Neutrosophic Structure of Sized Biased Exponential Distribution: Properties and Applications

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Abstract

This study presents a novel distribution derived from the exponential distribution, referred to as the neutrosophic size-biased exponential distribution (NSBED). Various characteristics of the proposed model, including moments, skewness, and kurtosis, are investigated. Plots depicting the cumulative distribution function, density function, and other relevant functions associated with the survival analysis hazard function under indeterminacy are provided. Parameter estimates for the proposed model within the neutrosophic framework are computed. To illustrate the statistical applications of the results in handling imprecise data, a motivation is provided. A simulation analysis is conducted to validate the theoretical aspects of the proposed NSBED. Results indicate that the new distribution exhibits right skewness and shares many properties with skewed distributions. Our novel distribution outperforms the size-biased exponential distribution. Finally, a real application of the proposed model is provided to illustrate the practical implications.

Keywords: Neutrosophic probability; skewed distribution; neutrosophic measures; estimation

1. Introduction

Researchers explore new distributions of data with statistical methods that are presented in different ways, as it these types of distributions play a major role in various academic fields. Weighted distributions find their application when one constructs statistics based on data coming from a stochastic process which is not evenly recorded; instead, such statistics are based upon weighted function. Length-biased distribution is found if the weight function gets impacted by the lengths of the units under consideration [3]. It has been widely reported that many continuous distributions are size-biased or length- or size-biased subjects for developing singular and desirable properties [4], [5]. In practical scenarios like probability proportional to sizes (PPS) designs, we can see that there are different chances of selecting distinct samples. Use of method conversed about in reference [6] lead to noticeable development in dealing with above mentioned problematic issue particularly among size biased distributions. A lot of study disciplines as econometrics, chemometrics, ecology, agriculture, linguistics, and medicinal science depend on those distributions. [7]. The length-biased approach was instrumental around sampling theory and greatly enhanced precision of data analysis in many fields. Researchers may make more dependable inferences and reach more accurate conclusions by considering the unequal chance of sample observations, as per this range [8-13].

DOI: https://doi.org/10.54216/IJNS.240323
Received: February 12, 2024 Revised: April 15, 2024 Accepted: May 09, 2024

To formulate size-biased distributions, let's assume we have a random variable Z with a probability density function (PDF) denoted as $f(z,\alpha)$, where α represents a precise value of the parameter. The corresponding weighted distribution function, denoted as $g(z,\alpha)$, is given by [14]:

$$g(z,\alpha)=\frac{w(z)f(z,\alpha)}{E(w(z))} \tag{1}$$

Where $w(z)$ is weight function and the integral in the denominator ensures normalization, making $g(z,\alpha)$ a valid probability distribution.

If we assume the weight function $w(z)=z^{\vartheta}$, the (1) is called size biased distribution of order ϑ. If the value of $\vartheta=1$ then it is called length biased distribution, denoted as follow [15]:

$$g(z,\alpha)=\frac{zf(z,\alpha)}{E(z)} \tag{2}$$

Length-biased models are typically used in numerous domain names such as public health, dependability design as well as demography to research study populaces with various prices of advancement. In the world of public health, these circulations play a crucial part in assessing health issues that advance at differing prices [16-18] Integrity design scientists use the failure times of system elements to evaluate larger parts that have longer life expectancies. This strategy is vital for understanding the probability of discovering failings. In demography length-biased circulations are used to research study human populaces especially in the context of examining fertility plus death. Scientist can acquire valuable understandings right into populace characteristics as well as projections by picking people with longer life expectancies or reproductive durations for example [19] Neutrosophic collections offer a much more comprehensive method in contrast with standard collection concept when taking care of uncertain, vague, as well as irregular information [20] These subscription features particularly truth-membership, indeterminacy-membership, plus falsity-membership supply a clear understanding of the way in which to which a component comes from the collection is not sure or does not come from the collection. This structure additionally integrates appropriate data and chance circulations to additional boost its performance in dealing with complex details [21-24] Neutrosophic stats as well as possibility circulations supply valuable techniques for examining information or unpredictability that is described by neutrosophic collections [25-27]. Neutrosophic probability distributions calculate likelihood of occurrences or outcomes in uncertainty situations, focusing on central tendency, dispersion, and other neutrosophic data characteristics [28]. Such an approach is adapted for making decisions, recognizing patterns, and developing artificial intelligence, especially in situations where normal approaches fail to cope with ambiguity and vagueness. However, there are few papers on the neutrosophic size-biased exponential distribution that could be used to analyze imprecise data values meaningfully calling for further study.

