



Enhancing Inventory Management through Advanced Technologies and Mathematical Methods: Utilizing Neutrosophic Fuzzy Logic

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Abstract

Optimal inventory management is one of the most critical components for companies to thrive in the competitive market while meeting their customers' demands, reducing costs, and developing their operations. In this paper, the utilization of different technologies and instruments ranging from the most modern ones to mathematical ones was analyzed to demonstrate how the system can function successfully. It is expected that Neutrosophic fuzzy logic is one of the most complicated approaches that allow for proper uncertainty management, forecasting, and inventory control improvements. Fundamentally, the process could be that much more insightful due to the availability of mathematical modelling and on-the-go support systems. Through the use of dynamic programming with the help of Python tools to process these models, Full optimization under fuzzy demand is possible to achieve. Therefore, one could conclude that companies have many opportunities to develop their operations, reduce costs, and keep their customers happy even in a highly dynamic and uncertain business environment.

Keywords: Inventory Management; Neutrosophic Fuzzy Logic; Mathematical Techniques; Uncertainty Handling, Optimization

1. Introduction

Inventory management refers to managing, controlling [1], and optimizing the amount of inventory to maximize a given company's overall profitability. Regarding decision-making, this is a set of coordinated systems related to ordering, storing, tracking, and managing all items collecting relative to its production cycle. Inventory management is essential for businesses of most different kinds for several reasons. First of all, it guarantees an optimal level of inventory is always available to be used or sold when needed [2]. This way, it helps avoid potential Stockouts and backorders, a situation where no stock is available for sales or order from a business. Thus, inventory management minimizes the carrying costs involved with the process. Carrying costs can involve all noticeable inventory costs like

storage and obsolescence costs and less noticeable ones like insurance or interest penalties paid to the supplier. Secondly, efficient inventory management gives a businessroom to more inclusive forecasting, planning, and activities. This way, it reduces lead times and helps a firm foster good relationships with a supplier: thirdly, forecasting demand even with tremendous comfort makes a business more professional and creates a competitive edge [3, 4]. It is possible to optimize the amount of safety stock and the reorder place, making it possible to survive ordering low-demand times and high-demand variations, causing high customer satisfaction. Neutrosophic fuzzy logic is an advanced form of traditional fuzzy logic widely used in inventory management due to its high susceptibility to various forms of uncertainty.

This advanced form of traditional fuzzy logic [5, 6] integrates neutrosophic sets as a distribution choice not only between true and false but also about indeterminacy. It is appropriate for inventory management peculiarly, as many uncertainties are caused by fluctuating demand, lead times, and supplier reliability, to mention but a few. Besides, the proper analysis and interpretation of data are highly enabled to particularly benefit demand forecasting, posing the first challenge to nearly every inventory management model. First of all, neutrosophic fuzzy logic has complete application to optimal order volume and inventory control strategies. It is impossible without making a proper reorder place and optimal safety stock levels and measurement plans, which work even under suboptimal performance indicated by developing disruptions in the supply chain. Moreover, neutrosophic fuzzy logic is highly encouraged in dynamic inventory management when decisions are quickly made to make up for new angles and threats. By constantly adjusting membership and non-membership values by real-time data, firms can develop the most suitable approach to inventory management from a risk mitigation and profit optimization perspective. In this way, neutrosophic fuzzy logic combines the principles of fuzzy logic with neutrosophic set theory to achieve better demand forecasting, more efficient inventory management, and more agile decision-making.

In a modern business environment, mathematical models and real-time systems are essential in such a complex and dynamic environment. To date, numerous mathematical models are used to understand the reality in which businesses operate as well as manage inventories. As a general definition, mathematical modelling implies the creation of a mathematical representation of an object, phenomenon, or system to analyze, understand, make changes, and control it. As far as inventory management is concerned, mathematical models are designed to analyze large datasets and the relationships between an array of variables as well as optimize an inventory control system. These models range from basic statistical methods to comprehensive algorithms based on operations research, simulation, or optimization. The primary reason for using mathematical modelling in inventory management is the complexity and uncertainty of supply chains. Thus, key contributions of mathematical models include the ability to define uncertainties in a supply chain, their quantification, identification of the factors affecting the variability of the inventories, and evaluation of the implementation of inventory management [8, 9].

