Neutrosophic Structure of the Log-logistic Model with Applications to Medical Data

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Abstract

In practical scenarios, it is common to encounter fuzzy data that contains numerous imprecise observations. The uncertainty associated with this type of data often leads to the use of interval statistical measures and the proposal of neutrosophic versions of probability distributions to better handle such data. This study introduces a new generalized design of the log-logistic distribution within a neutrosophic framework, building upon encouraging applications of this distribution in fields such as economics, engineering, survival analysis, and lifetime modeling. The proposed neutrosophic log-logistic distribution (NLLD) is analyzed in terms of statistical properties, including moments, shape coefficients, and various survival characteristics. To evaluate the performance of the predicted neutrosophic parameters, an estimation procedure is conducted. Finally, the practical application of the proposed model is demonstrated using a sample dataset consisting of 128 bladder cancer patients.

Keywords: Neutrosophic probability; uncertain data; estimation; log-logistic model

1. Introduction

Statistical distributions are a powerful tool for describing and predicting real-world events [1]. In a wide range of fields, classical distributions have played a crucial role in representing data over the past few decades [2]. The statistics literature alone contains hundreds of continuous univariate distributions [3]. In economics, the log-logistic distribution (LLD), which is also called the Fisk distribution when studying income inequality, is widely used in many fields, such as economics, actuarial studies, survival analysis, dependability, hydrology, and engineering [4]. The LLD is useful because it can be utilized to develop functions for growing hazard rates. This makes it a good choice to the log-normal distribution in many situations. This benefit is especially clear when working with censored observations, which is a famous type of data in these areas [5]. The LLD is a random variable distribution that doesn't have a negative value, and its logarithm follows the well-known logistic distribution. It was first used to model the growth of a population [6]. One of the best things about the LLD is that it shows the cumulative distribution function in a closed form. Because of this, it is a good choice for statistics analysis with limited data [7]. The LLD is a type of survival time parametric model in which the risk rate goes up at first, then down, and sometimes shows a hump-like pattern. Al-shomrani et al. [8] investigated that the LLD is a more reliable option to the Weibull distributed data.

But the LLD can only be used in certain situations because the risk function must be monotonic regardless of input values. Researchers like Ragab and Green [9] have looked at the LLD's order data and signified its characteristics. Kantam et al. [10] also used the LLD to come up with a way to sample acceptance sampling. Kantam et al. [11] developed a modified maximum likelihood (ML) procedure for this distribution to improve its estimation framework. Overall, the landscape of statistical distributions gives a lot of choices, and the LLD has shown itself to be a useful and flexible distribution

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that can be used in many different ways and has traits that are helpful in analyzing variety of data. As shown in [12], the idea of a Neutrosophic Set (NS) offers a bigger context that includes both fuzzy and classical sets. This extension, proposed by Smarandache is rooted in the notion of neutrosophy [13].

Real-world situations often involve uncertain data, and several researchers have addressed this challenge using the Neutrosophic Set approach [14, 15]. Neutrosophy logic [16] guides the examination of statements that possess degrees of uncertainty, neutrality, inconsistency, or ambiguity [16]-[19]. Recognizing the presence of inaccuracies in the variables under study, Smarandache pioneered the application of neutrosophic techniques in precalculus, calculus, and statistics [20]. Consequently, the exploration of neutrosophic statistics has emerged as a field investigating the impact of inconsistency on statistical modeling.

While the idea of statistical modeling from a neutrosophic standpoint has recently been presented in a few papers [21]-[23], discussions on neutrosophic descriptive statistics and probability measurement can be found in [24]. Notably, applications of neutrosophic decision-making in quality control have demonstrated significant effectiveness [25]. Salama et al. [26] have made notable contributions by introducing neutrosophic algebraic structures for probability models. However, it is worth noting that the research on neutrosophic statistics has predominantly focused on the application side of algebraic structures, with limited attention given to the logic of probability distributions. Consequently, despite the growing popularity of neutrosophic distributions, there remains a dearth of research aimed at comprehending the fundamental characteristics of probability models and their application to various real-world data scenarios.

This work introduces the neutrosophic Log-Logistic model with the primary objective of integrating ambiguous information regarding the study variables. It is essential to consider ambiguities in research parameters for practical analysis and incorporate them into the model used to represent the system. Notably, no previous research has explored the neutrosophic structure of the log-logistic model, which serves as a motivating factor for our continued efforts in this direction.

The remaining sections of this investigation are summarized as follows: Section 2 establishes the neutrosophic generalization of the log-logistic distribution. In Section 3, we explain the mathematical technique employed to identify unknown distributional characteristics. Section 4 details the implementation approach of the NLLD in a real-world situation. Finally, in Section 5, we provide a comprehensive summary of the research findings.

2. Proposed Model with Some Useful properties

The shorthand $\mathcal{Y}\sim$NLLD (δ_N, λ_N) denotes aneutrosophiclog-logistic distribution with a neutrosophicpositive scale$\delta_N = [\delta_l, \delta_u]$ and positive shape $\lambda_N = [\lambda_l, \lambda_u]$ parameters for the random variable $\mathcal{Y}$.The neutrosophic cumulative distribution function ($\widetilde{\text{CDF}}_N$) of a real random value$\mathcal{Y}$, or simply $\mathcal{Y}$ distribution function. The neutrosophic cumulative function $\widetilde{\text{CDF}}_N$ is shown in Figure 1 as the general statistical pattern of the NLLD. This $\widetilde{\text{CDF}}_N$ is used to calculate the likelihood that how many operational units will fail in less than or equal to the specific period of time.The LLND of $\widetilde{\text{CDF}}_N$ is given by:

$$\mathcal{Q}_N(\mathcal{Y}) = \frac{\left(\mathcal{Y}/\delta_N\right)^{\lambda_N}}{1+\left(\mathcal{Y}/\delta_N\right)^{\lambda_N}}, \mathcal{Y} > 0 \tag{1}$$

When $\delta_l = \delta_u = \delta$ and $\lambda_l = \lambda_u = \lambda$ NLLD convert to the current log logistic distribution make a note.However, Figure 1 shows the $\widetilde{\text{CDF}}_N$ curve with inaccurate values of δ_N and λ_N.

