# Sorting Out Interval Valued Neutrosophic Fuzzy Shortest Cycle Route Problem by Reduced Matrix Method

**S. Krishna Prabha[1*], M. Clement Joe Anand[2], V. Vidhya[3], G. Nagarajan[4], Utpal Saikia[5], Nivetha Martin[6], M. Santoshi Kumari[7], Mohit Tiwari[8]**

[1*] Department of Mathematics, PSNA College of Engineering and Technology, Dindigul – 624622, Tamil Nadu, India.
[2]Department of Mathematics, Mount Carmel College (Autonomous), Affiliated to Bengaluru City University, Bengaluru - 560052, Karnataka, India.
[3]Devision of Mathematics, Vellore Institute of Technology, Chennai - 600127, Tamil Nadu, India.
[4]Department of Mathematics, Panimalar Engineering College, Chennai - 600 123, Tamil Nadu, India.
[5]Department of Mathematics, Silapathar College, Dhemaji, Assam – 787059, India.
[6]Department of Mathematics, Arul Anandar College, Karumathur-625514, Tamil Nadu, India.
[7]Department of Mathematics, Chaitanya Bharathi Institute of Technology, Gandipet, Hyderabad - 500075, India.
[8]Department of Computer Science and Engineering, Bharati Vidyapeeth's College of Engineering, Delhi - 110063, India.

Emails: jvprbh1@gmail.com; arjoemi@gmail.com; vidhya.v@vit.ac.in; sridinnaga@gmail.com; utpalsaikiajorhat@gmail.com; nivetha.martin710@gmail.com; santoshinagaram@gmail.com; mohit.tiwari@bharatividyapeeth.edu .

**Abstract**

The assertiveness theory next addresses the difficulties of the travelling salesman after discussing the problem with transportation and assignment. The Shortest Cycling Route Problem (SCRP) finds the shortest route that stops in each city exactly once using a preset set of cities and their bilateral distances. The arc lengths in TSO are typically seen as representing travel time or travel expenses rather than actual distance. The precise arc length cannot be predicted because cargo, climate, road conditions, and other factors also can affect the journey time or cost. For handling the unpredictability in SCRP, fuzzy set theory provides a new tool. The shortest cyclic route problem with interval-valued neutrosophic fuzzy numbers as cost coefficients is solved using the simplified matrix techniques in this study. Reduced Matrix Method is used to solve a numerical problem and its efficacy is demonstrated.

**Keywords:** Interval-valued neutrosophic fuzzy; shortest cyclic route problem; reduced matrix.

## 1. Introduction

A salesman visits $n$ cities in the same route that he visits one city at random to begin, visits the remaining cities one at a time, and then travels his way back to the original starting city. As a consequence, the salesman's journey is mapped out as a comprehensive tour that includes all the cities. The objective is to locate the shortest Hamiltonian tour on a completely connected graph. The entire strategy is represented by a weighted complete graph G = (V, E), where V denotes sets of vertex cities and E denotes sets of edges that are completely connected to the nodes. Each edge (i, j) of type E has a weight $d_{ij}$ that corresponds to the space between i and j. The matrix, which is called adjacency, displays the cities and the distance between each pair of cities. Every city must be visited exactly once, and the starting city must be reached after all of the cities have been traveled. The ensuing SCRP now is to identify the shortest path at the most affordable price. The unbounded proliferation of different approaches in the quest for a solution space leads to the Travelling Salesman Problem (TSP). The first person to



Doi: https://doi.org/10.54216/IJNS.230208
Received: June 23, 2023 Revised: September 19, 2023 Accepted: November 24, 2023

initiate SCRP was Hamilton in the 19th century. A sizable SCRP has been identified as the solution by Dantzig and Fulkerson [9]. To solve SCRPs, the Ones Assignment Method was introduced by Hadi Basirzadeh [11] to tackle the traveling salesman problem. However, some problems could arise due to uncertainty in real life, including measurement errors, a lack of confidence, and computational flaws. Zadeh [27,28] introduced the concept of fuzziness. The FS notion only considers the value of the elements' membership; it overlooks the value of their non-membership. Atanssov [3,4], and Smarandache [25] later developed the intuitionistic fuzzy and Neutosophic fuzzy sets.

One particular usage of the NS is the expansion of the real number domain to include neutrosophic numbers. The membership values are between [0, 1] in real units. Therefore, when the information is confusing and indeterminate between particular ranges of appropriate behavior, the trapezoidal interval valued neutrosophic number (TrIVNN) is significant. Smith proposed a study of permutation crossover operators on the travelling salesman problem. The shortest path issue in interval-valued trapezoidal and triangular neutrosophic fuzzy sets was recursively studied by Broumi et al.[7,8] and Hayat et. al.[14] suggested novel single-valued neutrosophic matrices operations and their use in multi-criteria group decision-making. Biswas [5] developed a solution to the travelling salesman problem that favors reinforcement learning over genetic algorithms. Anila Gupta [1] used coefficients as LR fuzzy parameters to solve assignment and travelling salesman problems in 2012. Dhouib [10] employed the dhouibmatrix-TSP1 heuristic to resolve the travelling salesman problem on a single-valued triangular neutrosophic number Karambir [15] used a genetic algorithm to study the issue of traveling salesmen. The classical symmetric shortest cyclic route problem is unraveled by using the Zero Suffix Method by Sudhakar [26]. Various researches like Nirmala [16], Shweta [22], Oliver [17], Vimala [23] and Jeyalakshmi [13] have deliberated many results in solving the Traveling Salesman Problem under various fuzzy environments. Pramanik [18], Sudha [19], Sangal [20], Appadoo [24] and Giri [6] and different types of neutrosophic sets are dealt [29-34] have solved various optimization problems under **a** Neutrosophic Fuzzy environment.