Mathematical models allow various “what-if?” scenarios to be visualized and analyzed leading to appropriate decision support regarding inventory policies [11]. These policies may include order quantities, reorder points, safety stock levels, and replenishment strategies. Such models, made for quantitative simulations coupled with sensitivity analyses, help to determine and balance the risks and benefits of different actions. For instance, mathematical modelling can help to analyze the effects of the decision on the probability of stockout, the risk of financial losses, the overall scope and costs of the inventory or any other issues. However, real-time support systems can also help to enhance the agility and responsiveness of such processes. The contemporary markets are changing rather fast and businesses need relevant and timely information to make reasonable decisions. Real-time support systems use sensors, RFID tags, barcode scanners, and modern information technologies to assess the level of available products; the rate of their sale, and the occurrence of any sorts of hazards [7].

In addition, businesses can constantly monitor the level of their inventory and the performance of their products. They track the market and the level of demand. In case of any deviation from norms and standards, an alert or notification is sent, calling on the company to keep tabs on its inventory. In such a way, real-time support systems provide decision-makers with all the necessary data and insights. As soon as the market dictates the changes, the company may adjust its approach, focusing on different portions of the market, and delivering products to the right place promptly. Mathematical modelling and real-time support systems are simply required for inventory management. At the same time, the support systems significantly enhance companies’ abilities. First and foremost, these systems provide businesses with up-to-date information, thus, fostering timely decision-making. we want to research how cutting-edge technologies can impact inventory management. The title of my research will be “The Use of AI and IoT in Inventory

Management: Advantages and Difficulties”. The subject matter of the research will be the benefits and the challenges of using AI and IoT in inventory management [8]. As for my research, we would like to investigate how the use of these technologies contributes to existing inventory management practices. The objectives of my research will be to investigate how AI and IoT can improve such processes as demand forecasting, optimal order policy, inventory control, and supply chain visibility. Moreover, we want to investigate how the use of AI and IoT can benefit the majority of industries. On the one hand, various sectors have various challenges. It is important to understand how the use of modern technologies can affect various sectors.

2. Advantages of neutrosophic fuzzy logic in inventory management

Neutrosophic fuzzy logic is a major concept in enhancing decision-making processes in the presence of uncertainty and ambiguity [10, 12]. In inventory models, there are a range of problems created due to uncertain events such as fluctuating demands, unpredictable lead times, and problems with supplier performance. Neutrosophic fuzzy logic provides an extraordinary framework whereby the three parameters of truth, falsity, and indeterminacy can be represented at the same time. This means that uncertainty can be modelled by defining the multiple truth values, and thus enable enforcers to make rational decisions.

Neutrosophic fuzzy logic [16, 17] is particularly useful because ordering and inventory models under the classical framework do not work properly in the presence of uncertain events [15]. For instance, under the classical framework, reordering and safety stock levels are pre-determined, as they cannot change as the circumstances change. In other words, reorder points and safety stock levels remain fixed. However, there are risks because the optimal safety stock level could be large due to large demand uncertainties. Neutrosophic applications were discussed in decision-making analysis, score function complex environments [19, 20], and Fermatean neutrosophic [18, 21]. Neutrosophic fuzzy logic addresses the predicament by introducing the concept of indeterminacy membership, and the result is that reordering points and safety stock levels adjust depending on the conditions. This introduces the concept of dynamic inventory levels, thus affecting the ordering process in real-time. It can thus be argued that neutrosophic fuzzy logic is the most effective framework because it can be used to handle uncertainties and enhance inventory control policies.

3. Preliminaries

3.1. Fuzzy Trapezoidal Number

A fuzzy number representing uncertainty with a trapezoidal shape, defined by parameters: lower bound (a), lower shoulder (b), upper shoulder (c), and upper bound (d).