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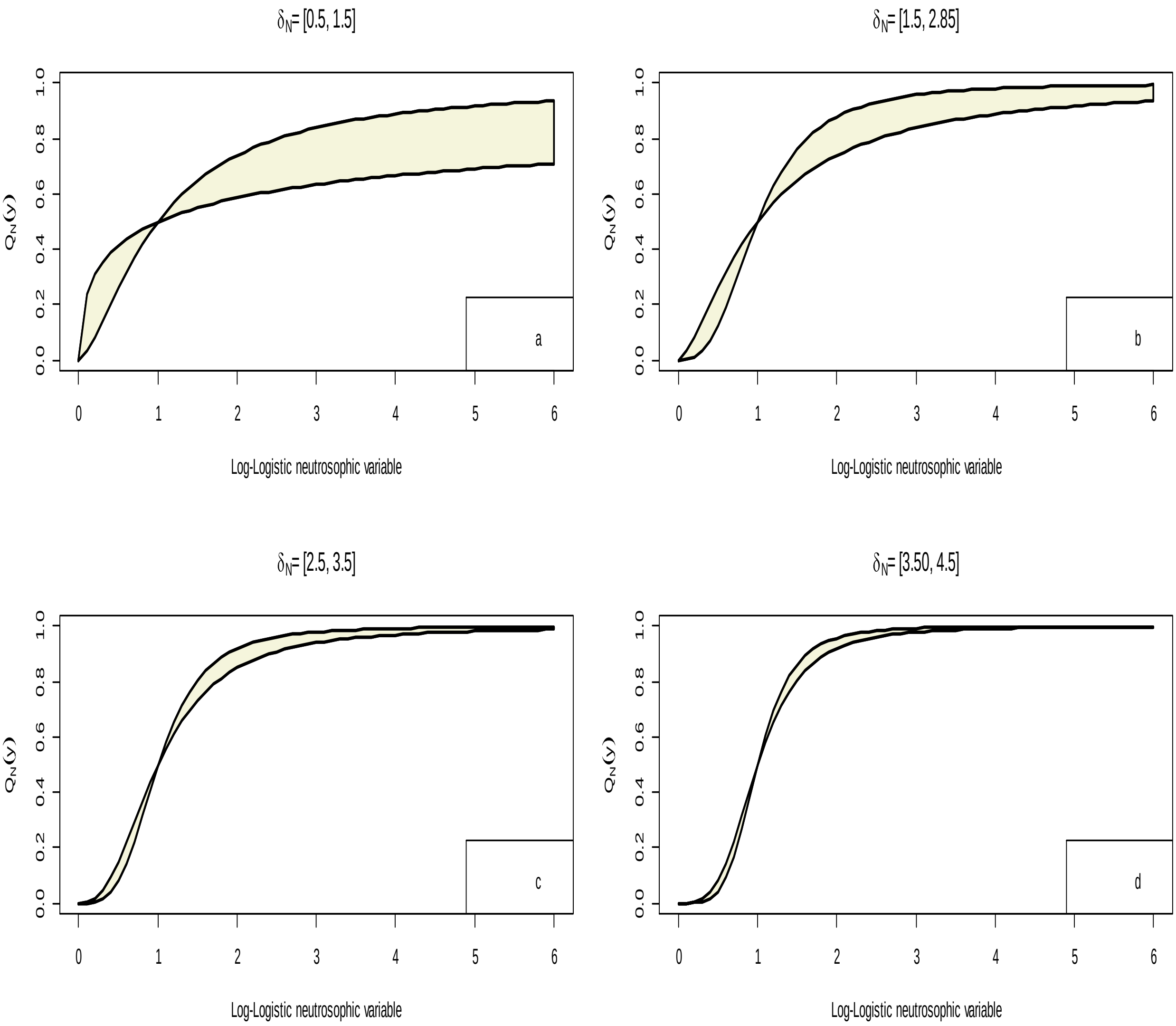

Figure 1: The curve of the $\widetilde{CDF}_N$ for the inaccurate values

Likewise, the other significant function of the NLLD is the neutrosophic density function ($\widetilde{\mathrm{PDF}}_{\mathrm{N}}$) is sketched in Figure 2. Figure 2 illustrates the basic statistical pattern of the $\widetilde{\mathrm{PDF}}_{\mathrm{N}}$ when the scale and shape parameters of the distribution are considered to be indeterminate. The area under the neutrosophic curve represent the interval in which the NLLD will fall. In figure 2 the shaded part illustrate the neutrosophic region due to uncertainties in the distribution defined parameters. During this period, the total area of the graph equals the probability of neutrosophic log-logistic variable occurring. The following theorems can be used to construct some more useful functions of the NLLD.

The probability density function of a NLL random variable $\mathcal{Y}$ with parameters δ_{N} and λ_{N} is:

$$q_{\mathrm{N}}(\mathcal{Y}) = \frac{\left(\lambda_{\mathrm{N}}/\delta_{\mathrm{N}}\right)\left(\mathcal{Y}/\delta_{\mathrm{N}}\right)^{\lambda_{\mathrm{N}}-1}}{\left(1+\left(\mathcal{Y}/\delta_{\mathrm{N}}\right)^{\lambda_{\mathrm{N}}}\right)^{2}}, \delta_{\mathrm{N}} > 0, \lambda_{\mathrm{N}} > 0, and\ \mathcal{Y} \geq 0. \quad (2)$$