In this study, an Interval-valued Valued Neutrosophic Fuzzy Shortest Cycle Route Problem is taken under consideration. The crisp Shortest Cycle Route problem is solved by utilizing the Reduced Matrix Method. We can lower the level of uncertainty of the constituents of a universe corresponding to an interval-valued Neutrosophic fuzzy set by using the interval-valued Neutrosophic fuzzy point operators. NSs may be able to cope with uncertainty better since indeterminacy is also taken effectively. The main challenge with TSP search algorithms is the ability to identify the best route rapidly and ensure that it is the optimal route. The amount of processing time needed to scam all potential solutions is exponential. The following is the organized version of this article. A brief introduction to SCRP and literature reviews are offered in Section 1. In Section 2, some basic definitions are provided. In Section 3, an algorithm for the interval-valued neutrosophic Fuzzy Shortest Cyclic Route Problem is described. In section 4, a numerical example is used to demonstrate the previously discussed process. Section 5 deals with the outcomes and conclusion of the work.

## 2. Preliminaries

### 2.1 Definition

Let U be a non-empty set. Then a fuzzy set τ on U is a set having the form τ = {(x, $\mu_\tau(x)$), x∈ U} where the function $\mu_\tau$:U→ [0, 1] is called the membership function and $\mu_\tau(x)$ represents the degree of membership of each element x∈ U.

### 2.2 Definition

Let U be a non-empty set. Then an intuitionistic fuzzy set (IFS) τ is an object having the form τ = {(x, $\mu_\tau(x), \gamma_\tau(x)$): x∈U}where the functions $\mu_\tau$:U→ [0, 1] and $\gamma_\tau$:U→ [0, 1] are called membership function and non-membership function respectively. $\mu_\tau(x)$ and $\gamma_\tau(x)$ represent the degree of membership and the degree of non-membership respectively of each element x∈ U and 0 ≤ $\mu_\tau(x)$ + $\gamma_\tau(x)$≤ 1 for each x∈ U. We denote the class of all intuitionistic fuzzy sets on U by IFSU.

### 2.3 Definition

Let U be a non-empty set. Then a neutrosophic set (NS) Γ is an object having the form Γ={(x, $\mu_\tau(x), \gamma_\tau(x), \delta_\tau(x)$): x∈U} where the functions $\mu_\tau, \gamma_\tau, \delta_\tau$:U→] $^-0,1^+$[ and $^-0 \le \mu_\tau(x) + \gamma_\tau(x) + \delta_\tau(x) \le 3^+$, From philosophical point of view, the neutrosophic set takes the value from



Doi: https://doi.org/10.54216/IJNS.230208
Received: June 23, 2023 Revised: September 19, 2023 Accepted: November 24, 2023

real standard or non-standard subsets of $]^-0, 1^+[$. But in real life applications in scientific and engineering problems it is difficult to use neutrosophic sets with value from real standard or nonstandard subsets of $]^-0, 1^+[$. Hence, we consider the neutrophic set which takes the value from the subset of [0, 1] i.e; $0 \leq \mu_\tau(x) + \gamma_\tau(x) + \delta_\tau(x) \leq 3$ where $\mu_\tau$, $\gamma_\tau$ and $\delta_\tau$ are called truth membership function, indeterminacy membership function and falsity function respectively. We denote the class of all neutrosophic sets on U by $NS^U$.

**2.4 Definition**

Let $x$ be TrIVNN. Then its truth, indeterminacy and falsity MFs aregiven by

$$T_x(Z) = \begin{cases} \frac{(Z-a)t_x}{(b-a)} & a \leq z < b \\ t_x & b \leq z \leq c \\ \frac{(d-z)t_x}{(c-d)} & c < z \leq d \\ 0 & otherwise \end{cases}$$

Its indeterminacy MF is $$I_x(Z) = \begin{cases} \frac{(b-z)(Z-a)i_x}{(b-a)} & a \leq z < b \\ i_x & b \leq z \leq c \\ \frac{(Z-c)(d-z)i_x}{(c-d)} & c < z \leq d \\ 0 & otherwise \end{cases}$$

Its falsity MF is $$F_x(Z) = \begin{cases} \frac{(b-z)(Z-a)f_x}{(b-a)} & a \leq z < b \\ f_x & b \leq z \leq c \\ \frac{(Z-c)(d-z)f_x}{(c-d)} & c < z \leq d \\ 0 & otherwise \end{cases}$$

where $0 \leq T_x(z) \leq 1$, $0 \leq I_x(z) \leq 1$ and $0 \leq F_x(z) \leq 1$, also $t_x$, $i_x$, $f_x$ are subset of [0,1] and $0 \leq a \leq b \leq c \leq d \leq 1$, $0 \leq \sup(t_x) + \sup(i_x) + \sup(f_x) \leq 3$; Then x is called an interval trapezoidal neutrosophic number x = {[a, b, c, d]; $t_x$, $i_x$, $f_x$ }. We take $t_x = [\underline{t}, \overline{t}]$, $i_x = [\underline{i}, \overline{i}]$ and $f_x = [\underline{f}, \overline{f}]$.

**2.5 Definition**

Let U be a non empty set. Then an interval valued neutrosophic set (IVNS) Γ is an object having the form Γ= [(x, $[\inf \mu_\tau(x), \sup\mu_\tau(x)], [\inf\gamma_\tau(x), \sup\gamma_\tau(x)], [\inf\delta_\tau(x), \sup\delta_\tau(x)]$): $x \in U\}$ where the functions $\mu_\tau$, $\gamma_\tau$ and $\delta_\tau$ :U→int ([0, 1]) and $0 \leq \sup\mu_\tau(x) + \sup\gamma_\tau(x)) + \sup\delta_\tau(x)) \leq 3$. We denote the class of all interval valued neutrosophic sets on U by IVNSU.

