Stationary Factor of Non-Stationary Random Process Based on Differential Transformation

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Abstract

This research studies the deviation of the output signal from stationary state by calculating stationary factor in differential filter with constant and non-stochastic coefficients, a stationary process is applied on its input. We show that this deviation is related to the degree of transformation the study range length and the form of a correlation function of the process applied on the input and the special solution of the equation LY=X and its correlation or un-correlation with that process.

Keywords: Differential operator; stationary process; stable process; linear

1. Introduction:

Stable stochastic processes [1,2,3] (stationary Random processes) are processes of some of their characteristic probabilistic dependencies that do not change depending on the displacement of time or one of the space factors or for a clique or a half clique of linear transformations [1]. this stability recipe makes it play a fundamental role in practical applications of the theory of stochastic processes, therefore the study of the deviation of unstable stochastic processes from the stability mode is of great importance.

He explained the correlation of the concepts of stability and the concept of self-coupling of a finite linear effect on Hilbert space H through a theorem expressing the necessary and sufficient condition for the process X $X(t) = e^{itA}x_0$ to be semi-stable is to be dim $G_A = \rho < \infty$ where $G_A = (2I_m A)H$, which is the subspace of H where A is not a Hermitian on it but a Hermitian on $H \ominus G_A$ [1,4]

Definition 1 (second-order operation): [3]

Are those operations $X(t)$ that achieve the relationship $E|X(t)|^2 < \infty$

Definition 2 (stable process): [3]

A stochastic process $X(t)$ is said to be stable in the broad sense if the follower of its expectation has a constant correlation $K(t,s)$ for any two segments of it at two different moments t and s relates to the difference of the two moments, and the follower of variation is constant.

Observation:

By stability in this work we will mean stability in the broad sense, which is sufficient for the study of second-order processes.

Definition 3 (stability factor) [1]:

Let $X(t) = e^{itA}x_0$ be an unstable linear random process and $W(t,s)$ is the derivative of its dependent correlation, the rank of the quadratic formula $\left(2I_m A\, X(t), X(t)\right) = \sum_{k,j}^{n} W(t_k,t_j)\,\overline{\xi_k\,\xi_j}$ is called the stability factor of the random process.

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Definition 4 (differential filter) [5]:

We call the filter $Ly(t) = SX(t)$ where the two random operations $X(t)$, $y(t)$ are two derivable random operations of Order n, m and L, S are polynomials of Order n, m with respect to $\frac{d}{dt}$.

Research methodology:

The research is mainly based on the descriptive method of analysis, statistics, probability and random processes, and also relies on random effects and differential equations and on a reference survey of some relevant books and research.

The importance of research and its goals:

The importance of this research comes from the wide use of stable random processes in the study of many physical, biological, economic, Communications and other phenomena, but many phenomena are described by unstable random processes, so we will face great difficulty in studying the phenomena that describe the behavior of those processes, hence the importance of studying their proximity or distance from stability, which is the goal of this research, by studying the stability factor of a random process generated from a stable random process and discussing several cases under imposed conditions.

Results and discussion:

In this work, we study the stability factor of a random process $Y(t)$ generated by a linear transformation of a stable random process $X(t)$ [6 ,7] according to the equation:

$$L_t \mathrm{Y(t)} = \mathrm{X(t)}; \mathrm{Y(t)}\lfloor \mathrm{t}_0 = \mathrm{y}_0 \qquad (1)$$

Where $L_t = \sum_{k=0}^{n} a_k(t) \frac{d^k}{dt^k}$ assuming that $E\, X(t) = 0$ and

$a_k(t) = const.$

The solution of Equation (1) has the form [7]:

$$Y(t) = \int_{t_0}^{t} G(t-s)X(s)ds + \sum_{k=1}^{n} c_k \varphi_k(t_0) \qquad (2)$$

Where $G(t,s)$ is green's dependent [4,5,7], and $\varphi_k(t_0)$ is a dependent that seeks zero quickly when $\mathrm{t}_0 \to -\infty$

In this work we will deal with the following cases:

Case 1: for $\mathrm{t}_0 \to -\infty$ we find that $Y(t)$ is a stable process and this is expressed by the following theorem:

Theorem 1: the random process $Y(t)$ generated by a stable process $X(t)$ by transformation (1) for $a_k(t) = const$ on the domain $(-\infty, t)$ is a stable random process, that is, its stability factor is equal to zero.

