \sum_{i=1}^{n} 1 + \left(\gamma_i^2 \ell^{2+}_{\mathcal{P}(e_i)}(\flat_j) \cdot \gamma_i^{''2} \ell^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right]\right|} \geq$$
$$\frac{\left|\left[\sum_{i=1}^{n}\left(\gamma_i^{'2} \ell^{2-}_{\mathcal{Q}(e_i)}(\flat_j) - \gamma_i^{''2} \ell^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)\ ,\ \sum_{i=1}^{n}\left(\gamma_i^{'2} \ell^{2+}_{\mathcal{Q}(e_i)}(\flat_j) - \gamma_i^{''2} \ell^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right)\right]\right|}{\left|\left[\sum_{i=1}^{n} 1 + \left(\gamma_i^{'2} \ell^{2-}_{\mathcal{Q}(e_i)}(\flat_j) \cdot \gamma_i^{''2} \ell^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right), \sum_{i=1}^{n} 1 + \left(\gamma_i^{'2} \ell^{2+}_{\mathcal{Q}(e_i)}(\flat_j) \cdot \gamma_i^{''2} \ell^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)\right]\right|}$$

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implies that

$$\left|\left[\frac{\sum_{i=1}^{n}\left({\gamma_i}^2\ell^{2-}_{\mathcal{P}(e_i)}(\flat_j)-{\gamma_i''}^2\ell^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)}{\sum_{i=1}^{n}1+\left({\gamma_i}^2\ell^{2+}_{\mathcal{P}(e_i)}(\flat_j)\cdot{\gamma_i''}^2\ell^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)}\ ,\ \frac{\sum_{i=1}^{n}\left({\gamma_i}^2\ell^{2+}_{\mathcal{P}(e_i)}(\flat_j)-{\gamma_i''}^2\ell^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right)}{\sum_{i=1}^{n}1+\left({\gamma_i}^2\ell^{2-}_{\mathcal{P}(e_i)}(\flat_j)\cdot{\gamma_i''}^2\ell^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right)}\right]\right| \geq$$
$$\left|\left[\frac{\sum_{i=1}^{n}\left({\gamma_i'}^2\ell^{2-}_{\mathcal{Q}(e_i)}(\flat_j)-{\gamma_i''}^2\ell^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)}{\sum_{i=1}^{n}1+\left({\gamma_i'}^2\ell^{2+}_{\mathcal{Q}(e_i)}(\flat_j)\cdot{\gamma_i''}^2\ell^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)}\ ,\ \frac{\sum_{i=1}^{n}\left({\gamma_i'}^2\ell^{2+}_{\mathcal{Q}(e_i)}(\flat_j)-{\gamma_i''}^2\ell^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right)}{\sum_{i=1}^{n}1+\left({\gamma_i'}^2\ell^{2-}_{\mathcal{Q}(e_i)}(\flat_j)\cdot{\gamma_i''}^2\ell^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right)}\right]\right|$$

and

$$\sqrt{\left|\left[\frac{\sum_{i=1}^{n}\left({\gamma_i}^2\ell^{2-}_{\mathcal{P}(e_i)}(\flat_j)-{\gamma_i''}^2\ell^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)}{\sum_{i=1}^{n}1+\left({\gamma_i}^2\ell^{2+}_{\mathcal{P}(e_i)}(\flat_j)\cdot{\gamma_i''}^2\ell^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)}\ ,\ \frac{\sum_{i=1}^{n}\left({\gamma_i}^2\ell^{2+}_{\mathcal{P}(e_i)}(\flat_j)-{\gamma_i''}^2\ell^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right)}{\sum_{i=1}^{n}1+\left({\gamma_i}^2\ell^{2-}_{\mathcal{P}(e_i)}(\flat_j)\cdot{\gamma_i''}^2\ell^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right)}\right]\right|} \geq$$
$$\sqrt{\left|\left[\frac{\sum_{i=1}^{n}\left({\gamma_i'}^2\ell^{2-}_{\mathcal{Q}(e_i)}(\flat_j)-{\gamma_i''}^2\ell^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)}{\sum_{i=1}^{n}1+\left({\gamma_i'}^2\ell^{2+}_{\mathcal{Q}(e_i)}(\flat_j)\cdot{\gamma_i''}^2\ell^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)}\ ,\ \frac{\sum_{i=1}^{n}\left({\gamma_i'}^2\ell^{2+}_{\mathcal{Q}(e_i)}(\flat_j)-{\gamma_i''}^2\ell^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right)}{\sum_{i=1}^{n}1+\left({\gamma_i'}^2\ell^{2-}_{\mathcal{Q}(e_i)}(\flat_j)\cdot{\gamma_i''}^2\ell^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right)}\right]\right|}$$