The paper's structure is as follows: Section 2 gives a size-biased form of the exponential distribution. Section 3: Statistic properties of this modified distribution with simulations in section 4. Besides, in section 4 information on how to find mode and harmonic mean can be found. Showing real-life application of NSBED under consideration. Section 5 uses random numbers generation method. Section 6 demonstrates implementation of the suggested approach in practice. Section 7 finalizes the research report by giving concluding remarks about it.

2. Proposed Model

In this section, we propose the NSBED distribution. We provide graphical representations of the proposed model in the form of its probability density function (PDF) and cumulative distribution function (CDF).

The PDF of the neutrosophic exponential distribution and its CDF function are defined in (3) and (4) respectively:

$$f(z)=\mathrm{b_N}\mathrm{e}^{-\mathrm{b_N}z}, z>0, \mathrm{b_N}>0 \tag{3}$$

$$\mathcal{F}(z)=1-\mathrm{e}^{-\mathrm{b_N}z}, z>0, \mathrm{b_N}>0 \tag{4}$$

The exponential distribution has been widely used in different fields due to its versatility and practicality, as shown in previous studies [1]. For example, it has been utilized in modeling the lifetimes of electronic components and radioactive particles, as well as in queuing theory and

DOI: https://doi.org/10.54216/IJNS.240323
Received: February 12, 2024 Revised: April 15, 2024 Accepted: May 09, 2024

reliability analysis. Additionally, its simple mathematical properties make it a convenient choice for many statistical analyses.

Using (3) and (4) in (2) provides the density function of the proposed model.

$$g(z) = \mathrm{b}^2{}_{\mathrm{N}} z e^{-\mathrm{b}_{\mathrm{N}} z}, \ \mathrm{b}_{\mathrm{N}} > 0, z > 0. \quad (5)$$

The CDF of NSBED has the form

$$\mathcal{G}(z) = 1 - (1 + \mathrm{b}_{\mathrm{N}} z) e^{-\mathrm{b}_{\mathrm{N}} z} \quad (6)$$

Figure 1 shows probability density function (PDF) and figure 2 displays cumulative distribution function (CDF) of the proposed model. Probability Density Function is an important concept in probability theory and statistics, it characterizes the distribution of continuous random variables. It states how probable it is for various outcomes to happen within a given range. In this case, probability density function (PDF) cannot be negative and its integral over all possible values of variable amounts to one. Summing up the entire area covered by PDF leaves a total of 1 after integrating the PDF over all possible values of variable. Alternatively, a CDF integrates the associated PDF to establish probabilities for random variables less than or equal to a given value. The CDF begins at zero and goes up to one as the variable approaches' infinity. In the sphere of probability theory and statistics, this basic mathematical tool is used to calculate probabilities and make statistical inferences for random variables. When employed alongside the PDF though, the CDF provides a clear understanding of the characteristics and distributions of continuous random variables.