Proof: the derivative $\varphi_k(t_0)$ is proportional to the amount $e^{\omega_k t_0}$ where ω_k is the spectrum of the operation $X(t)$, we find that the derivative $e^{\omega_k t_0}$ ends in zero when $\mathrm{t}_0 \to -\infty$ and $\omega_k > 0$ so Solution (2) has the following form:

$$Y(t) = \int_{-\infty}^{t} G(t-s)X(s)ds \qquad (3)$$

Thus $E\, Y(t) = 0$ and the continued correlation of $Y(t)$ in the Hilbert space at two different moments is given as follows:

$$K_{YY}(t,s) = \big(Y(t), Y(s)\big) = E \int_{-\infty}^{t} G(t-\sigma_1)X(\sigma_1)d\sigma_1 \int_{-\infty}^{s} G(t-\sigma_2)\overline{X(\sigma_2)}d\sigma_2 = \iint_0^{\infty} \mathrm{G(r)}.\overline{\mathrm{G(q)}}\mathrm{K_{XX}}(t-s+q-r)dr.dq = K(t-s)$$

Case 2:

Assuming that $t_0 = 0$ in relation (2), then the solution takes the following form:

$$Y(t) = Y_0(t) + \int_0^t G(t-s)X(s)ds \qquad (4)$$

$$Y_0(t) = \sum_{k=1}^{n} c_k \theta_k(t) \qquad (5)$$

Where $Y_0(t)$ is a special solution of Equation (1) and c_k are constants that have a non-independent random character, $\theta_k(t)$ are non-random nodal dependencies and$\overline{Y}_0(t) = 0.\ E\, Y(t)$ and $E\, \overline{X(t)} Y_0(t) = 0$ and here we distinguish two cases, according to the value of the dependent correlation of $X(t)$:

I- if the link follower has the form [1]:

$$K_{XX}(t-s) = \emptyset(t).\overline{\emptyset(s)} \qquad (6)$$

We find that $Y(t)$ is an unstable random process and its stability factor fulfills the following theorem:

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Theorem 2: let $X(t)$ be a stable random process and the conditions $\overline{Y_0(t)} = 0$. $E\,X(t)$ and $E\,\overline{X(t)}Y_0(t) = 0$ and its continued connection has the form $K_{XX}(t-s) = \emptyset(t).\overline{\emptyset(s)}$[1] then the solution of Equation (1) is the process $Y(t) = Y_0(t) + \int_0^t G(t-s)X(s)ds$ is unstable and its stability factor:

It does not exceed 2($n+1$) $a_{k\,l} = E\,c_k\overline{c_l} = \xi_k.\delta_{k\,l}$ where $\delta_{k\,l}$ is the Kronecker symbol.

Does not exceed $2(n^2-n+1)$ when $a_{k\,l} = \overline{a_{l\,k}}$

Proof:

Proof a):

By calculating the correlation function of the process $Y(t)$ of the relation (4), we find that:

$$K_{YY}(t-s) = \big(Y(t), Y(s)\big) = E\,y(t)\overline{Y(s)} = K_{Y_0Y_0}(t,s) + \int_0^t\int_0^s G(t-r).\overline{G(s-q)}\,K_{XX}(r-q)dr.dq$$

Returning to the relationship (5) is:

$$K_{Y_0Y_0}(t,s) = \sum_{k,l=1}^{n} a_{k\,l}\theta_k(t)\overline{\theta_l(s)} \qquad (7)$$

And we find that the correlation continuation of the second

$\tilde{Y}(t) = \int_0^t G(t-r)X(s)ds$ for cases 1 and 2 is the same and is given by the relation:

We derive the relation (8) according to the differential operator

$B = -\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial s}\right)$ so we find that [1]:

$-W_{\overline{Y}}(t,s) = -\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial s}\right)K_{\overline{Y}\,\overline{Y}}(t,s)$ and using the special green's dependent $G(0) = \overline{G(0)} = 0$ is:

$-W_{\overline{Y}}(t,s) = \int_0^t\int_0^s [G_t'(t-r).\overline{G(s-q)} + G(t-r)\overline{G_s'(s-q)}]\,K_{XX}(r-q)dr.dq$

Using the relation

$K_{XX}(t-s) = \emptyset(t)\overline{\emptyset(s)}$ assuming that $\psi(t) = \int_0^t G(t,r)\phi(r)dr$ we find:

$$K_{\overline{Y}}(t,s) = \psi(t)\overline{\psi(s)}$$

And be:

$$K_{\overline{Y}}(t,s) = -[\psi'(t).\overline{\psi(s)} + \psi(t).\overline{\psi'(s)}$$
$$= \sum_{\alpha,\beta=1}^{2} \varphi_\alpha(t).\Im_{\alpha\beta}.\overline{\varphi_\beta(s)} \qquad (9)$$

Where and $\varphi_2(t) = \psi'(t)$, $\Im = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ $\varphi_1(t) = \psi(t)$ and therefore from definition 3 we find that the rank of the process $\tilde{Y}(t)$ does not exceed 2, that is $\tilde{Y}(t)$ is an unstable process.

To show the stability of the process $Y_0(t)$ in both cases, we will consider the beginning of the first item:

We find that the correlation function has the following form $K_{Y_0}(t,s) = \sum_{k=1}^{n} \xi_{k\,l}\theta_k(t)\overline{\theta_k(s)}$ and we derive this function for the exponent B to be:

$$K_{Y_0}(t,s) = \sum_{\alpha,\beta=1}^{2n} \varphi_\alpha(t).\Im_{\alpha\beta}.\overline{\varphi_\beta(s)} \qquad (10)$$

Where $\varphi_{2k}(t) = \sqrt{\xi_k}\theta_k'(t)$ and $\varphi_{2k-1}(t) = \sqrt{\xi_k}\theta_k(t)$ where k=1,2,....,n and I_αβ are the elements of the Matrix

$$\Im = \begin{bmatrix} 0 & -1 & & & 0 \\ -1 & 0 & \cdots & & \vdots \\ \vdots & & \ddots & 0 & -1 \\ 0 & & \cdots & -1 & 0 \end{bmatrix}$$

It is of measure $2n\times 2n$, that is, $Y_0(t)$ is an unstable process whose rank does not exceed 2n, and therefore from Relations (9) and (10) we find that the rank of $Y(t)$ does not exceed *2(n+1).*

Proof of item B) of theorem 2:

That the derivative of the correlation dependent of the operation $Y_0(t)$ has the following form:

$$W_0(t,s) = \sum_{k=1}^{n} a_{kk}\left[\theta_k'(t)\overline{\theta_k(s)} + \theta_k(t)\overline{\theta_k'(s)}\right] - \sum_{k\neq l=1}^{n} a_{kl}\left[\theta_k'(t)\overline{\theta_l(s)} + \theta_k(t)\overline{\theta_l'(s)}\right] =$$
$$= W_{Y_0}^{(1)}(t,s) + W_{Y_0}^{(2)}(t,s)$$

The first term of this relationship can be written as:

$$W_{Y_0}^{(1)}(t,s) = -\sum_{k=1}^{n} a_{kk}\left[\sum_{\alpha,\beta=1}^{2} \varphi_k^{(\alpha)}(t).\Im_{\alpha\beta}^{(k)}\overline{\varphi_k^{(\beta)}(s)}\right] \qquad (11)$$

Where $\varphi_1^{(k)}(t) = \theta_k(t)$ and $\varphi_2^{(k)}(t) = \theta_k'(t)$ and $\Im = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

From here we find that the rank $Y_0^{(1)}$ is equal to 2.