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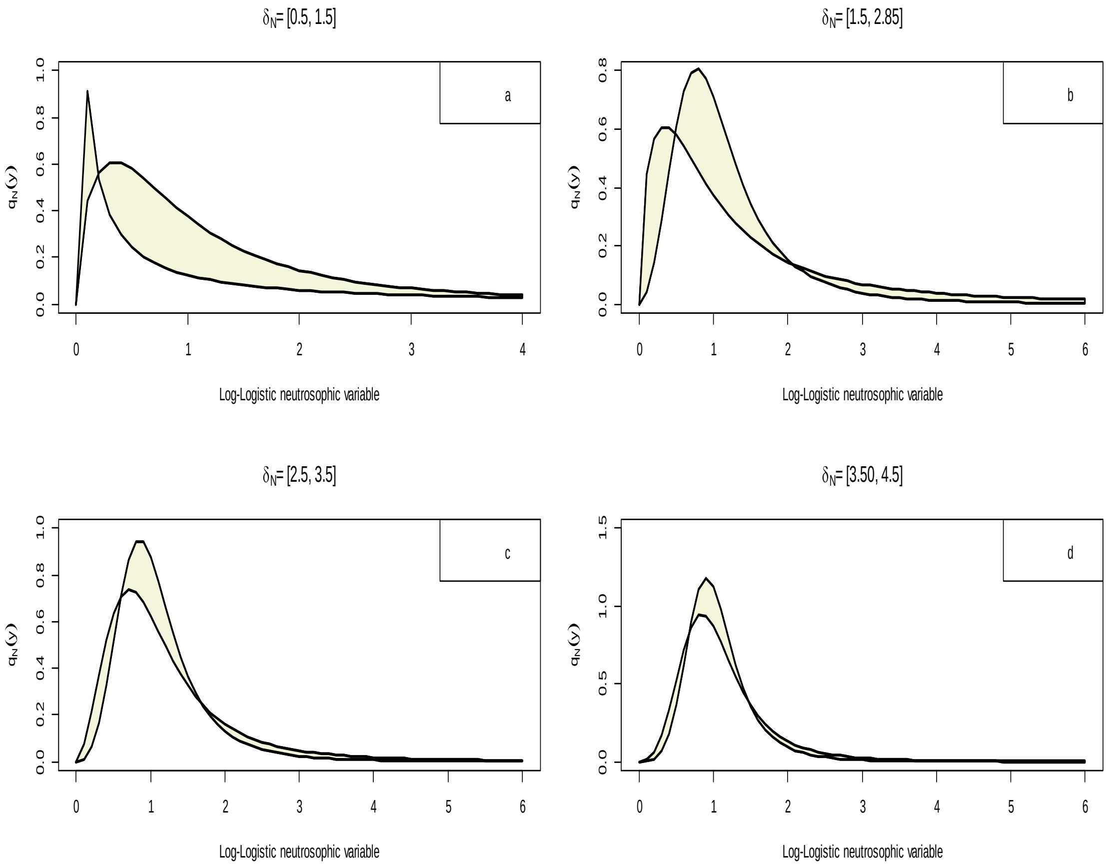

Figure 2: NLLD density function with neutrosophic parameters

The survival function, also known as the reliability function, is denoted by $\mathcal{S}_N(\mathcal{Y})$. It is the probability that the time of failure (random event) $\mathcal{Y}$ occurs after. The NLL model takes form the:

$$\mathcal{S}_N(\mathcal{Y}) = 1 - g_N(\mathcal{Y}) = \frac{1}{1+\left(\mathcal{Y}/\delta_N\right)^{\lambda_N}} \tag{3}$$

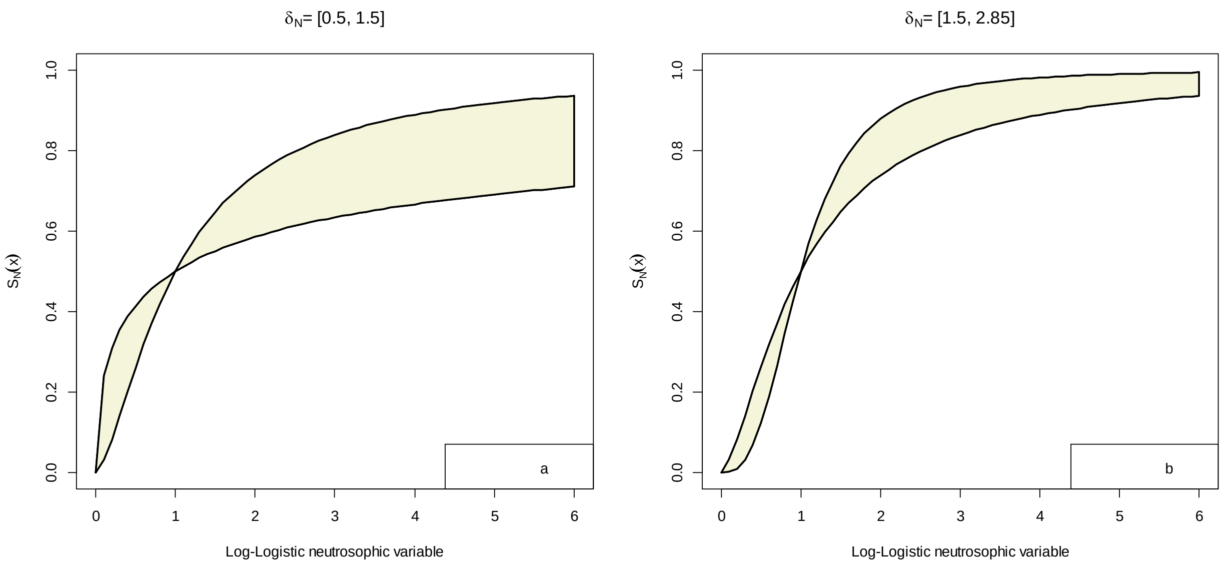

Figure 3: Survival curves for the proposed model

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The hazard rate is a component of hazard function (HF), which is a larger equation.The hazard function may be described as the probability of an event of interest occurring in a short period of time. The mathematical definition of the HF of simply:

$$\mathcal{H}_N(\mathcal{Y}) = \frac{\mathcal{F}_N(\mathcal{Y})}{\mathcal{S}_N(\mathcal{Y})} = \frac{\left(\lambda_N/\delta_N\right)\left(\mathcal{Y}/\delta_N\right)^{\lambda_N - 1}}{1+\left(\mathcal{Y}/\delta_N\right)^{\lambda_N}} \tag{4}$$