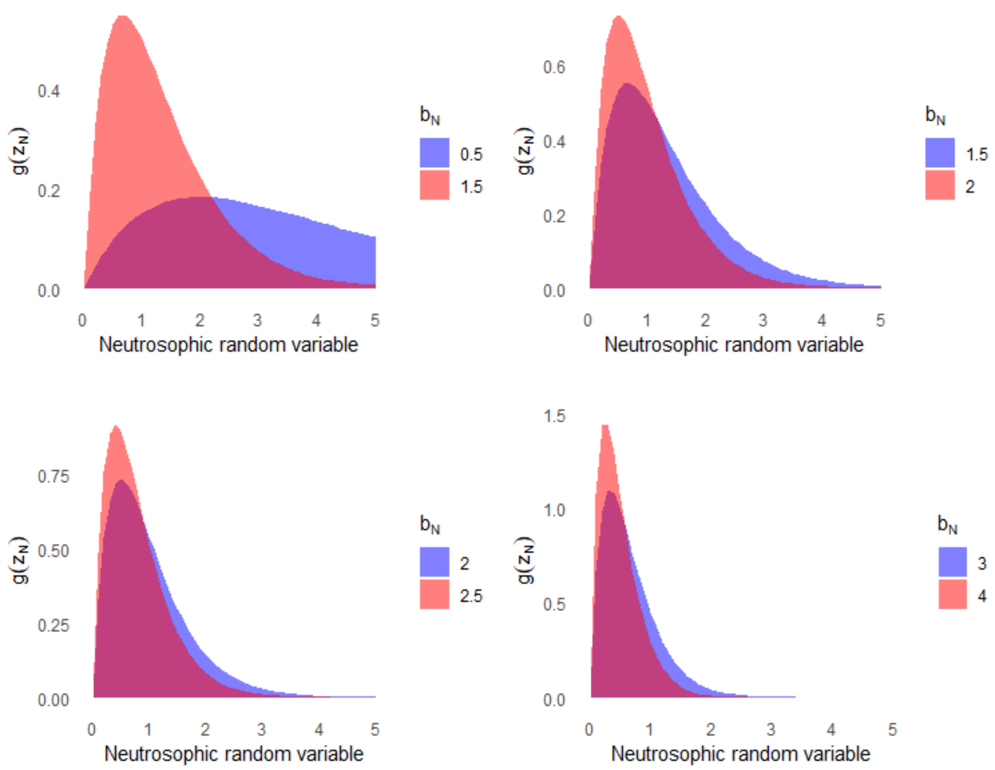

Figure 1: Pdf plot of the proposed distribution with different values of neutrosophic parameter

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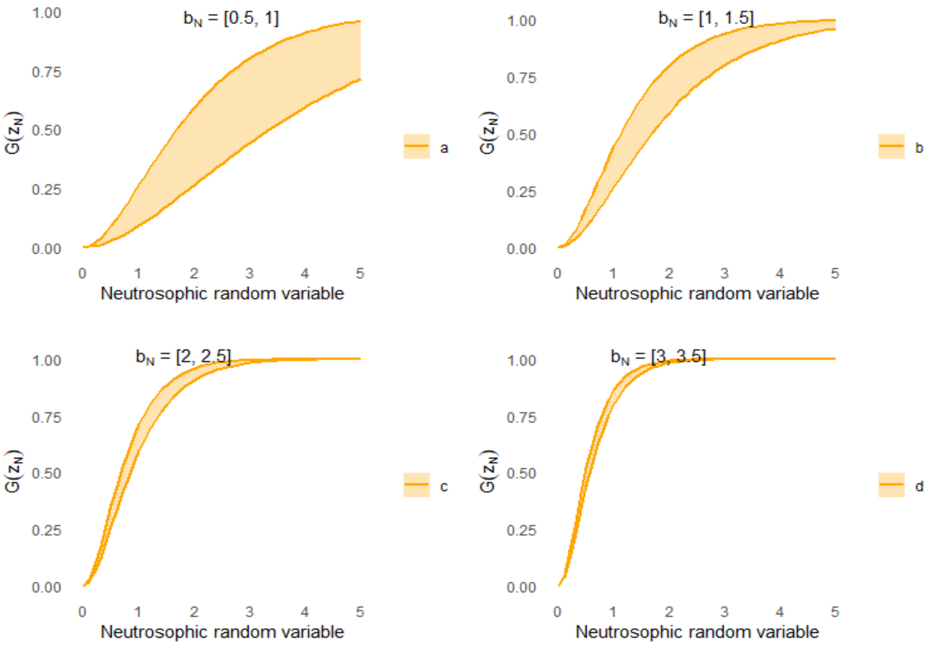

Figure 2: CDF plot of the proposed distribution with indeterminate parameter values