**2.6 Definition**

Let Φ , Π, be two interval neutrosophic sets on U . Then

(a) Φ is called a subset of Π , denoted by Φ ⊆ Π

if $\inf\mu_\Phi(x) \leq \inf\mu_\Pi(x), \sup\mu_\Phi(x) \leq \sup\mu_\Pi(x),$

$\inf\gamma_\Phi(x) \leq \inf\gamma_\Pi(x), \sup\gamma_\Phi(x) \leq \sup\gamma_\Pi(x),$

$\inf\delta_\Phi(x) \leq \inf\delta_\Pi(x), \sup\delta_\Phi(x) \leq \sup\delta_\Pi(x).$

(b) The intersection of Φ and Π is denoted by Φ ∩ Π and is defined by



Doi: https://doi.org/10.54216/IJNS.230208
Received: June 23, 2023 Revised: September 19, 2023 Accepted: November 24, 2023

$\Phi \cap \Pi$ = {([min($inf\mu_{\Phi}(x), inf\mu_{\Pi}(x)$), min ($sup\mu_{\Phi}(x) \leq sup\mu_{\Pi}(x)$)],
[max ($inf\gamma_{\Phi}(x), inf\gamma_{\Pi}(x)$), max ($sup\gamma_{\Phi}(x) \leq sup\gamma_{\Pi}(x)$)],
[max ($inf\delta_{\Phi}(x), inf\delta_{\Pi}(x)$), max ($sup\delta_{\Phi}(x) \leq sup\delta_{\Pi}(x)$)]): x ∈ $U$}

(c) The union of Φ and Π is denoted by Φ ∪ Π and is defined by

$\Phi \cup \Pi$ = {([max ($inf\mu_{\Phi}(x), inf\mu_{\Pi}(x)$), max ($sup\mu_{\Phi}(x) \leq sup\mu_{\Pi}(x)$)],
[min($inf\gamma_{\Phi}(x), inf\gamma_{\Pi}(x)$), min ($sup\gamma_{\Phi}(x) \leq sup\gamma_{\Pi}(x)$)],
[min ($inf\delta_{\Phi}(x), inf\delta_{\Pi}(x)$), min ($sup\delta_{\Phi}(x) \leq sup\delta_{\Pi}(x)$)]): x ∈ $U$}

(d) The complement of Φ is denoted by $\Phi^c$ and is defined by

$$\emptyset^C = \{(x, [inf\delta_\tau(x), sup\delta_\tau(x)][1 - sup\gamma_\tau(x), 1 - inf\gamma_\tau(x)], [inf\mu_\tau(x), sup\mu_\tau(x)]), x \in U\}$$

### 2.1 Ranking Technique for Trapezoidal Interval Valued Neutrosophic Numbers (TRIVNN)

Let $\tilde{\alpha}$ and $\tilde{r}$ be two TrIVNNs, the ranking of $\tilde{\alpha}$ and $\tilde{r}$ by score function and accuracy function is described as follows:

(i) if s$(\hat{r})^N <$ s$(\hat{s})^N$ then $(\hat{r})^N < (\hat{s})^N$

(ii) if s$(\hat{r})^N \approx$ s$(\hat{s})^N$

and if

(a) a$(\hat{s})^N <$ a$(\hat{s})^N$ then $(\hat{s})^N < (\hat{s})^N$

(b) a$(\hat{r})^N >$ a$(\hat{s})^N$ then $(\hat{s})^N > (\hat{s})^N$

(c) a$(\hat{r})^N \approx$ a$(\hat{s})^N$then $(\hat{s})^N \approx (\hat{s})^N$

### 2.2 Score Function of Trapezoidal Interval Valued Neutrosophic Number

Let x = ( [a, b, c, d]; [$\underline{t}$, $\overline{t}$], [$\underline{i}$, $\overline{i}$], [$\underline{f}$, $\overline{f}$] ) be a TrIVNN then its score function is defined by S(x) = $\frac{1}{16}$(a + b + c + d )( 2 + $\underline{t}$ + $\overline{t}$− $\underline{i}$ −$\overline{i}$− $\underline{f}$ − $\overline{f}$ ) and S(x) ϵ [0, 1].---------------------------------------------------------------------(1)

Here we take $0 \leq a \leq b \leq c \leq d \leq 1$, $t_x$ , $i_x$, , $f_x$ are subset of [0, 1] ,

where $t_x$ = [$\underline{t}$, $\overline{t}$], $i_x$ = [$\underline{i}$, $\overline{i}$], and $f_x$ = [$\underline{f}$, $\overline{f}$]

### 2.3 Accuracy Function of Trapezoidal Interval Valued Neutrosophic Number

Let x = ([a, b, c, d]; [$\underline{t}$, $\overline{t}$], [$\underline{i}$, $\overline{i}$], [$\underline{f}$, $\overline{f}$]) be a TrIVNN then its accuracy function is defined by $A_c$(x) = $\frac{1}{8}$ (c + d − a − b) (2 + $\underline{t}$ − $\overline{t}$ − $\underline{f}$ − $\overline{f}$) -----------------------------------------------------------------------------------------------(2)



Doi: https://doi.org/10.54216/IJNS.230208
Received: June 23, 2023 Revised: September 19, 2023 Accepted: November 24, 2023

and $A_c(x) \in [0, 1]$. Here we take $0 \leq a \leq b \leq c \leq d \leq 1$ and , $t_x$ , $i_x$, , $f_x$ are subset of [0, 1] where $t_x = [\underline{t}, \overline{t}]$, $i_x = [\underline{i}, \overline{i}]$, and $f_x = [\underline{f}, \overline{f}]$.

## 3. Shortest Cyclic Route Problem

Given a list of n cities $\{C_1, C2, \ldots, C_n\}$ and the associated distances between cities $C_i$ and $C_j$ , denoted by $d_{ij}$ , the SCRP aims to find an ordering σ of $\{1, 2, \ldots, n\}$ such that the tour cost, given by $c = \sum_{i=1}^{n-1} d_{\sigma(i),\sigma(i+1)} + d_{\sigma(n),\sigma(1)}$ is minimized. For the Euclidean SCRP, for instance, $d_{ij} = \|x_i - x_j\|_2$, where $x_i \in R$ d is the position of $C_i$. In general, however, the distance matrix $D = (d_{ij})$ does not have to be symmetric. The ordering σ can be represented as a unique permutation matrix P. Note, however, that due to the underlying cyclic symmetry, multiple orderings – corresponding to different permutation matrices – have the same cost.

