On Neutrosophic D –Topological Spaces

Amer Khrija Abed[1,*], Ekram Abd Ali[2], Ahmed Salam Razzaq[3], Qays Hatem Imran[4], Said Broumi[5]

[1,2,3,4]Department of Mathematics, College of Education for Pure Science, Al-Muthanna University, Samawah, Iraq
[5]Laboratory of Information Processing, Faculty of Science Ben M'Sik, University Hassan II, B.P 7955, Morocco.
Emails: amer.khrija@mu.edu.iq; ekramalimth@mu.edu.iq; Ahmed.salam@mu.edu.iq; qays.imran@mu.edu.iq; broumisaid78@gmail.com
*Correspondence: amer.khrija@mu.edu.iq

Abstract

In this paper, we redefine the dense neutrosophic set operations and, by using them, we introduce new definition for D − topological space in neutrosophic topological space and also gives the basic specifications for the new definitions of neutrosophic D − topological space in neutrosophic topological space, we also obtained some properties that show the relationship between neutrosophic D −topological space and neutrosophic semi-open sets in the neutrosophic topological space, we also studied image characterization and preimages of neutrosophic D − topological space .

Keywords: Neutrosophic dense set; neutrosophic semi-open; neutrosophic D –topological space; neutrosophic semi-continuous (irresolute) function.

1. Introduction

In 1965, Zadeh [1] introduced " fuzzy set theory " as a mathematical tool for dealing with uncertainties where each element had a degree of membership. Later in 1968, [2] fuzzy topology was studied by Chang. Atanassov In 1983, [3] generalized fuzzy sets as intuitionistic fuzzy sets, which have the degree of membership and degree of non-membership. After, Coker in 1997 [4] defined intuitionistic fuzzy topological spaces using the concept of intuitionistic fuzzy sets. In 1999, Smarandache [5] developed neutrosophic sets with degree of membership, degree of indeterminacy and degree of non-membership. In 2010 [6] as a generalization of the intuitive fuzzy set idea, the Neutrosophic set was introduced by F. Smarandache. In 2012, Salama, Alblowi [7], introduced the concept of Neutrosophic topological spaces. They introduced neutrosophic topological space as a generalization of Intuitionistic fuzzy topological space and a Neutrosophic set besides the degree of membership, the degree of indeterminacy and the degree of non-membership of each element. The concepts of neutrosophic dense sets and some of its properties, introduced by Dhavaseelan, R. Devi and S. Jafari [8] in 2018. On the other hand, the note of generalized homeomorphism with respect to the neutrosophic concept was founded by PAGE et al. [14]. Subsequently, the neutrosophic generalized alpha generalized continuous functions, and neutrosophic generalized semi generalized closed sets were deliberate by Imran et al. [15-16].
In this paper, we used the concept and properties of neutrosophic dense sets mentioned above. We intend to introduce difintion and study some of the basic properties of a topological neutrosophic space we have named neutrosophic D −topological space. In the second part of this paper we attempt to characterize , in genral a setting as we can manage . In the third part we give some properties and characterization of various kinds of functions, in this part we also prove some properties and characterization of neutrosophic D −topological space. In the latter part we wrots conclusions for this paper . End of the prove denoted by ▪ .

Doi: https://doi.org/10.54216/IJNS.220301
Received: April 20, 2023 Revised: July 02, 2023 Accepted: September 28, 2023

2. Preliminaries

This part of the paper is allocated to some definitions and vital observations used in subsequent sections.

Definition 2.1 [5]:
Let Y be a non-void fixed set. A neutrosophic set B [brevity N.S.] is an object having the form $B = < \mu_B(y), \sigma_B(y), \gamma_B(y) >$, where $\mu_B(y), \sigma_B(y)$ and $\gamma_B(y)$ which represent the degree of member ship function, the degree of indeterminacy, and the degree of non-membership, respectively of each element $y \in Y$ to the set B.

Definition 2.2 [7]:
A neutrosophic topology on a non-void neutrosophic set Y is a family τ of neutrosophic subsets of Y which satisfies the following three conditions:

1. $0_N, 1_N \in \tau$.
2. If $M_1, M_2 \in \tau$, their $M_1 \cap M_2 \in \tau$.
3. If $M_i \in \tau$, for each $i \in I$, then $\bigcup_{i \in I} M_i \in \tau$.

