Necessary and Sufficient Conditions for a Stability of the Concepts of Stable Interior and Stable Exterior via Neutrosophic Crisp Sets

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Abstract

The general direction for generating any stable neutrosophic crisp topology is through base or the stable neutrosophic crisp interior concept, which is closed in the finite intersection process and not closed in the union process, likewise the stable neutrosophic crisp exterior is closed in the finite union process but not closed in finite intersection. Our research deals with finding necessary and sufficient condition for the finite union and finite intersection to be closed respectively using the concept of confused crisp sets.

Keywords: Stable Neutrosophic Crisp Topological Spaces; Stable Neutrosophic Crisp Interior Set; Stable Neutrosophic Crisp Exterior Set; Confused Crisp Set.

1. Introduction

The problems that researchers stumble on in mathematics are represented by finding equivalents for some properties or constructing sufficient and necessary conditions upon which they are based. Hence, we can start building new mathematical concepts that help us by the tools available [6]. Imran et al. [12,13] provided the view of new concepts of weakly neutrosophic crisp separation axioms, and neutrosophic crisp generalized sg-closed sets and their continuity. Finally, the senses of generalized alpha generalized closed sets in neutrosophic crisp topological spaces and neutrosophic generalized alpha generalized separation axioms were informed by Abdulkadhim et al. [14-15].

At a first sight to the neutrosophic crisp sets [2-5], it comes to mind that the work is smooth and synonymous with set theory, but in light of the analytical study, we found that the process is within the three type of family and types of union and intersection

(($\mathbb{L}^N \cup_1 \mathrm{K}^N =< \mathbb{L}_1 \cup \mathrm{K}_1, \mathbb{L}_2 \cup \mathrm{K}_2, \mathbb{L}_3 \cap \mathrm{K}_3 >, L^N \cup_2 \mathrm{K}^N =< \mathbb{L}_1 \cup \mathrm{K}_1, \mathbb{L}_2 \cap \mathrm{K}_2, \mathbb{L}_3 \cap \mathrm{K}_3 > \mathbb{L}^N \cap_1 \mathrm{K}^N =< \mathbb{L}_1 \cap \mathrm{K}_1, \mathbb{L}_2 \cap \mathrm{K}_2, \mathbb{L}_3 \cup \mathrm{K}_3 >$ and $\mathbb{L}^N \cap_2 \mathrm{K}^N =< \mathbb{L}_1 \cap \mathrm{K}_1, \mathbb{L}_2 \cup \mathrm{K}_2, \mathbb{L}_3 \cup \mathrm{K}_3 >$),

the complement(($(\mathrm{K}^N)^{C1} =< \mathrm{K}_1^C, \mathrm{K}_2^C, \mathrm{K}_3^C >, (\mathrm{K}^N)^{C2} =< \mathrm{K}_3, \mathrm{K}_2, \mathrm{K}_1 > and (C^N)^{C3} =< \mathrm{K}_3, \mathrm{K}_2^C, \mathrm{K}_1 >$)),

the empty and universal sets ($\emptyset_1^N =< \emptyset, \emptyset, X >, \emptyset_2^N =< \emptyset, X, \emptyset >, \emptyset_3^N =< \emptyset, X, X >$,

$\emptyset_4^N =< \emptyset, \emptyset, \emptyset >, X_1^N =< X, \emptyset, \emptyset >, X_2^N =< X, X, \emptyset >, X_3^N =< X, \emptyset, X >$ and $X_4^N =< X, X, X >$

but the subset (($\mathbb{L}^N \subseteq_1 \mathrm{K}^N \leftrightarrow \mathbb{L}_1 \subseteq \mathrm{K}_1, \mathbb{L}_2 \subseteq \mathrm{K}_2, \mathbb{L}_3 \supseteq \mathrm{K}_3$), ($\mathbb{L}^N \subseteq_2 \mathrm{K}^N \leftrightarrow$

$\mathbb{L}_1 \subseteq \mathrm{K}_1, \mathbb{L}_2 \supseteq \mathrm{K}_2, \mathbb{L}_3 \supseteq \mathrm{K}_3$)). See [1] and the type of NC-point ($P^{N1} =< \{P_1\}, \{P_2\}, \{P_3\} >$,

$\{P_1\} \neq \{P_2\} \neq \{P_3\}$ where $P_1, P_2, P_3 \in X$, $P^{N2} =< \{P\}, \emptyset, \{P\}^C >$, $P^{N3} =< \emptyset, \{P\}, \{P\}^C >$)

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$P^{N4} =< \{P\}, \emptyset, \emptyset >, P^{N5} =< \emptyset, \{P\}, \emptyset >$ and $\{P\}$ is singleton [7] and we defined an additional sixth point as follows: ($P^{N6} =< \mathbb{A}_1, \mathbb{A}_2, \mathbb{A}_3 >, \mathbb{A}_i \neq \emptyset, i$=1 or 2 or 3)). The SNC-sets it is an important mathematical concept with broad scientific resonance, and at the same time it is considered one of the influential topics in practical and engineering life. A specialist in this field, as well as someone who has knowledge and experience, can generalize many mathematical concepts, especially stable neutrosophic crisp topological ones, using it.

2. Preliminaries

Here in this section, we will briefly review on give the basic concepts and their results on which our research was based. For more details, see [6,8].