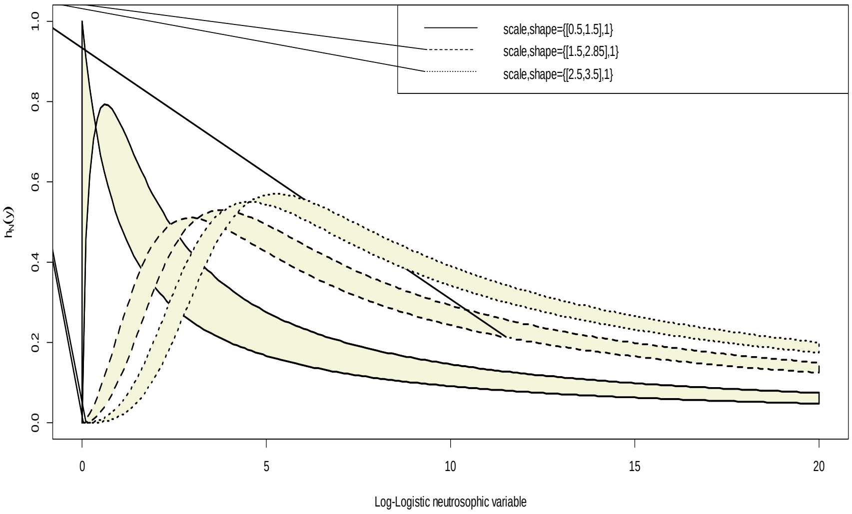

Figure 4: Hazard curve for the proposed model

Definition 1: If $\mathcal{Y}$ follows the NLL then $E(\mathcal{Y}) = \delta_N.\Gamma\left(\frac{\lambda_N+1}{\lambda_N}, \frac{\lambda_N-1}{\lambda_N}\right)$

Proof: By definition the mean of the NLLD is:

$$E(\mathcal{Y}) = \int_{-\infty}^{\infty} \frac{\left(\lambda_N/\delta_N\right)\left(\mathcal{Y}/\delta_N\right)^{\lambda_N - 1}}{\left(1+\left(\mathcal{Y}/\delta_N\right)^{\lambda_N}\right)^2} d\mathcal{Y} \tag{5}$$

By substituting $\mathcal{X} = \left(\mathcal{Y}/\delta_N\right)^{\lambda_N}$ in (5) yielded and applying gamma function:

$$\left(\lambda_l/\delta_l\right)\int_0^{\infty} \frac{\mathcal{X}^{-1}}{(1+\mathcal{X})^2} d\mathcal{X} = \delta_l.\Gamma\left(\frac{\lambda_l+1}{\lambda_l}, \frac{\lambda_l-1}{\lambda_l}\right)$$

and

$$\left(\lambda_u/\delta_u\right)\int_0^{\infty} \frac{\mathcal{X}^{-1}}{(1+\mathcal{X})^2} d\mathcal{X} = \delta_u.\Gamma\left(\frac{\lambda_u+1}{\lambda_u}, \frac{\lambda_u-1}{\lambda_u}\right)$$

So,

$\left[\delta_l.\Gamma\left(\frac{\lambda_l+1}{\lambda_l}, \frac{\lambda_l-1}{\lambda_l}\right), \delta_u.\Gamma\left(\frac{\lambda_u+1}{\lambda_u}, \frac{\lambda_u-1}{\lambda_u}\right)\right] = \delta_N.\Gamma\left(\frac{\lambda_N+1}{\lambda_N}, \frac{\lambda_N-1}{\lambda_N}\right)$, hence proved.

Definition 2: If $\mathcal{Y}$ follow the NLL, then variance is

$$V'(\mathcal{Y}) = \delta_N^2.\Gamma\left(\frac{\lambda_N+2}{\lambda_N}, \frac{\lambda_N-2}{\lambda_N}\right) - \left(\delta_N.\Gamma\left(\frac{\lambda_N+1}{\lambda_N}, \frac{\lambda_N-1}{\lambda_N}\right)\right)^2$$

Proof: By definition the variance of the NLL is:

$$V'(\mathcal{Y}) = E(\mathcal{Y}^2) - [E(\mathcal{Y})]^2 \tag{6}$$

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$$\mathrm{E}(\mathcal{Y}^2) = \int_{-\infty}^{\infty} \mathcal{Y}^2 \frac{\left(\lambda_N/\delta_N\right)\left(\mathcal{Y}/\delta_N\right)^{\lambda_N - 1}}{\left(1+\left(\mathcal{Y}/\delta_N\right)^{\lambda_N}\right)^2} d\mathcal{Y}$$

$$\left[\int_{-\infty}^{\infty} \mathcal{Y}^2 \frac{\left(\lambda_l/\delta_l\right)\left(\mathcal{Y}/\delta_l\right)^{\lambda_l - 1}}{\left(1+\left(\mathcal{Y}/\delta_l\right)^{\lambda_l}\right)^2} d\mathcal{Y}, \int_{-\infty}^{\infty} \mathcal{Y}^2 \frac{\left(\lambda_u/\delta_u\right)\left(\mathcal{Y}/\delta_u\right)^{\lambda_u - 1}}{\left(1+\left(\mathcal{Y}/\delta_u\right)^{\lambda_u}\right)^2} d\mathcal{Y}\right]$$

Further simplification provides;

$$\left(\lambda_l/\delta_l\right)\int_0^{\infty} \mathcal{X}^2 \frac{\mathcal{X}^{-1}}{(1+\mathcal{X})^2} d\mathcal{X} = \delta_l^2 . \Gamma\left(\frac{\lambda_l+2}{\lambda_l}, \frac{\lambda_l-2}{\lambda_l}\right)$$

And

$$\left(\lambda_u/\delta_u\right)\int_0^{\infty} \mathcal{X}^2 \frac{\mathcal{X}^{-1}}{(1+\mathcal{X})^2} d\mathcal{X} = \delta_u^2 . \Gamma\left(\frac{\lambda_u+2}{\lambda_u}, \frac{\lambda_u-2}{\lambda_u}\right)$$

