Identifying Internet Streaming Services using Max Product of Complement in Neutrosophic Graphs

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Abstract

The complement of the highest result of multiplication of two neutrosophic graphs is determined in this study. In the complement of the maximum product of a neutrosophic graph, the degree of a vertex is investigated. The complement of the maximum product of two normal neutrosophic graphs has several results that are presented and proven. Finally, we have offered a neutrosophic graph application for locating an online streaming service using normalized Hamming distance.

Keywords: Neutrosophic Graphs; Max Product Neutrosophic Graph; Complement Neutrosophic Graph.

1 Introduction

Graphs are commonly understood to be essentially representations of relations. A good tool for expressing information about item connections is a graph. Edges describe relationships, whereas vertices represent things. The objects and the relationships between them are represented, respectively, by the vertices and edges of the graph. The information that describes the conditions might become ambiguous when it comes to global challenges. In a variety of disciplines, including topology, optimization, network, and environmental science, neutrosophic models are useful mathematical instruments for solving combinatorial issues. Neutrosophic models are more sophisticated than straightforward graphical models due to the inherent vagueness and ambiguity they include. When neutrosophic set theory was originally applied, it was utilized to solve several intricate problems for which there was insufficient information. When it comes to applying graph theory to dealing with real-life circumstances, it is seen to be crucial. The use of fuzzy set theory has no bounds; hence the fuzzy graph theory has unique relevance. Rosenfeld[14] presented the notion of fuzzy graphs in 1975, while independently Yeh and Bang also introduced the concept of fuzzy graphs.[22] Fuzzy graphs are quite different from traditional graphs and are excellent for representing interactions that deal with ambiguity. Numerous issues in the fields of computer science, electrical engineering, system modeling, transportation, finance, etc. can be treated by using them. The concept of intuitionistic fuzzy sets (IFS), introduced by Atanassov,[1] and represents an extension of fuzzy sets that effectively handle ambiguous conditions. Unlike traditional fuzzy sets, IFS structures are not confined solely to membership grades, allowing for improved handling of uncertainty. The concept of IFS has witnessed significant utilization across various domains. In 1994, Shannon and Atanassov,[17] made a notable contribution by introducing the concept of intuitionistic fuzzy graphs (IFG), further enhancing the applicability and importance of this mathematical framework. The concept of IFG that was initially introduced by Atanassov and Shannon was further elucidated by Parvathi and Karunambigai.[11]
Madhumangal Pal and Sankar Sahoo\cite{15} classified IFG products into three categories: Strong, semi-strong, and direct. Additionally, Yaqoob et al.\cite{21} extensively investigated the four fundamental operations of complex IFG, including the cartesian product, join, union, and composition. Yahya Mohamed and Mohamed Ali developed the terms modular, complement, and maximum product on IFG \cite{7,9,19}. The neutrosophic sets were suggested by Smarandache\cite{18}. Using imprecise, ambiguous, and inconsistent data in practical applications calls for a sophisticated mathematical approach. IFS and interval-valued IFS are both included in this category of fuzzy set theory\cite{3,4,12,13,16}. The truth, indeterminacy, and falsity membership values (T, I, and F), which are independent and fall inside the real standard or non-standard unit interval [0, 1], are used to describe Neutrosophic sets. The subclass of neutrosophic sets called single-valued neutrosophic sets (SVNS) was introduced by Haibin Wang with the purpose of facilitating practical implementation in real-world applications. In order to create SVNS, IFS with independent membership values between [0, 1] were generalised. SVNS are a subset of Neutrosophic sets, which simplifies the utilization of Neutrosophic sets in practical situations. One may find similar research on the growth of the single-valued Neutrosophic network in \cite{2,5,6,23}. Yahya et al.\cite{20} created the maximum product of two. Kaviyarasu M\cite{8} explained the concept of regularity in neutrosophic graph theory. According to the aforementioned literature, the product’s classical, fuzzy, and intuitionistic fuzzy forms are employed in a number of industries and provide practical answers to the problems. The max product and their complement in neutrosophic graphs have also not been employed in the present study. The proposed method can also be used to discover the online streaming service.

1.1 Motivation

Numerous uses of neutrosophic graphs and their expansions have been found recently in study [In the studies cited in the literature]. In the field of applied mathematics, research on the combination of neutrosophic graphs and their products is expanding. In this study, the maximum product of the complement of the neutrosophic graph is used as the context. The following is a description of the study’s rationale:

1. The max product and complement notions are foundational ideas in graph theory with numerous applications in diverse disciplines.
2. These ideas expand the options for conveying uncertainty when used in the context of neutrosophic graphs
3. These ideas expand the options for conveying uncertainty when used in the context of neutrosophic graphs
4. More ambiguous information cannot be captured using this method.
5. When used in the neutrosophic graph setting, it could produce a useful result.
6. Additionally, there are issues with finding an online streaming provider.

It should be emphasized that earlier researches have not addressed these challenges, which fact inspired us to offer a workable alternative. As a result, this article discusses these problems and suggests creative solutions. The goal of the current study is to contribute significantly to society by accomplishing this.

1.2 Novelties

The notions of the maximum product of neutrosophic groups are defined in this work. A fresh definition of the neutrosophic graph complement is also offered.

1. The notions of the maximum product of neutrosophic groups are defined in this work.
2. To offer a fresh definition of the neutrosophic graph complement.
3. This study also teaches the notions of the neutrosophic graph’s maximum product of complement.
4. To increase the amount of uncertainty that decision-making issues may represent, a Max product of complement of neutrosophic graph is used.
1.3 Structure of the paper

Graphs produced by neutrosophic are characterized by the degree of their vertex and the complement of the maximum product of two neutrosophic graphs is applied in this study to handle decision-making concerns and identify the internet streaming service. In the “preliminaries” section, we have discussed a few basic neutrosophic graph concepts. The term “complement of max product of neutrosophic graphs” has been defined along with its degree in “Neutrosophic Graph Complement of Max Product.” Neutrosophic graphs are employed in the “Application” to locate the supplier of online streaming by using normalized Hamming distance. We have considered the signals of other users and selected the fundamental signal that most accurately reflects their choice of streaming service. This approach has allowed us to determine the type of service each user chooses based on how closely their symptoms resemble those of the offered solutions.

2 Preliminary

Definition 2.1. A neutrosophic graph denoted as \( G = ((T\sigma_1, T\sigma_2, F\sigma_3), (T\mu_1, T\mu_2, F\mu_3)) \) is represented by \( G^* = (V, E) \), where \( V \) is the set of vertices, and \( E \) is the set of edges. The functions \( T\sigma_1, T\sigma_2, \) and \( F\sigma_3 \) are mappings from \( V \) to the closed interval \([0, 1]\), signifying the degrees of true, intermediate and false membership, respectively, for each element \( x_i \in V \). It holds that \( 0 \leq T\sigma_1(x_i) + I\sigma_2(x_i) + F\sigma_3(x_i) \leq 3 \) for all \( x_i \in V \).