With neutrosophic probability theory, the range of uncertainty can be well modelled by use of PDF and CDF when the parameters of uncertainty remain to be indeterminate, as seen in Figure 1 and Figure 2. The possibilistic solution using the falsehood-membership enables the physical meaning of uncertainty to be comprehended more perfectly, especially when dealing with the vague and ambiguous data. The neutrosophic CDF is a mathematical tool for estimating the probability that the neutrosophic random variable would be no more than or equal to some constant in the light of the uncertain factors. Neutrosophic probability theory offers the sound and flexible methodology to process the uncertain data with different fields by putting the uncertain factors into both PDF and CDF. The survival function denoted briefly by $\mathcal{S}(z)$ is an important piece of any probabilistic model. The word 'probability' refers to the chance that the random variable would be more than or equal to some value, i.e. the probability that the event would not occur. The survival function in this mathematical model represents the probability of surviving beyond a specific point in time.

$$\mathcal{S}(z) = 1 - \mathcal{G}(z) = 1 - (1 - b_N)\mathrm{e}^{-\mathrm{b_N}z}, z > 0 \qquad (7)$$

The survival function is a fundamental concept employed in survival analysis and reliability engineering to ascertain the chance of an event not occurring within a given timeframe or threshold. Comprehending this function is crucial for forecasting the timing of occurrences or anticipating potential breakdowns. Figure 3 depicts the procedure for generating the survival function.

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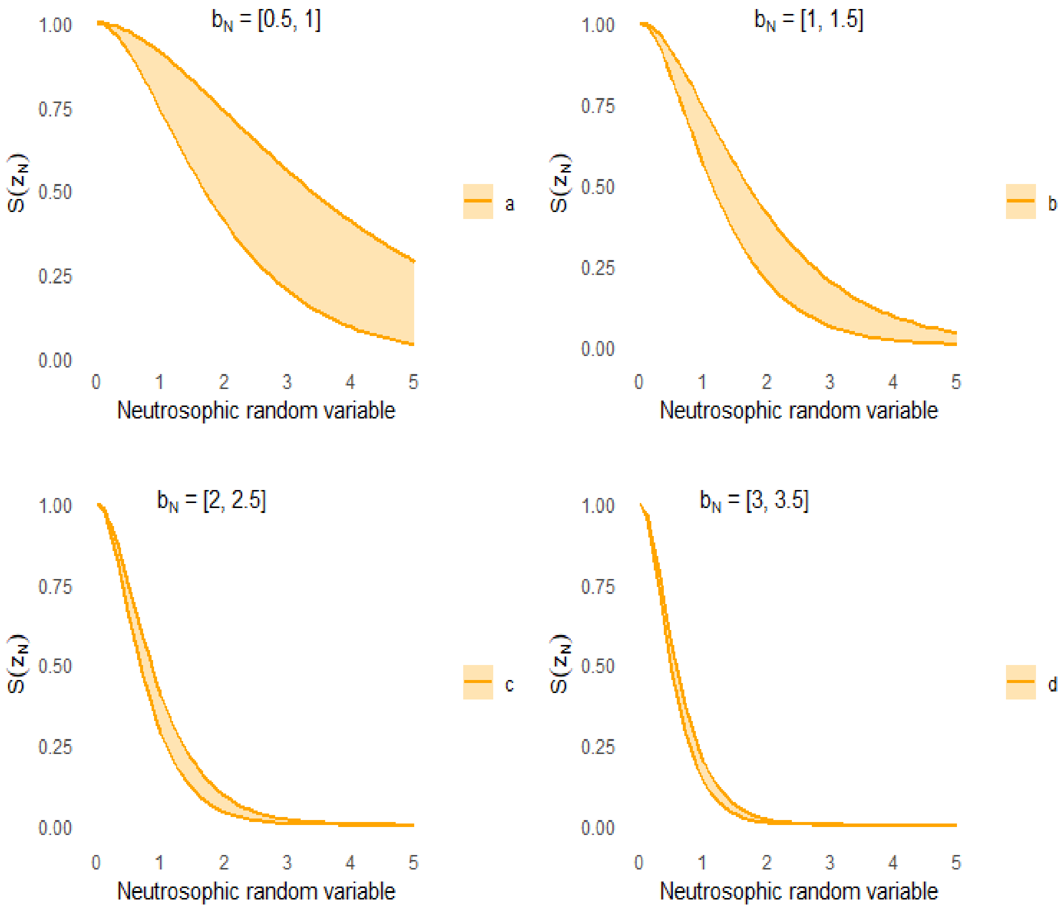