SCRP can be classified into the following categories:

(i) Symmetric Shortest Cycle Route Problem (*S - SCRP*): Let $V = v_1, v_2,\ldots, v_n$,be a set of cities, $A = (p, q) : p, q \in V$ be the set of edges, and $d_{pq} = d_{qp}$ be a cost measure associated with the edge $(p, q) \in A$ which is symmetric. The s-TSP is the problem of finding then, a minimal length closed tour that visits each city once. In this case cities $v_i \in V$ are given by their coordinates $(x_i, y_i)$ and $d_r$'s is the Euclidean distance between r and s then we have an Euclidean TSP.

(ii) Asymmetric Shortest Cycle Route Problem (*A - SCRP*): From the above definition, if the cost measure $d_{pq} \neq d_{qp}$ for at least one (p, q) then the TSP becomes a SCRP.

(iii) Multiple Shortest Cycle Route Problem (*M - SCRP*): Given a set of nodes, let there be m salesmen located at a single depot node. The remaining nodes (cities) that are to be visited are intermediate nodes. Then, the m SCRP consists of finding tours for all m salesmen, who all start and end at the same deport, such that each intermediate node is visited exactly once and the total cost of visiting all nodes is minimized.

### 3.1 Procedure for Framing The Reduced Matrix

This Method is similar to Branch and Bound method used for solving SCRP. This strategy can be compared to the Branch and Bound method. Here, in this method, the matrix reduction approach is used to determine the path's cost and the constraint. The presumptions for a reduced matrix are listed below:

- A particular row or column of the cost adjacency matrix is said to be reduced iff it contains at least one zero element and all remaining entries in that row or column ≥ 0.
- The overall matrix is said to be reduced if all the rows and columns are reduced.
- Latest Tour length (new) = Previous Tour length - Total value reduced.
- All diagonal entries are replaced from 0 to Infinity in the original cost adjacency matrix.

The fundamental concept underlying to find the solution of the problem: The smallest possible cost for the travelling salesman problem is used as the cost to reduce the matrix initially. Now, at each step, we need to determine the minimum possible cost if that path is taken, i.e., a path from vertex u to v is followed.

We may achieve this by replacing the costs for the $u^{th}$ row and $v^{th}$ column with *infinity*, further lowering the matri x, and then addiing the previously determined minimal path cost by the additional costs for reduction and cost of edge (u, v). When at least one path has been identified, its cost is employed as the upper bound of cost to apply the branch and bound strategy to the other routes. The upper bound is revised consequently when a path with lower cost is found. Follow the illustration below for a better understanding.

### 3.1 Algorithm for Reduced Matrix Method

Convert the interval valued neutrosophic cost to crisp by using the score function. The processes required to carry out the above method are outlined below:

**Step 1:** Create a class termed as "Node" in step one that can contain the reduced matrix, cost, the current city's number, level (the total number of cities visited), and the path taken up to this point.

**Step 2:** Construct a queue based on priority to store the live nodes with the minimum cost at the top.



Doi: https://doi.org/10.54216/IJNS.230208
Received: June 23, 2023 Revised: September 19, 2023 Accepted: November 24, 2023

**Step 3:** Reduce the matrix after setting the start index's level to 0. By first lowering the row, then the column, determine the cost of the provided matrix. The price is determined in the manner described below:

**Row reduction** - identify and retain the minimum value for each row. After identifying the minimum element in each row subtract it from every single component in that particular row.

**Column reduction** - identify and save the minimum value for each column. Take the minimum element from each column and deduct it from all the other components in that particular column. The matrix has now been shrunk. In order to determine the cost, add all the minimal components to the row and column you already identified.

**Step 4:** Insert the element into the Priority Queue with all the data that Node needs.

**Step 5:** Continue the actions listed below until the priority queue is emptied.

- Eliminate the element with the minimum value from the priority queue.
- Verify that the level of the current node matches the number of nodes/cities for each pop operation.
- If so, print the path and provide the lowest cost.
- If the response is no, proceed to employ the formula to determine the cost for each and every child node of the current node.
- Child Cost = Parent_matrix_cost + Cost_from_parent to child + Child_reduced Matrix_cost
- It is possible to determine the cost of a reduced matrix by setting all of its row and column values to infinity and setting the index Matrix [Col][row] = infinity.
- After that, reorder the priority queue to include the current node.

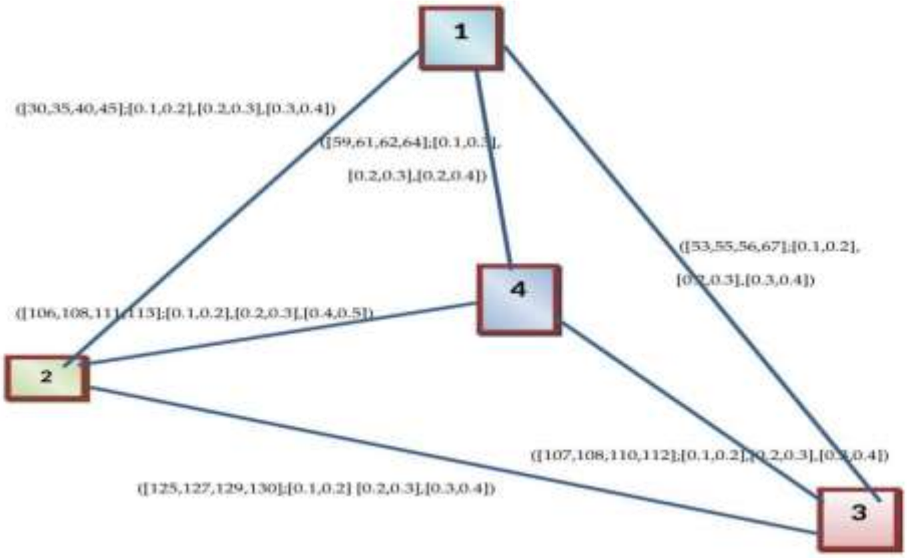
**Step 6:** Continue Step 5 until we do not reach the level equal to Number of nodes minus 1.

## 4. Numerical Example

Consider the shortest cycle route problem with 4 nodes. The cost functions are given as Interval Valued Neutrosophic Fuzzy Shortest Cycle Route Problem.