and

$$\left[1-\sqrt{\left|\left[\frac{\sum_{i=1}^{n}\left({\gamma_i}^2\ell^{2-}_{\mathcal{P}(e_i)}(\flat_j)-{\gamma_i''}^2\ell^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)}{\sum_{i=1}^{n}1+\left({\gamma_i}^2\ell^{2+}_{\mathcal{P}(e_i)}(\flat_j)\cdot{\gamma_i''}^2\ell^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)}\ ,\ \frac{\sum_{i=1}^{n}\left({\gamma_i}^2\ell^{2+}_{\mathcal{P}(e_i)}(\flat_j)-{\gamma_i''}^2\ell^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right)}{\sum_{i=1}^{n}1+\left({\gamma_i}^2\ell^{2-}_{\mathcal{P}(e_i)}(\flat_j)\cdot{\gamma_i''}^2\ell^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right)}\right]\right|}\,\right] \leq$$
$$\left[1-\sqrt{\left|\left[\frac{\sum_{i=1}^{n}\left({\gamma_i'}^2\ell^{2-}_{\mathcal{Q}(e_i)}(\flat_j)-{\gamma_i''}^2\ell^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)}{\sum_{i=1}^{n}1+\left({\gamma_i'}^2\ell^{2+}_{\mathcal{Q}(e_i)}(\flat_j)\cdot{\gamma_i''}^2\ell^{2+}_{\mathcal{R}(e_i)}(\flat_j)\right)}\ ,\ \frac{\sum_{i=1}^{n}\left({\gamma_i'}^2\ell^{2+}_{\mathcal{Q}(e_i)}(\flat_j)-{\gamma_i''}^2\ell^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right)}{\sum_{i=1}^{n}1+\left({\gamma_i'}^2\ell^{2-}_{\mathcal{Q}(e_i)}(\flat_j)\cdot{\gamma_i''}^2\ell^{2-}_{\mathcal{R}(e_i)}(\flat_j)\right)}\right]\right|}\,\right].$$

Therefore $\widehat{S}\left(\widehat{\mathcal{P}}(e)(\flat_j), \widehat{\mathcal{R}}(e)(\flat_j)\right) \leq \widehat{S}\left(\widehat{\mathcal{Q}}(e)(\flat_j), \widehat{\mathcal{R}}(e)(\flat_j)\right)$ ....(9)

Minimum of Equations (3), (6), (9) of lower bound sides with adding, also Maximum of Equations (3), (6), (9) of upper bound sides with adding for each $j = 1, 2, ..., m$.
Finally, divided by m on both sides, we get $\varphi(\widehat{\mathcal{P}}, \widehat{\mathcal{R}}) \leq \varphi(\widehat{\mathcal{Q}}, \widehat{\mathcal{R}})$ ....(10)
By Equation (*), Clearly $\left|p(e) - r(e)\right| \geq \left|q(e) - r(e)\right|$ and $\left|p(e) + r(e)\right| \leq \left|q(e) + r(e)\right|$.
Hence $\sum_{i=1}^{n} \left|p(e_i) - r(e_i)\right| \geq \sum_{i=1}^{n} \left|q(e_i) - r(e_i)\right|$.
Thus, $-\sum_{i=1}^{n} \left|p(e_i)-r(e_i)\right| \leq -\sum_{i=1}^{n} \left|q(e_i)-r(e_i)\right|$ ....(11)
and $\sum_{i=1}^{n} \left|p(e_i) + r(e_i)\right| \leq \sum_{i=1}^{n} \left|q(e_i) + r(e_i)\right|$ ....(12)
Equation (11) is divided by (12), we get

$$-\frac{\sum_{i=1}^{n}\left|p(e_i)-r(e_i)\right|}{\sum_{i=1}^{n}\left|p(e_i)+r(e_i)\right|} \leq -\frac{\sum_{i=1}^{n}\left|q(e_i)-r(e_i)\right|}{\sum_{i=1}^{n}\left|q(e_i)+r(e_i)\right|}$$

and $1 - \frac{\sum_{i=1}^{n}\left|p(e_i)-r(e_i)\right|}{\sum_{i=1}^{n}\left|p(e_i)+r(e_i)\right|} \leq 1 - \frac{\sum_{i=1}^{n}\left|q(e_i)-r(e_i)\right|}{\sum_{i=1}^{n}\left|q(e_i)+r(e_i)\right|}$.
Hence $\psi(p, r) \leq \psi(q, r)$. ....(13)
Multiply by Equations (10) and (13), $\varphi(\widehat{\mathcal{P}}, \widehat{\mathcal{R}}) \cdot \psi(p, r) \leq \varphi(\widehat{\mathcal{Q}}, \widehat{\mathcal{R}}) \cdot \psi(q, r)$.
Hence $Sim(\widehat{\mathcal{P}}_{\tau_1,p}, \widehat{\mathcal{R}}_{\tau_3,r}) \leq Sim(\widehat{\mathcal{Q}}_{\tau_2,q}, \widehat{\mathcal{R}}_{\tau_3,r})$.

