A Review of Neutrosophic Linear Programming Problems Under Uncertain Environments

Shubham Kumar Tripathi, and Ranjan Kumar *
School of Advanced Sciences, VIT-AP University, Inavolu, Beside AP Secretariat, Amaravati AP, India
Emails: shubhamt.vit22@gmail.com, and ranjank.nit52@gmail.com *

Abstract

This review article focuses on the integration of Neutrosophic Set Theory and Extended Fuzzy Set Theory in the context of Linear Programming (LP) problems. Neutrosophic set theory deals with uncertain, imprecise, and indeterminate information, while Extended Fuzzy Set Theory extends the classical fuzzy set theory to handle more complex and nuanced membership degrees. The combination of these two frameworks provides a powerful toolset for modeling and solving LP problems in environments where uncertainty and ambiguity are prevalent. This review aims to analyze and summarize the existing literature on Neutrosophic Linear Programming problems in extended fuzzy environments, exploring the theoretical foundations and practical applications. The review article seeks to contribute to the understanding of these integrated approaches and their potential for addressing decision-making problems under complex and uncertain conditions.

Keywords: Operational research; Linear Programming problems; Uncertainty principle; fuzzy LP problems; Membership function; Extended fuzzy; NLPP.

1 Introduction

Operational research, which emerged during World War II between 1939 and 1944, was pioneered by Watt and Rowe in response to the pressing need for effective resource management. This discipline employs a diverse array of mathematical and analytical techniques, such as optimization, mathematical modeling, statistical analysis, simulation, and computer programming. Its application spans numerous domains, including manufacturing, logistics, supply chain management, transportation, finance, healthcare, and project management. Notable applications of operational research encompass production planning, inventory management, scheduling, facility location, network optimization, resource allocation, risk analysis, and decision support systems. Through the utilization of mathematical modeling and optimization methods, operational research aims to enhance organizational decision-making processes by boosting efficiency, cutting costs, increasing productivity, and addressing resource scarcity. As A.H. Taha[1] stated, operations research represents a scientific approach to decision-making, particularly in scenarios that necessitate the optimal design and operation of systems grappling with limited resources. In general, Operations Research holds significant importance in decision-making and problem-solving across diverse industries and sectors. It serves as a vital tool for optimizing processes and addressing various decision-making scenarios. One prominent optimization technique within Operational Research is Linear Programming (LP), which proves highly effective in tackling complex problems. The following paragraph provides a detailed discussion of LP.

Linear programming (LP) is a mathematical optimization technique used to solve problems that involve max or min a linear objective function subject to a set of linear constraints. The goal of linear programming is to

https://doi.org/10.54216/IJNS.210410
Received: January 26, 2023 Revised: May 27, 2023 Accepted: July 18, 2023

find the values of the decision variables that satisfy all the constraints while optimizing the objective function. According to V.K. Kapoor,[2] "Linear Programming is the process of optimizing a linear function subject to linear constraints." Linear programming has a wide range of applications, including production planning, transportation and distribution problems, resource allocation, portfolio optimization, scheduling, and many others. It provides a powerful tool for decision-making, allowing businesses and organizations to make efficient use of their resources and achieve their objectives. There are several types of linear programming models, including the standard, slack, canonical, surplus, dual, transportation, assignment, network flow, integer linear programming models, and so on. In the field of optimization, Yaghini et al.,[3] employed a simulated annealing algorithm within a simplex framework to tackle the complex train formation problem. On a different note, Li et al.,[4] conducted a study focusing on feature-learning techniques specifically designed for inputs with graph structures. Shifting gears, Liu et al.[5] delved into a problem involving a single machine that needs to handle multiple tasks while adhering to a common due date, with a particular emphasis on product delivery processes within the logistics domain. This problem necessitates efficient joint multitasking scheduling and due date assignment strategies. In the realm of resource allocation, Jhawar et al.[6] set out to evaluate the effectiveness of Game Theory through a two-by-two symmetric game model, providing insights into water resource allocation. Lastly, Bouman et al.[7] introduced novel exact solution methods based on dynamic programming for solving the TSP with Drones, complemented by an empirical analysis that compares these approaches. Overall, Various researchers have addressed different challenges, such as the complex train formation problem, feature learning with graph structures, multitasking scheduling with due date constraints, water resource allocation using Game Theory, and exact solutions for the TSP with Drones.