Doi: https://doi.org/10.54216/IJNS.230208
Received: June 23, 2023 Revised: September 19, 2023 Accepted: November 24, 2023




1 ---- 2 = ([30, 35, 40, 45] ;[0.1,0.2],[0.2,0.3],[0.3,0.4])

1---- 4 = ([59, 61, 62, 64] ;[0.1,0.3], [0.2,0.3],[0.2,0.4])

2---- 4 = ([106,108,111,113] ;[0.1,0.2],[0.2,0.3],[0.4,0.5])

2---- 3 = ([125,127,129,130] ;[0.1,0.2] [0.2,0.3],[0.3,0.4])

3---- 4 = ([107,108,110,112] ;[0.1,0.2],[0.2,0.3],[0.3,0.4])

1---- 3 = ([53, 55, 56, 57] ;[0.1,0.3], [0.2,0.3],[0.3,0.4])

Figure 1: Interval Valued **Neutrosophic** Fuzzy Shortest Cycle Route Problem.

Defuzzifying the score function using (1), $S(x) = \frac{1}{16}(a + b + c + d)(2 + \underline{t} + \overline{t} - \underline{i} - \overline{i} - \underline{f} - \overline{f})$

$1 \leftrightarrow 2$ = ([30, 35, 40, 45] ;[0.1,0.2],[0.2,0.3],[0.3,0.4])

$= \frac{1}{16}(30 + 35 + 40 + 45)(2 + 0.1 + 0.2 - 0.2 - 0.3 - 0.3 - 0.4)$

$= \frac{1}{16}(150)(1.1) = 10.3 \approx 10.$

$1 \leftrightarrow 4$ = ([59, 61, 62, 64];[0.1,0.3],[0.2,0.3],[0.2,0.4])

$= \frac{1}{16}(59 + 61 + 62 + 64)(2 + 0.1 + 0.3 - 0.2 - 0.3 - 0.2 - 0.4)$

$= \frac{1}{16}(246)(1.3) = 19.98 \approx 20,$

Proceeding like this, we can defuzzify all the cost values by above score function.

$1 \leftrightarrow 3 = 15.19 \approx 15,$

$2 \leftrightarrow 4 = 24.6 \approx 25,$

$2 \leftrightarrow 3 = 35.13 \approx 35,$

$3 \leftrightarrow 4 = 30.04 \approx 30.$

The cost matrix of the given shortest cycle route problem is given below**,**



Doi: https://doi.org/10.54216/IJNS.230208
Received: June 23, 2023 Revised: September 19, 2023 Accepted: November 24, 2023

Table 1: Cost Matrix after Defuzzifying

| ROW/COLUMN | N1 | N 2 | N 3 | N 4 |
|---|---|---|---|---|
| N 1 | ∞ | 10 | 15 | 20 |
| N 2 | 10 | ∞ | 35 | 25 |
| N 3 | 15 | 35 | ∞ | 30 |
| N 4 | 20 | 25 | 30 | ∞ |

The row minimum for the corresponding rows is given as $R_1 \rightarrow 10$, $R_2 \rightarrow 10$, $R_3 \rightarrow 15$, $R_4 \rightarrow 20$. After reducing the corresponding elements from each row, the outcome is given below.

Table 2: Row Deduction

| ROW/COLUMN | N1 | N 2 | N 3 | N 4 |
|---|---|---|---|---|
| N 1 | ∞ | 0 | 5 | 10 |
| N 2 | 0 | ∞ | 25 | 15 |
| N 3 | 0 | 20 | ∞ | 15 |
| N 4 | 0 | 5 | 10 | ∞ |

The column minimum for the corresponding column is $C_1 \rightarrow 10$, $C_2 \rightarrow 0$, $C_3 \rightarrow 5$, $C_4 \rightarrow 10$. After reducing the corresponding elements from each column, the outcome is given below. After row and column reduction the matrix will be:

Table 3: Column Reduction

| ROW/COLUMN | N1 | N 2 | N 3 | N 4 |
|---|---|---|---|---|
| N 1 | ∞ | 0 | 0 | 0 |
| N 2 | 0 | ∞ | 20 | 5 |
| N 3 | 0 | 20 | ∞ | 5 |



Doi: https://doi.org/10.54216/IJNS.230208
Received: June 23, 2023 Revised: September 19, 2023 Accepted: November 24, 2023

| | | | | |
|---|---|---|---|---|
| **N 4** | 0 | 5 | 5 | ∞ |

So the cost reduction of the matrix is (10 + 10 + 15 + 20 + 5 + 10) = 70. Now let us consider movement from 1 to 2. Initially after substituting the 1st row and 2nd column to infinity, the matrix will be:

Table 4: Cost Matrix- movement from 1 to 2

| ROW/COLUMN | N1 | N 2 | N 3 | N 4 |
|---|---|---|---|---|
| **N 1** | ∞ | ∞ | ∞ | ∞ |
| **N 2** | ∞ | ∞ | 20 | 5 |
| **N 3** | 0 | ∞ | ∞ | 5 |
| **N 4** | 0 | ∞ | 5 | ∞ |

The row minimum for the corresponding rows are given as, $R_2 \to 5$, $R_3 \to 0$, $R_4 \to 0$. After reducing the corresponding elements from each row, the outcome is given below.

