Neutrosophic Beta-Lindley distribution: Mathematical properties and modeling bladder cancer data

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Abstract

The beta-Lindley distribution is used in the field of survival analysis to imitate techniques employed with human lifetime data. The neutrosophic beta-Lindley distribution (NBL) is designed to characterize a range of survival statistics with indeterminacies. The established distribution is used, for instance, to describe unknown data that is roughly favorably skewed. The evolved NBL's three main statistical characteristics—the neutrosophic moments, hazard, and survival functions are covered in this article. Additionally, The well-known maximum likelihood estimation method is used to estimate the neutrosophic parameters. To check if the predicted neutrosophic parameters were met, a simulation study was done. Notably, talks of prospective NBL uses in the real world have made use of actual data. Actual data were utilized to show how well the suggested model performed in compared to the current distributions.

Keywords: Bladder cancer; survival analysis; beta- Lindley distribution; neutrosophic statistics; hazard function.

1. Introduction

The analysis of ambiguous, fuzzy, imprecise, or uncertain observations can be done using neutrosophic statistics, which is an extension of classical statistics. The analysis of neutrosophic numbers and events, neutrosophic regression, neutrosophic probability distribution, and neutrosophic estimate are all carried out using neutrosophic statistical methods [1] to create neutrosophy, which enables the depiction of uncertainty, ambiguity, and contradiction. Traditional statistics usually make the assumption that the data is unambiguous, in which case each observation is assigned a certain value. However, sometimes facts from the real world are unclear or insufficient. Neurosophic statistics offers a paradigm for handling confusing, inadequate, and inconsistent data in order to get beyond these limitations [2-4].

Non-empirical statistics consider three variables: membership in truth, membership in indeterminacy, and membership in falsity. The degree of authenticity, ambiguity, or falsity associated with an observation or a hypothesis is represented by each factor. Membership functions are used to represent these degrees in a manner akin to fuzzy sets [2, 3].

Neutrophic statistics is used in several fields, including decision-making, pattern recognition, data mining, and image processing [4-9]. It provides a flexible mathematical tool for simulating and examining complex systems that are very uncertain and imprecise.

One of the most important applications of neutrosophic information is the analysis of survival statistics. The survival analysis statistical technique estimates the time until an event occurs [10]. The probability distributions of the temporal data serve as the framework for the overall survival study. Survival analysis and neutrosophic reasoning are used to create a neutrosophic survival probability distribution. The likelihood of an event happening at different dates is represented by the survival probability distribution in the context of neutrosophic. Neutosophic reasoning is used to account for the ambiguity and uncertainty of the survival statistics. It allows for

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the portrayal of only having a limited or foggy understanding of events. The degree of truth, falsity, and ambiguity associated with the survival probability at various time points must be taken into consideration by both neutrosophic parameters and the already available survival data. This can be done using mathematical models and techniques particular to neutosophic logic. Numerous studies discuss the neutrosophic probability distribution [10-22].
There are many uses for the beta-Lindley distribution, including in survival analysis. In this work, the applications of the beta-Lindley distribution when the data is in interval form and contains some neutrosophical indeterminacy were expanded. A variety of qualities are studied under the newly proposed distribution and their applications are explored with the help of simulated and real data applications based on bladder cancer.

2. Neutrosophic beta- Lindley distribution (NBL)

The goal of the neutrosophic probability distribution (NPD) is to manage uncertain information with limited knowledge. It is an extension of classical probability theory. The idea of indeterminacy is included, which states that the likelihood of an event can be true, untrue, or undetermined all at once. Three parameters are used by the NPD: truth-membership, indeterminacy-membership, and falsity-membership. The NPD provides for a more subtle portrayal of uncertainty, which makes it fascinating to understand. Neutosophic probability, which considers the possibility of indeterminacy in addition to typical crisp probabilities, enables a more expansive interpretation of uncertainty.
The beta-Lindley distribution which proposed by Merovci and Sharma [23] can be defined in terms of the cumulative distribution function (CDF) and probability density function (PDF), respectively, by:

$$F(x;\theta,\alpha,\beta) = \frac{\left(1-((\theta+1+\theta x)/(\theta+1))e^{-\theta x}\right)^{\alpha}}{\alpha B(\alpha,\beta)} \times {}_2F_1\left(\alpha, 1-\beta; \alpha+1; 1-\frac{\theta+1+\theta x}{\theta+1}e^{-\theta x}\right) \quad (1)$$

$$f(x;\theta,\alpha,\beta) = \frac{\theta^2(\theta+1+\theta x)^{\beta-1}(1+x)e^{-\theta\beta x}}{B(\alpha,\beta)(\theta+1)^{\beta}} \times \left[1-\frac{\theta+1+\theta x}{\theta+1}e^{-\theta x}\right]^{\alpha-1}, x>0, \theta,\beta,\alpha>0 \quad (2)$$

Smarandache [2] was the first to introduce the idea of neutrosophic probability as a function $NP :\to [0,1]^3$ where U is a neutrosophic sample space and defined the probability mapping to take the form $NP(S) = (ch(S), ch(neut\ S), ch(anti\ S)) = (\eta, \beta, \tau)$
$where\ 0 \le \eta, \beta, \tau \le 1\ and\ 0 \le \eta+\beta+\tau \le 3.$

The term Λ represents the set of sample space, R represents the set of real numbers, and Υ denotes a sample space event, X_N and Y_N denote neutrosophic r.v. Additionally, we present certain well-known concepts and features of neutrosophic probability and logic that are crucial for developing this model of neutrosophic probability.

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Definition 1 Consider the real-valued crisp r.v.W, which has the following definition: $W : \Lambda \to R$

where Λ is the event space and W_N neutrosophic r.v. as follows:

$$W_N : \Lambda \to R(I)$$

and

$$W_N = W + I$$

The term I represents indeterminacy.

Theorem 1 Let the neutrosophic r.v. $W_N = W + I$ and the CDF of W is $F_W(w) = P(W \le w)$ [15]. The following assertions are correct:

$$FW_N(w) = F_W(w - I),$$

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$f_{W_N}(w) = f_W(w - I),$

where F_{W_N} and f_{W_N} are the *CDF* and *PDF* of a neutrosophic r.v. W_N, respectively.

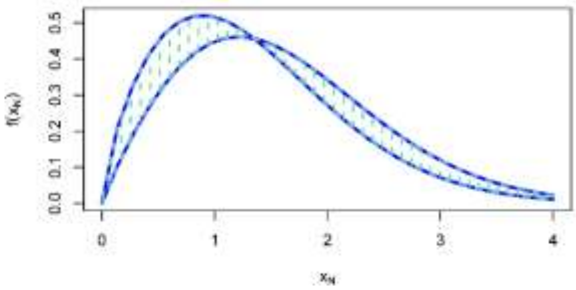
Theorem 2 Let $W_N = W + I$, is the neutrosophic r.v., then the expected value and variance can be derived as follows: $E(W_N) = E(W) + I$ and $V(W_N) = V(W)$ [15].

Suppose the neutrosophic variable could be expressed as: $w_N = w_L + w_U I_N$ where $I_N \in \{I_L, I_U\}$ and w_L and $w_U I_N$ denote the determined and indeterminate parts, respectively.

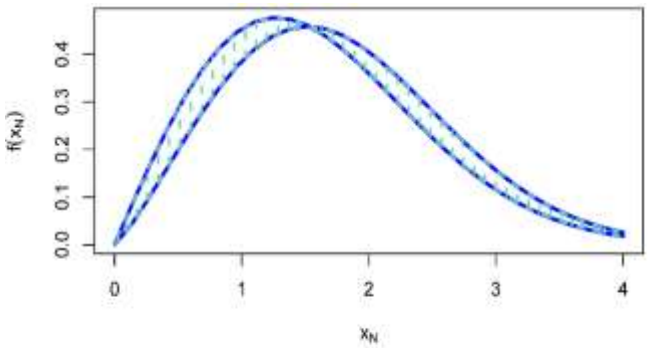
Assume that the neutrosophic random variable $x_N \in \{x_L, x_U\}$ follows the beta-Lindley having neutrosophic parameters: $\theta_N \in \{\theta_L, \theta_U\}$, $\beta_N \in \{\beta_L, \beta_U\}$, $\alpha_N \in \{\alpha_L, \alpha_U\}$ where the letters L and U are the lower values and the upper values, respectively. Then, the neutrosophic CDF and PDF of neutrosophic beta-Lindley (NBL) is given by