So,

$$\left[\delta_l^2 . \Gamma\left(\frac{\lambda_l+2}{\lambda_l}, \frac{\lambda_l-2}{\lambda_l}\right), \delta_u^2 . \Gamma\left(\frac{\lambda_u+2}{\lambda_u}, \frac{\lambda_u-2}{\lambda_u}\right)\right] = \delta_N^2 . \Gamma\left(\frac{\lambda_N+2}{\lambda_N}, \frac{\lambda_N-2}{\lambda_N}\right)$$

Equation (6) becomes:

$V'(\mathcal{Y}) = \delta_N^2 . \Gamma\left(\frac{\lambda_N+2}{\lambda_N}, \frac{\lambda_N-2}{\lambda_N}\right) - \left(\delta_N . \Gamma\left(\frac{\lambda_N+1}{\lambda_N}, \frac{\lambda_N-1}{\lambda_N}\right)\right)^2$, hence proved.

Definition 4: Show that j^{th} raw moment of the NLLD is $\delta_N^j . \Gamma\left(\frac{\lambda_N+j}{\lambda_N}, \frac{\lambda_N-j}{\lambda_N}\right)$

Proof: By applying gamma function:

$$\mu'_{jN}(\mathcal{Y}) = \int_{-\infty}^{\infty} e^{j\mathcal{Y}} \mathcal{F}(\mathcal{Y}) d\mathcal{Y}$$

$$= \int_0^{\infty} e^{j\mathcal{Y}} \frac{\left(\lambda_N/\delta_N\right)\left(\mathcal{Y}/\delta_N\right)^{\lambda_N - 1}}{\left(1+\left(\mathcal{Y}/\delta_N\right)^{\lambda_N}\right)^2} d\mathcal{Y}$$

$$= \left[\int_0^{\infty} e^{j\mathcal{Y}} \frac{\left(\lambda_l/\delta_l\right)\left(\mathcal{Y}/\delta_l\right)^{\lambda_l - 1}}{\left(1+\left(\mathcal{Y}/\delta_l\right)^{\lambda_l}\right)^2} d\mathcal{Y}, \int_0^{\infty} e^{j\mathcal{Y}} \frac{\left(\lambda_u/\delta_u\right)\left(\mathcal{Y}/\delta_u\right)^{\lambda_u - 1}}{\left(1+\left(\mathcal{Y}/\delta_u\right)^{\lambda_u}\right)^2} d\mathcal{Y}\right] \quad (7)$$

Further simplification Eq. (7) provides:

$$\int_0^{\infty} e^{j\mathcal{Y}} \frac{\left(\lambda_l/\delta_l\right)\left(\mathcal{Y}/\delta_l\right)^{\lambda_l - 1}}{\left(1+\left(\mathcal{Y}/\delta_l\right)^{\lambda_l}\right)^2} d\mathcal{Y} = \delta_l^j . \Gamma\left(\frac{\lambda_l+j}{\lambda_l}, \frac{\lambda_l-j}{\lambda_l}\right)$$

$$\int_0^{\infty} e^{j\mathcal{Y}} \frac{\left(\lambda_u/\delta_u\right)\left(\mathcal{Y}/\delta_u\right)^{\lambda_u - 1}}{\left(1+\left(\mathcal{Y}/\delta_u\right)^{\lambda_u}\right)^2} d\mathcal{Y} = \delta_u^j . \Gamma\left(\frac{\lambda_u+j}{\lambda_u}, \frac{\lambda_l-j}{\lambda_u}\right)$$

Thus Eq. (7)

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$$\mu'_{jN}(\mathcal{Y}) = \left[\delta_l^j.\Gamma\left(\frac{\lambda_l + j}{\lambda_l}, \frac{\lambda_l - j}{\lambda_l}\right), \delta_u^j.\Gamma\left(\frac{\lambda_u + j}{\lambda_u}, \frac{\lambda_l - j}{\lambda_u}\right)\right]$$

Hence

$\mu'_{jN}(\mathcal{Y}) = \delta_N^j.\Gamma\left(\frac{\lambda_N+j}{\lambda_N}, \frac{\lambda_N-j}{\lambda_N}\right)$ where $j = 1,2, \ldots$, the j^{th} moment about the origin of the NLLD distribution.

$$\mu_{1N} = \mu'_{1N} = \delta_N.\Gamma\left(\frac{\lambda_N + 1}{\lambda_N}, \frac{\lambda_N - 1}{\lambda_N}\right)$$

$$\mu_{2N} = \mu'_{2n} - (\mu'_{1n})^2 = \delta_N^2.\Gamma\left(\frac{\lambda_N + 2}{\lambda_N}, \frac{\lambda_N - 2}{\lambda_N}\right) - \left(\delta_N.\Gamma\left(\frac{\lambda_N + 1}{\lambda_N}, \frac{\lambda_N - 1}{\lambda_N}\right)\right)^2$$

$$\mu_{3N} = \mu'_{3n} - 3\mu'_{2n}\mu'_{1n} + 2(\mu'_{1n})^3 = \delta_N^3.\Gamma\left(\frac{\lambda_N+3}{\lambda_N}, \frac{\lambda_N-3}{\lambda_N}\right) - 3\delta_N^2.\Gamma\left(\frac{\lambda_N+2}{\lambda_N}, \frac{\lambda_N-2}{\lambda_N}\right)\delta_N.\Gamma\left(\frac{\lambda_N+1}{\lambda_N}, \frac{\lambda_N-1}{\lambda_N}\right)$$

$$+2\left(\delta_N.\Gamma\left(\frac{\lambda_N+1}{\lambda_N}, \frac{\lambda_N-1}{\lambda_N}\right)\right)^3$$