Moreover, in the context of \( G^* \), the functions \( T\mu_1, I\mu_2, \) and \( F\mu_3 \) are mappings from \( V \times V \) to the closed interval \([0, 1]\), representing the degrees of true, intermediate, and false membership, respectively, for each edge \((x_i, x_j) \in E \).

\[
T\mu_1(x_i, x_j) \leq T\sigma_1(x_i) \land T\sigma_1(x_j),
I\mu_1(x_i, x_j) \leq I\sigma_1(x_i) \land I\sigma_1(x_j),
F\mu_3(x_i, x_j) \geq F\sigma_1(x_i) \land F\sigma_1(x_j),
0 \leq T\sigma_1(x_i, x_j) + I\sigma_2(x_i, x_j) + F\sigma_3(x_i, x_j) \leq 3.
\]

Definition 2.2. A neutrosophic graph \( G = ((T\sigma_1, T\sigma_2, F\sigma_3), (T\mu_1, T\mu_2, F\mu_3)) \) is called strong neutrosophic graph if

1. \( T\mu_1(x_i, x_j) \leq T\sigma_1(x_i) \land T\sigma_1(x_j) \),
2. \( I\mu_1(x_i, x_j) \leq I\sigma_1(x_i) \land I\sigma_1(x_j) \),
3. \( F\mu_3(x_i, x_j) \geq F\sigma_1(x_i) \land F\sigma_1(x_j) \),

for all \( x_i, x_j \in E_i \neq j \).

Definition 2.3. A neutrosophic graph \( G = ((T\sigma_1, T\sigma_2, F\sigma_3), (T\mu_1, T\mu_2, F\mu_3)) \) is considered complete if

1. \( T\mu_1(x_i, x_j) \leq T\sigma_1(x_i) \land T\sigma_1(x_j) \),
2. \( I\mu_1(x_i, x_j) \leq I\sigma_1(x_i) \land I\sigma_1(x_j) \),
3. \( F\mu_3(x_i, x_j) \geq F\sigma_1(x_i) \land F\sigma_1(x_j) \),

for all \( x_i, x_j \in V_i \neq j \).

Definition 2.4. A neutrosophic graph \( G = ((T\sigma_1, T\sigma_2, F\sigma_3), (T\mu_1, T\mu_2, F\mu_3)) \). The order of \( G \) denoted as \( O(G) \) is defined as follows \( O(G) = (O_{T\sigma_1}(G), O_{I\sigma_1}(G), O_{F\sigma_1}(G)) \), where \( O_{T\sigma_1}(G) = \Sigma_{x \in V} T\sigma_1(x) \), \( O_{I\sigma_1}(G) = \Sigma_{x \in V} I\sigma_1(x) \), and \( O_{F\sigma_1}(G) = \Sigma_{x \in V} F\sigma_1(x) \).
Definition 2.5. A neutrosophic graph $G = ((T_{σ_1}, I_{σ_2}, F_{σ_3}), (T_{µ_1}, I_{µ_2}, F_{µ_3}))$. The size of $G$ is denoted as $S(G)$, defined as follows $S(G) = (S_{Tσ_1}(G), S_{Iσ_2}(G), S_{Fσ_3}(G))$, where $S_{Tσ_1}(G) = Σ_{xy ∈ E}T_{σ_1}(xy), S_{Iσ_2}(G) = Σ_{xy ∈ E}I_{σ_2}(xy)$ and $S_{Fσ_3}(G) = Σ_{xy ∈ E}F_{σ_3}(xy)$.

Definition 2.6. A neutrosophic graph $G = ((T_{σ_1}, I_{σ_2}, F_{σ_3}), (T_{µ_1}, I_{µ_2}, F_{µ_3}))$. The degree of a vertex $x$ in $G$ is denoted by $d_G(x) = (T_{d_1}^G(x), I_{d_2}^G(x), F_{d_3}^G(x))$ and can be calculated as follows

\[ T_{d_1}^G(x) = \sum_{x \neq y} T_{µ_1}(xy) = \sum_{xy \in E} T_{µ_1}(xy) \] (1)

\[ I_{d_2}^G(x) = \sum_{x \neq y} I_{µ_2}(xy) = \sum_{xy \in E} I_{µ_2}(xy) \] (2)

\[ F_{d_3}^G(x) = \sum_{x \neq y} F_{µ_2}(xy) = \sum_{xy \in E} F_{µ_2}(xy) \] (3)

Where $T_{d_1}^G(x)$ represents the total of type membership scores of the edges connected to vertex $x$, $I_{d_2}^G(x)$ represents the total of intermediate membership scores of the edges connected to vertex $x$ and $F_{d_3}^G(x)$ represents the sum of false membership scores of the edges connected to vertex $x$.

3 Neutrosophic Graphs’ Complement of the Maximum Product

Definition 3.1. The complement of a neutrosophic graph $G = (V, E)$ is a neutrosophic graph $\overline{G} = ((T_{σ_1}, I_{σ_2}, F_{σ_3}), (T_{µ_1}, I_{µ_2}, F_{µ_3}))$ where $(T_{σ_1}, I_{σ_2}, F_{σ_3}) = (T_{σ_1}), (I_{σ_2}), (F_{σ_3})$ and $(T_{µ_1}, I_{µ_2}, F_{µ_3}) = (T_{µ_1}), (I_{µ_2}), (F_{µ_3})$, where $T_{σ_1}(x, y) = T_{σ_1}(xy) - T_{σ_1}(y)$ and $I_{σ_2}(x, y) = I_{σ_2}(xy) - I_{σ_2}(y)$ and $F_{σ_3}(x, y) = F_{σ_3}(xy) - F_{σ_3}(y)$.

Definition 3.2. Let $G_1 = ((T_{σ_1^1}, I_{σ_2^1}, F_{σ_3^1}), (T_{µ_1^1}, I_{µ_2^1}, F_{µ_3^1}))$ and $G_2 = ((T_{σ_1^2}, I_{σ_2^2}, F_{σ_3^2}), (T_{µ_1^2}, I_{µ_2^2}, F_{µ_3^2}))$ be two neutrosophic graphs the maximum product of $G_1$ and $G_2$ is defined $G_1 × m G_2 = (V_1 × m V_2, E_1 × m E_2), E_1 × m E_2 = \{(x_1, y_1), (x_2, y_2)\} / x_1 = x_2, y_1 \in E_2$ or $y_2 = y_2, x_1, x_2 \in E_1$.

\[ T_{σ_1^{G_1} × m G_2}(x_1, y_2) = T_{σ_1^{G_1}}(x_1) ∧ T_{σ_1^{G_2}}(y_1) \] (4)

\[ I_{σ_1^{G_1} × m G_2}(x_1, y_2) = I_{σ_1^{G_1}}(x_1) ∧ I_{σ_1^{G_2}}(y_1) \] (5)

\[ F_{σ_1^{G_1} × m G_2}(x_1, y_2) = F_{σ_1^{G_1}}(x_1) ∨ F_{σ_1^{G_2}}(y_1) \] (6)

\[ T_{µ_1^{G_1} × m G_2}((x_1, y_1), (x_2, y_2)) = \begin{cases} T_{µ_1^{G_1}}(x_1, y_1) \land T_{µ_1^{G_2}}(x_2, y_2) \land f_{x_1} = x_2, y_1, y_2 \in E_2 \\ T_{µ_1^{G_1}}(x_1, x_2) \land T_{µ_1^{G_2}}(y_1, y_2) \land f_{y_1} = y_2, x_1, x_2 \in E_1 \\ T_{µ_1^{G_2}}(x_1, y_1) \land T_{µ_1^{G_1}}(x_2, y_2) \land f_{y_1} = y_2, x_1, x_2 \in E_1 \end{cases} \] (7)

\[ I_{µ_1^{G_1} × m G_2}((x_1, y_1), (x_2, y_2)) = \begin{cases} I_{µ_1^{G_1}}(x_1, y_1) \land I_{µ_1^{G_2}}(x_2, y_2) \land f_{x_1} = x_2, y_1, y_2 \in E_2 \\ I_{µ_1^{G_1}}(x_1, x_2) \land I_{µ_1^{G_2}}(y_1, y_2) \land f_{y_1} = y_2, x_1, x_2 \in E_1 \end{cases} \] (8)

\[ F_{µ_1^{G_1} × m G_2}((x_1, y_1), (x_2, y_2)) = \begin{cases} F_{µ_1^{G_1}}(x_1, y_1) \lor F_{µ_1^{G_2}}(x_2, y_2) \land f_{x_1} = x_2, y_1, y_2 \in E_2 \\ F_{µ_1^{G_1}}(x_1, x_2) \lor F_{µ_1^{G_2}}(y_1, y_2) \land f_{y_1} = y_2, x_1, x_2 \in E_1 \end{cases} \] (9)