Figure 3: Survival function of the proposed model

Figure 3 represents a survival function for the proposed model. Neutrosophic theory introduces a new perspective on survival function, departing from the usual standard of probability theory and statistics. In this sense, the survival function measures the likelihood or ability of indefinite or uncertain occurrences or claims to survive beyond a threshold, the inherent uncertainty and indeterminacy that plays a role in it. In neutrosophic theory, the survival function is of great importance, with it being possible to study and represent systems and phenomena that surpass conventional probability. It represents a rigorous way measure uncertain occurrences and claims by considering the notions of indeterminacy, ambiguity, and missing knowledge. Given the complexity of real-world situations, in which uncertainty prevails, it was important to have this powerful tool to portray the complexity of the situations. The hazard function is another linked function of the probability distribution, commonly employed in reliability theory.

$$h(z) = \frac{g(z)}{S(z)} = b_N z \tag{8}$$

The hazard function is a crucial tool in analyzing complex systems prone to failure, enabling researchers to predict failure patterns, evaluate interventions, and make informed decisions across fields like epidemiology, medicine, engineering, and finance.

3. Basic Characteristics

This section explores several statistical properties associated with the proposed model. We have established the basic characteristics of the proposed model in terms of following theorems:

Theorem 1. If $z \backsim$ NSBED (b_N) then the r^{th} moment is given by as:

$$E(Z^r) = \left(\frac{1}{b_N}\right)^r \Gamma(2 + r) \tag{9}$$

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Proof. The r^{th} moments of LBPHRD can be attained by

$$E(Z^r) = \int_0^\infty z^r g(z)\, d z,$$

From (5), then

$$E(Z^r) = \int_0^\infty \frac{b_N z^{r+a_N+1} e^{-\frac{b_N}{a_N+1} z^{a_N+1}}}{\left(\frac{a_N+1}{b_N}\right)^{\frac{1}{a_N+1}} \Gamma\left(\frac{a_N+2}{a_N+1}\right)} \tag{10}$$

Let $u = \frac{b_N}{a_N+1} z^{a_N+1}$, $du = b_N z^{a_N}$. Upon simplification, (10) leads to

$$E(Z^r) = \left(\frac{1}{b_N}\right)^r \Gamma(2+r) \tag{11}$$

The mean and variance for NSBED can be calculated from (11) as follows.

Theorem 2. Derive mean and variance of the NSBED

Setting $r = 1$, in (11),

$$\mathrm{E}(Z) = \left(\frac{1}{b_N}\right) \Gamma(2+1). \tag{12}$$

Putting $r = 2$, in (11),

$$E(Z^2) = \left(\frac{1}{b_N}\right)^2 \Gamma(2+2) \tag{13}$$

Therefore, variance of NSBED is

$$\mathrm{Var}(Z) = \left(\frac{1}{b_N}\right)^2 [\Gamma 4 - (\Gamma 3)^2]. \tag{14}$$

Theorem 3. Derive the shape coefficients of the NSBED

The shape characteristics of the probability distribution, skewness and kurtosis play an important role. These can be derived from theorem 1, using the following relations.

$$S_k = \frac{\mu_3' - 3\mu_1'\mu_2' + 2\mu_1'^3}{(\mu_2' - \mu_1')^{3/2}}, \quad K_u = \frac{\mu_4' - 4\mu_1'\mu_3' + 6\mu_1'^2\mu_2' - 3\mu_1'^4}{(\mu_2' - \mu_1')^2},$$

where $\mu_r' = E(Z^r)$.

Theorem 4. Derive the expression for mode of the proposed distribution.