Definition 2.1[6]: Let X be a fixed set that is not empty, a (SNCT-space) is a family $\wr$ satisfies the following condition:

1. $\emptyset_1^N, X_1^N \in \wr$
2. $\forall \mathbb{A}^N, \mathbb{B}^N \in \wr, \exists\ \mathbb{K}^N \in \wr, \ni\ \mathbb{K}^N \subseteq_1 \mathbb{A}^N \cap_1 \mathbb{B}^N$
3. $\forall\ \mathbb{A}_i^N \in \wr, \exists\ \mathbb{F}^N \in \wr\ \ni\ \mathbb{F}^N \subseteq_1 {}_{i=1}^{n}\cup_2 \mathbb{A}_i^N$

Then $(X, \wr)$is a (SNCT-space). For any $\mathbb{A}^N \in \wr$ is a stable neutrosophic crisp open set and its denoted by $(SNCO - set)$, the complement of type 2 for $(SNCO - set)$ is stable neutrosophic crisp closed set and denoted by $(SNCC - set)$.

Definition 2.2[6]: Let $(X, \wr)$ be a (SNCT-space), $\mathbb{A}^N$is a NC- set, then the stable neutrosophic crisp interior of $\mathbb{A}^N$ denoted by $Si_{ij}(\mathbb{A}^N)$ and define as: $Si_{ij}(\mathbb{A}^N) = \cup_i \{\mathbb{S}^N \in \wr, \mathbb{S}^N \subseteq_j \mathbb{A}^N\}$, $i, j = 1{,}2$. It can be noted that the index i is an indication of the type of union and the index j is an indication of the type of the subsets.

Definition 2.3 [8]: Let (X, $\wr$) be a SNCT-space and $L^N be\ a$ NC-set. Then, the stable neutrosophic crisp exterior of L^N denoted $by\ Se_{ij}(L^N)$ and define as: $Se_{ij}(L^N) = Si_{ij}((L^N)^{C_2})$ $\quad i, j = 1{,}2$

generally, the idea of the stable neutrosophic crisp exterior is not $SNCC - set$ under the process of intersection, and the stable neutrosophic crisp interior is not $SNCC - set$ under the process of union in any SNCT-space.

Example 2.4: Let X=$\{f, l, g, s, n\}$, (X,$\wr$) be a SNCT − space where
$\wr = \{A^N, B^N, C^N, D^N, K^N, L^N, M^N, N^N, \emptyset_1^N, X_1^N\}$ such that:
$A^N =< \{f, l\}, \{g, n\}, \{s\} >, B^N =< \{f\}, \{g\}, \{s\} >, C^N =< \{g\}, \{l, n\}, \{s, g\} >,$
$D^N =< \{g\}, \{n\}, \emptyset >, K^N =< \{g\}, \emptyset, \{s\} >, L^N =< \emptyset, \emptyset, \{s, g\} >, M^N =< \emptyset, \emptyset, \{s\} >$
$N^N =< \emptyset, \emptyset, \{g\} >$. Now let
$O^N =< \{g, s\}, \{l, n\}, \{g\} >, P^N =< \emptyset, \emptyset, \{l\} >, E^N =< \{s\}, \{g, n\}, \{f, g\} >, F^N =< \{n\}, \emptyset, \emptyset >$
$Si_{21}(O^N) \cup_1 Si_{21}(P^N) = < \{g\}, \emptyset, \{g\} >$ and $Si_{21}(O^N \cup_1 P^N)$=$< \{g\}, \emptyset, \emptyset >$.
$Se_{21}(E^N) \cap_1 Se_{21}(F^N) = \emptyset_1^N$and $Se_{21}(E^N \cup_1 F^N)$=$\emptyset_4^N$. We can see that
$Si_{21}(O^N) \cup_i Si_{21}(P^N) \neq_i Si_{21}(O^N \cup_i P^N)$ and $Se_{21}(E^N) \cap_1 Se_{21}(F^N) \neq_i Se_{21}(E^N \cup_1 F^N)$.
Since establishing this equality is crucial, one of the main goals of this research is to resolve this issue by offering some essential and adequate criteria in both theoretical and practical domains. The researcher believes that some new concepts were added, which can be linked with basic topological mathematical concepts useful in the applied fields making it easier for us to solve some practical problems and have a comprehensive view.

Definition 2.5[8]: Take (X, $\wr$) be a SNCT-space and $L^N be\ a$ SNC-set, then the confused crisp set of L^N denoted by $\text{Ꭓ}_{ij}(L^N)$ and define as: $\text{Ꭓ}_{ij}(L^N) =_i Si_{ij}(L^N) \cup_i Se_{ij}(L^N), i, j = 1{,}2$

3. The main theorem

Theorem 3.1: Take (X, $\wr$) be a SNCT-space, V^N, $Ḥ^N are\ a\ NC - sets$ of any type. Then the next two conditions are equivalent for $i, j = 1{,}2$

1. $Si_{ij}(V^N \cup_i Ḥ^N) =_i Si_{ij}(V^N) \cup_i Si_{ij}(Ḥ^N)$
2. $\text{Ꭓ}_{ij}(V^N \cup_i Ḥ^N) =_i [\text{Ꭓ}_{ij}(V^N) \cup_i Si_{ij}(Ḥ^N)] \cap_i [\text{Ꭓ}_{ij}(Ḥ^N) \cup_i Si_{ij}(V^N)]$