Example 4.2. Calculate the similarity measure between two Type-I EDioNSIVSSs namely $\widehat{\mathcal{P}}_{\tau_1,p}$ and $\widehat{\mathcal{Q}}_{\tau_2,q}$. We choose $\mathcal{X} = \{\flat_1, \flat_2, \flat_3\}$ and parameter $E = \{e_1, e_2, e_3\}$ can be defined below:

$\mathcal{P}_p(e)$	e_1	e_2
$\mathcal{P}(e)(\flat_1)$	$[0.5, 0.6]$ $[0.4, 0.65]$ $[0.7, 0.75], (0.5, 0.35, 0.15)$	$[0.7, 0.75]$ $[0.5, 0.7]$ $[0.6, 0.65], (0.4, 0.3, 0.3)$
$\mathcal{P}(e)(\flat_2)$	$[0.6, 0.8]$ $[0.5, 0.55]$ $[0.4, 0.45], (0.25, 0.3, 0.45)$	$[0.8, 0.9]$ $[0.55, 0.65]$ $[0.45, 0.5], (0.35, 0.15, 0.35)$
$\mathcal{P}(e)(\flat_3)$	$[0.5, 0.65]$ $[0.5, 0.65]$ $[0.35, 0.7], (0.2, 0.2, 0.4)$	$[0.35, 0.5]$ $[0.4, 0.55]$ $[0.45, 0.55], (0.1, 0.25, 0.5)$
$p(e)$	$(0.5, 0.5, 0.4)$	$(0.4, 0.6, 0.3)$

$\mathcal{P}_p(e)$	e_3
$\mathcal{P}(e)(\flat_1)$	$[0.6, 0.7]$ $[0.5, 0.55]$ $[0.55, 0.7], (0.6, 0.2, 0.1)$
$\mathcal{P}(e)(\flat_2)$	$[0.5, 0.65]$ $[0.55, 0.75]$ $[0.3, 0.4], (0.4, 0.2, 0.35)$
$\mathcal{P}(e)(\flat_3)$	$[0.15, 0.2]$ $[0.6, 0.75]$ $[0.55, 0.65], (0.3, 0.15, 0.55)$
$p(e)$	$(0.6, 0.45, 0.2)$

$\mathcal{Q}_q(e)$	e_1	e_2
$\mathcal{Q}(e)(\flat_1)$	$[0.35, 0.5]$ $[0.55, 0.6]$ $[0.4, 0.65], (0.45, 0.35, 0.2)$	$[0.5, 0.6]$ $[0.3, 0.5]$ $[0.45, 0.55], (0.5, 0.2, 0.1)$
$\mathcal{Q}(e)(\flat_2)$	$[0.65, 0.7]$ $[0.65, 0.75]$ $[0.5, 0.55], (0.4, 0.45, 0.15)$	$[0.55, 0.8]$ $[0.85, 0.9]$ $[0.4, 0.5], (0.45, 0.35, 0.2)$
$\mathcal{Q}(e)(\flat_3)$	$[0.55, 0.6]$ $[0.4, 0.45]$ $[0.2, 0.25], (0.2, 0.3, 0.45)$	$[0.45, 0.5]$ $[0.4, 0.55]$ $[0.3, 0.35], (0.4, 0.2, 0.4)$
$q(e)$	$(0.45, 0.55, 0.45)$	$(0.55, 0.15, 0.65)$

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$\mathcal{Q}_q(e)$	e_3
$\mathcal{Q}(e)(\flat_1)$	$[0.4, 0.45]$ $[0.6, 0.75]$ $[0.4, 0.45], (0.35, 0.45, 0.1)$
$\mathcal{Q}(e)(\flat_2)$	$[0.45, 0.7]$ $[0.75, 0.85]$ $[0.6, 0.7], (0.3, 0.4, 0.3)$
$\mathcal{Q}(e)(\flat_3)$	$[0.5, 0.6]$ $[0.5, 0.55]$ $[0.4, 0.45], (0.1, 0.25, 0.5)$
$q(e)$	$(0.65, 0.25, 0.55)$

Now, $T_1\left(\widehat{\mathcal{P}}(e)(\flat_1), \widehat{\mathcal{Q}}(e)(\flat_1)\right) = \left[\frac{0.159775}{0.190559}, \frac{0.22365}{0.264324}\right] = \Big[0.838455, 0.846122\Big]$,
$T_2\left(\widehat{\mathcal{P}}(e)(\flat_1), \widehat{\mathcal{Q}}(e)(\flat_1)\right) = \left[\frac{0.001536}{0.003849}, \frac{0.004102}{0.009916}\right] = \Big[0.39914, 0.413651\Big]$ and
$S\left(\widehat{\mathcal{P}}(e)(\flat_1), \widehat{\mathcal{Q}}(e)(\flat_1)\right) = 1 - \sqrt{\left|\left[\frac{0.0245}{3.000339}, \frac{0.045556}{3.000141}\right]\right|} = 1 - \Big[0.090365, 0.123226\Big] = \Big[0.876774, 0.909635\Big]$.
Hence,