The pair (Y, τ) [shortly Y] is called a neutrosophic topological space [simply N.T.S.]. Any member of (Y, τ) named is neutrosophic open set [N.O.S.] and the complement of neutrosophic open set is called a neutrosophic closed set [N.C.S.]. For a N.S. M, the neutrosophic closure, neutrosophic interior and complement of M in N.T.S. (Y, τ) is notation $Ncl(M)$, $Nint(M)$ & $M^C = 1_N - M$, respectively.

Definition 2.3 [9]:
Let Y be a N.T.S. and Y^* be a non-void subset of Y, then a neutrosophic relative topology on Y is defined by $T_Y = \{B \cap Y^* : B \in T\}$, then, (Y^*, T_Y) is called a neutrosophic subspace of (Y, T)[briefly, N.S.T.S.].

Definition 2.4 [10]:
Let B be a N.S. in a N.T.S. Y, then B is said to be a neutrosophic semi-open set [shortly, N_S.O.S.] in Y if there exists a N.O.S. O such that $O \subseteq B \subseteq Ncl(O)$ or $[B \subseteq Ncl((Nint(B))]$. B^C called neutrosophic semi-closed set [shortly, N_S.C.S.] in Y. Obviously, every N.O.S. is N_S.O.S. For example:
If $Y = \{a, b\}$ and then $T = \{0_N, B_1, B_2, B_3, B_4, 1_N\}$ is N.T.S. on Y such that $B_1 = \langle(a, 0.3, 0.5, 0.4), (b, 0.6, 0.2, 0.5)\rangle$, B_2 $\langle(a, 0.2, 0.6, 0.7), (b, 0.5, 0.3, 0.1)\rangle$, $B_3 = \langle(a, 0.3, 0.6, 0.4), (b, 0.6, 0.3, 0.1)\rangle$, $B_4 = \langle(a, 0.2, 0.5, 0.7), (b, 0.5, 0.2, 0.5)\rangle$. Define two N_S.O.S. subset as follows: $M_1 = \langle(a, 0.4, 0.6, 0.4), (b, 0.8, 0.3, 0.4)\rangle$ & $M_2 = \langle(a, 1, 0.9, 0.2), (b, 0.5, 0.7, 0)\rangle$.

Lemma 2.5 [10]:
Any N.S. B in a N.T.S. Y *is a* N_S.O.S. if and only if $A \subseteq Ncl(Nint(B))$.

1. A N.S. B in a N.T.S. Y is a N_S.C.S. if there exists N.C.S. K such that $Nint(K) \subseteq B \subseteq K$.
2. Any N.S. B in a N.T.S. Y is a N_S.C.S. if and only if $Nint(Ncl(B)) \subseteq B$.

Definition 2.6 [10]:
Let B be a N.S. in N.T.S. Y.

1. $N_S int(B)$ notation for neutrosophic semi-interior of B. That is, $N_S int(B) = \cup\{M : M$ is a N_S.O.S. and $M \subseteq B\}$.
2. $N_S cl(B)$ notation for neutrosophic semi-closure of B. That is, $N_S cl(B) = \cap\{K : K$ is a N_S.C.S. and $B \subseteq K\}$.

Lemma 2.7 [10]:
Let Y be a N.T.S., then for any N.S. B_1 and B_2 in Y,

1. B_1 is a N_S.O.S. [N_S.C.S.] in Y if and only if $N_S int(B_1) = B_1$ [$N_S cl(B_1) = B_1$]
2. $N_S int(N_S int(B_1)) = N_S int(B_1)$. [$N_S cl((N_S cl(B_1)) = N_S cl(B_1)$] .
3. If $B_1 \subseteq B_2$, then $N_S int(B_1) \subseteq N_S int(B_2)$ [$N_S cl(B_1) \subseteq N_S cl(B_2)$].
4. $N_S cl(B_1) \subseteq Ncl(B_1)$.