Upon reviewing the introduction, it has become apparent that there are significant gaps in the study of classical LP and FLPP. Consequently, we are compelled to investigate the application of the Fuzzy extension principle to LPP and provide an updated analysis of methodologies and applications. We aim to address these gaps and highlight the lack of research pertaining to FLP-related issues in real-world applications. This scarcity of research motivates us to present recent trends and advancements in FLP. Based on the aforementioned review, our study objective is to provide insights into evolutionary methods commonly associated with Fuzzy linear programming problems. This study emphasizes fundamental aspects of linear programming, including the assignment problem, game theory, and dynamic programming. By doing so, we aim to help researchers and students develop a comprehensive understanding of both classical and EFLPP through a single review paper. The summary of the EFLPP model will assist researchers in discovering and promoting advancements and developments in the field of Linear Programming Problems (LPP). Our primary goal is to offer valuable insights into evolutionary methods commonly associated with Linear programming problems, with a specific focus on essential aspects of management such as Linear programming and decision-making in fuzzy extension scenarios. Through this comprehensive review paper, we strive to support researchers and students in developing a profound understanding of Extended Fuzzy Linear Programming Problems (EFLPP), including Neutrosophic, Pythagorean, Intuitionistic, and Hesitant fuzzy models, ultimately contributing to advancements in the field of EFLPP.

The paper is organized as, Section 2 Discussion difficulties in the classical LP Model: A reflective dialogue, while Section 3 Fuzzy Theory: Key Concepts and Definitions and Section 4 Some important definitions, and introduction related to Fuzzy Extended Theory. In Section 5, we present an updated review of methodologies and applications of the LPP under the Neutrosophic Principle., highlighting the latest trends and advancements. Finally, we summarize our conclusions regarding the extension of fuzzy in LPP.

1.1 List of Abbreviations used throughout this paper

FLP stands for “Fuzzy Linear Programming”
FLPP stands for “Fuzzy Linear Programming Problem”
TFNs stands for “Triangular fuzzy numbers”
WTrFn stands for “Weighted Triangular Fuzzy Numbers”
TSP stands for “Traveling Salesman Problem”
TIFNs stands for “Triangular Intuitionistic fuzzy number”
IFS stands for “Intuitionistic fuzzy Set”
IFNs stands for “Intuitionistic fuzzy numbers”
EFLPP stands for “Extended fuzzy linear programming problem”

https://doi.org/10.54216/IJNS.210410

Received: January 26, 2023 Revised: May 27, 2023 Accepted: July 18, 2023

APP stands for "Aggregate Production Planning"
SVNNs stands for "Single-Valued Neutrosophic Numbers"
SVTNNs stands for "Single-Valued Triangular Neutrosophic Numbers"
WTpFn stands for "Weighted Trapezoidal Fuzzy Number"
TrIFN stands for "Trapezoidal Intuitionistic Fuzzy Number"
NLFP stands for "Neutrosophic Linear Fractional Programming"
TFNN-GRA stands for "Triangular Fuzzy Neutrosophic Numbers Grey Relational Analysis"

2 Discussion Difficulties in the Classical LP Model: A Reflective Dialogue

The classical linear programming problem assumes that all input parameters are known with certainty. However, in many real-world situations, the input parameters are uncertain or subject to change over time, which can make it difficult to use the classical linear programming approach. Fuzzy linear programming (FLP) is a generalization of classical linear programming that uses fuzzy logic to deal with uncertainties and vagueness in input data. Some advantages of fuzzy linear programming over classical linear programming such as Flexibility, Robustness, Better decision-making, and Inclusiveness. Fuzzy linear programming can incorporate imprecise or vague data into the optimization problem, allowing for a more inclusive approach that considers a broader range of possibilities.

In certain decision-making scenarios involving multiple factors, some of which may be challenging to quantify, fuzzy logic offers a more adaptable and nuanced approach compared to an LP model. This is due to its ability to consider a broader range of factors and incorporate subjective or qualitative information. Consequently, fuzzy logic has proven to be superior to Linear Programming models. It was initially introduced by Lotfi A. Zadeh[8] in 1965.