$$\begin{bmatrix} \min\left\{T_1^-\left(\widehat{\mathcal{P}}(e)(\flat_1), \widehat{\mathcal{Q}}(e)(\flat_1)\right), T_2^-\left(\widehat{\mathcal{P}}(e)(\flat_1), \widehat{\mathcal{Q}}(e)(\flat_1)\right), S^-\left(\widehat{\mathcal{P}}(e)(\flat_1), \widehat{\mathcal{Q}}(e)(\flat_1)\right)\right\}, \\ \max\left\{T_1^+\left(\widehat{\mathcal{P}}(e)(\flat_1), \widehat{\mathcal{Q}}(e)(\flat_1)\right), T_2^+\left(\widehat{\mathcal{P}}(e)(\flat_1), \widehat{\mathcal{Q}}(e)(\flat_1)\right), S^+\left(\widehat{\mathcal{P}}(e)(\flat_1), \widehat{\mathcal{Q}}(e)(\flat_1)\right)\right\} \end{bmatrix} = \Big[0.39914, 0.909635\Big].$$

Similarly, $T_1\left(\widehat{\mathcal{P}}(e)(\flat_2), \widehat{\mathcal{Q}}(e)(\flat_2)\right) = \Big[0.937257, 0.974499\Big]$,
$T_2\left(\widehat{\mathcal{P}}(e)(\flat_2), \widehat{\mathcal{Q}}(e)(\flat_2)\right) = \Big[0.301431, 0.353245\Big]$ and
$S\left(\widehat{\mathcal{P}}(e)(\flat_2), \widehat{\mathcal{Q}}(e)(\flat_2)\right) = \Big[0.875106, 0.950599\Big]$.
Hence,

$$\begin{bmatrix} \min\left\{T_1^-\left(\widehat{\mathcal{P}}(e)(\flat_2), \widehat{\mathcal{Q}}(e)(\flat_2)\right), T_2^-\left(\widehat{\mathcal{P}}(e)(\flat_2), \widehat{\mathcal{Q}}(e)(\flat_2)\right), S^-\left(\widehat{\mathcal{P}}(e)(\flat_2), \widehat{\mathcal{Q}}(e)(\flat_2)\right)\right\}, \\ \max\left\{T_1^+\left(\widehat{\mathcal{P}}(e)(\flat_2), \widehat{\mathcal{Q}}(e)(\flat_2)\right), T_2^+\left(\widehat{\mathcal{P}}(e)(\flat_2), \widehat{\mathcal{Q}}(e)(\flat_2)\right), S^+\left(\widehat{\mathcal{P}}(e)(\flat_2), \widehat{\mathcal{Q}}(e)(\flat_2)\right)\right\} \end{bmatrix} = \Big[0.301431, 0.974499\Big].$$

Similarly, $T_1\left(\widehat{\mathcal{P}}(e)(\flat_3), \widehat{\mathcal{Q}}(e)(\flat_3)\right) = \Big[0.646299, 0.71779\Big]$,
$T_2\left(\widehat{\mathcal{P}}(e)(\flat_3), \widehat{\mathcal{Q}}(e)(\flat_3)\right) = \Big[0.882655, 0.946827\Big]$ and
$S\left(\widehat{\mathcal{P}}(e)(\flat_3), \widehat{\mathcal{Q}}(e)(\flat_3)\right) = \Big[0.729815, 0.83812\Big]$.
Hence,

$$\begin{bmatrix} \min\left\{T_1^-\left(\widehat{\mathcal{P}}(e)(\flat_3), \widehat{\mathcal{Q}}(e)(\flat_3)\right), T_2^-\left(\widehat{\mathcal{P}}(e)(\flat_3), \widehat{\mathcal{Q}}(e)(\flat_3)\right), S^-\left(\widehat{\mathcal{P}}(e)(\flat_3), \widehat{\mathcal{Q}}(e)(\flat_3)\right)\right\}, \\ \max\left\{T_1^+\left(\widehat{\mathcal{P}}(e)(\flat_3), \widehat{\mathcal{Q}}(e)(\flat_3)\right), T_2^+\left(\widehat{\mathcal{P}}(e)(\flat_3), \widehat{\mathcal{Q}}(e)(\flat_3)\right), S^+\left(\widehat{\mathcal{P}}(e)(\flat_3), \widehat{\mathcal{Q}}(e)(\flat_3)\right)\right\} \end{bmatrix} = \Big[0.646299, 0.946827\Big].$$

Thus, $\varphi(\widehat{\mathcal{P}}, \widehat{\mathcal{Q}}) = \left[\frac{0.39914+0.301431+0.646299}{3}, \frac{0.909635+0.974499+0.946827}{3}\right] = \Big[0.448957, 0.943654\Big]$.
Also, $\psi(p, q) = 0.792683$.
Hence, $Sim(\widehat{\mathcal{P}}_{\tau_1,p}, \widehat{\mathcal{Q}}_{\tau_2,q}) = \Big[0.448957 \times 0.792683, 0.943654 \times 0.792683\Big] = \Big[0.3558802,\ 0.748018\Big]$.