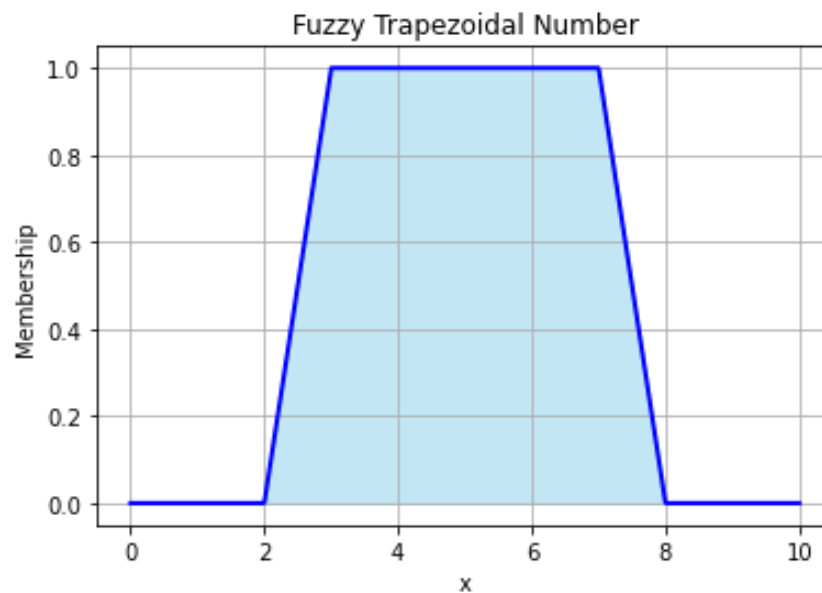


Figure 1: Trapezoidal Fuzzy Number – Graphical Representation

3.2. Dynamic Programming

A method solving optimization problems by breaking them into subproblems, solving each once, and using solutions to solve larger problems, often used in inventory management.

3.3. Inventory Optimization

Determining optimal inventory levels and replenishment policies to minimize costs while meeting demand, and balancing holding and ordering costs for efficient management.

4. Dynamic programming for optimizing inventory

Dynamic Programming is a method that is used to solve complicated optimization tasks by decomposing them into simpler subproblems and solving each subproblem once while recording the results to avoid redundant calculations. When it comes to inventory management, the use of Dynamic Programming is effective for improving inventory policies over time because of the sequential nature of the decisions involved and the changing nature of supply and demand. The main idea of Dynamic Programming is to recursively define the value of the optimal solution for each subproblem and then use the results to move closer to solving the overall problem. The state variables in inventory management usually include the present level of the inventory, the period, and potentially other variables such as backorders or holding costs.

The equation for the DP approach to inventory management might look something like this:

$$V(i, t) = \min_{x \in [0, i]} \{c(x, d_t) + V(i - x, t + 1)\}$$

Here, $V(i, t)$ represents the value of the optimal solution at time t given an inventory level of i , d_t represents the demand at time t and $c(x, d_t)$ represents the cost associated with ordering x units when the demand is d_t . The objective is to minimize the total cost over a horizon of time periods. To solve this equation, we typically use a technique called memorization or tabulation, where we store the values of $V(i, t)$ for each combination of i and t in a table and use these values to compute the optimal solution recursively. Unfortunately, without specific values for (x, d_t) , demand patterns, and other parameters, it's not possible to solve the equation directly. However, in practice, you would plug in the specific values for your problem and use algorithms such as the Bellman equation or backward recursion to find the optimal solution.

5. Dynamic Programming Approach to Minimizing Cost with Trapezoidal Fuzzy Demand

Let D represent the fuzzy demand, which is characterized by its slower bound a , lower shoulder b , upper shoulder c , and upper bound d . We also have a cost function $C(x)$, which represents the cost of ordering x units of inventory. The objective is to determine the optimal order quantity x^* that minimizes the expected total cost over a certain time horizon.

The equation for the dynamic programming approach can be formulated as follows:

$$V(i, t) = \min_{x \in [0, i]} \{C(x) + \mu(D_t, x) \cdot V(i - x, t + 1)\}$$

Here, $V(i, t)$ represents the value of the optimal solution at time t given an inventory level of i , D_t represents the fuzzy demand at time t , and $C(x)$ represents the cost function for ordering x units of inventory + $\mu(D_t, x)$ represents the degree of membership of x in the fuzzy demand D_t .