$$F(x_N;\theta_N,\alpha_N,\beta_N) = \frac{\left(1-\left((\theta_N+1+\theta_N x_N)/(\theta_N+1)\right)e^{-\theta_N x_N}\right)^{\alpha_N}}{\alpha B(\alpha,\beta)} \times {}_2F_1\left(\alpha_N, 1-\beta_N; \alpha_N+1; 1-\frac{\theta_N+1+\theta_N x_N}{\theta_N+1}e^{-\theta_N x_N}\right). \quad (3)$$

$$\text{f}(x_N;\theta_N,\alpha_N,\beta_N) = \frac{\theta_N{}^2(\theta_N+1+\theta_N x_N)^{\beta_N-1}(1+x_N)e^{-\theta_N\beta_N x_N}}{\beta_N(\alpha_N,\beta_N)(\theta_N+1)^{\beta_N}} \times \left[1-\frac{\theta_N+1+\theta_N x_N}{\theta_N+1}e^{-\theta_N x_N}\right]^{\alpha_N-1}. \quad (4)$$



Figures 1: The pdf of NBL when $\theta_N \in \{2, 2.2\}$, $\beta_N \in \{1.5, 1.7\}$, $\alpha_N \in \{1.6, 1.9\}$



Figures2: The pdf of NBL when $\theta_N \in \{1.8, 2\}$, $\beta_N \in \{1.2, 1.5\}$, $\alpha_N \in \{1.3, 1.5\}$

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3. Statistical Properties of NPL

3.1 The neutrosophic Survival and hazard functions

The survival function, $S(x)$, which is the probability of a member is surviving prior to some time, is defined for

$$S(x_N, \theta_N, \alpha_N, \beta_N) = 1 - \frac{\left(1-\left((\theta_N+1+\theta_N x_N)/(\theta_N+1)\right)e^{-\theta_N x_N}\right)^{\alpha_N}}{\alpha B(\alpha,\beta)} \times {}_2F_1\left(\alpha_N, 1-\beta_N; \alpha_N+1; 1-\frac{\theta_N+1+\theta_N x_N}{\theta_N+1}e^{-\theta_N x_N}\right) \quad (5)$$

The hazard function, $h(x)$, is defined as:

$$h(x_N, \theta_N, \alpha_N, \beta_N) = \& \frac{\theta_N{}^2(\theta_N+1+\theta_N x_N)^{\beta_N-1}(1+x_N)e^{-\theta_N\beta_N x_N}}{B_N(\alpha_N,\beta_N)(\theta_N+1)^{\beta}{}_N} \times\left[1-\frac{\theta_N+1+\theta_N x_N}{\theta_N+1}e^{-\theta_N x_N}\right]^{\alpha_N-1}\Bigg) \times\left(1-\frac{\left(1-\left((\theta_N+1+\theta_N x_N)/(\theta_N+1)\right)e^{-\theta_N x_N}\right)^{\alpha_N}}{\alpha_N\beta_N(\alpha_N,\beta_N)}\times {}_2F_1(\alpha_N, 1-\beta_N; \alpha_N+1; 1-\frac{\theta_N+1+\theta_N x_N}{\theta_N+1}e^{-\theta_N x_N}\Bigg)\Bigg)^{-1}. \quad (6)$$