$$\begin{aligned}\mu_{4N} = \mu'_{4n} &- 4\mu'_{3n}\mu'_{1n} - 3(\mu'_{2n})^2 + 12\mu'_{2n}(\mu'_{1n})^2 - 6(\mu'_{1n})^4 \\ &= \delta_N^4.\Gamma\left(\frac{\lambda_N + 4}{\lambda_N}, \frac{\lambda_N - 4}{\lambda_N}\right) \\ &- 4\delta_N^3.\Gamma\left(\frac{\lambda_N + 3}{\lambda_N}, \frac{\lambda_N - 3}{\lambda_N}\right)\delta_N.\Gamma\left(\frac{\lambda_N + 1}{\lambda_N}, \frac{\lambda_N - 1}{\lambda_N}\right) \\ &- 3\left(\delta_N^2.\Gamma\left(\frac{\lambda_N + 2}{\lambda_N}, \frac{\lambda_N - 2}{\lambda_N}\right)\right)^2 \\ &+ 12\delta_N^2.\Gamma\left(\frac{\lambda_N + 2}{\lambda_N}, \frac{\lambda_N - 2}{\lambda_N}\right)\left(\delta_N.\Gamma\left(\frac{\lambda_N + 1}{\lambda_N}, \frac{\lambda_N - 1}{\lambda_N}\right)\right)^2 \\ &- 6\left(\delta_N.\Gamma\left(\frac{\lambda_N + 1}{\lambda_N}, \frac{\lambda_N - 1}{\lambda_N}\right)\right)^4\end{aligned}$$

Definition 5: Show that the coefficient of skewness for the NLLD is

$$\frac{\left[\Gamma\left(\frac{\lambda_N+3}{\lambda_N}, \frac{\lambda_N-3}{\lambda_N}\right) - 3\Gamma\left(\frac{\lambda_N+2}{\lambda_N}, \frac{\lambda_N-2}{\lambda_N}\right)\Gamma\left(\frac{\lambda_N+1}{\lambda_N}, \frac{\lambda_N-1}{\lambda_N}\right) + 2.\Gamma\left(\frac{\lambda_N+1}{\lambda_N}, \frac{\lambda_N-1}{\lambda_N}\right)^3\right]}{\left(\Gamma\left(\frac{\lambda_N+2}{\lambda_N}, \frac{\lambda_N-2}{\lambda_N}\right).\Gamma\left(\frac{\lambda_N+1}{\lambda_N}, \frac{\lambda_N-1}{\lambda_N}\right)^2\right)^{3/2}}$$

Proof: The coefficient of skewness for NLLD is given by:

$$\eta_{1N} = \frac{\mu_{3N}}{(\mu_{2N})^{3/2}} \tag{8}$$

Where $\mu_{2N} = \delta_N^2.\Gamma\left(\frac{\lambda_N+2}{\lambda_N}, \frac{\lambda_N-2}{\lambda_N}\right)$ and

$\mu_{3N} = \delta_N^3.\Gamma\left(\frac{\lambda_N+3}{\lambda_N}, \frac{\lambda_N-3}{\lambda_N}\right) - 3\delta_N^2.\Gamma\left(\frac{\lambda_N+2}{\lambda_N}, \frac{\lambda_N-2}{\lambda_N}\right)\delta_N.\Gamma\left(\frac{\lambda_N+1}{\lambda_N}, \frac{\lambda_N-1}{\lambda_N}\right) + 2\delta_N.\Gamma\left(\frac{\lambda_N+1}{\lambda_N}, \frac{\lambda_N-1}{\lambda_N}\right)^3$

Substituting (8) provides;

$$\eta_{1N} = \frac{\left[\Gamma\left(\frac{\lambda_N+3}{\lambda_N}, \frac{\lambda_N-3}{\lambda_N}\right) - 3\Gamma\left(\frac{\lambda_N+2}{\lambda_N}, \frac{\lambda_N-2}{\lambda_N}\right)\Gamma\left(\frac{\lambda_N+1}{\lambda_N}, \frac{\lambda_N-1}{\lambda_N}\right) + 2.\Gamma\left(\frac{\lambda_N+1}{\lambda_N}, \frac{\lambda_N-1}{\lambda_N}\right)^3\right]}{\left(\Gamma\left(\frac{\lambda_N+2}{\lambda_N}, \frac{\lambda_N-2}{\lambda_N}\right).\Gamma\left(\frac{\lambda_N+1}{\lambda_N}, \frac{\lambda_N-1}{\lambda_N}\right)^2\right)^{3/2}}$$

where $\eta_{1N} \in [\eta_{1l}, \eta_{1u}]$.

Definition 6: Show that the coefficient of kurtosis for NLLD is

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$$\frac{\left[\Gamma\left(\frac{\lambda_N+4}{\lambda_N},\frac{\lambda_N-4}{\lambda_N}\right)-4\Gamma\left(\frac{\lambda_N+3}{\lambda_N},\frac{\lambda_N-3}{\lambda_N}\right)\Gamma\left(\frac{\lambda_N+1}{\lambda_N},\frac{\lambda_N-1}{\lambda_N}\right)-3\left(\Gamma\left(\frac{\lambda_N+2}{\lambda_N},\frac{\lambda_N-2}{\lambda_N}\right)\right)^2+12\Gamma\left(\frac{\lambda_N+2}{\lambda_N},\frac{\lambda_N-2}{\lambda_N}\right)\left(\delta_N.\Gamma\left(\frac{\lambda_N+1}{\lambda_N},\frac{\lambda_N-1}{\lambda_N}\right)\right)^2-6\left(\Gamma\left(\frac{\lambda_N+1}{\lambda_N},\frac{\lambda_N-1}{\lambda_N}\right)\right)^4\right]}{\left[\Gamma\left(\frac{\lambda_N+2}{\lambda_N},\frac{\lambda_N-2}{\lambda_N}\right)-\left(\delta_N.\Gamma\left(\frac{\lambda_N+1}{\lambda_N},\frac{\lambda_N-1}{\lambda_N}\right)\right)^2\right]^2}$$