Table 5: Cost Matrix after row reduction

| ROW/COLUMN | N1 | N 2 | N 3 | N 4 |
|---|---|---|---|---|
| **N 1** | ∞ | ∞ | ∞ | ∞ |
| **N 2** | ∞ | ∞ | 15 | 0 |
| **N 3** | 0 | ∞ | ∞ | 5 |
| **N 4** | 0 | ∞ | 5 | ∞ |

and the column minimum will be 0, 5, 0. The column minimum for the corresponding column is $C_2 \to 0$, $C_3 \to 0$, $C_4 \to 0$. After reducing the corresponding elements from each column, the outcome is given below. After row and column reduction the matrix will be

Table 6: Cost Matrix after Column reduction

| ROW/COLUMN | | N1 | N 2 | N 3 | N 4 |
|---|---|---|---|---|---|
| **N 1** | | ∞ | ∞ | ∞ | ∞ |



Doi: https://doi.org/10.54216/IJNS.230208
Received: June 23, 2023 Revised: September 19, 2023 Accepted: November 24, 2023

| | | | | |
|---|---|---|---|---|
| **N 2** | ∞ | ∞ | 10 | 0 |
| **N 3** | 0 | ∞ | ∞ | 5 |
| **N 4** | 0 | ∞ | 0 | ∞ |

So the cost reduction of the matrix is 70 + cost (1, 2) + 5 + 5 = 70 + 0 + 5 + 5 = 80. Keep performing this until the traverse is finished, and then calculate the least expensive route. The optimal path is given by *1* – 2 – 4 – 3 – 1.

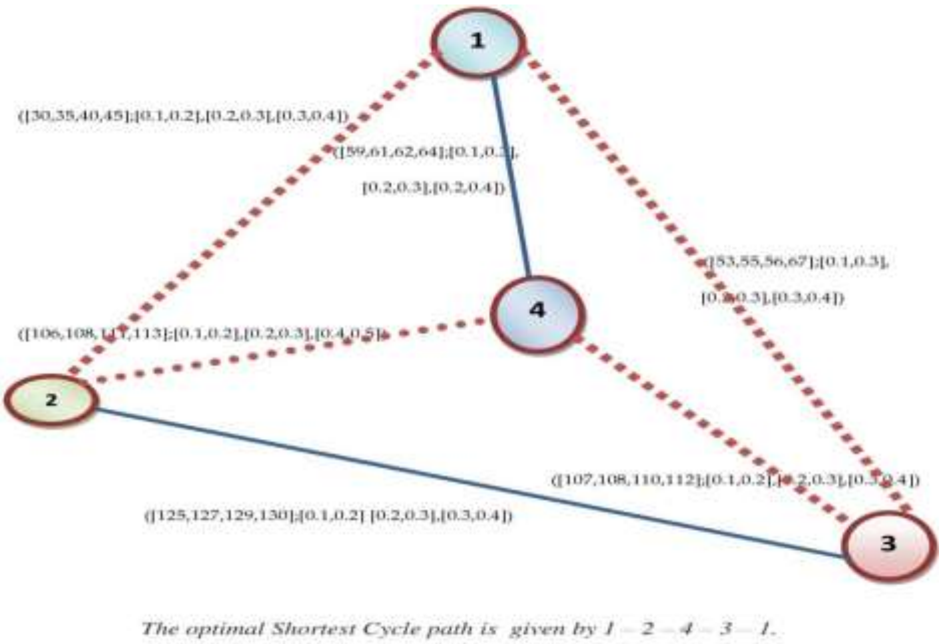



Figure 2: Optimal Path

The recursion tree method is a visual representation of an iteration method that is in the form of a tree where at e ach level nodes are expanded. In general, we consider the second term in recurrence as root. 3. It is useful when t he divide & conquer algorithm is used. Sometimes it's hard to make an accurate prediction. Each root and child of a recursion tree indicates the cost of a single sub problem. We add the costs for each level of the tree to get a list of pre-level costs, which we then add together to get the total cost for all recursive levels. The Substitution Method should be used to obtain a solid guess from a recursive tree. Initially we'll examine node 1 with the objective to create a state-space tree. As seen in the diagram below, we can travel from node 1 to nodes 2, 3, or 4 respectively. The cost of node 1 would be the cost of 70, which is what we were able to achieve in the above-reduced matrix. The upper bound is kept in this case as well. The upper bound started out as infinite. The boundaries and the framework of the recursion tree is provided below,



Doi: https://doi.org/10.54216/IJNS.230208
Received: June 23, 2023 Revised: September 19, 2023 Accepted: November 24, 2023

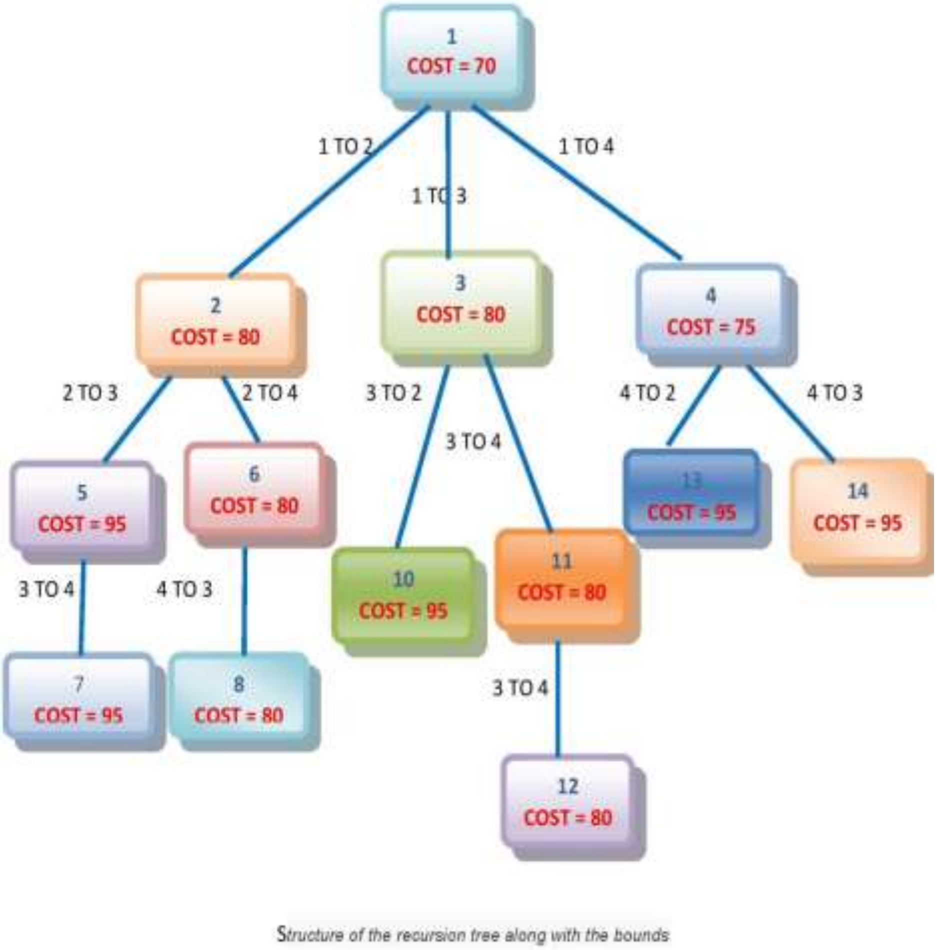



Figure 3: Framework of the Recursion Tree

The Time Complexity and various space complexity of SCRP using various algorithms are presented below.