Doi: https://doi.org/10.54216/IJNS.220301
Received: April 20, 2023 Revised: July 02, 2023 Accepted: September 28, 2023

Definition 2.8 [11]:
A N.T.S. Y is called neutrosophic semi-disconnected [neutrosophic disconnected] space if there exist two $N_S.O.S$ $[N.O.S]$ B_1, B_2 in Y & $B_1 \neq 0_N$, $B_2 \neq 0_N$ such that $B_1 \cup B_2 = 1_N$ and $B_1 \cap B_2 = 0_N$. If Y is not a neutrosophic semi-disconnected [neutrosophic disconnected] space, then it is said to be a neutrosophic semi-connected [neutrosophic connected] space. It is clear that every neutrosophic semi-connected is neutrosophic connected. For example: If $Y = \{a, b\}$ with $T = \{0_N, B_1, B_2, B_3, B_4, 1_N\}$, where $B_1 = \langle(a, 0.6, 0.4, 0.2), (b, 0.3, 0.5, 0.4)\rangle$, $B_2 = \langle(a, 0.5, 0.3, 0.4,)\ (b, 0.6, 0.2, 0.5,)\rangle$ $B_3 = \langle(a, 0.4, 0.1, 0.7), (b, 0.8, 0, 0.5)\rangle$ & $B_4 = \langle(a, 0.6, 0.4, 0.2), (b, 0.8, 0.5, 0.4)\rangle$. Wherefore by definition B_1 is a neutrosophic semi-connected set.

Definition 2.9 [11, 12]:
A neutrosophic function $f : Y^* \rightarrow Y^{**}$ is called
1. neutrosophic semi-continuous (semi-irresolute) function [briefly, $N_S.C.F.$ ($N_S.I.F.$)] if the inverse image of every $N.O.S.$ ($N_S.O.S.$) in Y^{**} is a $N_S.O.S.$ ($N_S.O.S.$) in Y^*.
2. neutrosophic semi-open (pre semi-open) function [briefly, $N_S.O.F.$ ($N_{PS}.O.F.$)], if the pre image of every $N.O.S.$ ($N_S.O.S.$) in Y^* is a $N_S.O.S.$ ($N_S.O.S.$) in Y^{**}.

For example:
Let $Y^* = \{a^*, b^*\}$, & $T^* = \{0_N, B^*, 1_N\}$ is N.T.S. on Y such that, $B^* = \langle(a, 0.6, 0.3, 0.2), (b, 0.5, 0.2, 0.3)\rangle$. $Y^{**} = \{a^{**}, b^{**}\}$ & $T^{**} = \{0_N, B^{**}{}_1, B^{**}{}_2, 1_N\}$ is N.T.S. on Y, $B^{**}{}_1 = \langle(a^{**}, 0.5, 0.4, 0.3), (b^{**}, 0.4, 0.2, 0.3)\rangle$, $B^{**}{}_2 = \langle(a^{**}, 0.8, 0.2, 0.1), (b^{**}, 0.7, 0.2, 0.3)\rangle$. Then $f : Y^* \rightarrow Y^{**}$ is a function defined by $f(a^*) = b^{**}$ & $f(b^*) = a^{**}$, f is $N_S.C.F.$.

Definition 2.10 [12]:
A N.S. M of N.T.S. Y is called
1. Neutrosophic regular open set [shortly, $N_R.O.S.$] if $Nint(Ncl(M)) = M$, the complement of neutrosophic regular open set is called a neutrosophic regular closed set [shortly, $N_R.C.S.$]
2. Neutrosophic regular semi-open set [shortly, $N_{RS}.O.S.$] if there exists a $N_R.O.S.$ O such that $O \subseteq M \subseteq Ncl\ (O)$, the complement of neutrosophic regular semi-open set is called a neutrosophic regular semi-closed set [shortly, $N_{RS}.C.S.$].

   For example: Let $Y = \{a, b, c\}$ & $T = \{0_N, 1_N, B_{1*}, B_{2*}, B_{3*}\}$ be N.T.S. on Y, such that $B_{1*} = \langle(a, 0.4, 0.5, 0.6), (b, 0.7, 0.5, 0.3), (c, 0.5, 0.5, 0.5)\rangle$, $B_{2*} = \langle(a, 0.6, 0.5, 0.4), (b, 0.3, 0.5, 0.7), (c, 0.5, 0.5, 0.5)\rangle$, $B_{3*} = \langle(a, 0.6, 0.5, 0.4), (b, 0.7, 0.5, 0.3), (c, 0.5, 0.5, 0.5)\rangle$. Then B_{1*}, B_{2*} are two $N_{RS}.O.S.$

Definition 2.11 [8]:
A N.S. B of N.T.S. Y is called neutrosophic dense set [in short, N.D.S.] if there exists no N.C.S. K in N.T.S. Y such that $B \subseteq K \subseteq 1_N$. That is $Ncl\ (B) = 1_N$.
For example: in example (2.7), B_1 & B_2 are two N.D.S.