Proof: By definition the coefficient of kurtosis is given by:

$$\eta_{2N}=\frac{\mu_{4N}}{\mu_{2N}{}^2} \tag{9}$$

where

$$\mu_{2N}=\delta_N^2.\Gamma\left(\frac{\lambda_N+2}{\lambda_N},\frac{\lambda_N-2}{\lambda_N}\right)-\left(\delta_N.\Gamma\left(\frac{\lambda_N+1}{\lambda_N},\frac{\lambda_N-1}{\lambda_N}\right)\right)^2 \text{ and}$$

$$\begin{aligned}\mu_{4N}=\ &\delta_N^4.\Gamma\left(\frac{\lambda_N+4}{\lambda_N},\frac{\lambda_N-4}{\lambda_N}\right)-4\delta_N^3.\Gamma\left(\frac{\lambda_N+3}{\lambda_N},\frac{\lambda_N-3}{\lambda_N}\right)\delta_N.\Gamma\left(\frac{\lambda_N+1}{\lambda_N},\frac{\lambda_N-1}{\lambda_N}\right)\\&-3\left(\delta_N^2.\Gamma\left(\frac{\lambda_N+2}{\lambda_N},\frac{\lambda_N-2}{\lambda_N}\right)\right)^2\\&+12\delta_N^2.\Gamma\left(\frac{\lambda_N+2}{\lambda_N},\frac{\lambda_N-2}{\lambda_N}\right)\left(\delta_N.\Gamma\left(\frac{\lambda_N+1}{\lambda_N},\frac{\lambda_N-1}{\lambda_N}\right)\right)^2\\&-6\left(\delta_N.\Gamma\left(\frac{\lambda_N+1}{\lambda_N},\frac{\lambda_N-1}{\lambda_N}\right)\right)^4\end{aligned}$$

Substituting in (9) provides;

$$\eta_{2N}=\frac{\left[\begin{array}{c}\Gamma\left(\frac{\lambda_N+4}{\lambda_N},\frac{\lambda_N-4}{\lambda_N}\right)-4\Gamma\left(\frac{\lambda_N+3}{\lambda_N},\frac{\lambda_N-3}{\lambda_N}\right)\Gamma\left(\frac{\lambda_N+1}{\lambda_N},\frac{\lambda_N-1}{\lambda_N}\right)-3\left(\Gamma\left(\frac{\lambda_N+2}{\lambda_N},\frac{\lambda_N-2}{\lambda_N}\right)\right)^2\\+12\Gamma\left(\frac{\lambda_N+2}{\lambda_N},\frac{\lambda_N-2}{\lambda_N}\right)\left(\delta_N.\Gamma\left(\frac{\lambda_N+1}{\lambda_N},\frac{\lambda_N-1}{\lambda_N}\right)\right)^2-6\left(\Gamma\left(\frac{\lambda_N+1}{\lambda_N},\frac{\lambda_N-1}{\lambda_N}\right)\right)^4\end{array}\right]}{\left[\Gamma\left(\frac{\lambda_N+2}{\lambda_N},\frac{\lambda_N-2}{\lambda_N}\right)-\left(\delta_N.\Gamma\left(\frac{\lambda_N+1}{\lambda_N},\frac{\lambda_N-1}{\lambda_N}\right)\right)^2\right]^2}$$

where $\eta_{2N}\in[\eta_{2l},\eta_{2u}]$

The extension of other statistical properties to a neutrosophic framework can be achieved through a similar and coherent approach.

3. Neutrosophic Estimation

The neutrosophic maximum likelihood estimation of the two parameters NLLD, as well as their large sample characteristics, are explained in this section. If $\mathcal{Y}_j=\mathcal{Y}_1,\mathcal{Y}_2,\ldots\mathcal{Y}_n$ is an observed sample size n from NLL model, then $L(\mathcal{Y},\delta_N,\lambda_N)$ is the neutrosophic log likelihood function:

$$T_N(\delta_N,\lambda_N|\mathcal{Y})=\prod_{j=1}^{n}\mathcal{F}_N(\mathcal{Y}_j) \tag{10}$$

The probability function for (10) is as follow:

$$T_N(\delta_N,\lambda_N|\mathcal{Y})=n\log(\delta_N)-n\log(\lambda_N)+(\delta_N-1)\sum_{j=1}^{n}\log(\mathcal{Y}_j)-n(\delta_N-1)\log(\lambda_N)-2\sum_{j=1}^{n}\log\left[1+\left(\frac{\mathcal{Y}_j}{\lambda_N}\right)^{\delta_N}\right] \tag{11}$$

Maximize (11) directly with respect to δ_N and λ_N to obtain the neutrosophic maximum likelihood estimation:

$$\frac{\partial\mathcal{Y}_N}{\partial\delta_N}=\frac{n}{\delta_N}+\sum_{j=1}^{n}\log(\mathcal{Y}_j)-n\log(\lambda_N)-2\sum_{j=1}^{n}\frac{\left(\frac{\mathcal{Y}_j}{\lambda_N}\right)^{\delta_N}\log\left(\frac{\mathcal{Y}_j}{\lambda_N}\right)}{1+\left(\frac{\mathcal{Y}_j}{\lambda_N}\right)^{\delta_N}}=0 \tag{12}$$

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$$\frac{\partial \mathcal{Y}_N}{\partial \lambda_N} = -\frac{n}{\lambda_N} - \frac{n(\delta_N - 1)}{\lambda_N} + \frac{2\delta_N}{\lambda_N}\sum_{j=1}^{n}\frac{\left(\frac{y_j}{\lambda_N}\right)^{\delta_N}}{1+\left(\frac{y_j}{\lambda_N}\right)^{\delta_N}} = 0 \qquad (13)$$

Thus Eq.(12) and Eq. (13) provide the neutrosophic parameters of the proposed model.

4. Real Life Application

The computational approach of the proposed NLLD is demonstrated using an actual dataset of remission durations (in months) from a random sample of 128 bladder cancer patients. Bladder cancer is a common disease characterized by unregulated and aberrant cell proliferation in urinary bladder tissues. Smoking increases the risk of acquiring bladder cancer substantially. The presence of blood in the urine and pain while urination are common symptoms and indicators of bladder cancer. Lee and Wang published the dataset utilized in this investigation [27].The lengths of 128

bladder cancer patients' remissions (measured in months) were gathered from a variety of sources, including epidemiological studies about bladder cancer. The precise source of the data is not made known in a clear manner. The reported results are subject to uncertainty because the precision of remission period measurements is not well-defined. The dataset, as shown in Table 1 includes uncertainty rather than providing exact figures.