Table 7: Time Complexity and various Space Complexity

| | **Greedy Approach** | **Dynamic Programming** | **Back Tracking** | **Reduced Matrix** |
|---|---|---|---|---|
| **Time Complexity** | $O(n^2*log_2n)$ | $\mathbf{O(n^2 * 2^n)}$ | O(n!) | $O(2^n*n^2)$ |
| **Auxiliary Space** | $O(n)$ | $\mathbf{O(n * 2^n)}$ | O(n) | $O(n^2)$ |

## 5. Conclusion

This attempt investigates an Interval Valued Neutrosophic Fuzzy SCRP. The results show that the suggested procedures can successfully dismantle the Interval Valued Neutrosophic Fuzzy. This plan is simple to understand and provides a methodical approach to dissecting SCRP. The least expensive optimal path is found and a recursion tree is drawn based on the results obtained by traversing the path. This method is more efficient since the time complexity is given by $O(2^N*N^2)$ and Space complexity is given by $O(N^2)$ where N = number of nodes / City. Many Heuristic methods can be tested by utilizing under this fuzzy environment. Numerous real-world applications, Applications of SCRP includes proper deployment of cloud computing resources, optimal path search for transportation, computational modeling of proteins, microchip production, X-ray crystallography, scheduling the resources, and recently SCRP can be implemented in drone routing, all involve combinatorial optimization problems.



Doi: https://doi.org/10.54216/IJNS.230208
Received: June 23, 2023 Revised: September 19, 2023 Accepted: November 24, 2023

**References**


[1] Kumar, A., Gupta, A., "Assignment and Travelling Salesman Problems with Coefficients as *LR* Fuzzy Parameters", *International Journal of Applied Science and Engineering,* vol. 10, no. 3, pp. 155-170, 2012. https://doi.org/10.6703/IJASE.2012.10(3).155

[2] Angelo. P.P., "Optimization in an intuitionistic fuzzy environment". *Fuzzy Sets and Systems,* Vol.86, pp.299-306, 1997. https://doi.org/10.1016/S0165-0114(96)00009-7

[3] Atanassov, K. T., Gargov, G., "Interval valued intuitionistic fuzzy sets," *Fuzzy Sets and S*ystems, Vol. 31, No. 3, pp. 343–349.1989. https://doi.org/10.1016/0165-0114(89)90205-4

[4] Atanassov. K. T, "Intuitionistic fuzzy sets," *Fuzzy Sets and Systems",* 1986,Vol. 20, No. 1, pp. 87–96,1986. https://doi.org/10.1016/S0165-0114(86)80034-3

[5] Biswas, B., Mitra, A., Sengupta. S., " A Study of Travelling Salesman Problem using Reinforcement Learning Over Genetic Algorithm", *Turkish Journal of Computer and Mathematics Education* ,Vol.11, No.02 pp.963-981, 2020.

[6] Biswas, P., Pramanik, S., and Giri.C., "Aggregation of triangular fuzzy neutrosophic set information and its application to multi attribute decision making," *Neutrosophic Sets and Systems,* vol. 12, No.1,pp. 20–40,2016. https://digitalrepository.unm.edu/nss_journal/vol12/iss1/4

[7] Broumi, S., Nagarajan, D., and Bakali. A., "The shortest path problem in interval valued trapezoidal and triangular neutrosophic environment", *Complex Intell. Syst*. Vol.5, pp. 391–402.2019. https://doi.org/10.1007/s40747-019-0092-5

[8] Broumi, S., Bakali,A., Talea, M., Smarandache, F., and Vladareanu, L., "Shortest path problem under triangular fuzzy neutrosophic information", in Proceedings of the 2016, 10th *International Conference on Software, Knowledge, Information Management & Applications (SKIMA), Chengdu,* China.2016.

[9] Dantzig, G. B., Fulkerson, D. R., and Johnson, S.M., "Solution of a large- scale traveling salesman problem", *Operations Research*, vol. 2, pp. 393-410, 1954.

[10] Dhouib, S., "Optimization of travelling salesman problem on single valued triangular neutrosophic number using dhouibmatrix-TSP1 heuristic", *International Journal of Engineering,*Vol. 34, No. 12, pp. 1–6, 2021. https://doi.org/10.5829/ije.2021.34.12C.09.

[11] Hadi Basirzadeh., "Ones Assignment Method for Solving Traveling Salesman Problem", *Journal of mathematics and computer science,* Vol.10, pp.258-265, 2014.

[12] Hitchcock, F. L., "The distribution of a product from several sources to numerous localities", *Journal of mathematics and physics,* Vol.20 (1-4), pp.224-230, 1941. http://dx.doi.org/10.1002/sapm1941201224

[13] Jeyalakshmi, K., and Prabha, S. K., "Unraveling Type-3 Fuzzy Shortest Cycle Route Problem By RMM*", Journal Of Critical Reviews,*Vol. **7**,No. 15, pp.4854-4858, 2020.

[14] Karaaslan, F., and Hayat, K., "Some new operations on single valued neutrosophic matrices and their applications in multicriteria group decision making," *Applied Intelligence*, Vol. 48, No. 12, pp. 4594–4614.2018. https://doi.org/10.1007/s10489-018-1226-y

[15] Kumar, N., Karambir and Rajiv Kumar., "A study of genetic algorithm approach to study travelling salesman problem", *Journal of Global Research in Computer Science*, Vol. 3, 3,pp.33-37, 2012.