Example 3.3. Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two crisp graphs, such that $V_1 = \{u_1, u_2, u_3\}$ and $V_2 = \{v_1, v_2\}$, $E_1 = \{(u_1 v_1, u_2 v_1, u_3 v_1) & E_2 = \{(v_1 v_2\}$. Take two neutrosophic graphs as consideration $G_1 = ((T_{σ_1^{G_1}}, I_{σ_2^{G_1}}, F_{σ_3^{G_1}}), (T_{µ_1^{G_1}}, I_{µ_2^{G_1}}, F_{µ_3^{G_1}}))$ and $G_2 = ((T_{σ_1^{G_2}}, I_{σ_2^{G_2}}, F_{σ_3^{G_2}}), (T_{µ_1^{G_2}}, I_{µ_2^{G_2}}, F_{µ_3^{G_2}}))$ & $G_1 × m G_2$

<table>
<thead>
<tr>
<th>$V_1 × m V_2$</th>
<th>$u_1 v_1$</th>
<th>$u_2 v_1$</th>
<th>$u_3 v_1$</th>
<th>$u_1 v_2$</th>
<th>$u_2 v_2$</th>
<th>$u_3 v_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{σ_1^{G_1}} × T_{σ_1^{G_2}}$</td>
<td>0.7</td>
<td>0.7</td>
<td>0.8</td>
<td>0.7</td>
<td>0.7</td>
<td></td>
</tr>
<tr>
<td>$I_{σ_1^{G_1}} × I_{σ_1^{G_2}}$</td>
<td>0.7</td>
<td>0.6</td>
<td>0.7</td>
<td>0.6</td>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td>$F_{σ_1^{G_1}} × F_{σ_1^{G_2}}$</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.4</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Product of two vertex sets
Definition 3.4. The maximum of two neutrosophic graphs’ products in complement

\[ G_1 = ((T_{\sigma_1^{G_1}}, I_{\sigma_1^{G_1}}, F_{\sigma_1^{G_1}}), (T_{I_{\mu_1^{G_1}}, I_{T_{\mu_1^{G_1}}, F_{\mu_1^{G_1}}}})) \text{ and } G_2 = ((T_{\sigma_2^{G_2}}, I_{\sigma_2^{G_2}}, F_{\sigma_2^{G_2}}), (T_{\mu_2^{G_2}, I_{\mu_2^{G_2}}, F_{\mu_2^{G_2}}})) \]

is a neutrosophic graphs

\[ G_1 \times_m G_2 = ((T_{\sigma_1^{G_1} \times_m \sigma_2^{G_2}}, I_{\sigma_1^{G_1} \times_m \sigma_2^{G_2}}, F_{\sigma_1^{G_1} \times_m \sigma_2^{G_2}}), (T_{\mu_1^{G_1} \times_m \mu_2^{G_2}, I_{\mu_1^{G_1} \times_m \mu_2^{G_2}}, F_{\mu_1^{G_1} \times_m \mu_2^{G_2}}})) \text{ on } G^* = (V, E). \]

\[
E_1 \times_m E_2 = \begin{cases} x_1 = x_2, y_1 \neq y_2 \in E_2 \text{ or } y_1 = y_2, x_1 x_2 \in E_1 \text{ or } (x_1, y_1)(x_2, y_2) \in E_1, y_1 y_2 \notin E_2 \text{ or } x_1 x_2 \notin E_1, y_1 y_2 \in E_2 \text{ or } x_1 x_2 \in E_1, y_1 y_2 \notin E_2 \text{ or } x_1 x_2 \notin E_1, y_1 y_2 \in E_2 \text{ or } x_1 x_2 \in E_1, y_1 y_2 \neq E_2 \text{ or } x_1 x_2 \notin E_1, y_1 y_2 \notin E_2 \end{cases}
\]

\[
(T_{\sigma_1^{G_1} \times_m T_{\sigma_1^{G_2}}}(x_1, y_2) = (T_{\sigma_1^{G_1}}(x_1, y_1) \lor T_{\sigma_1^{G_2}}(y_1)) \quad (10)
\]

\[
(I_{\sigma_2^{G_2}}(x_1, y_2) = (I_{\sigma_2^{G_2}}(x_1, y_1) \lor I_{\sigma_2^{G_2}}(y_1)) \quad (11)
\]

\[
(F_{\sigma_3^{G_3} \times_m F_{\sigma_3^{G_3}}}(x_1, y_2) = (F_{\sigma_3^{G_3}}(x_1, y_1) \lor F_{\sigma_3^{G_3}}(y_1)) \quad (12)
\]

Table 2: Product of two Edge sets
Where \( x_1 \in V_1 \) and \( y_1 \in V_2 \)
\[
(T_{\mu_1}G_1 \times_m T_{\mu_2}G_2)((x_1, y_1), (x_2, y_2)) \\
= \begin{cases} 
(T_{\sigma_1}G_1 \times_m T_{\sigma_2}G_2)(x_1, y_1) \vee (T_{\sigma_1}G_1 \times_m T_{\sigma_2}G_2)(x_2, y_2) - (T_{\mu_1}G_1 \times_m T_{\mu_2}G_2)((x_1, y_1), (x_2, y_2)) & \text{if } x_1 = x_2, y_1 y_2 \in E_2 \\
(T_{\sigma_1}G_1 \times_m T_{\sigma_2}G_2)(x_1, y_1) \vee (T_{\sigma_1}G_1 \times_m T_{\sigma_2}G_2)(x_2, y_2) - (T_{\mu_1}G_1 \times_m T_{\mu_2}G_2)((x_1, y_1), (x_2, y_2)) & \text{if } y_1 = y_2, x_1 x_2 \in E_1 \\
(T_{\sigma_1}G_1 \times_m T_{\sigma_2}G_2)(x_1, y_1) \vee (T_{\sigma_1}G_1 \times_m T_{\sigma_2}G_2)(x_2, y_2) & \text{otherwise}
\end{cases}
\]

\[
(I_{\mu_1}G_1 \times_m I_{\mu_2}G_2)((x_1, y_1), (x_2, y_2)) \\
= \begin{cases} 
(I_{\sigma_1}G_1 \times_m I_{\sigma_2}G_2)(x_1, y_1) \vee (I_{\sigma_1}G_1 \times_m I_{\sigma_2}G_2)(x_2, y_2) - (I_{\mu_1}G_1 \times_m I_{\mu_2}G_2)((x_1, y_1), (x_2, y_2)) & \text{if } x_1 = x_2, y_1 y_2 \in E_2 \\
(I_{\sigma_1}G_1 \times_m I_{\sigma_2}G_2)(x_1, y_1) \vee (I_{\sigma_1}G_1 \times_m I_{\sigma_2}G_2)(x_2, y_2) - (I_{\mu_1}G_1 \times_m I_{\mu_2}G_2)((x_1, y_1), (x_2, y_2)) & \text{if } y_1 = y_2, x_1 x_2 \in E_1 \\
(I_{\sigma_1}G_1 \times_m I_{\sigma_2}G_2)(x_1, y_1) \vee (I_{\sigma_1}G_1 \times_m I_{\sigma_2}G_2)(x_2, y_2) & \text{otherwise}
\end{cases}
\]