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Proof: 2⇔ 1) $Ϫ_{ij}\ (V^N \cup_i H^N) =_i [Ϫ_{ij}\ (V^N) \cup_i Si_{ij}(H^N)] \cap_i [Ϫ_{ij}\ (H^N) \cup_i Si_{ij}(V^N)]$

$=_i [Ϫ_{ij}\ (V^N) \cap_i Ϫ_{ij}(H^N)] \cup_i \left[Ϫ_{ij}\ (V^N) \cap_i Si_{ij}\left({V^N}^N\right)\right] \cup_i [Ϫ_{ij}\ (H^N) \cap_i Si_{ij}(H^N\ )]$

$\cup_i [Si_{ij}(V^N) \cap_i Si_{ij}(H^N)] =_i ([Ϫ_{ij}\ (V^N) \cup_i Ϫ_{ij}(H^N)])^{C_2} \cup_i Si_{ij}(V^N) \cup_i Si_{ij}(H^N)$

Now $Si_{ij}(V^N \cup_i K^N) =_i \left(Se_{ij}(V^N \cup_i H^N)\right)^{C_2} \cap_i Ϫ_{ij}\ (V^N \cup_i H^N)$

$=_i \left(Se_{ij}(V^N)\right)^{C_2} \cup_i \left(Se_{ij}(H^N)\right)^{C_2} \cap_i Ϫ_{ij}\ (V^N \cup_i H^N)$

$=_i \left[\left(Se_{ij}(V^N)\right)^{C_2} \cap_i Ϫ_{ij}\ (V^N \cup_i H^N)\right] \cup_i \left[\left(Se_{ij}(H^N)\right)^{C_2} \cap_i Ϫ_{ij}\ (V^N \cup_i H^N)\right]$

$=_i \left[\left(Se_{ij}(V^N)\right)^{C_2} \cap_i \left(([Ϫ_{ij}\ (V^N) \cup_i Ϫ_{ij}(H^N)])^{C_2} \cup_i Si_{ij}(V^N) \cup_i Si_{ij}(H^N)\right)\right] \cup_i$

$\left[\left(Se_{ij}(H^N)\right)^{C_2} \cap_i \left(\left(([Ϫ_{ij}\ (V^N) \cup_i Ϫ_{ij}(H^N)])^{C_2} \cup_i Si_{ij}(V^N) \cup_i Si_{ij}(H^N)\right)\right)\right]$

$=_i Si_{ij}(V^N) \cup_i Si_{ij}(H^N)$

1 ⇔ 2) $[Ϫ_{ij}\ (V^N) \cup_i Si_{ij}(H^N)] \cap_i [Ϫ_{ij}\ (H^N) \cup_i Si_{ij}(V^N)] =_i$

$[Ϫ_{ij}\ (V^N) \cap_i Ϫ_{ij}(H^N)] \cup_i [Ϫ_{ij}\ (V^N) \cap_i Si_{ij}(V^N)] \cup_i [Ϫ_{ij}\ (H^N) \cap_i Si_{ij}(H^N)]$

$\cup_i (Si_{ij}(V^N) \cap_i Si_{ij}(H^N)) =_i [Ϫ_{ij}\ (V^N) \cap_i Ϫ_{ij}(H^N)] \cup_i (Si_{ij}(V^N) \cup_i Si_{ij}(H^N))$

$=_i \left[\left(Si_{ij}(V^N) \cup_i Se_{ij}(V^N)\right) \cap_i \left(Si_{ij}(K^N) \cup_i Se_{ij}(H^N)\right)\right] \cup_i (Si_{ij}(V^N) \cup_i Si_{ij}(H^N))$

$=_i \left(Si_{ij}(V^N) \cap_i Si_{ij}(H^N)\right) \cup_i \left(Si_{ij}(V^N) \cap_i Se_{ij}(H^N)\right) \cup_i \left(Se_{ij}(V^N) \cap_i Si_{ij}(H^N)\right)$

$\cup_i \left(Se_{ij}(V^N) \cap_i Se_{ij}(H^N)\right) \cup_i (Si_{ij}(V^N) \cup_i Si_{ij}(H^N))$

$=_i \left(Se_{ij}(V^N) \cap_i Se_{ij}(H^N)\right) \cup_i \left(Si_{ij}(V^N) \cup_i Si_{ij}(H^N)\right) =_i Si_{ij}(V^N \cup_i H^N) \cup_i Se_{ij}(V^N \cup_i H^N)$

$=_i Ϫ_{ij}\ (V^N \cup_i H^N)$

The following two conditions are sufficient for subsets $L^N and\ H^N$ to satisfy the condition (b) in Theorem 3.1, and hence, to satisfy the relation $Si_{ij}(V^N \cup_i H^N) =_i Si_{ij}(V^N) \cup_i Si_{ij}(H^N)$ for $i = 1,2$

Corollary 3.2: Let $V^N, H^N are\ a\ NC - sets$ of SNCT-space $(X,\ \mathcal{Z})$, if V^N

and H^N satisfies the following both conditions:

1. $[Ϫ_{ij}\ (V^N) \cup_i Si_{ij}(H^N)] =_i Se_{ij}(V^N) \cup_i Si_{ij}(V^N \cup_i H^N)$;
2. $[Ϫ_{ij}\ (H^N) \cup_i Si_{ij}(H^N)] =_i Se_{ij}(H^N) \cup_i Si_{ij}(V^N \cup_i H^N)$;

Then it hold that $Si_{ij}(V^N \cup_i H^N) =_i Si_{ij}(V^N) \cup_i Si_{ij}(H^N)$ for $i = 1,2$

Proof: To achieve proof $Si_{ij}(V^N \cup_i H^N) =_i Si_{ij}(V^N) \cup_i Si_{ij}(H^N)$, we must prove that conditions 1 and 2 are met in theorem 3.1, to fulfill the proof.