[16] Nirmala, G., and Anju, R., "Travelling Salesman Problem (SCRP) Using Fuzzy Quantifier", *International Journal of Science and Research* ,Vol.3, No.12, pp.184-186, 2012. www.ijsr.net/archive/v3i12/U1VCMTQxNjI=.pdf

[17] Oliver, I.M., Smith, D.J., and Holland, J.R.C., "A Study of Permutation Crossover Operators on the Travelling Salesman Problem". In J.J. Grefenstette (ed.). Genetic Algorithms and Their Applications: Proceedings of the 2nd International Conference on Genetic Algorithms. Lawrence Erlbaum Associates, Hilladale, NJ, 1987.

[18] Pramanik, S., Dey, P.P., "Multi-level linear programming problem with neutrosophic numbers: a goal programming strategy", *Neutrosophic Sets Syst,* 29:242–254, 2020. https://digitalrepository.unm.edu/nss_journal/vol29/iss1/19

[19] Sudha. S, Nivetha Martin, M. Clement Joe Anand, P. G. Palanimani,T. Thirunamakkani, B. Ranjitha. "MACBETH-MAIRCA Plithogenic Decision-Making on Feasible Strategies of Extended Producer's Responsibility towards Environmental Sustainability." International Journal of Neutrosophic Science, Vol. 22, No. 2, 2023 ,PP. 114-130.

[20] Saini, R.K., and Sangal, A.,"Application of single valued trapezoidal neutrosophicnumbers in transportation problem",*NeutrosophicSetsandSystem,*Vol.35,pp.563583,2020.https://digitalrepository.unm.edu/nss_journal/vol35/iss1/33




Doi: https://doi.org/10.54216/IJNS.230208
Received: June 23, 2023 Revised: September 19, 2023 Accepted: November 24, 2023

[21] Saloni Gupta, Poonam Panwar., “Solving Travelling Salesman Problem Using Genetic Algorithm”, International Journal of Advanced Research in Computer Science and Software Engineering , Volume 3, No. 6,pp.376-380, June 2013.

[22] Shweta, R., and Saurabh Ranjan. S., “Solving Travelling Salesman Problem Using Improved Genetic Algorithm”, Indian Journal of Science and Technology,Vol.10,No.30,pp.1-6,2017. 10.17485/ijst/2017/v10i30/115512

[23] Sikkannan, K.P., Vimala. S.,“Unraveling neutrosophic transportation problem using costs mean and Complete contingency cost table,” *Neutrosophic Sets and Systems*, vol. 29, pp. 165–173,2019. https://digitalrepository.unm.edu/nss_journal/vol29/iss1/13

[24] Singh.A, Kumar. A, and Appadoo. S. S., “Modified approach for optimization of real life transportation problem in neutrosophic environment,” *Mathematical Problems in Engineering*, Article ID 2139791, pp.1-10, pages, 2017. https://doi.org/10.1155/2017/2139791

[25] Smarandache, F., “A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set”, *Neutrosophic Probability and Statistics*, InfoLearnQuest, Philadelphia, PA, USA ,6th edition.2007. https://digitalrepository.unm.edu/math_fsp/163/

[26] Sudhakar, V.J., Navaneetha Kumar, V., “A New Approach to Solve the Classical Symmetric Shortest Cyclic Route Problem by Zero Suffix Method”,*Int. J. Contemp. Math. Sciences*, Vol. 6, no. 23,pp. 1111 – 1120, 2011.

[27] Zadeh, L.A., “Fuzzy sets as a basis for a theory of possibility”, *Fuzzy sets and systems*, Vol.1,No.1,pp. 3-28,1978. https://doi.org/10.1016/0165-0114(78)90029-5

[28] Zadeh. L.A., “Fuzzy sets”, *Information and Computation”*, vol. 8.pp. 338–353,1965. http://dx.doi.org/10.1016/S0019-9958(65)90241-X

[29] Manshath,E. Kungumaraj,E. Lathanayagam,M. C. Joe Anand, Nivetha Martin,Elangovan Muniyandy,S. Indrakumar. "Neutrosophic Integrals by Reduction Formula and Partial Fraction Methods for Indefinite Integrals." International Journal of Neutrosophic Science, Vol. 23, No. 1, 2024 ,PP. 08-16.

[30] Rajesh, Sharmila Rathod, Jyoti Kundale, Nilesh Rathod, M. Clement Joe Anand, Utpal Saikia, Mohit Tiwari, Nivetha Martin. "A Study on Interval Valued Temporal Neutrosophic Fuzzy Sets." International Journal of Neutrosophic Science, Vol. 23, No. 1, 2024 ,PP. 341-349.

[31] Broumi, S., Mohanaselvi, S., Witczak, T., Talea, M., Bakali, A., & Smarandache, F. (2023). Complex fermatean neutrosophic graph and application to decision making. Decision Making: Applications in Management and Engineering, 6(1), 474-501.

[32] Broumi, S., Raut, P. K., & Behera, S. P. (2023). Solving shortest path problems using an ant colony algorithm with triangular neutrosophic arc weights. International Journal of Neutrosophic Science, 20(4), 128-28.

[33] Bharatraj J., Clement Joe Anand, M., “Power harmonic weighted aggregation operator on single-valued trapezoidal neutrosophic numbers and interval-valued neutrosophic sets”. In: Fuzzy Multi-criteria Decision-Making Using Neutrosophic Sets. Springer International Publishing, Cham, pp. 45–62, 2019.

[34] Clement Joe Anand M, Janani Bharatraj, “Interval-Valued Neutrosophic Numbers with WASPAS”. In: Kahraman, C., Otay, İ. (eds) Fuzzy Multi-criteria Decision-Making Using Neutrosophic Sets. Studies in Fuzziness and Soft Computing, Springer, Cham, pp.435-453, 2019




Doi: https://doi.org/10.54216/IJNS.230208
Received: June 23, 2023 Revised: September 19, 2023 Accepted: November 24, 2023