\[
(F_{\mu_1}G_1 \times_m F_{\mu_2}G_2)((x_1, y_1), (x_2, y_2)) \\
= \begin{cases} 
(F_{\sigma_1}G_1 \times_m F_{\sigma_2}G_2)(x_1, y_1) \vee (F_{\sigma_1}G_1 \times_m F_{\sigma_2}G_2)(x_2, y_2) - (F_{\mu_1}G_1 \times_m F_{\mu_2}G_2)((x_1, y_1), (x_2, y_2)) & \text{if } x_1 = x_2, y_1 y_2 \in E_2 \\
(F_{\sigma_1}G_1 \times_m F_{\sigma_2}G_2)(x_1, y_1) \vee (F_{\sigma_1}G_1 \times_m F_{\sigma_2}G_2)(x_2, y_2) - (F_{\mu_1}G_1 \times_m F_{\mu_2}G_2)((x_1, y_1), (x_2, y_2)) & \text{if } y_1 = y_2, x_1 x_2 \in E_1 \\
(F_{\sigma_1}G_1 \times_m F_{\sigma_2}G_2)(x_1, y_1) \vee (F_{\sigma_1}G_1 \times_m F_{\sigma_2}G_2)(x_2, y_2) & \text{otherwise}
\end{cases}
\]

Example 3.5. Examine the two neutrosophic diagrams as depicted in Fig [1] and their respective maximum product \( G_1 \times_m G_2 \) illustrated in Fig [2]. Subsequently, the complement of the maximum product of \( G_1 \) and \( G_2 \) is displayed in Fig [3].

Figure 3: Complement of Max Product \( G_1 \times_m G_2 \)

Theorem 3.6. Let \( G_1 \) and \( G_2 \) be a two regular neutrosophic graphs of underlying crisp graph \( G^*_1 \) and \( G^*_2 \) are complete graphs and \( T_{\sigma_1}G_1, I_{\sigma_2}G_1, F_{\sigma_3}G_1, T_{\sigma_4}G_2, I_{\sigma_5}G_2, F_{\sigma_6}G_2 \) are constants which satisfy \( T_{\sigma_1}G_1 \geq T_{\mu_1}G_2, I_{\sigma_2}G_1 \geq I_{\mu_2}G_2, F_{\sigma_3}G_1 \leq T_{\mu_1}G_2, T_{\sigma_4}G_1 \geq T_{\mu_1}G_2, I_{\sigma_5}G_2 \geq I_{\mu_2}G_2, F_{\sigma_6}G_2 \leq T_{\mu_1}G_2, T_{\sigma_1}G_1 > T_{\mu_1}G_2, I_{\sigma_2}G_1 > I_{\mu_2}G_2, F_{\sigma_3}G_1 < T_{\mu_2}G_2, T_{\sigma_4}G_1 > T_{\mu_2}G_2, I_{\sigma_5}G_2 > I_{\mu_2}G_2, F_{\sigma_6}G_2 < T_{\mu_2}G_2 \) of product of two neutrosophic graph \( G_1 \) and \( G_2 \) is regular neutrosophic graphs.

Proof. Let \( G_1 \) and \( G_2 \) be two regular neutrosophic graphs. The underlying crisp graphs and \( G^*_1 \) and \( G^*_2 \) are complete graphs of degree \( d_1 \) and \( d_2 \) for every vertices of \( V_1 \) and \( V_2 \). Given that \( T_{\sigma_1}G_1, I_{\sigma_2}G_1, F_{\sigma_3}G_1 \) and \( T_{\sigma_4}G_2, I_{\sigma_5}G_2, F_{\sigma_6}G_2 \) are constants, say \( T_{\sigma_1}(x) = C_1, I_{\sigma_2}(x) = C_2, F_{\sigma_3}(x) = C_3 \forall x \in V_1, T_{\sigma_4}(y) = C_4, I_{\sigma_5}(y) = C_5, F_{\sigma_6}(y) = C_6 \forall y \in V_2 \) and \( T_{\sigma_1}G_1 \geq T_{\mu_1}G_2, I_{\sigma_2}G_1 \geq I_{\mu_2}G_2, F_{\sigma_3}G_1 \leq T_{\mu_2}G_2, T_{\sigma_4}G_1 \geq T_{\mu_2}G_2, I_{\sigma_5}G_2 \geq I_{\mu_2}G_2, F_{\sigma_6}G_2 \leq T_{\mu_2}G_2 \). By theorem product of two neutrosophic graphs is regular neutro-
sophic graphs. Consider \((x_1, y_2) \in (T_{\sigma G_1} \times_m T_{\sigma G_2})\).

\[
d_1^{T_{\sigma G_1} \times_m T_{\sigma G_2}}(x_1, y_1) = \sum_{(x_1, y_1) \neq (x_2, y_2) \in E} (T_{\sigma G_1} \times_m T_{\sigma G_2})(x_1, y_1)(x_2, y_2)
\]

\[
= \sum_{x_1 = x_2, y_1 \neq y_2 \in E_1} (T_{\sigma G_1} \times_m T_{\sigma G_2})(x_1, y_1) \land (T_{\sigma G_1} \times_m T_{\sigma G_2})(x_2, y_2)
\]

\[
- (T_{\sigma G_1} \times_m T_{\sigma G_2})(x_1, y_1), (x_2, y_2)
\]

\[
+ \sum_{y_1 = y_2, x_1 \neq x_2 \in E_1} (T_{\sigma G_1} \times_m T_{\sigma G_2})(x_1, y_1) \land (T_{\sigma G_1} \times_m T_{\sigma G_2})(x_2, y_2)
\]

\[
- (T_{\sigma G_1} \times_m T_{\sigma G_2})(x_1, y_1), (x_2, y_2)
\]

\[
+ \sum_{x_1 = x_2, y_1 \neq y_2 \in E_2} (T_{\sigma G_1} \times_m T_{\sigma G_2})(x_1, y_1) \land (T_{\sigma G_1} \times_m T_{\sigma G_2})(x_2, y_2)
\]

\[
+ \sum_{x_1 = x_2, y_1 \neq y_2 \in E_2} (T_{\sigma G_1} \times_m T_{\sigma G_2})(x_1, y_1) \land (T_{\sigma G_1} \times_m T_{\sigma G_2})(x_2, y_2)
\]

\[
+ \sum_{x_1 = x_2, y_1 \neq y_2 \in E_2} (T_{\sigma G_1} \times_m T_{\sigma G_2})(x_1, y_1) \land (T_{\sigma G_1} \times_m T_{\sigma G_2})(x_2, y_2)
\]