$Ϫ_{ij}\ (V^N \cup_i H^N) =_i Si_{ij}(V^N \cup_i H^N) \cup_i Se_{ij}(V^N \cup_i H^N)$

$=_i Si_{ij}(V^N \cup_i H^N) \cup_i (Se_{ij}(V^N) \cap_i Se_{ij}(H^N))$

$=_i (Si_{ij}(V^N \cup_i H^N) \cup_i (Se_{ij}(V^N)) \cap_i (Si_{ij}(V^N \cup_i H^N) \cup_i (Se_{ij}(H^N))$

$=_i [Ϫ_{ij}\ (V^N) \cup_i Si_{ij}(H^N)] \cap_i [Ϫ_{ij}\ (H^N) \cup_i Si_{ij}(V^N)]$

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and the assertion of this corollary immediately follows from theorem 3.1.

Preposition 3.3: Let (X, Ƨ) be a SNCT-space, $\mathrm{V}^N, Ⱨ^N$ *are a NC − sets* of any type. Then the following two conditions are equivalent for $i = 1,2$

a) $Se_{ij}(\mathrm{V}^N \cap_i Ⱨ^N) =_i Se_{ij}(\mathrm{V}^N) \cup_i Se_{ij}(Ⱨ^N)$

b) $Ӿ_{ij}(\mathrm{V}^N \cap_i Ⱨ^N) =_i [Ӿ_{ij}(\mathrm{V}^N) \cap_i Ӿ_{ij}(Ⱨ^N)] \cup_i [Se_{ij}(\mathrm{V}^N) \cup_i Se_{ij}(Ⱨ^N)]$

Proof: b⇔ $a)$ $Se_{ij}(\mathrm{V}^N \cap_i Ⱨ^N) =_i \left(Si_{ij}(\mathrm{V}^N \cap_i Ⱨ^N)\right)^{C_2} \cap_i Ӿ_{ij}(\mathrm{V}^N \cap_i Ⱨ^N)$

$=_i \left(Si_{ij}(\mathrm{V}^N \cap_i Ⱨ^N)\right)^{C_2} \cap_i \left([Ӿ_{ij}(\mathrm{V}^N) \cap_i Ӿ_{ij}(Ⱨ^N)] \cup_i [Se_{ij}(\mathrm{V}^N) \cup_i Se_{ij}(Ⱨ^N)]\right)$

$=_i \left[\left(Si_{ij}(\mathrm{V}^N \cap_i Ⱨ^N)\right)^{C_2} \cap_i \left([Ӿ_{ij}(\mathrm{V}^N) \cap_i Ӿ_{ij}(Ⱨ^N)]\right)\right]$

$\cup_i \left[\left(\left(Si_{ij}(\mathrm{V}^N \cap_i Ⱨ^N)\right)^{C_2}\right) \cap_i [Se_{ij}(\mathrm{V}^N) \cup_i Se_{ij}(Ⱨ^N)]\right]$

$=_i \left[\left(Si_{ij}(\mathrm{V}^N \cap_i Ⱨ^N)\right)^{C_2} \cap_i \left([Ӿ_{ij}(\mathrm{V}^N) \cap_i Ӿ_{ij}(Ⱨ^N)]\right)\right] \cup_i [Se_{ij}(\mathrm{V}^N) \cup_i Se_{ij}(Ⱨ^N)]$

$=_i \left[\left(\left(Si_{ij}(\mathrm{V}^N)\right)^{C_2} \cup_i \left(Si_{ij}(\mathrm{V}^N)\right)^{C_2}\right) \cap_i \left([Ӿ_{ij}(\mathrm{V}^N) \cap_i Ӿ_{ij}(Ⱨ^N)]\right)\right] \cup_i [Se_{ij}(\mathrm{V}^N) \cup_i Se_{ij}(Ⱨ^N)]$

$=_i \left[\left(\left(Si_{ij}(\mathrm{V}^N)\right)^{C_2}\right) \cap_i \left([Ӿ_{ij}(\mathrm{V}^N) \cap_i Ӿ_{ij}(Ⱨ^N)]\right)\right] \cup_i \left[\left(\left(Si_{ij}(Ⱨ^N)\right)^{C_2}\right) \cap_i \left([Ӿ_{ij}(\mathrm{V}^N) \cap_i Ӿ_{ij}(Ⱨ^N)]\right)\right]$

$\cup_i [Se_{ij}(\mathrm{V}^N) \cup_i Se_{ij}(Ⱨ^N)]$

$=_i \left[\left(\left(Si_{ij}(\mathrm{V}^N)\right)^{C_2} \cap_i Ӿ_{ij}(\mathrm{V}^N)\right) \cap_i [Ӿ_{ij}(Ⱨ^N)]\right] \cup_i \left[\left(\left(Si_{ij}(Ⱨ^N)\right)^{C_2} \cap_i Ӿ_{ij}(\mathrm{V}^N)\right) \cap_i [Ӿ_{ij}(Ⱨ^N)]\right]$