Taking the logarithm of (5), we have

$$\ln g(z) = \ln(b_N) + \ln(z) - b_N Z - \ln\left[\frac{1}{b_N}\right]. \tag{15}$$

Differentiate (15) w.r.t. z and equating it zero,

$$\frac{d}{dz} \ln g(z) = \frac{1}{b_N} - b_N z = 0, \tag{16}$$

Therefore

$$z = \frac{1}{b_N}$$

Again differentiate (16),

$$\frac{d^2}{dz^2} \ln g(z) = -\frac{1}{z^2} - b_N z^{-1} = -\frac{1+b_N z}{z^2},$$

At $z = \frac{1}{b_N}$,

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$$\frac{d^2}{dz^2} ln g(z) = -\frac{1}{z^2} < 0.$$

Therefore, the mode is $z = \frac{1}{b_N}$.

Theorem 5. Derive the expression for percentile of the proposed model.

Using the following relation, p^{th} percentile can be obtained.

$$1 - (1 - b_N)e^{-b_N z} = p. \quad (17)$$

Substituting from (6) into (17), z_p satisfies the equation

$$z = -\frac{1}{b_N}\frac{(1-p)}{(1-b_N)}, p \in [0,1] \quad (18)$$

Likewise other statistical properties of the proposed model can be derived in neutrosophic framework.

4. Sample Estimation

In this section, the probability density function and its related important statistical properties of the proposed model are discussed in neutrosophic framework. To derive the MLE of b_N for the PDF, we first need to construct the likelihood function and then maximize it with respect to b_N.

The likelihood function $L(b_N)$ is given by the product of the PDF evaluated at each observed data point. Let $z_1, z_2, \dots z_n$ are n independent and identically distributed observations:

$$L(b_N) = \prod_{i}^{n} g(z_i)$$

Taking the logrithem of the likelihood function, we obtain the log likelihood function $l(b_N)$:

$$l(b_N) = \sum_i^n ln\big(g(z_i)\big) \quad (19)$$

Subtituting the given PDF in (19), we have

$$l(b_N) = 2n ln(b_N) + \sum_i^n ln(z_i) - b_N \sum_i^n z_i \quad (20)$$

To find the MLE of b_N we differentiate (2) by b_N and equating the expression to zero, we get

$$b_N = \frac{2n}{\sum_i^n z_i}$$

where n is the number of observations and z_i is the time until failure for each transistor. This is required neutrosophic statistic for the unknown value of b_N.

To understand this MLE procedure, let us consider the following example. In the field of reliability engineering, engineers are studying the time to failure of a specific electronic component that is assumed to follow NSBED. For example, a manufacturer produces transistors for electronic devices and engineers are collecting data on the time until failure for a sample of these transistors under constant stress. This data will help them analyze and predict the reliability of these transistors in real-world applications. To simplify the process, data was gathered on the time until failure for 10 transistors. The recorded times were imprecisely defined as: 100-120 hours, 150-170 hours, 200-205 hours, 220-230 hours, 250-255 hours, 280-290 hours, 300-315 hours, 320-330 hours, 350-380 hours, and 380-400 hours. Now, to estimate the parameter b_N governing the failure rate of these transistors, they can use Maximum Likelihood Estimation (MLE). The MLE of b_N provides an estimate of the rate at which these transistors are failing.

Using the MLE formula given in (20)

$$b_N = \frac{10}{[2550, 2695]} = [0.0037, 0.0039]$$

This implies that, typically, a transistor is anticipated to malfunction around every with failing rate [0.0037, 0.0039] hours when subjected to the specified stress factors. Such data is essential for

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evaluating reliability, planning maintenance tasks, and enhancing product development tactics. It enables engineers to make educated choices about designing products, setting usage recommendations, and providing customer assistance.

5. Quantile Function

The pth quantile of the NSBED can be derived as:

$$z = -\frac{1}{b_N} ln\left(\frac{1-u}{1+\theta z}\right)$$

The (a) does not follow the closed form, we can use an iterative method like the inverse transform sampling to generate the random sample. A code written has been utilized to find the random numbers of data.