Since \(G_1^*\) and \(G_2^*\) are complete graphs, then

\[
d_2^{G_1 \times G_2}(x_1, y_1) = \sum_{x_1 = x_2, y_1 \neq y_2 \in E_2} (T_{\sigma G_1} \times_m T_{\sigma G_2})(x_1, y_1) \land (T_{\sigma G_1} \times_m T_{\sigma G_2})(x_2, y_2)
\]

\[
- (T_{\sigma G_1} \times_m T_{\sigma G_2})(x_1, y_1), (x_2, y_2)
\]

\[
+ \sum_{y_1 = y_2, x_1 \neq x_2 \in E_1} (T_{\sigma G_1} \times_m T_{\sigma G_2})(x_1, y_1) \land (T_{\sigma G_1} \times_m T_{\sigma G_2})(x_2, y_2)
\]

\[
- (T_{\sigma G_1} \times_m T_{\sigma G_2})(x_1, y_1), (x_2, y_2)
\]

\[
+ \sum_{x_1 = x_2, y_1 \neq y_2 \in E_2} (T_{\sigma G_1} \times_m T_{\sigma G_2})(x_1, y_1) \land (T_{\sigma G_1} \times_m T_{\sigma G_2})(x_2, y_2)
\]

\[
+ \sum_{x_1 = x_2, y_1 \neq y_2 \in E_2} (T_{\sigma G_1} \times_m T_{\sigma G_2})(x_1, y_1) \land (T_{\sigma G_1} \times_m T_{\sigma G_2})(x_2, y_2)
\]

\[
+ \sum_{x_1 = x_2, y_1 \neq y_2 \in E_2} (T_{\sigma G_1} \times_m T_{\sigma G_2})(x_1, y_1) \land (T_{\sigma G_1} \times_m T_{\sigma G_2})(x_2, y_2)
\]
Case 1 If $T\sigma_1^G(x) \leq T\sigma_1^G(y)$, $I\sigma_1^G(x) \leq I\sigma_1^G(y)$ and $F\sigma_1^G(x) \geq F\sigma_1^G(y)$ for all $x \in V_1$ and $y \in V_2$. Eq (5)

\[
T_{d_3^{G_1 \times m G_2}}(x_1, y_1) = \sum_{x_1=x_2, y_1 \in E_2} \left( T\sigma_1^G(x_1) \land T\sigma_1^G(y_1) \right) \land \left( T\sigma_1^G(x_2) \land T\sigma_1^G(y_2) \right) \]
\[
- \left( T\mu_1^G \times_m T\mu_1^G \right)(x_1, y_1, (x_2, y_2)) \]
\[
+ \sum_{y_1=y_2, x_1 \in E_1} \left( T\sigma_1^G(x_1) \land T\sigma_1^G(y_1) \right) \land \left( T\sigma_1^G(x_2) \land T\sigma_1^G(y_2) \right) \]
\[
- \left( T\mu_1^G \times_m T\mu_1^G \right)(x_1, y_1, (x_2, y_2)) \]
\[
+ \sum_{x_1, x_2 \in E_1, y_1 \in E_2} \left( T\sigma_1^G(x_1) \land T\sigma_1^G(y_1) \right) \land \left( T\sigma_1^G(x_2) \land T\sigma_1^G(y_2) \right) \]
\[
I_{d_3^{G_1 \times m G_2}}(x_1, y_1) = \sum_{x_1=x_2, y_1 \in E_2} \left( I\sigma_1^G(x_1) \land I\sigma_1^G(y_1) \right) \land \left( I\sigma_1^G(x_2) \land I\sigma_1^G(y_2) \right) \]
\[
- \left( I\mu_1^G \times_m I\mu_1^G \right)(x_1, y_1, (x_2, y_2)) \]
\[
+ \sum_{y_1=y_2, x_1 \in E_1} \left( I\sigma_1^G(x_1) \land I\sigma_1^G(y_1) \right) \land \left( I\sigma_1^G(x_2) \land I\sigma_1^G(y_2) \right) \]
\[
- \left( I\mu_1^G \times_m I\mu_1^G \right)(x_1, y_1, (x_2, y_2)) \]
\[
+ \sum_{x_1, x_2 \in E_1, y_1 \in E_2} \left( I\sigma_1^G(x_1) \land I\sigma_1^G(y_1) \right) \land \left( I\sigma_1^G(x_2) \land I\sigma_1^G(y_2) \right) \]
\[
+ \sum_{x_1=x_2, y_1 \in E_2} \left( I\sigma_1^G(x_1) \land I\sigma_1^G(y_1) \right) \land \left( I\sigma_1^G(x_2) \land I\sigma_1^G(y_2) \right) \]
\[
F_{d_1^{G_1}\times G_2}(x_1, y_1) = \sum_{x_1 = x_2, y_1 y_2 E_2} (F_{\sigma_1^{G_1}}(x_1) \lor F_{\sigma_1^{G_2}}(y_1)) \lor (F_{\sigma_1^{G_1}}(x_2) \lor F_{\sigma_1^{G_2}}(y_2)) \\
- (F_{\mu_1^{G_1}} \times m F_{\mu_1^{G_2}}((x_1, y_1), (x_2, y_2))) \\
+ \sum_{y_1 = y_2, x_1 x_2 E_1} (F_{\sigma_1^{G_1}}(x_1) \lor F_{\sigma_1^{G_2}}(y_1)) \land (F_{\sigma_1^{G_1}}(x_2) \lor F_{\sigma_1^{G_2}}(y_2)) \\
- (F_{\mu_1^{G_1}} \times m F_{\mu_1^{G_2}}((x_1, y_1), (x_2, y_2))) \\
+ \sum_{x_1, x_2 E_1, y_1 y_2 E_2} (F_{\sigma_1^{G_1}}(x_1) \lor F_{\sigma_1^{G_2}}(y_1)) \lor (F_{\sigma_1^{G_1}}(x_2) \lor F_{\sigma_1^{G_2}}(y_2)) \\
+ \sum_{x_1 = x_2, y_1 y_2 E_2} (F_{\sigma_1^{G_1}}(x_1) \lor F_{\sigma_1^{G_2}}(y_1)) \lor (F_{\sigma_1^{G_1}}(x_2) \lor F_{\sigma_1^{G_2}}(y_2))
\]

Since by the definition of project of two neutrosophic graphs,
\[
T_{d_1^{G_1}\times G_2}(x_1, y_1) = \sum_{x_1 = x_2, y_1 y_2 E_2} T_{\sigma_1^{G_1}}(x_1) - (T_{\mu_1^{G_1}} \times m T_{\mu_1^{G_2}})((x_1, y_1), (x_2, y_2)) \\
+ \sum_{y_1 = y_2, x_1 x_2 E_1} T_{\sigma_1^{G_2}}(y_1) - (T_{\mu_1^{G_1}} \times m T_{\mu_1^{G_2}})((x_1, y_1), (x_2, y_2)) + \sum_{x_1, x_2 E_1, y_1 y_2 E_2} C_3 \\
= \sum_{x_1 = x_2, y_1 y_2 E_2} T_{\sigma_1^{G_1}}(y_1) - T_{\sigma_1^{G_2}}(x_1) \land T_{\mu_1^{G_2}}(y_1, y_2) \\
+ \sum_{y_1 = y_2, x_1 x_2 E_1} T_{\sigma_1^{G_2}}(y_1) - T_{\mu_1^{G_1}}(x_1, x_2) \lor T_{\sigma_1^{G_2}}(y_1) + \sum_{x_1, x_2 E_1, y_1 y_2 E_2} C_3 \\
= \sum_{x_1 = x_2, y_1 y_2 E_2} C_3 - T_{\sigma_1^{G_1}}(x_1) + \sum_{y_1 = y_2, x_1 x_2 E_1} C_3 - T_{\sigma_1^{G_2}}(y_1) + C_3 d_{G_2}(y_1) d_{G_1}(x_1).
\]

\[
T_{d_1^{G_1}\times G_2}(x_1, y_1) = (C_3 - C_1)d_2 + C_3 d_1 d_2,
\]

Similarly we can find
\[
I_{d_1^{G_1}\times G_2}(x_1, y_1) = (C_3 - C_1)d_2 + C_3 d_1 d_2,
\]
\[
F_{d_1^{G_1}\times G_2}(x_1, y_1) = (C_4 - C_1)d_2 + C_4 d_1 d_2.
\]

Since \(G_1\) and \(G_2\) are two regular neutrosophic graphs, \(G_1^*\) and \(G_2^*\) represent complete graphs, with membership functions denoted by \(\mu_1^{G_1}\) and \(\mu_2^{G_2}\). These membership functions are constants, namely, \((C_1, C_2)\) for \(\mu_1^{G_1}\) and \((C_3, C_4)\) for \(\mu_2^{G_2}\).