$\cup_i [Se_{ij}(\mathrm{V}^N) \cup_i Se_{ij}(Ⱨ^N)]$

$=_i \left[Se_{ij}(\mathrm{V}^N) \cap_i [Ӿ_{ij}(Ⱨ^N)]\right] \cup_i \left[Se_{ij}(Ⱨ^N) \cap_i [Ӿ_{ij}(Ⱨ^N)]\right] \cup_i [Se_{ij}(\mathrm{V}^N) \cup_i Se_{ij}(Ⱨ^N)]$

$=_i [Se_{ij}(\mathrm{V}^N) \cup_i Se_{ij}(Ⱨ^N)] \cap_i \left[Se_{ij}(\mathrm{V}^N) \cup_i [Ӿ_{ij}(Ⱨ^N)]\right] \cap_i \left[[Ӿ_{ij}(Ⱨ^N)] \cup_i Se_{ij}(Ⱨ^N)\right]$

$\cap_i [Ӿ_{ij}(\mathrm{V}^N) \cup_i Ӿ_{ij}(Ⱨ^N)]$

$\cup_i [Se_{ij}(\mathrm{V}^N) \cup_i Se_{ij}(Ⱨ^N)] =_i [Se_{ij}(\mathrm{V}^N) \cup_i Se_{ij}(Ⱨ^N)]$

$a \Leftrightarrow b)$ $Ӿ_{ij}(\mathrm{V}^N \cap_i Ⱨ^N) =_i Si_{ij}(\mathrm{V}^N \cap_i Ⱨ^N) \cup_i Se_{ij}(\mathrm{V}^N \cap_i Ⱨ^N)$

$=_i (Si_{ij}(\mathrm{V}^N) \cap_i Si_{ij}(Ⱨ^N)) \cup_i (Se_{ij}(\mathrm{V}^N) \cup_i Se_{ij}(Ⱨ^N))$

$=_i \left[Si_{ij}(\mathrm{V}^N) \cup_i \left(Se_{ij}(\mathrm{V}^N) \cup_i Se_{ij}(Ⱨ^N)\right)\right] \cap_i \left[Si_{ij}(Ⱨ^N) \cup_i \left(Se_{ij}(\mathrm{V}^N) \cup_i Se_{ij}(Ⱨ^N)\right)\right]$

$=_i \left[Ӿ_{ij}(\mathrm{V}^N) \cup_i \left(Se_{ij}(Ⱨ^N)\right)\right] \cap_i \left[Ӿ_{ij}(Ⱨ^N) \cup_i \left(Se_{ij}(\mathrm{V}^N)\right)\right]$

$=_i [Ӿ_{ij}(\mathrm{V}^N) \cap_i Ӿ_{ij}(Ⱨ^N)] \cup_i [Ӿ_{ij}(\mathrm{V}^N) \cap_i Se_{ij}(\mathrm{V}^N)] \cup_i [Ӿ_{ij}(Ⱨ^N) \cap_i Se_{ij}(Ⱨ^N)]$

$\cup_i [Se_{ij}(Ⱨ^N) \cap_i Se_{ij}(\mathrm{V}^N)] =_i [Ӿ_{ij}(\mathrm{V}^N) \cap_i Ӿ_{ij}(Ⱨ^N)] \cup_i [Se_{ij}(\mathrm{V}^N) \cup_i Se_{ij}(Ⱨ^N)]$

Which complete this prove. The sufficient (b) in preposition 3.3 can be substituted into two equivalent conditions, as in the following result.

Corollary 3.4: Let $\mathrm{V}^N, Ⱨ^N$ *are a NC − sets* of $(X, Ƨ)$, if V^N and $Ⱨ^N$ satisfies the following both conditions:

1. $[Ӿ_{ij}(\mathrm{V}^N) \cup_i Se_{ij}(Ⱨ^N)] =_i Si_{ij}(Ⱨ^N) \cup_i Se_{ij}(\mathrm{V}^N \cap_i Ⱨ^N)$

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2. $\left[\text{Ꭓ}_{ij}\ (\mathrm{H}^N) \cup_i Se_{ij}(\mathrm{V}^N)\right] =_i Si_{ij}(\mathrm{V}^N) \cup_i Se_{ij}(\mathrm{V}^N \cap_i \mathrm{H}^N)$;

Then it hold that $Se_{ij}(\mathrm{V}^N \cap_i \mathrm{H}^N) =_i Se_{ij}(\mathrm{V}^N) \cup_i Se_{ij}(\mathrm{H}^N)$ for $i, j = 1,2$

Proof: we must only prove that conditions 1 and 2 are equivalent to condition b in theorem 3.1.