3.2. The neutrosophic moments

Theorem 3. The neutrosophic k^{th} moment, $E(x)^k$, of the NBL is given as

$$E(X_N{}^k) = \frac{\Gamma^2(\alpha_N+\beta_N)}{\Gamma(\beta_N)\theta_N{}^k} \times \sum_{j_N=0}^{\infty}\sum_{i_N=0}^{\infty}\Big(\big((-1)^{i_N+j_N} e^{(\beta_N+j_N)(\theta_N+1)}\big) \times(\Gamma(\alpha_N-j_N)\Gamma(i_N-1)\Gamma(\alpha_N+\beta_N-i_N) \times(\theta_N+1)^{\beta_N+j_N-i_N} j_N!(\beta_N+j_N)^{\alpha_N+\beta_N+k_N-i_N}\big)^{-1}\Big) \times[\theta_N\Gamma(k_N-i_N+\alpha_N+\beta_N, (\theta_N+1)(\beta_N+j_N)) + \frac{1}{(\beta_N+j_N)}\Gamma(k_N-i_N+\alpha_N+\beta_N+1, (\theta_N+1)(\beta_N+j_N))\Big]. \quad (7)$$

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Proof:

$$E\left(X_N{}^k\right)=\int_0^\infty x_N{}^k f\left(x_N\right)dx_N$$
$$=\frac{\theta_N}{B_N\left(\alpha_N,\beta_N\right)(\theta_N+1)^{\beta_N}}$$
$$\times\int_0^\infty\left(\frac{x_N}{\theta_N}\right)^k(\theta_N+1+x_N)^{\beta_N-1}\left(\frac{\theta_N+x_N}{\theta_N}\right)e^{-\beta_N x_N}$$
$$=\frac{\Gamma\left[1-\frac{\theta_N+1+x_N}{\theta_N+1}e^{-x_N}\right]^{\alpha_N-1}dx_N}{B_N\left(\alpha_N,\beta_N\right)\theta_N{}^k(\theta_N+1)^{\beta_N+j_N}} \tag{8}$$
$$\times\sum_{j=0}^\infty\frac{(-1)^{j_N}}{\Gamma\left(\alpha_N-j_N\right)j_N!}$$
$$\times\left[\theta_N\int_0^\infty x_N{}^k(\theta_N+1+x_N)^{\alpha_N+\beta_N-1}e^{-(\beta_N+j_N)x_N}dx_N\right.$$
$$\left.+\int_0^\infty x_N{}^{k+1}(\theta_N+1+x_N)^{\alpha_N+\beta_N-1}e^{-(\beta_N+j_N)x_N}dx_N\right]dx_N$$

$$=\frac{\Gamma\left(\alpha_N\right)}{B_N\left(\alpha_N,\beta_N\right)\theta_N{}^k}\sum_{j=0}^\infty\frac{(-1)^j{}_N}{\Gamma\left(\alpha_N-j_N\right)(\theta_N+1)^{\beta_N+j_N}j_N!}e^{(\beta_N+j_N)(\theta_N+1)}$$

$$\times\sum_{i=0}^\infty\frac{\Gamma\left(\alpha_N+\beta_N\right)}{\Gamma\left(\alpha_N+\beta_N-i_N\right)\Gamma\left(i_N-1\right)}(-1)^{i_N}(\theta_N+1)^{i_N}$$
$$\times\left[\theta_N\int_{\theta+1}^\infty x_N{}^{k_N-i_N+\alpha_N+\beta_N-1}e^{-(\beta_N+j_N)x_N}dx_N\right.$$
$$\left.+\int_{\theta+1}^\infty x_N{}^{k-i_N+\alpha_N+\beta_N}e^{-(\beta_N+j_N)x_N}dx_N\right],$$
$$\int_{\theta+1}^\infty x^{k-i_N+\alpha_N+\beta_N-1}e^{-(\beta_N+j_N)x_N}dx_N$$
$$=\frac{1}{(\beta_N+j_N)^{k-i_N+\alpha_N+\beta_N}}$$
$$\times\Gamma\left(k-i_N+\alpha_N+\beta_N,(\theta_N+1)\left(\beta_N+j_N\right)\right).$$