Lemma 2.12 [8]:
Let Y be a N.T.S., then a N.S. B is N.D.S. in Y if and only if $M \cap B \neq 0_N$ for any N.O.S. M in Y.

Definition 2.13:
An N.T.S. Y is said to be
1. neutrosophic Hausdorff space [N.H.T.S., for short] if and only if for any two neutrosophic point y_1 and y_2, whereas $y_1 \neq y_2$, there exist two N.O.S. B_1 and B_2 whereas $y_1 \in B_1$, $y_2 \in B_2$ and $B_1 \cap B_2 = 0_N$. **[13]**
2. neutrosophic semi-Hausdorff space [$N_S.H.T.S.$, for short] if and only if for any two neutrosophic point y_1 and y_2, whereas $y_1 \neq y_2$, there exist two $N_S.O.S.$ M_1 and M_2 whereas $y_1 \in M_1$, $y_2 \in M_2$ and $M_1 \cap M_2 = 0_N$.

For example: Let $Y = \{a, b\}$ & $T = \{0_N, 1_N, B_1, B_2\}$ be N.T.S. on Y, wherever $B_1 = \langle(a, 1, 0, 0), (b, 0, 1, 1)\rangle$ & $B_2 = \langle(a, 0, 1, 1), (b, 1, 0, 0)\rangle$. Obvious (Y, T) is an N.T.S. & it is a $N_S.H.T.S.$

Doi: https://doi.org/10.54216/IJNS.220301
Received: April 20, 2023 Revised: July 02, 2023 Accepted: September 28, 2023

3. Neutrosophic D- Spaces

In this section we study a characterization of neutrosophic D-topological space in terms of neutrosophic semi-open and neutrosophic regular semi-open sets as will a functions on neutrosophic D- space.

Definition 3.1:
A N.T.S. Y is called neutrosophic $D-$topological space [N.D.T.S., for short] whenever every non-void N.O.S. is N.D.S. in Y.
For example: Let $Y = \{c\}$ with $T = \{0_N, C_*, C_{**}, 1_N\}$ be N.T.S. on Y, where $C_* = \langle (c, 0.3, 0.5, 0.2) \rangle$, $C_{**} = \langle (c, 0.2, 0.4, 0.5) \rangle$, then Y is N.D.T.S.

Theorem 3.2:
Y is N.D.T.S. if and only if every two non – void two N.O.S. in Y has non – void intersection.
Proof: The proof comes directly from the definition of (2.11) additional to lemma (2.12).▪

Theorem 3.3:
A N.T.S. Y is N.D.T.S. if and only if for each non-void N_S.O.S. B in Y, $N_Scl(B) = 1_N$.
Proof: Necessarily, let B_1 is non-void N_S.O.S. in Y & $y \in 1_N$, let B_2 is N_S.O.S. such that $y \in B_2$, that is to say there exist two non-void N.O.S. O_1 & O_2 in Y such that $O_1 \subseteq B_1 \subseteq Ncl(O_1)$ & $O_2 \subseteq B_2 \subseteq Ncl(O_2)$, from hypothesis $1_N = Ncl(O_1) = Ncl(O_2)$, if $B_1 \cap B_2 = 0_N$, then $O_1 \cap O_2 = 0_N$, by theorem (3.2) Y is not N.D.S., this is contradiction, hence $B_1 \cap B_2 \neq 0_N$, thus $y \in N_Scl(B_1)$ also $1_N \subseteq N_Scl(B_1)$ & $N_Scl(B_1) \subseteq 1_N$, this implies to $N_Scl(B_1) = 1_N$.
Sufficiency, assume that for all non-void N_S.O.S. B_1 in Y & $N_Scl(B_1) = 1_N$, let O_1 be any non-void N.O.S. in Y, this means O_1 is N_S.O.S., by hypothesis $N_Scl(O_1) = 1_N$, by lemma (2.6) $N_Scl(O_1) \subseteq Ncl(O_1)$, it follows immediately that $Ncl(O_1) = 1_N$, thus Y is N.D.T.S. ▪