Table 1: Real dataset on bladder cancer patients with uncertain observations

Remission times
[0.088,0.618], [1.686,3.004], [3.008,3.665], [4.002,5.152], [6.014,7.035], [7.778,8.87]
[12.436,14.087], [22.68,23.926], [0.316,0.926], [1.653,3.016], [3.184,3.625], [4.633,5.22]
[6.95,7.241], [8.517,9.121], [12.419,13.408], [0.394,1.391], [2.188,3.246], [3.406,3.707]
[4.29,5.965], [6.355,7.666], [8.248,9.615],[13.334,14.25], [25.666,26.447], [0.149,0.583]
[1.701,2.799], [3.503,4.321], [4.693,5.407], [7.035,8.092], [9.412,9.685], [13.844,14.738]
[25.755,26.096], [0.284,0.702], [2.485,3.491], [3.03,4.022], [4.872,5.648], [7.179,7.308]
[9.668,10.287], [13.88,15.404], [25.556,26.906], [0.007,1.132], [1.638,3.511], [3.716,4.446]
[5.221,5.623], [6.521,7.708], [9.275,10.22], [14.761,15.633], [31.371,33.103], [1.911,3.204]
[15.124,16.826], [36.422,37.369], [0.696,1.81], [1.833,2.837], [3.376,4.588], [5.114,6.083]
[7.473,8.144], [10.046,11.1], [16.516,16.861], [42.976,43.068], [0.191,1.427], [2.715,3.64]
[3.25,4.21], [4.839,6.317], [7.233,7.625], [10.332,10.953], [14.378,14.938], [33.768,34.747]
[0.51,0.999], [2.225,2.851], [3.467,4.463], [5.285,5.924], [7.235,8.322], [0.143,0.826]

A method described in [28] was used to introduce these uncertainties. A graphical technique was used to investigate the best model for describing the data from the remission period. The log-logistic distribution's visual fit to the actual dataset of remission periods is shown in Figure 5.

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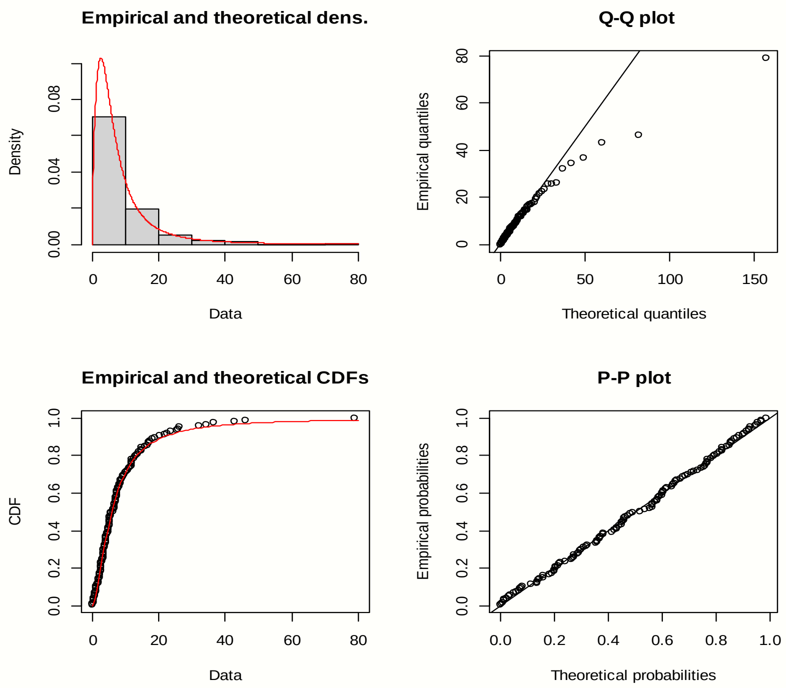

Figure 5: Basic plots of the LLD for the real dataset

The LLD provides a good fit for the remission period data, according to a visual analysis of popular plots like the frequency distribution, probability plot, quantile plot, and cumulative distribution function (CDF). This conclusion is made based on Figure 3, where it can be seen that the majority of genuine data points closely resemble the theoretical red lines, demonstrating a good match of the LLD to the remission times data. The remission data, which incorporates uncertainty and is shown in Table 2, should be noted that it cannot be properly examined using the standard LLD.

Table 2: Descriptive statistics of the proposed neutrosophic model

Neutrosophic Parameters	Estimated Values
Scale parameter	[5.35, 6.55]
Shape parameter	[1.45, 1.92]
Mean	[7.53, 11.13]
Variance	[46.67,168.24]

In Table 2, the fitted measures of mean remission period data are presented in the form of intervals for both the mean and variance, considering the estimated uncertainty in estimated parameters. It is worth noting that mean and variance are neutrosophic values because they are determined analytically and depend on uncertain estimated parameter values. As a result, the neutrosophic proposed model offers more valuable information and enables efficient analysis of data involving uncertainties, providing a comprehensive understanding of the dataset's statistical properties.

5. Conclusions

This paper offers a novel extended design of the log-logistic distribution inside a neutrosophic framework, with the goal of more successfully handling fuzzy data with imprecise observations. The statistical features, moments, form coefficients, and survival characteristics of the suggested NLLD have been carefully examined. An estimation process is used to evaluate the performance of the anticipated neutrosophic parameters, assuring the accuracy and reliability of the model. The potential utility of NLLD in domains such as economics, engineering, survival analysis, and lifetime modeling is demonstrated by its practical implementation on a sample dataset of 128 bladder cancer patients.

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Overall, the NLLD is a promising strategy for dealing with uncertain and imprecise data that might be useful for academics and practitioners working with such data in a variety of disciplines.

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