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The second term $W_{Y_0}^{(2)}(t,s)$ is given by the relation

$$W_{Y_0}^{(2)}(t,s) = -\sum_{k\neq l=1}^{n} a_{kl}\left[\theta_k'(t)\overline{\theta_l(s)} + \theta_k(t)\overline{\theta_l'(s)}\right] =$$

$$= \sum_{k\neq l=1}^{n} a_{kl}\left[\sum_{\alpha,\beta=1}^{4} \varphi_k^{(\alpha)}(t).\Im_{\alpha\beta}^{(k)}\overline{\varphi_l^{(\beta)}(s)}\right] \quad (12)$$

Where the dependencies $\varphi_k^{(1)}(t) = \theta_k(t)$ and $\varphi_k^{(2)}(t) = \theta_k'(t)$ and $\varphi_1^{(3)}(t) = \theta_l(t)$ and $\varphi_1^{(4)}(t) = \theta_l'(t)$ and

Matrix $\Im$ of the form $\Im = \begin{bmatrix} 0 & 0 & 0 & -a_{kl} \\ 0 & 0 & -a_{kl} & 0 \\ 0 & -a_{kl} & 0 & 0 \\ -a_{kl} & 0 & 0 & 0 \end{bmatrix}$

From Relations (10) and (11) we conclude that the rank of the random process $Y_0^{(2)}$ is $2(n^2 - n)$ and therefore the rank of $Y_0(t)$ will not exceed $2(n^2 - n + 1)$

II- To follow the correlation of $X(t)$ the following figure:

$$K_{XX}(t-s) = \sum_{l=1}^{m} \emptyset_l(t).\overline{\emptyset_l(s)} \quad (12)$$

Then the stability factor is given by the following theorem:

Theorem 3:

Let $X(t)$ be a stable process whose correlation continued at two different moments is given by relation (12), then the solution $Y(t)$ of Equation(1) under conditions $E\,X(T).\overline{Y_0(t)} = 0$. And $E\,X(T).\overline{Y_0(t)} = 0$ is an unstable random process and its stability factor does not exceed $2(n+m)$ when $a_{kl} = \delta_{kl}.\xi_k$ and $2(n^2 - n + m)$ when $a_{\mathrm{kl}} = \overline{a_{\mathrm{lk}}}$ where $a_{\mathrm{kl}} = EC_k\overline{C_l}$

Proof:

From relation (6) we find that:

$$K_{\overline{Y}}(t,s) = \int_0^t\int_0^s\sum_{l=1}^{m} \emptyset_l(\sigma_1).\overline{\emptyset_l(\sigma_2)}G(t,\sigma_1)\overline{G(\sigma_2,s)}d\sigma_1 d\sigma_2$$

$$= \sum_{l=1}^{m}(\int_0^t \emptyset_l(\sigma_1)G(t,\sigma_1)d\sigma_1)\,\overline{(\int_0^s \emptyset_l(\sigma_2)G(s,\sigma_2)d\sigma_2} = \sum_{i=1}^{m}\int_0^t \emptyset_l(\sigma_1)G(t,\sigma_1)d\sigma_1$$

Where $\psi_l(t) = \int_0^t \emptyset_l(\sigma_1)G(t,\sigma_1)d\sigma_1)$

Thus, the derivative of the dependent correlation for the effect

$B = -(\frac{\partial}{\partial T} + \frac{\partial}{\partial s})$ has the following form:

$$W_{\overline{Y}}(t,s) = -\sum_{l=1}^{m}[\psi_l'(t).\overline{\psi_l(s)} + \psi_l(t).\overline{\psi_l'(s)} =$$

$$= -\sum_{l=1}^{m}\sum_{\alpha,\beta=1}^{2} \varphi_l^{(\alpha)}(t).\Im_{\alpha\beta}^{(l)}.\overline{\varphi_l^{(\beta)}(s)}$$

Where $\varphi_1^{(\alpha)}(t) = \psi_l(t)$ and $\varphi_2^{(\alpha)}(t) = \psi_l'(t)$ and $\Im^{(1)} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ that is the rank of $\tilde{Y}(t)$ does not exceed 2m and therefore the stability factor of the process $Y(t)$ is not higher than 2(m+n) in the first case and $2(\mathrm{n}^2 - \mathrm{n} + \mathrm{m})$ in the second case.