Theorem 3.4:
Y is N.D.T.S. if and only if for each every two non-void N_S.O.S. has non – void intersection.
Proof: Necessarily, suppose that Y is N.D.T.S., let B_1 & B_2 be two non-void N_S.O.S. in Y, that is to say there exist two non-void N.O.S. O_1 & O_2 in Y such that $O_1 \subseteq B_1 \subseteq Ncl(O_1)$ & $O_2 \subseteq B_2 \subseteq Ncl(O_2)$, by theorem (3.2) $O_1 \cap O_2 \neq 0_N$ this means $B_1 \cap B_2 \neq 0_N$.
Sufficiency, let B_1 & B_2 be two non-void N_S.O.S. in Y & $B_1 \cap B_2 \neq 0_N$ & let $y \in 1_N$ & $y \in B_2$, thus $y \in N_Scl(B_1)$ also $1_N \subseteq N_Scl(B_1)$ & $N_Scl(B_1) \subseteq 1_N$, this implies to $N_Scl(B_1) = Y$, by theorem (3.3) Y is N.D.T.S. ▪

Theorem 3.5:
If Y is N.D.T.S., then Y is neutrosophic semi- connected.
Proof: Suppose that Y is N.D.T.S., if Y is not neutrosophic semi- connected, that is to say there exist two non-void N_S.O.S. M_1 & M_2 in Y such that $M_1 \cup M_2 = 1_N$ & $M_1 \cap M_2 = 0_N$, hence $N_Scl(M_1) \neq 1_N$, by theorem (3.3) Y is not N.D.T.S., which is a contradiction, this means a N.T.S. Y is neutrosophic semi- connected.▪

Corollary 3.6:
If Y is N.D.T.S., then Y is neutrosophic connected.

Theorem 3.7:
A N.T.S. Y is N.D.T.S. if and only if for each non-void N_S.O.S. in Y is neutrosophic semi-connected.
Proof: Necessarily, let M be any non-void N_S.O.S. in Y, if M is not semi-connected, there is two non-void N_S.O.S M_1 & M_2 in Y, such that $M = M_1 \cup M_2$ & $M_1 \cap M_2 = 0_N$, then M_1 & M_2 are two disjoint non-void N_S.O.S. in Y, but by theorem (3.3) Y is not N.D.T.S., this is a contradiction, hence a N.T.S. Y is neutrosophic semi- connected.
Sufficiency, suppose that Y is not N.D.T.S., buy theorem (3.2) there is non-void two N.O.S. O_1 & O_2 in Y such that $O_1 \cap O_2 = 0_N$, let $O_1 \cup O_2 = M$, then M is N_S.O.S. in Y so that O_1 & O_2 is N_S.O.S. in M, M is not neutrosophic semi- connected, this contradiction, show that Y is N.D.T.S. ▪

Doi: https://doi.org/10.54216/IJNS.220301
Received: April 20, 2023 Revised: July 02, 2023 Accepted: September 28, 2023

Corollary 3.8:
If Y is N. D. T. S., then every non-void N. O. S. in Y is neutrosophic connected.

Theorem 3.9:
Every N. S. T. S. Y^* of a N. D. T. S. Y is N. D. T. S.
Proof: Let ${O^*}_1$ & ${O^*}_2$ be any two N. O. S. in Y^*, then ${O^*}_1 = Y^* \cap B_1$ & ${O^*}_2 = Y^* \cap B_2$, such that B_2 & B_2 are N. O. S. in Y and ${O^*}_1 \cap {O^*}_2 = Y^* \cap (B_1 \cap B_2)$, since Y is N. D. T. S., then $B_1 \cap B_2 \neq 0_N$, hence ${O^*}_1 \cap {O^*}_2 \neq 0_N$, by theorem (3.2) Y^* is N. D. T. S. ▪

Theorem 3.10:
Let Y be N. T. S. and Y^* be N. D. S. & N. S. T. S. of Y with is N. D. T. S., then Y is N. D. T. S.
Proof: Let B_1 & B_2 be any non-void two N. O. S. in Y, with Y^* be N. D. S. & N. D. T. S. in Y, then $Y^* \cap B_1$ & $Y^* \cap B_2$ are non-void N. O. S. in Y^*, hence $Y^* \cap (B_1 \cap B_2) \neq 0_N$, this means $B_2 \cap B_2 \neq 0_N$ in Y, by theorem (3.2) we get Y is N. D. T. S. ▪

Theorem 3.11:
If Y^* N. S. T. S. of Y and Y^* is N. D. T. S., then $Ncl(Y^*)$ is also N. D. T. S. of Y.
Proof: It is clear that $Y^*$is N. D. S. of $Ncl(Y^*)$, so by theorem (3.10) we get $Ncl(Y^*)$ is N. D. T. S. ▪