The procedure involves the following steps in the iterative procedure:

1: start with initial guess for z, say z_0

2: Use the (a) to compute a new estimate of z as:$z_1 = -\frac{1}{b_N} ln\left(\frac{1-u}{1+\theta z_0}\right)$

Use z_1 as the new estimate and repeat step 2 until convergence or for a certain number of iterations.

The iterative number eventually converge to a value of z that satisfies the equation. We have simulated random numbers from the proposed model with $b_N = [1,4]$ and $\theta_N = [0.5, 0.5]$, results are shown in Table 1

Table 1: Random number generation from the proposed model

Random Numbers				
[1.161, 2.383]	[0.710, 3.018]]	[0.381,2.893]	[0.211, 2.369]	[0.904, 1.306]
[0.634, 1.690]	[1.332, 8.583]	[0.215, 1.563]	[0.402, 1.733]	[0.262, 1.733]
[0.202, 1.625]	[0.636, 2.856]	[0.239, 5.710]	[0.368, 2.804]	[0.125, 1.463]
[0.320, 3.11]	[0.822, 1.321]	[0.982, 1.799]	[0.001,1.723]	[1.360, 1.620]

Table 2: Some uncertainty measures of the proposed model

Statistical measures	Computed values
Mean	[3.099, 31.141]
Variance	[0.016, 1.795]
Mode	[0.091, 0.947]
Estimated parameter b_N	[1.055, 10.873]

The proposed model incorporates neutrosophic random numbers, a mathematical concept that extends traditional probabilistic frameworks. These random numbers represent uncertainty in three values: true, indeterminate, or false, and are useful in fields like artificial intelligence, economics, engineering, and social sciences. They offer a sophisticated tool for problem-solving under uncertainty and partial truth. The model's key characteristics can be studied using these random numbers, as shown in Table 2. Table 2 presents the proposed model's characteristics, based on analytical calculations. The variance ranges from 0.016 to 1.795, indicating significant uncertainty in the data's spread. The mode ranges from 0.091 to 0.947, indicating uncertainty caused by indeterminate data points. The wide range of values suggests multiple peaks or unclear data points in the distribution, making it difficult to pinpoint the most frequent value. The parameter range, which plays a key role in shaping the distribution, falls between 1.055 and 10.873. The wide range of values indicates a significant level of uncertainty in

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identifying this parameter, impacting statistical measures like mean, variance, and mode. A broad interval indicates high uncertainty in the data's distribution, affecting subsequent calculations.

6. Application to Real Data

In this section, we have utilized the proposed model on the global see level to analyze the uncertainty data. The see level is the average height of the entire ocean surface. The global mean sea level rise is primarily driven by land-based ice sheets and glaciers melting, along with the expansion of saltwater due to warming temperatures. An important indicator of climate change effects, the global mean sea level has been steadily increasing for decades. From 1993 onwards, the rate of this rise has doubled to 0.17 inches (0.44 centimeters) per year from 0.08 inches (0.20 centimeters) per year. Understanding and predicting future sea level trends is crucial for planners, making this data indispensable. The global see level data is shown in Figure 4 which is taken from the online source for understanding purpose [30].

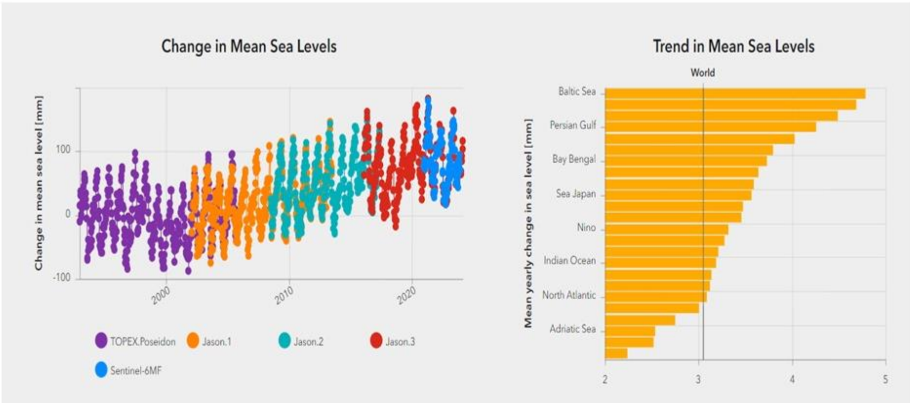

Figure 4: Mean see level change globally [30]