$\text{Ꭓ}_{ij}\ (\mathrm{V}^N \cap_i \mathrm{H}^N) =_i Si_{ij}(\mathrm{V}^N \cap_i \mathrm{H}^N) \cup_i Se_{ij}(\mathrm{V}^N \cap_i \mathrm{H}^N)$

$=_i (Si_{ij}(\mathrm{V}^N) \cap_i Si_{ij}(\mathrm{H}^N)) \cup_i Se_{ij}(\mathrm{V}^N \cap_i \mathrm{H}^N)$

$=_i (Si_{ij}(\mathrm{V}^N) \cup_i Se_{ij}(\mathrm{V}^N \cap_i \mathrm{H}^N)) \cap_i (Si_{ij}(\mathrm{H}^N) \cup_i Se_{ij}(\mathrm{V}^N \cap_i \mathrm{H}^N))$

$=_i \left[\text{Ꭓ}_{ij}\ (\mathrm{H}^N) \cup_i Se_{ij}(\mathrm{V}^N)\right] \cap_i \left[\text{Ꭓ}_{ij}\ (\mathrm{V}^N) \cup_i Se_{ij}(\mathrm{H}^N)\right]$

$=_i \left[\text{Ꭓ}_{ij}\ (\mathrm{V}^N) \cap_i \text{Ꭓ}_{ij}\ (\mathrm{H}^N)\right] \cup_i \left[Se_{ij}(\mathrm{V}^N) \cup_i Se_{ij}(\mathrm{H}^N)\right]$

And the assertion of this corollary immediately follows from preposition 3.3.

Preposition 3.5: If $\mathrm{V}^N and\ \mathrm{H}^N$ are a NC-sets of any type, then $Si_{ij}(\mathrm{V}^N \cap_i (\mathrm{H}^N)^{C_2}) =_i Si_{ij}(\mathrm{V}^N) \cap_i Se_{ij}(\mathrm{H}^N)$. In addition the two are equivalent for $i, j = 1,2$

$a) Se_{ij}(\mathrm{V}^N) \cup_i Si_{ij}(\mathrm{H}^N) =_i Se_{ij}(\mathrm{V}^N \cap_i (\mathrm{H}^N)^{C_2})$;

$b) Si_{ij}((\mathrm{V}^N)^{C_2} \cup_i \mathrm{H}^N) =_i Se_{ij}(\mathrm{V}^N) \cup_i Si_{ij}(\mathrm{H}^N)$.

Proof: We have $Si_{ij}(\mathrm{V}^N) \cap_i Se_{ij}(\mathrm{H}^N) =_i Si_{ij}(\mathrm{V}^N) \cap_i Si_{ij}((\mathrm{H}^N)^{C_2}) =_i Si_{ij}(\mathrm{V}^N \cap_i (\mathrm{H}^N)^{C_2})$

More ever if $\mathrm{V}^N and\ \mathrm{H}^N$ satisfy the condition (b),

then it follows that $Se_{ij}(\mathrm{V}^N \cap_i (\mathrm{H}^N)^{C_2}) =_i Si_{ij}((\mathrm{V}^N)^{C_2} \cup_i \mathrm{H}^N) =_i Si_{ij}((\mathrm{V}^N)^{C_2}) \cup_i Si_{ij}(\mathrm{H}^N)$.

Which implies that $Se_{ij}(\mathrm{V}^N) \cup_i Si_{ij}(\mathrm{H}^N) =_i Se_{ij}(\mathrm{V}^N \cap_i (\mathrm{H}^N)^{C_2})$.

on the other hand if assume that $Se_{ij}(\mathrm{V}^N) \cup_i Si_{ij}(\mathrm{H}^N) =_i Se_{ij}(\mathrm{V}^N \cap_i (\mathrm{H}^N)^{C_2})$, then we have

$$Se_{ij}(\mathrm{V}^N) \cup_i Si_{ij}(\mathrm{H}^N) =_i Se_{ij}(\mathrm{V}^N \cap_i (\mathrm{H}^N)^{C_2}) \qquad (1)$$

$$Se_{ij}(\mathrm{V}^N) \cup_i Si_{ij}(\mathrm{H}^N) =_i Si_{ij}((\mathrm{V}^N)^{C_2}) \cup_i Si_{ij}(\mathrm{H}^N) \qquad (2)$$

$$Se_{ij}(\mathrm{V}^N \cap_i (\mathrm{H}^N)^{C_2}) =_i Si_{ij}\left((\mathrm{V}^N)^{C_2} \cup_i (\mathrm{H}^N)\right) \qquad (3)$$

By using the last three equations, we conclude that (b) is true.

If $\mathrm{V}^N =_i \mathrm{H}^N$ and $\text{Ꭓ}_{ij}\ (\mathrm{V}^N) =_i \emptyset_1^N$, then $L^N and\ \mathrm{H}^N$ do not satisfy the condition (b). In this case,

$Se_{ij}(\mathrm{V}^N) \cup_i Si_{ij}(\mathrm{H}^N) \neq_i Se_{ij}(\mathrm{V}^N \cap_i (\mathrm{H}^N)^{C_2})$

We remark that the condition (b) of preposition 3.5 is equivalent to

$Se_{ij}(\mathrm{V}^N \cap_i (\mathrm{H}^N)^{C_2}) =_i Se_{ij}(\mathrm{V}^N) \cup_i Si_{ij}(\mathrm{H}^N)$. We now introduce some specific conditions by which the condition (b) of preposition 3.5 is satisfied. Under those conditions, the NC-sets $\mathrm{V}^N and\ \mathrm{H}^N$ of a SNCT-space satisfy the relation $Se_{ij}(\mathrm{V}^N \cap_i (\mathrm{H}^N)^{C_2}) =_i Se_{ij}(\mathrm{V}^N) \cup_i Si_{ij}(\mathrm{H}^N)$.