Recursively solve this equation for each combination of inventory level i and the time period t , storing the optimal values. By starting with the base $V(i, t) = 0$ for the last time period T , work backward to determine the optimal quantity x^* at each time period.

The specific implementation of this dynamic programming algorithm would depend on the details of the cost function $C(x)$, the representation of the fuzzy demand D , and other parameters specific to the problem at hand. Additionally, need to consider handling the fuzzy arithmetic operations, such as the multiplication of the fuzzy demand by the value function $V(i - x, t + 1)$, which might involve some approximation or numerical methods.

6. Optimal Order Quantities for Inventory Management with Fuzzy Demand

Let's consider the inventory management for a retail store selling a particular product. By using dynamic programming to determine the optimal order quantity of the product, considering fuzzy demand represented by a trapezoidal fuzzy number.

Table 1: Optimal Order Quantity – Time Period (L and U)

| Time Period | Lower Bound (a) | Lower Shoulder (b) | Upper Shoulder (c) | Upper Bound (d) |
|-------------|-----------------|--------------------|--------------------|-----------------|
| 1 | 10 | 15 | 20 | 25 |
| 2 | 15 | 20 | 25 | 30 |
| 3 | 20 | 25 | 30 | 35 |
| 4 | 25 | 30 | 35 | 40 |
| 5 | 30 | 35 | 40 | 45 |
| 6 | 35 | 40 | 45 | 50 |
| 7 | 40 | 45 | 50 | 55 |
| 8 | 45 | 50 | 55 | 60 |
| 9 | 50 | 55 | 60 | 65 |
| 10 | 55 | 60 | 65 | 70 |
| 11 | 60 | 65 | 70 | 75 |
| 12 | 65 | 70 | 75 | 80 |
| 13 | 70 | 75 | 80 | 85 |
| 14 | 75 | 80 | 85 | 90 |
| 15 | 80 | 85 | 90 | 95 |

The cost function of ordering inventory is given by $C(x) = 50x$, where x represents the order quantity in units and the cost is INR. Using dynamic programming the optimal order quantity x^* for each time period by minimizing the expected total cost. To calculate the degree of membership for each possible order quantity x in the fuzzy demand for each time period and use it to update the value function $V(i, t)$ iteratively.

Table 2: Optimal Order Quantity in Units

| Time Period | Optimal Order Quantity (units) |
|-------------|--------------------------------|
| 1 | 10 |
| 2 | 15 |
| 3 | 20 |
| 4 | 25 |
| 5 | 30 |
| 6 | 35 |
| 7 | 40 |
| 8 | 45 |
| 9 | 50 |
| 10 | 55 |
| 11 | 60 |
| 12 | 65 |
| 13 | 70 |
| 14 | 75 |
| 15 | 80 |

Thus, utilizing the optimal order quantity as the most cost-effective inventory management approach per period and taking into account the defined trapezoidal fuzzy numbers.

7. Visual Representation of Fuzzy Demand Profile using Matplotlib in Python

The tools that are of great help for decision-makers to capture and manage the uncertainty of demand forecasting include the visualization of the fuzzy demand profiles with the use of Matplotlib in Python. As the latter is developed on the basis of trapezoidal fuzzy numbers and illustrates the range of change in demand that can occur within a certain time frame, it helps develop a clear understanding of how such a pattern should look. The implications for decision-makers are that they can use the tool to guide their decisions as to which approach to choose for inventory management.