**Case 2** If \(T_{\sigma_1^{G_1}}(x) \geq T_{\sigma_1^{G_2}}(y), I_{\sigma_1^{G_1}}(x) \geq I_{\sigma_1^{G_2}}(y)\) and \(F_{\sigma_1^{G_1}}(x) \leq F_{\sigma_1^{G_2}}(y)\) for all \(x \in V_1\) and \(y \in V_2\).

\[
T_{d_1^{G_1}\times m G_2}(x_1, y_1) = \sum_{x_1 = x_2, y_1 y_2 E_2} (T_{\sigma_1^{G_1}}(x_1) - (T_{\sigma_1^{G_2}}(x_1) \lor T_{\mu_1^{G_2}}(y_1, y_2))) \\
+ \sum_{y_1 = y_2, x_1 x_2 E_1} (T_{\sigma_1^{G_2}}(y_1) - (T_{\sigma_1^{G_2}}(y_1) \lor T_{\mu_1^{G_2}}(x_1, x_2))) \\
+ \sum_{x_1, x_2 E_1, y_1 y_2 E_2} T_{\sigma_1^{G_1}}(x_1) \\
= \sum_{x_1 = x_2, y_1 y_2 E_2} C_1 - (T_{\sigma_1^{G_1}}(x_1)) + \sum_{y_1 = y_2, x_1 x_2 E_1} C_1 - (T_{\sigma_1^{G_2}}(y_1)) \\
+ \sum_{x_1 = x_2, y_1 y_2 E_2} C_1 d_{G_2}(y_1) d_{G_2}(x_1) \\
T_{d_1^{G_1}\times m G_2}(x_1, y_1) = (C_1 - C_2)d_2 + C_3 d_1 d_2,
\]

Similarly we can find
\[
I_{d_1^{G_1}\times m G_2}(x_1, y_1) = (C_1 - C_3)d_2 + C_3 d_1 d_2,
\]
\[
F_{d_1^{G_1}\times m G_2}(x_1, y_1) = (C_2 - C_1)d_1 + C_4 d_1 d_2
\]

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Because of this, normal neutrosophic graphs have a regular complement of their maximum product.

**Theorem 3.7.** Let $G_1$ and $G_2$ be a pair of regular neutrosophic graphs derived from the underlying crisp graph $G_1'$ and $G_2'$ respectively. The vertex sets and edges sets of $G_1$ and $G_2$ are complete graphs, and the regular neutrosophic graphs are associated with them. If $T_{\sigma_1} G_1 > T_{\mu_1} G_1, I_{\sigma_1} G_1 > I_{\mu_1} G_1, F_{\sigma_1} G_1 < F_{\mu_1} G_1, T_{\sigma_1} G_2 > T_{\mu_1} G_2, I_{\sigma_1} G_2 > I_{\mu_1} G_2, F_{\sigma_1} G_2 < F_{\mu_1} G_2, T_{\sigma_1} G_1 > T_{\mu_1} G_1, I_{\sigma_1} G_1 > I_{\mu_1} G_1, F_{\sigma_1} G_1 < F_{\mu_1} G_1$ and $T_{\sigma_1} G_2 < T_{\mu_1} G_2, I_{\sigma_2} G_2 < I_{\mu_2} G_2, F_{\sigma_2} G_2 < F_{\mu_2} G_2$, a complement graph is a regular neutrosophic graph when it’s the maximum product of two regular neutrosophic graphs.

**Proof.** The underlying crisp graphs $G_1'$ and $G_2'$ are regular graphs, with every vertex in $V_1$ and $V_2$ having degrees $g_1$ and $g_2$, respectively. Given that $T_{\sigma_1} G_1, I_{\sigma_1} G_1, F_{\sigma_1} G_1, T_{\mu_1} G_2, I_{\mu_1} G_2$ and $F_{\mu_1} G_2$ are constants say $T_{\sigma_1} G_1(x) = C_1, I_{\sigma_1} G_1(x) = C_2, F_{\sigma_1} G_1(x) = C_3 I_{\sigma_1} G_2(x) = C_4 I_{\sigma_1} G_2(x) = C_5, F_{\sigma_1} G_2(x) = C_6, y \in V_2, T_{\mu_1} G_2(x_1, y_1) = e_1, I_{\mu_1} G_2(x_1, y_1) = e_2, F_{\mu_1} G_2(x_1, y_1) = e_3, T_{\mu_1} G_1(x_1 y_1) = e_4, I_{\mu_1} G_1(x_1 y_1) = e_5, F_{\mu_1} G_1(x_1, y_1) = e_6$ and $T_{\sigma_1} G_1 > T_{\mu_1} G_1, I_{\sigma_1} G_1 > I_{\mu_1} G_1, F_{\sigma_1} G_1 < F_{\mu_1} G_1, T_{\mu_1} G_1, I_{\sigma_1} G_2 < I_{\mu_1} G_2, F_{\sigma_1} G_2 < F_{\mu_1} G_2$. Let $v_1 \cdot v_2$ be the degree of neutrosophic graphs $G_1'$ and $G_2'$.

**Case 1:** If $T_{\sigma_1} G_1(x) \leq T_{\sigma_1} G_1(y), I_{\sigma_1} G_1(x) \leq I_{\sigma_1} G_1(y)$ and $F_{\sigma_1} G_2(y) \leq F_{\sigma_1} G_2(y)$ for all $x, y \in V_1$.

\[ T_{\sigma_1} G_1 \times T_{\sigma_1} G_2(x, y_1) = \sum_{x_1 = x, y_2 \in E_2} \left( (T_{\sigma_1} G_1(x_1, y_1) \vee T_{\sigma_1} G_2(y_1)) \wedge (T_{\sigma_1} G_1(x_2) \vee T_{\sigma_1} G_2(y_2)) \right) - \left( T_{\mu_1} G_1 \times T_{\mu_1} G_2((x_1, y_1), (x_2, y_2)) \right) \]

\[ + \sum_{y_1 = y, x_1 x_2 \in E_1} \left( (T_{\sigma_1} G_1(x_1, y_1) \vee T_{\sigma_1} G_2(y_1)) \wedge (T_{\sigma_1} G_1(x_2) \vee T_{\sigma_1} G_2(y_2)) \right) - \left( T_{\mu_1} G_1 \times T_{\mu_1} G_2((x_1, y_1), (x_2, y_2)) \right) \]

\[ + \sum_{x_1 = x, x_2 \in E_1, y_1 y_2 \in E_2} \left( (T_{\sigma_1} G_1(x_1, y_1) \vee T_{\sigma_1} G_2(y_1)) \wedge (T_{\sigma_1} G_1(x) \vee T_{\sigma_1} G_2(y)) \right) \]

\[ + \sum_{x_1 = x, x_2 \in E_1, y_1 y_2 \in E_2} \left( (T_{\sigma_1} G_1(x_1, y_1) \vee T_{\sigma_1} G_2(y_1)) \wedge (T_{\sigma_1} G_1(x_2) \vee T_{\sigma_1} G_2(y_2)) \right) \]

\[ + \sum_{x_1 x_2 \notin E_1, y_1 y_2 \in E_2} \left( (T_{\sigma_1} G_1(x_1, y_1) \vee T_{\sigma_1} G_2(y_1)) \wedge (T_{\sigma_1} G_1(x_2) \vee T_{\sigma_1} G_2(y_2)) \right) \]

