The Mathematical Exploration of $\pi$ value and its Approximation

Prem Kumar Singh

Department of Computer Science and Engineering,
Gandhi Institute of Technology and Management-Visakhapatnam,
Andhra Pradesh 530045, India
ORCID: 0000-0003-1465-6572
*Correspondence: premsingh.csjm@gmail.com

Abstract

In this paper the author will illustrate some basic mathematics about $\pi$ and its computation in comparison of both Indian Knowledge System as well as others. Same time the infinite series computation and its application is discussed with a pseudo code for decision making process. It is believed that this computation will be helpful for early career researcher for various applications of pie in knowledge processing tasks.

Keyword: Decision making; Indian Knowledge System; Knowledge representation; Mathematics; Turiyam.

1. Indian Knowledge System and Pie value computation.

The computation of $\pi$ or infinite series was considered as one of the most interesting mathematical proof [1-3]. It is started by Baudhayana while measuring the area of circle using square as shown in Fig. 1 [4-5]. It means the area of circle is almost near to square of its radius which may be approx to 3 as per Baudhayana written in Sulbasutra [6-7]. Hence its mathematical exploration is required for Turiyam awareness for better understanding [8].

Figure 1: The Baudhayana circle area approximation.
It again started by Pingala as binary number system and its addition in term of Laghu(L) and Guru (G) in Sanskrit[9-11]. It is also depicted as Pingala's Chandaḥśāstra in Sanskrit literature [12-13]. Due to which the Sanskrit is considered as one of the scientific language for knowledge representation [14]. In other countries they known as Pascal Triangle as shown in Fig. 2.

Figure 2: The Pingala's Chandaḥśāstra to generate the series of Number

It is believed that the Pingala series concept is used in Indian Knowledge system to build Jyotirlinga as shown in Fig. 3. Fig. 4 represents some other geometry build on Pingala Series.

Figure 3: The Pingala's Chandaḥśāstra to generate the series of Number
The Aryabhata gave a formula to compute the Pie value as written in Sanskrit [15-16]:

caturadhikam śatamaṣṭagunaṃ dvāṣaṣṭistathā sahasrāṇām
ayutadvayaviskambhasyāsanno vṛttapariṇāhāh.

It says first add 4 with 100 as (4+100=104). Try to multiply it (i.e. 104) via 8 i.e. (104 × 8=832). In this multiplication add 62,000 i.e. 62832. The obtain number consider as circumference of a circle and divide it by 20000 considering a circle having diameter 20,000. It provides the value of π=3.1416. This he used as Spherical geometry construction as shown in Fig. 5.
It was extended by Narayana’s as cows sequence motivated from Pingala series [17]. He formulated the problem as let us suppose one cow at starting of the year. The cow give birth one calf in each year from the age of 4 years old. In this case try to approximate the number of calves produced every year. It gave third order sequence as $C_n = C_{n-1} + C_{n-3}$ where $n \geq 3$, $C_0 = 1$, $C_1 = 1$, $C_2 = 1$, $C_3 = 1$.

The same concept used in Egyptian Papyrus as distinct way [18]. They used division of circumference and its division by diameter as shown in Fig. 6. It is used as a game where 64 ball used to represent the circle and 9 ball on the diameter of circle used to divide them for computation of pie shown in Fig. 7. Later it is used to represent as Square and its area computation equivalent to circle as shown in Fig. 8. Its computation is shown in Fig. 9 in a mathematical way.
Fig 6: The Egyptian Papyrus to compute the Pie value

\[ \pi = \frac{\text{Circumference}}{\text{Diameter}} = 3.14159 \]

Figure 7: The Egyptian Papyrus as game of 64 Ball and its arrangement in the circle
(2r)X(2r) = Area of Circle+4 X Area of Triangle;
4r2=Area of circle+4X1/2X(2r/3)X(2r/3);
Πr2= 4r2-8r2/9;
Π= 28/9=3.11

The Madhava gave sine, cosine and arc tan(x) series and its computation at early 14th century as shown in Fig 10 [19].

\[
\sin \theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \cdots = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} \theta^{2k+1},
\]

\[
\cos \theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \cdots = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} \theta^{2k},
\]

\[
\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots = \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} x^{2k+1} \quad \text{where } |x| \leq 1.
\]
The Madhava wrote a formula computation of Pie in Sanskrit as shown in Fig. 11. It is illustrated mathematically in Fig. 12.

![Image](https://i.imgur.com/3Z5Q5Q.png)

**Figure 11:** The Madhava formula for computation of Pie

Just put \( \arctan(x) = 1 \) in Madhava series shown in Fig. 10. It will provide Leibniz formula for Pie as shown in Fig. 13. This series and its square multiplication provides approximate of Euler Pie value approximation as shown in Fig. 14 [20].

![Image](https://i.imgur.com/5Q5Q5Q.png)

**Figure 12:** The mathematical exploration of Formula Shown in Fig. 11
The Nilkantha gave a infinite series to compute the Pie value using the mathematical expression as shown in Fig. 15. It can be observed that the Nilkantha series contains multiplication of 3-consecuting numbers. The value of variable increases by two or every iteration for the denominator with sign changes. Hence it gave a pseudo code for pie and its implementation. First create three variable \( n=2 \), pie=3, and sign=1. We can create a infinite series as for loop based on our choice from [0, 100000]. Every iteration change the sign as sign*(-1). Compute the Pie value as = Pie + sign*(4/(n) * (n+1) * (n+2)). Increase the n value as 2 after the each iteration and print its value. In this way this code may take maximum \( \mathcal{O}(n\cdot\log(n)\cdot(\log\log(n))) \). Hence the exploration of Pie value and its computation is totally based on Turiyam consciousness rather than only Indigenous or non-indigenous. In future the author will focus on exploring the pie value based on Buffon Needle Problem as shown in Fig. 16.
In this paper the computation of Pie and infinite series computation based on Indian Knowledge system is explored with an example. Same time the comparative study of pie value computation by Indian Knowledge System with Egypt Papyrus and other mathematician is also compared. It is believed that the current paper will help to early career researcher and data visualization student. In near future the author will focus on other mathematical concept and its exploration with Indian Knowledge system.

2. Conclusion

This paper provide a mathematical exploration of Pie value and its computation in context of Indian Knowledge System as well as comparison with Egypt papyrus and others. It is also discussed that the computation of pie value and its implementation takes $O(n^{\log(n)}(\log\log(n))$ time as per Nilkantha work. In this way this paper will be helpful for various decision making process and applications of pie.

Acknowledgements: Author thanks the editorial team for their valuable time.

Funding: Author declares that, there is no funding for this paper.

Conflicts of Interest: Author declares that, there is no conflict of interest for the given paper.

Ethics approval: This article does not contain any studies with human or animals participants.

References

Doi: https://doi.org/10.54216/JNFS.080101
Received: October 22, 2023 Revised: December 19, 2023 Accepted: January 22, 2024