Corollary 3.6: Let $\mathrm{V}^N, \mathrm{H}^N are\ a\ SNC-sets$ of SNCT-space $(X, \mathfrak{Z})$, if $\mathrm{V}^N and\ \mathrm{H}^N$ satisfies the following both conditions:

1. $\left[\text{Ꭓ}_{ij}\ (\mathrm{V}^N) \cup_i Si_{ij}(\mathrm{H}^N)\right] =_i Si_{ij}(\mathrm{V}^N) \cup_i Si_{ij}((\mathrm{V}^N)^{C_2} \cup_i \mathrm{H}^N)$;

2. $\left[\text{Ꭓ}_{ij}\ (\mathrm{H}^N) \cup_i Se_{ij}(\mathrm{V}^N)\right] =_i Se_{ij}(\mathrm{H}^N) \cup_i Si_{ij}((\mathrm{V}^N)^{C_2} \cup_i \mathrm{H}^N)$

Then it hold that $Se_{ij}(\mathrm{V}^N) \cup_i Si_{ij}(\mathrm{H}^N) =_i Se_{ij}(\mathrm{V}^N \cap_i (\mathrm{H}^N)^{C_2})$ for $i, j = 1,2$

Proof: We know that $\text{Ꭓ}_{ij}\ (\mathrm{V}^N) =_i \text{Ꭓ}_{ij}\ ((\mathrm{V}^N)^{C_2})$

And we have $\left[\text{Ꭓ}_{ij}\ (\mathrm{H}^N) \cup_i Se_{ij}(\mathrm{V}^N)\right] =_i \text{Ꭓ}_{ij}\ (\mathrm{H}^N) \cup_i Si_{ij}((\mathrm{V}^N)^{C_2})$

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Now $\mathrm{Ꜫ}_{ij}\ ((\mathrm{V}^N)^{C_2}) \cup_i Si_{ij}(\mathrm{Ң}^N) =_i\ \mathrm{Ꜫ}_{ij}\ (\mathrm{V}^N) \cup_i Si_{ij}(\mathrm{Ң}^N)\ =_i Si_{ij}(\mathrm{V}^N) \cup_i Si_{ij}((\mathrm{V}^N)^{C_2} \cup_i \mathrm{Ң}^N)$

And $[\mathrm{Ꜫ}_{ij}\ (\mathrm{Ң}^N) \cup_i Se_{ij}(\mathrm{V}^N)] =_i Se_{ij}(\mathrm{Ң}^N) \cup_i Si_{ij}((\mathrm{V}^N)^{C_2} \cup_i \mathrm{Ң}^N)$

From corollary 3.2 we get

$Si_{ij}((\mathrm{V}^N)^{C_2} \cup_i \mathrm{Ң}^N) =_i Si_{ij}((\mathrm{V}^N)^{C_2}) \cup_i Si_{ij}(\mathrm{Ң}^N)$. Which completes.

Preposition 3.7: Let $\mathrm{V}^N, \mathrm{Ң}^N$ *are SNC − sets* of $(X, Ƨ)$ containing at least two point, if L^N *and* K^N satisfies the following both conditions:

a) $K^N \cap_i \mathrm{V}^N \cap_i [\mathrm{Ꜫ}_{ij}\ (\mathrm{Ң}^N)]^{C_2} =_i \emptyset_1^N$;

b) V^N is $SNCO - set$;

Then it hold that Then $\mathrm{Ң}^N \cap_i \mathrm{V}^N$ is SNC$O - set$ for $i, j = 1{,}2$

Proof: The assertion is obviously true for the case $\mathrm{Ң}^N \cap_i \mathrm{V}^N =_i \emptyset_1^N$ or $\mathrm{Ң}^N \cap_i \mathrm{V}^N =_i X_1^N$. We now assume that $\mathrm{Ң}^N \cap_i \mathrm{V}^N \neq_i \emptyset_1^N$ and $\mathrm{Ң}^N \cap_i \mathrm{V}^N \neq_i X_1^N$, and $p^{Nz} \in_i \mathrm{Ң}^N \cap_i \mathrm{V}^N$(z=1..6) . Then the assumption (a) implies that $p^{Nz} \notin_i [\mathrm{Ꜫ}_{ij}\ (K^N)]^{C_2}$,and since $p^{Nz} \in_i K^N$ it follows that $p^{Nz} \in_i Si_{ij}(\mathrm{Ң}^N)$, which means that $\mathrm{Ң}^N \cap_i \mathrm{V}^N \subseteq_i Si_{ij}(\mathrm{Ң}^N) \cap_i \mathrm{V}^N$ or $\mathrm{Ң}^N \cap_i \mathrm{V}^N =_i Si_{ij}(\mathrm{Ң}^N) \cap_i \mathrm{V}^N$. Finally, by (b), $\mathrm{Ң}^N \cap_i \mathrm{V}^N =_i Si_{ij}(\mathrm{Ң}^N) \cap_i \mathrm{V}^N$is $SNCO - set$. In the following theorem, by using 1.5, we will introduce some sufficient conditions for the intersection of subsets to be $SNCO - set$. The first part of this theorem is analogous to the first part of the proof of preceding theorem.