Then

$$E\left(X_N{}^k\right)=\frac{\Gamma^2\left(\alpha_N+\beta_N\right)}{\Gamma\left(\beta_N\right)\theta_N{}^{k_N}}$$
$$\times\sum_{j=0}^\infty\sum_{i=0}^\infty\left(\left((-1)^{i_N+j_N}e^{(\beta_N+j_N)(\theta_N+1)}\right)\right.$$
$$\times(\Gamma\left(\alpha_N-j_N\right)\Gamma\left(i_N-1\right)\Gamma\left(\alpha_N+\beta_N-i_N\right)$$
$$\left.\times(\theta_N+1)^{\beta_N+j_N-i_N}j_N!(\beta_N+j_N)^{\alpha_N+\beta_N+k_N-i_N}\right)^{-1}\Big)$$
$$\times[\theta_N\Gamma\left(k_N-i_N+\alpha_N+\beta_N,(\theta_N+1)\left(\beta_N+j_N\right)\right)$$
$$\left.+\frac{1}{\left(\beta_N+j_N\right)}\Gamma\left(k_N-i_N+\alpha_N+\beta_N+1,(\theta_N+1)\left(\beta_N+j_N\right)\right)\right].$$

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4. Parameter Estimation of NBL

The Maximum Likelihood Estimator (MLE) is a popular statistical method for estimating a probability distribution's parameters from observed data. Finding the parameter values that maximize the likelihood function, which gauges the likelihood of witnessing the provided data under various parameter settings, is the notion underlying MLE. The MLE offers estimates that are most likely to have produced the observed data by maximizing the likelihood function. It is a well-known and commonly applied statistical method with applications in many different industries.
The MLE of the parameters $\theta_N, \alpha_N, \beta_N$ of the NBL distribution is mostly used. Maximum likelihood estimation (MLE) method in estimating NBL parameters. Suppose $x_{N1}, x_{N2}, \ldots, x_{Nn}$ be a random sample of size n from the NBL, the log-likelihood function is then given by:

$$\begin{aligned} \ell = \ln L = \; & n(2\log(\theta_N) - \log\Gamma(\alpha_N) - \log\Gamma(\beta_N) + \log\Gamma(\alpha_N+\beta_N) \\ & -\beta_N \log(\theta_N+1)) \\ & +\sum_{i=1}^{n}\log\left(1+x_{N_i}\right) + (\beta_N-1)\log\left(\theta_N+1+\theta_N x_{N_i}\right) \\ & -\theta_N\beta_N\sum_{i=1}^{n} x_{N_i} + (\alpha_N-1)\sum_{i=1}^{n}\log\left(1-\frac{\theta_N+1+\theta_N x_{N_i}}{\theta_N+1}e^{-\theta_N x_{N_i}}\right). \end{aligned} \quad (8)$$

Therefore, MLE of $\hat{\theta}_N$, $\hat{\alpha}_N$, and $\hat{\beta}_N$, respectively, can be obtained by solving the following log-likelihood equations:

$$\frac{\partial \ln L}{\partial \theta_N} = \frac{2m}{\theta_N} - \frac{m\beta_N}{\theta_N+1} + (\beta_N-1)\sum_{i=1}^{n}\frac{1+x_{Ni}}{\theta_N+1+\theta_N x_{Ni}} - \beta_N\sum_{i=1}^{m} x_{Ni} + (\alpha_N-1)$$

$$\times\sum_{i=1}^{n}\frac{e^{-\theta_N x_N} x_{Ni}\left((\theta_N+1)(\theta_N+1+\theta_N x_{Ni})-1\right)}{(\theta_N+1)^2\left(1-\left((\theta_N+1+\theta_N x_{Ni})/(\theta_N+1)\right)e^{-\theta_N x_{Ni}}\right)} = 0 \quad (9)$$

$$\frac{\partial \ln L}{\partial \alpha_N} = n\psi(\alpha_N+\beta_N) - n\psi(\alpha_N) + \sum_{i=1}^{m}\ln\left(1-\frac{(\theta_N+1+\theta_N x_{Ni})e^{-\theta_N x_{Ni}}}{\theta_N+1}\right) = 0 \quad (10)$$

$$\frac{\partial \ln L}{\partial \beta_N} = n\psi(\alpha_N+\beta_N) - n\psi(\beta_N) - n\log(\theta_N+1) + \sum_{i=1}^{n}\log(\theta_N+1+\theta_N x_{Ni}) - \theta_N\sum_{i=1}^{n} x_{Ni} = 0 \quad (11)$$