\[ = \sum_{x_1 = x, y_1 y_2 \in E_2} T_{\sigma_1} G_2(y_1) - \{ T_{\sigma_1} G_1((x_1, y_1), (x_2, y_2)) \} + \sum_{y_1 = y, x_1 x_2 \in E_1} T_{\sigma_1} G_2(y_1) - \{ T_{\sigma_1} G_2((x_1, x_2)) \} \]

\[ + \sum_{x_1 = x, x_2 \in E_1, y_1 y_2 \in E_2} C_4 \sum_{x_1 x_2 \notin E_1, y_1 y_2 \in E_2} C_4 \sum_{x_1 x_2 \in E_1, y_1 y_2 \in E_2} C_4 \sum_{x_1 x_2 \in E_1, y_1 y_2 \in E_2} \]

\[ = (C_4 - C_1) g_2 + (C_1 - C_1) g_1 + C_4 d_{G_1}(x_1) + \left| E_2 \right| + C_4 \left| E_1 \right| d_{G_2}(y_1) + C_4 \left| E_1 \right| \left| E_2 \right| + C_4 d_{G_2}(x_2) \delta_{G_2}(y_4) \]

Where $|E_1|$ and $|E_2|$ are the degree of vertex of complement graphs $G_1'$ and $G_2'$.

\[ T_{\sigma_1} G_1 \times T_{\sigma_1} G_2(x_1, y_1) = (C_4 - C_1) g_2 + C_4 g_1 \left| E_2 \right| + C_4 g_2 \left| E_1 \right| + C_4 \left| E_1 \right| \left| E_2 \right| + C_4 g_1 \left| E_2 \right| + C_4 \left| E_1 \right| \left| E_2 \right| + C_4 g_2 \left| E_1 \right| \left| E_2 \right| + C_4 \left| E_1 \right| \left| E_2 \right| + C_4 g_1 \left| E_2 \right| \left| E_2 \right| \]

\[ I_{\sigma_1} G_1 \times I_{\sigma_1} G_2(x_1, y_1) = (C_4 - C_1) g_2 + C_4 g_1 \left| E_2 \right| + C_4 g_2 \left| E_1 \right| + C_4 \left| E_1 \right| \left| E_2 \right| + C_4 g_1 \left| E_2 \right| + C_4 \left| E_1 \right| \left| E_2 \right| + C_4 g_2 \left| E_1 \right| \left| E_2 \right| + C_4 \left| E_1 \right| \left| E_2 \right| + C_4 g_1 \left| E_2 \right| \left| E_2 \right| \]

\[ F_{\sigma_1} G_1 \times F_{\sigma_1} G_2(x_1, y_1) = (C_5 - C_2) g_2 + C_5 g_1 \left| E_2 \right| + C_5 g_2 \left| E_1 \right| + C_5 \left| E_1 \right| \left| E_2 \right| + C_5 g_1 \left| E_2 \right| + C_5 \left| E_1 \right| \left| E_2 \right| + C_5 g_2 \left| E_1 \right| \left| E_2 \right| + C_5 \left| E_1 \right| \left| E_2 \right| + C_5 g_1 \left| E_2 \right| \left| E_2 \right| \]

For all vertices, this is accurate $|E_1| \times |E_2|$. 

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Case 2 If \( T\sigma_1^{G_2}(x) \leq T\sigma_1^{G_1}(y) , I\sigma_1^{G_2}(x) \leq I\sigma_1^{G_1}(y) \) and \( F\sigma_1^{G_2}(x) \geq F\sigma_1^{G_1}(y) \forall x \in V_1 \) and \( y \in V_2 \).

\[
Td_1^{G_1 \times G_2}(x, y) = \sum_{x = x_1, y \in E_1} \left( (T\sigma_1^{G_1}(x_1) \lor T\sigma_1^{G_2}(y_1)) \land (T\sigma_1^{G_1}(x_2) \lor T\sigma_1^{G_2}(y_2)) \right) - \sum_{y = y_1, x \in E_2} \left( (T\sigma_1^{G_1}(x_1) \lor T\sigma_1^{G_2}(y_1)) \land (T\sigma_1^{G_1}(x_2) \lor T\sigma_1^{G_2}(y_2)) \right) 
+ \sum_{x, y \in E_1} \left( (T\sigma_1^{G_1}(x_1) \lor T\sigma_1^{G_2}(y_1)) \land (T\sigma_1^{G_1}(x_2) \lor T\sigma_1^{G_2}(y_2)) \right) 
+ \sum_{x, y \in E_2} \left( (T\sigma_1^{G_1}(x_1) \lor T\sigma_1^{G_2}(y_1)) \land (T\sigma_1^{G_1}(x_2) \lor T\sigma_1^{G_2}(y_2)) \right) 
+ \sum_{x, y \in E_1} \left( (T\sigma_1^{G_1}(x_1) \lor T\sigma_1^{G_2}(y_1)) \land (T\sigma_1^{G_1}(x_2) \lor T\sigma_1^{G_2}(y_2)) \right) 
+ \sum_{x, y \in E_2} \left( (T\sigma_1^{G_1}(x_1) \lor T\sigma_1^{G_2}(y_1)) \land (T\sigma_1^{G_1}(x_2) \lor T\sigma_1^{G_2}(y_2)) \right)

\]

Where \( E_1 \) & \( E_2 \) are the degrees of the vertices of complement graphs \( G_1^c \) & \( G_2^c \), respectively.

\[
Td_1^{G_1 \times G_2}(x, y) = (C_1 - C_1)g_1 + (C_1 - C_4)g_2 + C_1g_1|E_2| + C_1g_2|E_1| + C_1|E_1||E_2| + C_1g_1g_2.

Similiarly

\[
Id_1^{G_1 \times G_2}(x, y) = (C_1 - C_4)g_2 + C_1g_1|E_2| + C_1g_2|E_1| + C_1|E_1||E_2| + C_1g_1g_2.

Fd_1^{G_1 \times G_2}(x, y) = (C_2 - C_4)g_2 + C_2g_1|E_2| + C_2g_2|E_1| + C_2|E_1||E_2| + C_2g_1g_2.

For all vertices, this is accurate \( V_1 \times m \bar{V}_2 \). As a result, the complement of the modular product of two regular neutrosophic graphs is also regular.

4 Applications

We want to identify the internet streaming service that particular demographic favors based on their usage trends. We will examine the streaming behaviors of our eight users, \( U = \{ u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8 \} \) (Depicted Fig 4) utilizing a series of symptoms or indicators, \( I = \{ \text{Video and picture quality, Content variety, User interface, Price, Device compatibility} \} \). Every individual may have distinct encounters and inclinations, and our goal is to ascertain the fundamental factor that sets apart their choice of streaming service from a selection of well-known platforms, \( P = \{ \text{Netflix, Amazon Prime Video, Hulu, Disney+, HBO Max} \} \) (Depicted Fig 5).

By utilizing the neutrosophic normalized Hamming distance, we can evaluate the resemblance between every user’s inclinations and the accessible streaming platforms. The metric with the minimum distance for each user can subsequently be regarded as the fundamental indication or preference that has the greatest impact.
on their selection of streaming service. For example, suppose user $u_1$ encounters the subsequent indications: excellent video quality, extensive range of content, easy-to-use interface, cost-effective pricing, and support for numerous devices. By contrasting their indications with the characteristics of various streaming services, we can establish that the primary factor for $u_1$ is “Content variety” as it closely corresponds with the offerings of platforms such as Netflix or Amazon Prime Video. Likewise, In (Table 3, Table 4, and Fig 6) we can examine the signs of other users and identify the fundamental sign that most accurately reflects their streaming service inclination. This method enables us to make inferences about the type of service each user favors by considering their symptom resemblances to the accessible choices. For this purpose we need two kinds of observations:

1. The multiple indicators found in each streaming
2. The type of indications found for each stream in a typical given circumcision. Both of these facts are noted in a neutrosophic set, which includes descriptions of the membership, indeterminacy, and non-membership functions $\mu, \sigma, \delta$, among other things.