Example:

Let $Y(t)$ be a random operation that achieves the equation

$$\frac{\mathrm{dY(t)}}{\mathrm{dt}} = \mathrm{X(t)}$$

With the Cauchy condition $Y(t) = Y_0 + \int_0^t X(t)ds$

If the conditions $E\,X(t) = a_0 = const$ and $EY_0.\overline{X(t)} = 0$ and X (t) is a stable operation, then the derivative of the dependent correlation W(t , s) is given by the relation:

$$W_Y(t,s) = -\frac{\partial}{\partial\tau}K_Y(t+\tau, s+\tau)\Big|_{\tau=0} = -\frac{\partial}{\partial\tau}\int_0^{t+\tau}\int_0^{s+\tau} K_{XX}(\sigma_1 - \sigma_2)d\sigma_1 d\sigma_1 \mathrm{I}_\tau$$

$$= -\int_0^s K_X(t-\sigma_2)d\sigma_2 - \int_0^t K_X(\sigma_1 - s)d\sigma_1$$

We change the integration variable in the integrals $t - \sigma_2 = \vartheta_1$ and $\sigma_2 - s = \vartheta_2$ so that

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$$W_Y(t,s) = \int_0^{t-s} K_{XX}(\vartheta_1)d\vartheta_1$$

$$-\int_{-s}^{t-s} K_{XX}(\vartheta_2)d\vartheta_2 = \int_t^0 K_{XX}(\vartheta_1)d\vartheta_1 - \int_{-s}^0 K_{XX}(\vartheta_2)d\vartheta_2 = \emptyset(t).\emptyset(-s)$$

Then this relationship can be written as follows:

$$W_Y(t,s) = \sum_{\alpha,\beta=1}^{4} \varphi_\alpha(t).\mathfrak{I}^{(l)}_{\alpha,\beta}.\overline{\varphi_\beta(s)}$$

Where $\varphi_4(t) = 1$, $\varphi_3 = -\emptyset(t), \varphi_2 = -\emptyset(t), \varphi_1(t) = 1$
And The Matrix

$$\mathfrak{I} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

That is, the rank of $Y(t)$ does not exceed the 4th.
As for if the conditions $E\,Y_0 \neq 0$, E X(t) $\neq 0$ and $E\,Y_0\overline{X(t)} \neq 0$ are met
We find that the correlation dependent is given by the following relation:

$$K_Y(t+\tau, s+\tau)E|Y_0|^2 + \int_0^s E\left(Y_0\overline{X}(\sigma_2)\right)d\sigma_2$$

$$+\int_0^t E\left(X(\sigma_1).\overline{Y_0}\right)d\sigma_1 + \int_0^t\int_0^s K_X(\sigma_1-\sigma_1)d\sigma_1 d\sigma_2$$

From it we find that

$$W_Y(t,s) = Z_1(t) - Z_2(t) = \sum_{\alpha,\beta=1}^{4} \varphi_1(t)\mathfrak{I}_{\alpha\,\beta}\overline{\varphi_\beta}(s)$$

Where

$$\varphi_3(t) = Z_2(t), \varphi_1(t) = Z_1(t), \varphi_2(t) = \varphi_4(t) = 1$$

$$Z_1(t) = E\,X(t).\overline{Y_0} + \int_t^0 K_X(u)\,du$$

$$Z_2(t) = E\,Y_0.\overline{X(t)} + \int_{-t}^0 K_X(u)\,du$$

And we have

$$\mathfrak{I} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

2. Conclusions and recommendations:

1. The stability criterion of a random critical signal concerns the correlation sequence between the random input signal and the output of a linear differential filter.
2. The calculation of the stability coefficient is easy for inertial random processes located at the entrance to the system.
3. We recommend calculating the stability factor for other rows of stochastic unstable, inertial processes.

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