5 Real life applications

We offer an application in this paper that uses the suggested similarity measure of Type-I EDioNSIVSSs to solve a practical problem. This method of measuring the similarity between two Type-I EDioNSIVSS can be used to determine if a patient is unwell or not. First, we provide the definition as follows:

Definition 5.1. Let $\widehat{\mathcal{P}}_{\tau_1,p}$ and $\widehat{\mathcal{Q}}_{\tau_2,q}$ be two Type-I EDioNSIVSS's over the same soft universe $(\mathcal{X}, E)$. We call the two Type-I EDioNSIVSS's to be significantly similar if $Sim^+(\widehat{\mathcal{P}}_{\tau_1,p}, \widehat{\mathcal{Q}}_{\tau_2,q}) > 0.70$.

With the assistance of a medical professional, we first create a Type-I EDioNSIVSS for the illness and a Type-I EDioNSIVSS for the ill individual. Next, we determine the degree of similarity between two Type-I EDioNSIVSSs. We deduce that the person may have a sickness if they are noticeably similar, and not if they are not.

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5.1 Algorithm

Step 1. Input the Type-I EDioNSIVSS.
Step 2. Input the set of parameters $U \subseteq E$.
Step 3. Compute $T_1(\flat_j)$, $T_2(\flat_j)$ and $S(\flat_j)$ and $1 \leq j \leq m$.
Step 4. Calculate $\varphi = \frac{1}{m}\sum_{j=1}^{m}\left[\min\{T_1^-(\flat_j), T_2^-(\flat_j), S^-(\flat_j)\}, \max\{T_1^+(\flat_j), T_2^+(\flat_j), S^+(\flat_j)\}\right]$.
Step 5. Determine $\psi(p,q) = 1 - \frac{\sum_{i=1}^{n}\left|p(e_i)-q(e_i)\right|}{\sum_{i=1}^{n}\left|p(e_i)+q(e_i)\right|}$.
Step 6. Compute the similarity measure = $\varphi \cdot \psi$.
Step 7. Select similarity measure, when suitable criteria for significantly similar.
Step 8. Finally, decision to the problem.
Step 9. End.

5.2 Data Analysis

Suppose that there are five patients $\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3$ in a hospital with certain symptoms of viral fever. Let the universal set contain only three elements. That is $\mathcal{X} = \{\flat_1$: severe, $\flat_2$: mild, $\flat_3$: no$\}$. Were the set of parameters E is the set of certain symptoms of viral fever is represented by $E = \{e_1$: fatigue, e_2: weakness, e_3 : headache$\}$.

Table 1 is represented by the viral fever prepared with the help of a medical person.

Table 1
Type-I EDioNSIVSS model for pneumonia (viral fever).

$\widehat{\mathcal{M}_{p(e)}}$	e_1	e_2	e_3
$\widehat{\mathcal{M}(e)(\flat_1)}$	$[0.85, 0.9]$ $[0.75, 0.85]$ $[0.8, 0.85], (0.45, 0.4, 0.1)$	$[0.7, 0.85]$ $[0.75, 0.8]$ $[0.65, 0.75], (0.5, 0.35, 0.15)$	$[0.75, 0.85]$ $[0.85, 0.9]$ $[0.6, 0.8], (0.35, 0.45, 0.15)$
$\widehat{\mathcal{M}(e)(\flat_2)}$	$[0.4, 0.6]$ $[0.65, 0.85]$ $[0.45, 0.5], (0.3, 0.2, 0.4)$	$[0.55, 0.7]$ $[0.8, 0.85]$ $[0.5, 0.55], (0.4, 0.3, 0.2)$	$[0.6, 0.75]$ $[0.85, 0.9]$ $[0.4, 0.45], (0.5, 0.25, 0.15)$
$\widehat{\mathcal{M}(e)(\flat_3)}$	$[0.35, 0.45]$ $[0.65, 0.75]$ $[0.45, 0.55], (0.4, 0.35, 0.15)$	$[0.4, 0.55][0.75, 0.8]$ $[0.65, 0.7], (0.35, 0.3, 0.25)$	$[0.6, 0.65]$ $[0.55, 0.65]$ $[0.6, 0.75], (0.45, 0.35, 0.15)$
$\widehat{p(e)}$	$(1, 1, 1)$	$(1, 1, 1)$	$(1, 1, 1)$

We construct the Type-I EDioNSIVSS's for three patients under consideration as in **Table 2**, **Table 3** and **Table 4**.

Table 2
Type-I EDioNSIVSS model for the ill person $\mathcal{P}_1$.