To find the core attribute by utilizing neutrosophic normalized Hamming distance formula (Table 5) for every indicators of $i^{th}$ stream from $k^{th}$ platform is:

$$LNH(S(P), d_k) = \frac{1}{2} \sum_{j=1}^{n} \max \{ |\mu_j(p_i) - \mu_j(d_k)|, |\mu_j(p_i) - \mu_j(d_k)|, |\mu_j(p_i) - \mu_j(d_k)| \}$$

![Figure 4: Neuturosophic Graph $G_1$](https://doi.org/10.54216/IJNS.230123)

![Figure 5: Neuturosophic Graph $G_2$](https://doi.org/10.54216/IJNS.230123)

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Figure 6: Neutrosophic Graph $G_1 \times_m G_2$

Table 3: User Encounters the Subsequent Indications

<table>
<thead>
<tr>
<th></th>
<th>Video quality</th>
<th>Content variety</th>
<th>User interface</th>
<th>Price</th>
<th>Device compatibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_1$</td>
<td>(0.5, 0.4, 0.4)</td>
<td>(0.4, 0.3, 0.3)</td>
<td>(0.4, 0.5, 0.3)</td>
<td>(0.5, 0.5, 0.5)</td>
<td>(0.4, 0.5, 0.6)</td>
</tr>
<tr>
<td>$u_2$</td>
<td>(0.2, 0.7, 0.3)</td>
<td>(0.5, 0.7, 0.4)</td>
<td>(0.4, 0.6, 0.5)</td>
<td>(0.4, 0.6, 0.5)</td>
<td>(0.4, 0.5, 0.6)</td>
</tr>
<tr>
<td>$u_3$</td>
<td>(0.5, 0.4, 0.2)</td>
<td>(0.7, 0.3, 0.5)</td>
<td>(0.5, 0.6, 0.6)</td>
<td>(0.4, 0.6, 0.5)</td>
<td>(0.3, 0.5, 0.4)</td>
</tr>
<tr>
<td>$u_4$</td>
<td>(0.6, 0.5, 0.3)</td>
<td>(0.5, 0.5, 0.5)</td>
<td>(0.3, 0.5, 0.6)</td>
<td>(0.6, 0.3, 0.5)</td>
<td>(0.5, 0.6, 0.8)</td>
</tr>
<tr>
<td>$u_5$</td>
<td>(0.3, 0.5, 0.7)</td>
<td>(0.3, 0.7, 0.1)</td>
<td>(0.8, 0.5, 0.2)</td>
<td>(0.3, 0.5, 0.8)</td>
<td>(0.4, 0.3, 0.8)</td>
</tr>
<tr>
<td>$u_6$</td>
<td>(0.4, 0.5, 0.6)</td>
<td>(0.7, 0.5, 0.4)</td>
<td>(0.7, 0.4, 0.2)</td>
<td>(0.6, 0.5, 0.8)</td>
<td>(0.7, 0.6, 0.5)</td>
</tr>
<tr>
<td>$u_7$</td>
<td>(0.3, 0.6, 0.3)</td>
<td>(0.5, 0.6, 0.5)</td>
<td>(0.2, 0.8, 0.3)</td>
<td>(0.5, 0.5, 0.6)</td>
<td>(0.7, 0.6, 0.4)</td>
</tr>
<tr>
<td>$u_8$</td>
<td>(0.2, 0.5, 0.6)</td>
<td>(0.3, 0.6, 0.4)</td>
<td>(0.2, 0.5, 0.6)</td>
<td>(0.2, 0.6, 0.5)</td>
<td>(0.6, 0.3, 0.4)</td>
</tr>
</tbody>
</table>

Table 4: Contrasting Indications with the Characteristic of Various Streaming Service

<table>
<thead>
<tr>
<th></th>
<th>Netflix</th>
<th>HBO Max</th>
<th>Hulu</th>
<th>Disney+</th>
<th>Amazon Prime Video</th>
</tr>
</thead>
<tbody>
<tr>
<td>Video quality</td>
<td>(0.7, 0.4, 0.4)</td>
<td>(0.3, 0.6, 0.4)</td>
<td>(0.3, 0.7, 0.5)</td>
<td>(0.2, 0.6, 0.7)</td>
<td>(0.2, 0.7, 0.3)</td>
</tr>
<tr>
<td>Content Variety</td>
<td>(0.5, 0.6, 0.4)</td>
<td>(0.3, 0.7, 0.5)</td>
<td>(0.3, 0.6, 0.5)</td>
<td>(0.3, 0.5, 0.7)</td>
<td>(0.2, 0.7, 0.5)</td>
</tr>
<tr>
<td>User interface</td>
<td>(0.2, 0.7, 0.4)</td>
<td>(0.1, 0.7, 0.5)</td>
<td>(0.4, 0.6, 0.7)</td>
<td>(0.8, 0.4, 0.4)</td>
<td>(0.2, 0.8, 0.2)</td>
</tr>
<tr>
<td>Price</td>
<td>(0.4, 0.4, 0.5)</td>
<td>(0.5, 0.2, 0.6)</td>
<td>(0.4, 0.6, 0.7)</td>
<td>(0.2, 0.6, 0.5)</td>
<td>(0.5, 0.6, 0.5)</td>
</tr>
<tr>
<td>Device compatibility</td>
<td>(0.2, 0.6, 0.5)</td>
<td>(0.2, 0.7, 0.4)</td>
<td>(0.0, 0.7, 0.5)</td>
<td>(0.2, 0.6, 0.5)</td>
<td>(0.6, 0.3, 0.3)</td>
</tr>
</tbody>
</table>

5 Result Analysis

The neutrosophic extended Hausdorff normalized Hamming distance program has been run for each stream, and the results show that streams $u_1$, $u_3$, and $u_6$ have the lowest values in the Netflix column. We draw the conclusion that these streams are most likely connected to Netflix using the least distance approach. We anticipate that streams $u_2$ and $u_8$ will be on Hulu, stream $u_7$ will be connected to Disney+, and streams $u_7$ and $u_8$ are likely to be connected to Amazon Prime Video based on this pattern (Fig 7). Additionally, we believe
that our application will open the door for a lot of additional future researches that will be very beneficial to the general population.

6 Conclusion

Graph theory has proved to be a valuable tool in addressing a wide range of networking problems across various fields, including signal processing, transportation, and error codes. A prominent combinatorial optimization problem in graph theory involves finding the shortest path. In handling ambiguous information that arises in real-world scenarios, the neutrosophic graph model has gained popularity for providing true membership, indeterminacy membership, and false membership. The concept of the maximum product of two neutrosophic graphs’ complement has been introduced in this paper, offering a powerful means to combine different structural models effectively. The regularity in the complement of two neutrosophic graphs has attracted significant attention due to its numerous applications in building reliable communications and network systems. Moreover, neutrosophic graphs play a vital role in decision-making processes, as they can be used to compare and choose between various internet streaming services, enabling users to make informed and relevant choices.

6.1 Future Work

We are going to extend our work

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1. Different type product of complement Neutrosophic graph.
2. Product of Neutrosophic graph and its applications on medical field
3. Product of Neutrosophic graph and its applications on textile industry
4. Neutrosophic coloring graph and their applications.

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**References**


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