Theorem 3.12:
Let Y be N. T. S., then Y is N. D. T. S. if and only if Y has no proper N_R. O. S.
Proof: Necessarily, let M be any N_R. O. S. in a N. T. S. Y, then $Nint(Ncl(M)) = M$, M is N. O. S. & M N. D. S., this comes to $Nint(1_N) = M$, Accordingly $1_N = M$, hence Y has no proper N_R. O. S.
Sufficiency, suppose that Y is not N. D. T. S., then there exist N. O. S. M is not N. D. S., $Ncl(M) \neq 1_N$ & $Ncl(M)$ is N_R. C. S. in Y this means $(Y - Ncl(M)) \neq 1_N$ is N_R. O. S., this contradiction, therefore Y is N. D. T. S. ▪

Theorem 3.13:
A N. T. S. Y is N. D. T. S. if and only if Y has no proper N_{SR}. O. S.
Proof: Necessarily, if Y has a proper N_{SR}. O. S. M^*, then there exists N_{SR}. O. S. O^*such that $O^* \subseteq M^* \subseteq Ncl(O^*)$, thus mean $O^*$is proper of a N. T. S. Y, by theorem (3.14) Y is not N. D. T. S., this contradiction shows that does not have any proper N_{SR}. O. S.
Sufficiency, suppose that Y is not N. D. T. S., bay theorem (3.14) there is a proper N_{SR}. O. S. M^*, clearly M^* N_{SR}. O. S. in a N. T. S. Y which is a contradiction, thus Y is N. D. T. S. ▪

Lemma 3.14:
Let Y be N. D. T. S. and M_1 and M_2 be two non-void N. O. S. in Y, then $Ncl(M_1 \cap M_2) = Ncl(M_1) \cap Ncl(M_2)$.

Theorem 3.15:
Let Y be N. D. T. S. and M_1 and M_2 be two non-void N_S. O. S. in Y, then $M_1 \cap M_2$ is N_S. O. S.
Proof: Let M_1, M_2 be two N_S. O. S., then $O_1 \subseteq M_1 \subseteq Ncl(O_1)$ & $O_2 \subseteq M_2 \subseteq Ncl(O_2)$, it follows immediately that $O_1 \cap O_2 \subseteq M_1 \cap M_2 \subseteq Ncl(O_1) \cap Ncl(O_2)$, by lemma (3.12) we have $O_1 \cap O_2 \subseteq M_1 \cap M_2 \subseteq Ncl(O_1 \cap O_2)$, thus mean M_1, M_2 are N_S. O. S. ▪

Theorem 3.16:
If $f: Y^* \to Y^{**}$ is a surjective N_S. C. F. and Y^* is N. D. T. S., then Y^{**} is N. D. T. S.
Proof: Let $f: Y^* \to Y^{**}$ be surjective N_S. C. F. where Y^* is a N. D. T. S., suppose that Y^{**} is not N. D. T. S., then there exist two non – void N. O. S. O^* &O^{**} in Y^{**} such that $O^* \cap O^{**} = 0_N$, then $f^{-1}(O^*)$ & $f^{-1}(O^{**})$ are non – void N_S. O. S. in Y^*, so by theorem (3.4) Y^* is not a N. D. T. S., but this is contradiction, hence Y^{**} is a N. D. T. S.▪

Corollary 3.17:
If $f: Y^* \to Y^{**}$ is a surjective N_S. I. F. and Y^* is N. D. T. S., then Y^{**} is N. D. T. S.

Theorem 3.18:
Let $f: Y^* \to Y^{**}$ be one to one and N_S. O. F. and if Y^{**} is N. D. T. S., then Y^* is N. D. T. S.

Doi: https://doi.org/10.54216/IJNS.220301
Received: April 20, 2023 Revised: July 02, 2023 Accepted: September 28, 2023

Proof: Suppose that Y^* is not N.D.T.S., then by theorem (3.2) there exist two non – void N.O.S. O^* &O^{**} in Y^* such that $O^* \cap O^{**} = 0_N$, then $f(O^*)$ & $f(O^{**})$ are non – void N_S.O.S. in Y^{**}, so by theorem (3.4)Y^{**} is not a N.D.T.S., but this is contradiction to our hypothesis, also Y^* is a N.D.T.S. ▪