$\widehat{\mathcal{P}1_{p_1(e)}}$	e_1	e_2
$\widehat{\mathcal{P}_1(e)(\flat_1)}$	$[0.4, 0.75]$ $[0.7, 0.75]$ $[0.45, 0.6], (0.45, 0.35, 0.2)$	$[0.55, 0.65]$ $[0.75, 0.8][0.55, 0.65], (0.5, 0.2, 0.1)$
$\widehat{\mathcal{P}_1(e)(\flat_2)}$	$[0.5, 0.55]$ $[0.65, 0.7]$ $[0.4, 0.5], (0.5, 0.35, 0.15)$	$[0.65, 0.7][0.7, 0.8]$ $[0.3, 0.35], (0.4, 0.3, 0.3)$
$\widehat{\mathcal{P}_1(e)(\flat_3)}$	$[0.45, 0.5]$ $[0.35, 0.55]$ $[0.65, 0.7], (0.3, 0.45, 0.2)$	$[0.5, 0.55]$ $[0.6, 0.65]$ $[0.7, 0.75], (0.45, 0.35, 0.15)$
$\widehat{p_1(e)}$	$(0.6, 0.5, 0.55)$	$(0.5, 0.45, 0.5)$

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$\widehat{\mathcal{P}1_{p_1(e)}}$	e_3
$\widehat{\mathcal{P}_1(e)(\flat_1)}$	$[0.45, 0.75]$ $[0.7, 0.85]$ $[0.65, 0.7], (0.35, 0.45, 0.1)$
$\widehat{\mathcal{P}_1(e)(\flat_2)}$	$[0.55, 0.6]$ $[0.65, 0.7]$ $[0.4, 0.45], (0.35, 0.45, 0.15)$
$\widehat{\mathcal{P}_1(e)(\flat_3)}$	$[0.35, 0.45]$ $[0.45, 0.5]$ $[0.75, 0.8], (0.5, 0.3, 0.1)$
$\widehat{p_1(e)}$	$(0.5, 0.55, 0.4)$

Table 3

Type-I EDioNSIVSS model for the ill person $\mathcal{P}_2$.

$\widehat{\mathcal{P}1_{p_1(e)}}$	e_1	e_2
$\widehat{\mathcal{P}_1(e)(\flat_1)}$	$[0.45, 0.8]$ $[0.65, 0.8]$ $[0.8, 0.9], (0.45, 0.3, 0.15)$	$[0.6, 0.7]$ $[0.7, 0.85][0.7, 0.8], (0.5, 0.35, 0.1)$
$\widehat{\mathcal{P}_1(e)(\flat_2)}$	$[0.6, 0.65]$ $[0.75, 0.8]$ $[0.35, 0.45], (0.45, 0.3, 0.15)$	$[0.6, 0.65][0.8, 0.85]$ $[0.25, 0.3], (0.3, 0.25, 0.25)$
$\widehat{\mathcal{P}_1(e)(\flat_3)}$	$[0.45, 0.5]$ $[0.35, 0.55]$ $[0.65, 0.7], (0.35, 0.4, 0.2)$	$[0.5, 0.55]$ $[0.6, 0.65]$ $[0.7, 0.75], (0.5, 0.3, 0.15)$
$\widehat{p_1(e)}$	$(0.7, 0.6, 0.65)$	$(0.6, 0.7, 0.75)$

$\widehat{\mathcal{P}1_{p_1(e)}}$	e_3
$\widehat{\mathcal{P}_1(e)(\flat_1)}$	$[0.55, 0.65]$ $[0.8, 0.9]$ $[0.65, 0.85], (0.35, 0.45, 0.1)$
$\widehat{\mathcal{P}_1(e)(\flat_2)}$	$[0.5, 0.7]$ $[0.85, 0.9]$ $[0.35, 0.4], (0.3, 0.4, 0.1)$
$\widehat{\mathcal{P}_1(e)(\flat_3)}$	$[0.35, 0.45]$ $[0.45, 0.5]$ $[0.75, 0.8], (0.55, 0.3, 0.1)$
$\widehat{p_1(e)}$	$(0.75, 0.8, 0.7)$

Table 4

Type-I EDioNSIVSS model for the ill person $\mathcal{P}_3$.

$\widehat{\mathcal{P}1_{p_1(e)}}$	e_1	e_2
$\widehat{\mathcal{P}_1(e)(\flat_1)}$	$[0.35, 0.4]$ $[0.45, 0.55]$ $[0.6, 0.7], (0.35, 0.4, 0.15)$	$[0.45, 0.5]$ $[0.6, 0.7][0.75, 0.85], (0.55, 0.3, 0.1)$
$\widehat{\mathcal{P}_1(e)(\flat_2)}$	$[0.45, 0.7]$ $[0.25, 0.3]$ $[0.3, 0.5], (0.3, 0.5, 0.2)$	$[0.5, 0.75][0.1, 0.25]$ $[0.2, 0.35], (0.25, 0.6, 0.1)$
$\widehat{\mathcal{P}_1(e)(\flat_3)}$	$[0.25, 0.3]$ $[0.4, 0.45]$ $[0.15, 0.2], (0.25, 0.35, 0.2)$	$[0.2, 0.35]$ $[0.5, 0.6]$ $[0.35, 0.4], (0.45, 0.35, 0.15)$
$\widehat{p_1(e)}$	$(0.5, 0.5, 0.4)$	$(0.6, 0.65, 0.3)$