5. Simulation results

R software is used to execute a Monte Carlo simulation with various sample sizes, $n = 30, 50, 150, 300$ and neutrosophic parameters in four cases:

(1) $\theta_N \in \{1.8, 2\}$, $\beta_N \in \{1.2, 1.5\}$, and $\alpha_N \in \{1.3, 1.5\}$

(2) $\theta_N \in \{1, 1.5\}$, $\beta_N \in \{1, 1.3\}$, and $\alpha_N \in \{1.5, 1.8\}$

(3) $\theta_N \in \{2, 2.3\}$, $\beta_N \in \{1.8, 2\}$, and $\alpha_N \in \{2, 2.4\}$

(4) $\theta_N \in \{2.5, 3\}$, $\beta_N \in \{2, 2.5\}$, and $\alpha_N \in \{2.3, 2.7\}$

One thousand times are repeated in the simulation. Performance indicators like the estimators' neutrosophic average and the bias of that average (NAB), NBA $= \sum_{i=1}^{1000}(\hat{\delta}-\delta)/n$, and neutrosophic Mean Square Error (NMSE), NMSE $= \sum_{i=1}^{1000}(\hat{\delta}-\delta)^2/n$ are attained for all values of n . Tables 1–4 present the findings. These tables show that, as predicted, as sample sizes increase, both the NAB and NMSE for neutrosophic parameters decrease. The neutrosophic MLE for the NBL also provides accurate estimation with a larger sample size, according to the study's findings.

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Table 1: Average performance criteria for case 1

n	NAB			NMSE		
	α_N	β_N	θ_N	α_N	β_N	θ_N
30	[0.0183,0.0191]	[0.0201,0.0210]	[0.0275,0.0283]	[0.0373,0.0381]	[0.0414,0.0422]	[0.0485,0.0501]
50	[0.0121,0.0134]	[0.0144,0.0152]	[0.0225,0.0243]	[0.0331,0.0342]	[0.0364,0.0372]	[0.0443,0.0463]
150	[0.0110,0.0119]	[0.0149,0.0151]	[0.0212,0.0225]	[0.0321,0.0329]	[0.0349,0.0351]	[0.0318,0.0328]
300	[0.0044,0.0057]	[0.0075,0.0084]	[0.0147,0.0159]	[0.0254,0.0268]	[0.0273,0.0288]	[0.0264,0.0271]

Table 2: Average NAB and NMSE for case 2

N	NAB			NMSE		
	α_N	β_N	θ_N	α_N	β_N	θ_N
30	[0.015,0.0143]	[0.1057,0.1292]	[0.1175,0.1527]	[0.0207,0.0548]	[0.1515,0.2263]	[0.1870,0.3160]
50	[0.0065,0.0089]	[0.0642,0.0784]	[0.0713,0.0927]	[0.0121,0.0323]	[0.0821,0.1226]	[0.1014,0.1713]
150	[0.0023,0.0031]	[0.0213,0.0260]	[0.0236,0.0307]	[0.0039,0.0105]	[0.0253,0.0379]	[0.0313,0.0529]
300	[0.0015,0.0021]	[0.0138,0.0168]	[0.0153,0.0199]	[0.0023,0.0063]	[0.0150,0.0224]	[0.0185,0.0312]

Table 3: Average NAB and NMSE for case 3

N	NAB			NMSE		
	α_N	β_N	θ_N	α_N	β_N	θ_N
30	[0.0243,0.0197]	[0.1823,0.2370]	[0.1641,0.2005]	[0.1003,0.1105]	[0.3201,0.5409]	[0.2593,0.3873]
50	[0.0131,0.0109]	[0.0863,0.1122]	[0.0777,0.0950]	[0.0492,0.0546]	[0.1276,0.2156]	[0.1033,0.1544]
150	[0.0029,0.0025]	[0.0182,0.0236]	[0.0163,0.0200]	[0.0029,0.0025]	[0.0096,0.0107]	[0.0183,0.0278]
300	[0.0022,0.0019]	[0.0128,0.0166]	[0.0115,0.0140]	[0.0064,0.0071]	[0.0152,0.0257]	[0.0123,0.0184]