Current measurement technologies have limitations due to the natural variability of oceanic and atmospheric conditions. Integrating data from tide gauges and satellite altimetry can be complex. Regional differences and short-term fluctuations can obscure long-term trends, making it challenging to obtain a consistent global picture. Therefore, while sea level data offers valuable insights, it should be interpreted cautiously, recognizing the inherent limitations and potential for error in current measurement techniques. Some see mean see level measurements with uncertainties are shown in Table 3. Uncertainties in measurements are introduced according to random strategy adopted by [30].

Table 3 Global see level measurements recorded with uncertainty.

See level measurements					
[3.38,3.99]	[3.33, 3.84]	[2.63, 3.74]	[4.25, 4.36]	[3.32,4.25]	[1.08,2.05]
[2.47, 4.10]	[3.91, 4.66]	[2.62,3.71]	[4.13,4.48]	[1.77, 3.02]	[2.30, 4.07]
[2.54,3.11]	[2.72,3.51]	[2.46, 3.99	[2.22, 3.55]	[4.10, 4.51]	[2.78, 3.49]
[1.65, 2.36]	[3.06, 4.45]	[2.68, 3.75]	[2.33, 3.82]	[2.53, 3.60]	[2.00, 3.49]

Table 3 displays that conventional models for probability distributions cannot be utilized due to uncertainties present in recorded measurements. We proceed to analyze the data using the proposed model, and the results are presented in Table 4.

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Table 4: Some uncertainty measures of the proposed model

Statistical measures	Computed values
Mean	[2.768, 3.750]
Variance	[3.831, 7.034]
Mode	[1.384,1.875]
Estimated parameter b_N	[0.7224,0.533]

The findings presented in Table 4 offer valuable information regarding the statistical properties of the data when examined with the suggested model, considering measurement uncertainties. The mean values calculated fall between 2.768 and 3.750, highlighting the central tendency of the data as per the proposed model. This range implies that, on average, the dataset values are situated within this spectrum, showcasing how uncertainty impacts the central location of the data. The range of variance values, ranging from 3.831 to 7.034, shows how spread out the data is. Larger variance values indicate more variability in the dataset, likely due to uncertainties in the measurements. The mode values, ranging from 1.384 to 1.875, highlight the most common data points in the dataset. This interval signifies the peak regions of the data, showcasing where values occur most frequently amidst uncertainty. The outcomes demonstrate the success of the suggested model in addressing the inherent uncertainties within the data, leading to a more thorough statistical analysis compared to traditional models. The wide range of computed values emphasizes the fluctuation and average tendencies influenced by the uncertainties within the measurements.

7. Conclusions

This research presents the introduction of the neutrosophic size-biased exponential distribution (NSBED) as a unique method for managing data with uncertainties. Through the derivation of this distribution from the exponential distribution, we have analyzed different features of the NSBED such as moments, skewness, and kurtosis. The study is further enhanced with visual representations of the cumulative distribution function, density function, and other important functions in relation to survival analysis under uncertainty. The practical utility of the NSBED in handling imprecise data has been demonstrated through the computation of parameter estimates within the neutrosophic framework. Our simulation analysis further confirmed the right-skewed nature and advantageous properties of the NSBED, highlighting its superiority over the traditional size-biased exponential distribution. An analysis of uncertainty in global sea level data was performed using the proposed model. The continuous increase in global mean sea level, primarily caused by ice melting and seawater expansion, highlights the need for accurate predictions of future trends. Results indicate that the NSBED model is an effective tool for evaluating uncertain data, providing valuable information for climate researchers and planners.

Acknowledgment

This study is supported via funding from Prince Sattam bin Abdulaziz University project number (PSAU/2024/R/1445)

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