$\widehat{\mathcal{P}1_{p_1(e)}}$	e_3
$\widehat{\mathcal{P}_1(e)(\flat_1)}$	$[0.5, 0.55]$ $[0.65, 0.75]$ $[0.6, 0.7](0.25, 0.5, 0.15)$
$\widehat{\mathcal{P}_1(e)(\flat_2)}$	$[0.35, 0.6]$ $[0.3, 0.4]$ $[0.15, 0.25], (0.45, 0.3, 0.2)$
$\widehat{\mathcal{P}_1(e)(\flat_3)}$	$[0.4, 0.5]$ $[0.4, 0.6]$ $[0.6, 0.65], (0.35, 0.4, 0.1)$
$\widehat{p_1(e)}$	$(0.75, 0.55, 0.5)$

The experts have provided the Type-I EDioNSIVSS values in **Tables 2-4** based on their evaluation of the alternatives in relation to the criteria that are being considered. The similarity measure between the Type-I EDioNSIVSSs in Tables 2-4 and the one in **Table 1** should be computed in this case. The similarity measure calculations for $\mathcal{P}_1$ to $\mathcal{P}_3$ sick individuals are provided below the table.

	$T_1(\flat_1)$	$T_2(\flat_1)$	$S(\flat_1)$	$T_1(\flat_2)$	$T_2(\flat_2)$
$(\widehat{\mathcal{M}, \mathcal{P}_1})$	$[0.868818, 0.974786]$	$[0.889907, 0.927432]$	$[0.920563, 0.987335]$	$[0.901980, 0.913144]$	$[0.827319, 0.860648]$
$(\widehat{\mathcal{M}, \mathcal{P}_2})$	$[0.914754, 0.980701]$	$[0.94518, 0.958669]$	$[0.940123, 0.94887]$	$[0.827531, 0.879728]$	$[0.722286, 0.748288]$
$(\widehat{\mathcal{M}, \mathcal{P}_3})$	$[0.762696, 0.769463]$	$[0.866504, 0.910841]$	$[0.935565, 0.958095]$	$[0.854532, 0.945]$	$[0.280518, 0.642251]$

	$S(\flat_2)$	$T_1(\flat_3)$	$T_2(\flat_3)$	$S(\flat_3)$
$(\widehat{\mathcal{M}, \mathcal{P}_1})$	$[0.882593, 0.909096]$	$[0.919941, 0.968997]$	$[0.881693, 0.946270]$	$[0.925759, 0.988364]$
$(\widehat{\mathcal{M}, \mathcal{P}_2})$	$[0.872503, 0.893205]$	$[0.918921, 0.970872]$	$[0.79447, 0.88944]$	$[0.925759, 0.988364]$
$(\widehat{\mathcal{M}, \mathcal{P}_3})$	$[0.868656, 0.896285]$	$[0.81842, 0.875266]$	$[0.817625, 0.869728]$	$[0.880515, 0.900609]$

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	φ	ψ	$Similarity$
$(\widehat{\mathcal{M},\mathcal{P}_1})$	$[0.859276, 0.962948]$	0.666667	$[0.572851, 0.641965]$
$(\widehat{\mathcal{M},\mathcal{P}_2})$	$[0.810504, 0.95409]$	0.819672	$[0.6643472, 0.782041]$
$(\widehat{\mathcal{M},\mathcal{P}_3})$	$[0.62028, 0.934568]$	0.690909	$[0.428557, 0.645701]$

5.3 Results

According to the aforementioned data, the second patient $\mathcal{P}_2$ has similarity measure of $(\widehat{\mathcal{M},\mathcal{P}_2}) = \mathbf{0.782041} > 0.70$, but the similarity measure of the first and third patients is < 0.70. Therefore, there is a notable similarity between these two Type-I EDioNSIVSS. Consequently, we deduce that $\mathcal{P}_2$ is afflicted with viral fever. Therefore, we must consider $\mathcal{P}_2$ first.

6 Conclusion

Presenting a Type-I EDioNSIVSS and studying some of its features is the primary objective of this work. The use of the similarity measure between two Type-I EDioNSIVSS is examined. The theory of generalized bipolar neutrosophic fuzzy soft sets and generalized neutrosophic cubic soft sets will be applied in the future.

References

[1] L. A. Zadeh, Fuzzy sets, Information and control, 8(3), (1965), 338-353.

[2] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy sets and Systems, 20(1), (1986), 87-96.

[3] Ranjit Biswas, Vague Groups, International journal of Computational Cognition, 4(2),(2006), 20-23.

[4] R. R. Yager, Pythagorean membership grades in multi criteria decision-making, IEEE Trans. Fuzzy Systems, 22, (2014), 958-965.

[5] S. Ashraf, S. Abdullah, T. Mahmood, F. Ghani and T. Mahmood, Spherical fuzzy sets and their applications in multi-attribute decision-making problems, Journal of Intelligent and Fuzzy Systems, 36, (2019), 2829-284.