Preposition 3.8: Let $\mathrm{V}^N, \mathrm{Ң}^N$ *are a NC − sets* of SNCT-space (X, Ƨ) containing at least two point. If L^N *and* K^N satisfy the following properties:

a) $Si_{ij}((\mathrm{V}^N)^{C_2} \cup_i \mathrm{Ң}^N) =_i Se_{ij}(\mathrm{V}^N) \cup_i Si_{ij}(\mathrm{Ң}^N)$;

b) L^N is $SNCO - set$;

c) $Se_{ij}(\mathrm{V}^N) \cup_i \mathrm{Ң}^N$is $SNCO - set$;

Then it hold that $K^N \cap_i \mathrm{V}^N$ is $SNCO - set$

Proof: Obviously, the assertion holds for the case $\mathrm{Ң}^N \cap_i \mathrm{V}^N =_i \emptyset_1^N$ or $\mathrm{Ң}^N \cap_i \mathrm{V}^N =_i X_1^N$. Hence, we assume that $\mathrm{Ң}^N \cap_i \mathrm{V}^N \neq_i \emptyset_1^N$ and $\mathrm{Ң}^N \cap_i \mathrm{V}^N \neq_i X_1^N$,, and that $\mathrm{Ң}^N \cap_i \mathrm{V}^N$ were not $SNCO - set$. Let p^{Nz} be the point(z=1…6) with the property $p^{Nz} \in_i \mathrm{Ң}^N \cap_i \mathrm{V}^N$ and $p^{N1} \in_i [\mathrm{Ꜫ}_{ij}\ (\mathrm{Ң}^N \cap_i \mathrm{V}^N)]^{C_2}$...4

If $p^{Nz} \in_i [\mathrm{Ꜫ}_{ij}\ (\mathrm{V}^N)]^{C_2}$, then $p^{Nz} \notin_i \mathrm{V}^N$ because of (b), which is contrary to (4). Hence, we notice that $p^{Nz} \notin_i [\mathrm{Ꜫ}_{ij}\ (\mathrm{V}^N)]^{C_2}$. Since $p^{Nz} \in_i [\mathrm{Ꜫ}_{ij}\ (\mathrm{Ң}^N \cap_i \mathrm{V}^N)]^{C_2} \subseteq_i [\mathrm{Ꜫ}_{ij}\ (\mathrm{V}^N)]^{C_2} \cup_i [\mathrm{Ꜫ}_{ij}\ (\mathrm{Ң}^N)]^{C_2}$,

it should be $p^{Nz} \in_i [\mathrm{Ꜫ}_{ij}\ (\mathrm{Ң}^N)]^{C_2}$…5

we have $L^N \cap_i [\mathrm{Ꜫ}_{ij}\ (\mathrm{Ң}^N)]^{C_2} =_i \mathrm{V}^N \cap_i [Se_{ij}(\mathrm{Ң}^N) \cup_i Si_{ij}(\mathrm{Ң}^N)]^{C_2} \subseteq_i \mathrm{V}^N \cap_i [Si_{ij}(\mathrm{Ң}^N)]^{C_2}$ ...6

Furthermore, by (4), (5), and (6),

we get $p^{Nz} \in_i \mathrm{V}^N \cap_i [\mathrm{Ꜫ}_{ij}\ (\mathrm{Ң}^N)]^{C_2} \subseteq_i \mathrm{V}^N \cap_i [Si_{ij}(\mathrm{Ң}^N)]^{C_2} \subseteq_i [Si_{ij}((\mathrm{V}^N)^{C_2})]^{C_2} \cap_i [Si_{ij}(\mathrm{Ң}^N)]^{C_2}$…7

By Theorem 2.5, (a), and (7), we get $p^{Nz} \notin_i Se_{ij}(\mathrm{V}^N) \cup_i Si_{ij}(\mathrm{Ң}^N) =_i Se_{ij}(\mathrm{V}^N \cap_i (\mathrm{Ң}^N)^{C_2})$. Since $=_i Se_{ij}(\mathrm{V}^N \cap_i (\mathrm{Ң}^N)^{C_2})$ is the 'largest' $SNCO - set$ including $(\mathrm{V}^N \cap_i (\mathrm{Ң}^N)^{C_2})^{C_2}$ and, by (c), $Se_{ij}(\mathrm{V}^N) \cup_i \mathrm{Ң}^N$is $SNCO - set$ including $(\mathrm{V}^N \cap_i (\mathrm{Ң}^N)^{C_2})^{C_2}$,

we have $p^{Nz} \notin_i Se_{ij}(\mathrm{V}^N \cap_i (\mathrm{Ң}^N)^{C_2}) \subseteq_i Se_{ij}(\mathrm{V}^N) \cup_i \mathrm{Ң}^N$...8

However, in view of (4), we see that $p^{Nz} \in_i \mathrm{Ң}^N$, and hence, $p^{Nz} \in_i Se_{ij}(\mathrm{V}^N) \cup_i \mathrm{Ң}^N$

Which is contrary to (8). Therefore, $\mathrm{Ң}^N \cap_i \mathrm{V}^N$ should $SNCO - set$.

6. Conclusion

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Many important concepts are compactness in classical analysis, we can study them in SNCT-space, and we can also modify them with respect to gem set see [9]. The paracompactness was first interdused by Dieudonne in 1944 [16], which still retains general properties to compactness , so we can also study it in SNCT- space. For more generalizations, see [10,11].

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