Theorem 3.19:
Let $f: Y^* \to Y^{**}$ be onto and N_S.C.F., then Y^{**} is N.D.T.S. if and only if Y^* is N.D.T.S.
Proof: Sufficiency, suppose that Y^* is not N.D.T.S. then by theorem (3.2) there exist two non – void N_S.O.S. M^* &M^{**} in Y^* such that $M^* \cap M^{**} = 0_N$, then there exists two non-void N.O.S. O^* &O^{**} in Y^{**} such that $M^* = f^{-1}(O^*)$ & $M^{**} = f^{-1}(O^{**})$, now $f^{-1}(O^* \cap O^{**}) = f^{-1}(O^*) \cap f^{-1}(O^{**}) = M^* \cap M^{**} = 0_N$, since $f: Y^* \to Y^{**}$ onto, so $O^* \cap O^{**} = 0_N$ in Y^{**}, by theorem (3.2) Y^{**} is not N.D.S., but is a contradiction, hence Y^* is N.D.T.S.
Necessarily, similar to proof theorem (3.16). ▪

Theorem 3.20:
Let $f: Y^* \to Y^{**}$ be onto and N_S.I.F., then Y^{**} is N.D.T.S. if and only if Y^* is N.D.T.S.
Proof: Sufficiency, suppose that Y^* is not N.D.T.S. then by theorem (3.2) there exist two non – void N_S.O.S. B^* & B^{**} in Y^* such that $B^* \cap B^{**} = 0_N$, then there exists two non-void N_S.O.S. M^* & M^{**} in Y^{**} such that $B^* = f^{-1}(M^*)$ & $B^{**} = f^{-1}(M^{**})$, now $f^{-1}(M^* \cap M^{**}) = f^{-1}(M^*) \cap f^{-1}(M^{**}) = B^* \cap B^{**} = 0_N$, since $f: Y^* \to Y^{**}$ onto , so $B^* \cap B^{**} = 0_N$ in Y^{**}, by theorem (3.2) Y^{**} is not N.D.T.S., this leads us to a contradiction, hence is proved that Y^* is N.D.T.S.
Necessarily, similar to proof of corollary (3.17).▪

Theorem 3.21 :
If $f: Y^* \to Y^{**}$ is N_S.C.F., Y^* is N.D.T.S. and Y^{**} is N.H.T.S., then f is constant.
Proof: Let $f: Y^* \to Y^{**}$ be N_S.C.F., where Y^* is N.D.T.S. and Y^{**} is N.H.T.S., let e_1 & $e_2 \in Y^*$, such that $f(e_1) \neq f(e_2)$ in Y^{**}, since Y^{**} is N.H.T.S., then there is two N.O.S. ${O_1}^{**}$ & ${O_2}^{**}$ inY^{**}, $f(e_1) \in {O_1}^{**}$ & $f(e_2) \in {O_2}^{**}$ such that ${O_1}^{**} \cap {O_2}^{**} \neq 0_N$. Then $f^{-1}({O_1}^{**})$ & $f^{-1}({O_2}^{**})$ in Y^* are N_S.O.S. , and contains e_1 & e_2 respectively such that $f^{-1}({O_1}^{**}) \cap f^{-1}({O_2}^{**}) = 0_N$, consequently by theorem (3.3) Y^* is not N.D.T.S., but this is contradiction, thus f is a constant function.▪

Theorem 3.22:
If $f: Y^* \to Y^{**}$ is N_S.I.F., Y^* is N.D.T.S. and Y^{**} is N.H.T.S., then f is constant.
Proof: The steps of proof comes comparable to the same steps as the theorem (3.21).▪

4. Conclusions

In this paper, we have presented the idea neutrosophic D −topological space by depending on the definition dense neutrosophic set and learned about its master properties. Then, we used it this idea to find some results that connect this type of neutrosophic topological spaces with neutrosophic semi-open sets and neutrosophic regular semi-open set. Also, we have revealed the relationship between of neutrosophic D −topological space and neutrosophic connected and neutrosophic semi-connected spaces. In addition, we showed the relationship between neutrosophic semi-continuous (semi-irresolute) function and this type of neutrosophic topological spaces. Also our next works will concentrate on studying further neutrosophic topological concepts associated with the dense neutrosophic set .

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Doi: https://doi.org/10.54216/IJNS.220301
Received: April 20, 2023 Revised: July 02, 2023 Accepted: September 28, 2023

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Doi: https://doi.org/10.54216/IJNS.220301
Received: April 20, 2023 Revised: July 02, 2023 Accepted: September 28, 2023