[6] B.C. Cuong and V. Kreinovich, Picture fuzzy sets a new concept for computational intelligence problems, in Proceedings of 2013 Third World Congress on Information and Communication Technologies (WICT 2013), IEEE, (2013), 1-6.

[7] F. Smarandache, A unifying field in logics, Neutrosophy neutrosophic probability, set and logic, American Research Press, Rehoboth, (1999).

[8] R. Jansi, K. Mohana and F. Smarandache, Correlation Measure for Pythagorean Neutrosophic Sets with T and F as Dependent Neutrosophic Components Neutrosophic Sets and Systems,30, (2019), 202-212.

[9] G. Shahzadi, M. Akram and A. B. Saeid, An application of single-valued neutrosophic sets in medical diagnosis, Neutrosophic Sets and Systems, 18, (2017), 80-88.

[10] P.K. Singh, Single-valued neutrosophic context analysis at distinct multi-granulation. Comp. Appl. Math. 38, 80 (2019), 1-18.

[11] SG Quek, H Garg, G Selvachandran, M Palanikumar, K Arulmozhi,VIKOR and TOPSIS framework with a truthful-distance measure for the (t, s)-regulated interval-valued neutrosophic soft set, Soft Computing, 1-27, 2023.

https://doi.org/10.54216/IJNS.240411

Received: September 08, 2023 Revised: February 07, 2024 Accepted: March 02, 2024

[12] M Palanikumar, K Arulmozhi, A Iampan, Multi criteria group decision making based on VIKOR and TOPSIS methods for Fermatean fuzzy soft with aggregation operators, ICIC Express Letters 16 (10), 1129–1138, 2022.

[13] M Palanikumar, K Arulmozhi, MCGDM based on TOPSIS and VIKOR using Pythagorean neutrosophic soft with aggregation operators, Neutrosophic Sets and Systems,, 538-555, 2022.

[14] M Palanikumar, S Broumi, Square root $(\hbar, \varepsilon)$phantine neutrosophic normal interval-valued sets and their aggregated operators in application to multiple attribute decision making, International Journal of Neutrosophic Science, 4, 2022.

[15] D. Molodtsov, Soft set theory First results, Computers and Mathematics with Applications, 37, (1999), 19-31.

[16] P.K. Maji, R. Biswas and A.R. Roy, Fuzzy Soft Set, Journal of Fuzzy Mathematics, 9(3), (2001), 589-602.

[17] P.K. Maji, R. Biswas and A.R. Roy, On intuitionistic Fuzzy Soft Set, Journal of Fuzzy Mathematics, 9(3), (2001), 677-692.

[18] Y. Yang, C. Liang, S. Ji and T. Liu, Adjustable soft discernibility matrix based on picture fuzzy soft sets and its applications in decision making, Journal of Intelligent and Fuzzy Systems, 29, (2015), 1711-1722.

[19] X.D. Peng, Y. Yang, J.P. Song, Pythagorean fuzzy soft set and its application, Computer Engineering, 41(7), (2015), 224-229.

[20] P. Majumdar, S.K. Samantab, Generalised fuzzy soft sets, Computers and Mathematics with Applications, 59, (2010), 1425-1432.

[21] M Palanikumar, K Arulmozhi, Novel possibility Pythagorean interval valued fuzzy soft set method for a decision making, TWMS J. App. and Eng. Math. V.13, N.1, 2023, pp. 327-340.

[22] M. Palanikumar, K. Arulmozhi, and C. Jana, Multiple attribute decision-making approach for Pythagorean neutrosophic normal interval-valued aggregation operators, Comp. Appl. Math. 41(90), (2022), 1-27.

[23] M Palanikumar, N Kausar, H Garg, A Iampan, S Kadry, M Sharaf, Medical robotic engineering selection based on square root neutrosophic normal interval-valued sets and their aggregated operators, AIMS Mathematics, 8(8), 2023, 17402-17432.

[24] Heilat, A. S., Zureigat, H., Hatamleh, R., Batiha, B. New Spline Method for Solving Linear Two-Point Boundary Value Problems. European Journal of Pure and Applied Mathematics, 14(4), 1283–1294, 2021.

[25] T. Qawasmeh, R.Hatamleh, A new contraction based on H-simulation functions in the frame of extended b-metric spaces and application,International Journal of Electrical and Computer Engineering, 13 (4), 4212-4221, 2023.

[26] Hatamleh, R., Zolotarev, V. A. Triangular Models of Commutative Systems of Linear Operators Close to Unitary Operators. Ukrainian Mathematical Journal, 68(5), 791-811, 2016.

[27] Ayman Hazaymeh, Rania Saadeh, Raed Hatamleh, Mohammad W. Alomari, Ahmad Qazza, A Perturbed Milnes Quadrature Rule for n-Times Differentiable Functions with Lp-Error Estimates, Axioms-MDPI, , 12(9), 803, 2023.

Received: September 08, 2023 Revised: February 07, 2024 Accepted: March 02, 2024