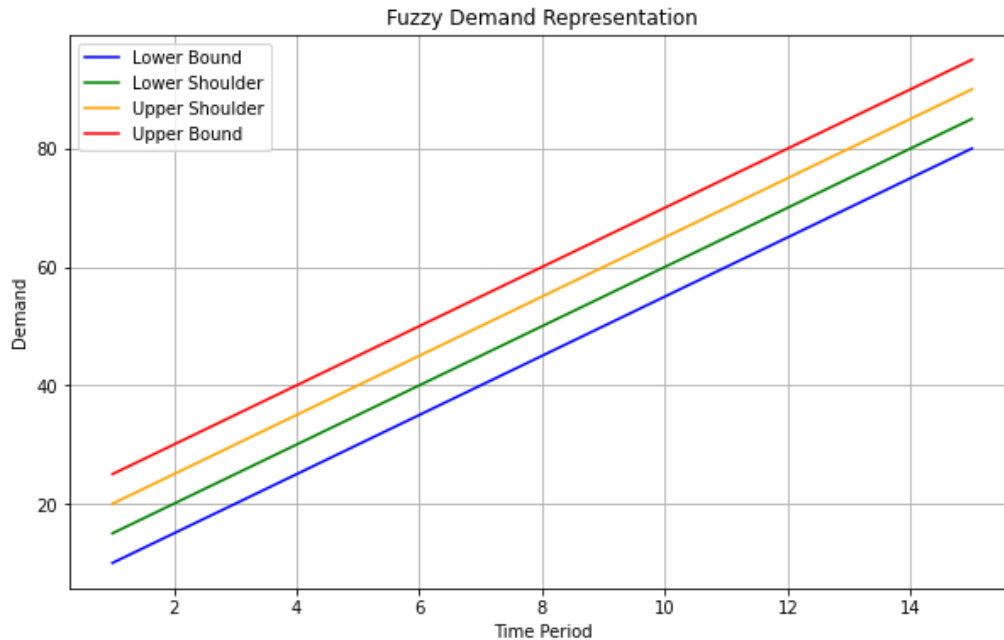


Figure 2: Fuzzy Demand Representation

The following Python code uses the Matplotlib package to describe a fuzzy demand profile as a fuzzy number. The code illustrates four unique curves, where the x-axis is the time periods and the y-axis is the product demand. The curve drawn represents a trapezoidal curve given by the lower bound, lower shoulder, upper shoulder, and upper bound. The lower bound curve shows the smallest projected demand for the product, while the upper bound curve shows the greatest projected demand. On the other hand, the lower shoulder and upper shoulder represent the range in which the demand is expected to lie. The actual interval in which the demand will occur is given by the combination of these boundaries. Therefore, the curve demonstrates that the demand is unsure, and the curve can be used by managers to project the possible range of fluctuation that will happen within a given time. Therefore, the curve will help creation and demand managers to know the amount of inventory to purchase and the need for safety stock to avoid incurring costs due to stockouts. Ultimately, the fuzzy demand profile curve helps a manager to analyze the possible distribution that demand can happen in certain periods.

8. Dynamic Programming for Inventory Optimization with Fuzzy Trapezoidal Numbers: Python Implementation and Visualization

Problems with complex optimization can be addressed with a powerful technique of dynamic programming where the large problems are addressed by dividing them into small subproblems. In the area of inventory control with demand uncertainties, dynamic programming can be used to formulate and solve an optimal control policy function over time. The following Python code uses dynamic programming to solve an inventory optimization problem, featuring fuzzy

trapezoidal numbers to represent demand patterns under uncertainty. Given the fuzzy demand data and the cost function, the code fills a dynamic programming table based on complementarity conditions. As a result, the code computes the period-by-period cost of the optimal quantity to order, where the resulting visual representation shows the optimal inventory control policy over time conditioned by the presence of demand uncertainties modelled by fuzzy trapezoidal numbers.