Table 4: Average performance criteria for case 4

n	NAB			NMSE		
	α_N	β_N	θ_N	α_N	β_N	θ_N
30	[0.0150,0.0200]	[0.1641,0.2005]	[0.0054,0.0084]	[0.0109,0.0339]	[0.7202,0.9250]	[0.0313,0.0824]
50	[0.0080,0.0109]	[0.0777,0.0950]	[0.0030,0.0048]	[0.0054,0.0168]	[0.2871,0.3687]	[0.0152,0.0405]
150	[0.0018,0.0025]	[0.0163,0.0200]	[0.0007,0.0011]	[0.0011,0.0033]	[0.0517,0.0664]	[0.0030,0.0079]
300	[0.0013,0.0019]	[0.0115,0.0140]	[0.0006,0.0009]	[0.0007,0.0022]	[0.0342,0.0439]	[0.0020,0.0053]

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6. Applications

In a practical application in this section, a real-world dataset was used to measure interest in the NBL distribution. The data under consideration is a compilation of 128 cancer patients' months-long remission durations. The remission periods reported here are based on a subset of data from bladder cancer studies that [24] published and are largely intended for descriptive purposes. The findings of the goodness of fit test based on the Kolmogorov-Smirnov (KS) test indicate that the Beta-Lindley distribution is one of the plausible distributions for the remission times. According to [14], this data demonstrates that interval reporting rather than precise reporting is used to offer remission times for specific cancer patients, including [7.26, 8.2], [12, 14.77], [15, 17.2], [5.3, 7.1], [75.02, 81], and [1.5, 3.2]. Two neutrosophic distributions, which were used previously for analyzing this data: The neutrosophic exponential distribution (NE) [14] and the neutrosophic inverse power Lindley distribution (NIPL) [25], are contrasted with the suggested NBL's model appropriateness. Which model fits the data the best is determined using the log-likelihood value (LogL), Akaike Information Criteria (AIC), Bayesian Information Criteria (BIC), and Kolmogorov-Smirnov (KS) tests. The highest LogL values and the lowest AIC, BIC, and KS statistic values identify the model that fits the data the best. A higher p-value also indicates that the model best matches the neutrosophic data. Table 5 provides a list of the neutrosophic maximum likelihood estimators and model sufficiency indicators. The outcomes show that for data, the NBL is superior to the NE and NIPL. The table's bold values display how effectively the suggested model performs.

Table 5: The criteria selection neutrosophic distributions for cancer patients' data

	NBL	NIPL	NE
Parameter	$\theta_N = [1.51, 1.68]$	$\beta_N = [1.202, 1.213]$	$\alpha_N = [0.1081, 0.10822]$
	$\alpha_N = [0.143,\ 0.155]$	$\alpha_N = [0.1532,\ 0.1578]$	
	$\alpha_N = [0.136,\ 0.144]$		
LogL	**[78.2591, 79.3684]**	[80.3025, 81.1497]	[10.352, 13.241]
AIC	**[152.2511, 153.7428]**	[156.605, 158.2994]	[63.508, 65.334]
BIC	**[150.2064, 151.4234]**	[154.8234, 156.5187]	[60.218, 61.229]
KS-value	**[0.118, 0.123]**	[0.124, 0.132]	[0.752, 0.774]
KS- p-value	**[0.961, 0.969]**	[0.955, 0.987]	[1.135×10-6, 1.188×10-6]

7. Conclusions

The neutrosophic beta-Lindley distribution (NBL) is advocated in this article. In a range of application data, this well-known distribution can be utilized to account for problems with survival and reliability. The key statistical properties of the developed NBL have all been explored, including the neutrosophic survival function, neutrosophic hazard rate, and neutrosophic moments. The developed neutrosophic MLEs have been used to demonstrate neutrosophic average bias and MSEs for a variety of sample sizes. To determine whether the computed neutrosophic parameters were met, a simulation study was conducted. The sample size and neutrosophic parametric value are crucial factors in accurately estimating an unknown parameter, according to simulation data. Another argument in favor of the use of the NBL in neutrosophic situations is the collecting of remission times from 128 cancer patients.

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