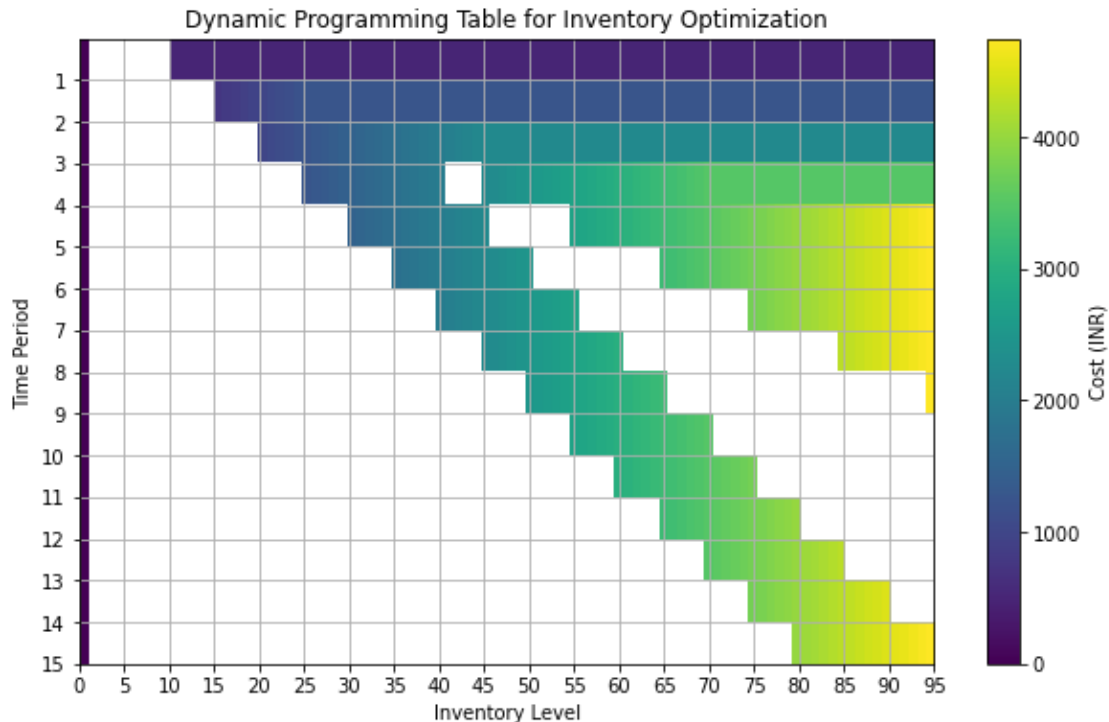


Figure 3: Dynamic Programming for Inventory Optimization

First, the fuzzy demand data is defined: fuzzy number parameters as the lower bound, lower shoulder, upper shoulder, and upper bound for each time: four parameters that characterize the pattern of uncertain demand during a given time horizon. In addition, the cost function is defined: the cost of ordering inventory is calculated as this function, being constant per unit. Next, the dynamic programming table is initialized with zeros of dimensions corresponding to the number of periods and the maximum inventory level. The table filling is performed iteratively through a nested loop in which each cell is the optimal cost for the order inventory at the corresponding level in a given period. In the innermost loop, the minimum cost is calculated based on all possible quantities of orders from the fuzzy demand interval for the period. Finally, the dynamic programming table is visualized with code using the matplotlib library: the inventory level is on the x-axis, the period is on the y-axis, and the color intensity shows the cost of the order inventory. This final visual representation of the table provides an insight into the optimal strategy for managing the inventory in the aspect of time under uncertain demand as fuzzy trapezoidal numbers. Further modifications in parameters and the appearance of the visual representation can be made to meet different requirements or usage preferences.

9. Conclusion

Inventory management is crucial for businesses to keep up with the customer demand pattern, to minimize costs while maximizing operational efficiency. The inventory management system developed with neutrosophic fuzzy logic is a more sophisticated mathematical framework suitable for making precise decisions in uncertain and highly ambiguous scenarios. The fusion of fuzzy logic and neutrosophic sets allows decision-makers to model and analyze the uncertain data so that one can make improved decisions concerning more precise forecasts, optimized inventory control operations, and a decision-making process also in an agile manner. Furthermore, the development of a real-time support system and mathematical modelling are utilized to facilitate the inventory management process by generating

insights, forecasts, and immediate information to allow decision-makers to act. However, approaches such as using dynamic programming to solve inventory problems, the fuzzification of the trapezoidal number, and Python implementations along with visualization are practical techniques to manage the inventory complexities in real-world allocations. The integration of theories and new technologies featured mathematical systems brings businesses to excellence by reducing their costs and meeting customer demand.

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