3 Fuzzy Theory: Key Concepts and Definitions

Definition 3.1. [8] Fuzzy Set : Zadeh introduced the fuzzy set idea in 1965. The framework provides the system boundaries that are ill-defined or vaguely described, or even incomplete information. As per the Zedah's definition: Fuzzy sets (FS) are defined as $H = \{\langle h, \mu_h(h)\rangle : h \in H\}$, where $\mu_h(h)$: A $\rightarrow [0, 1]$ is a membership function of h or degree of belonging to the set A, where A is a non-empty set of discourse. Therefore, the fuzzy set $H \subseteq A$ is a collection of objects with a degree of membership. As we know that there are few types of fuzzy membership function exists such as Triangular, Weighted triangular fuzzy and Weighted trapezoidal fuzzy which are defined as below:

Definition 3.2. [9] Triangular fuzzy Numbers : A triangular fuzzy number $\tilde{B}_s = (\Phi_{t_1s}, \Phi_{t_2s}, \Phi_{t_3s})$ by satisfying the following condition:

I. $\mu_{\tilde{B}_s}(H)$, a continuous function and strictly decreasing between the intervals $[\Phi_{t_2s}, \Phi_{t_3s}]$.

II. $\mu_{\tilde{B}_s}(H)$, a continuous function and strictly increasing between the intervals $[\Phi_{t_1s}, \Phi_{t_2s}]$.

III. $\mu_{\tilde{B}_s}(H)$, a continuous function under the interval [0,1].

A TFN can be written as $\tilde{B}_s = (\Phi_{t_1s}, \Phi_{t_2s}, \Phi_{t_3s})$ whose membership function is:

$$\mu_{\tilde{B}_{sTFN}}(H) = \left\{ \begin{array}{c} \frac{h-\Phi_{t_1s}}{\Phi_{t_2s}-\Phi_{t_1s}} \text{ if } \Phi_{t_1s} \leq h \leq \Phi_{t_2s} \\ 1 \text{ if h} = \Phi_{t_2s} \\ \frac{\Phi_{t_3s}-h}{\Phi_{t_3s}-\Phi_{t_2s}} \text{ if } \Phi_{t_2s} \leq h \leq \Phi_{t_3s} \\ 0 \text{ Elsewhere} \end{array} \right\}$$

https://doi.org/10.54216/IJNS.210410

Received: January 26, 2023 Revised: May 27, 2023 Accepted: July 18, 2023

Definition 3.3. [10] Weighted Trapezoidal Fuzzy Number : Let a GTpFn is $\tilde{S}_p = (\tilde{s}_1^p, \tilde{s}_2^p, \tilde{s}_3^p, \tilde{s}_4^p : \omega_l^p, \omega_r^p)$ where $\tilde{s}_1^p, \tilde{s}_2^p, \tilde{s}_3^p$and $\tilde{s}_4^p$ real number with $\tilde{s}_1^p \leq \tilde{s}_2^p \leq \tilde{s}_3^p \leq \tilde{s}_4^p$, $\omega_l^p, \omega_r^p$the left height and the right height of $\tilde{S}_p$ and the membership function is define as :

$$\mu_{\tilde{S}_p}(t) = \left\{ \begin{array}{c} 0 \text{ for t}\tilde{s}_1^p or \text{ t}\tilde{s}_4^p \\ \omega_l^p \cdot \frac{t-\tilde{s}_1^p}{\tilde{s}_2^p-\tilde{s}_1^p} \text{ for } \tilde{s}_1^p \leq t \leq \tilde{s}_2^p \\ \omega_l^p + (\omega_l^p - \omega_l^p)\frac{t-\tilde{s}_2^p}{\tilde{s}_3^p-\tilde{s}_2^p} \text{ for } \tilde{s}_2^p \leq t \leq \tilde{s}_3^p \\ \omega_r^p . \frac{t-\tilde{s}_4^p}{\tilde{s}_3^p-\tilde{s}_4^p} \text{ for } \tilde{s}_3^p \leq t \leq \tilde{s}_4^p \end{array} \right\}$$

Where $0 \prec \omega_l^p \leq 1$ and $0 \prec \omega_r^p \leq 1$; If $\omega_l^p = \omega_r^p$ then $\tilde{S}_p$become $\tilde{S}_p = (\tilde{s}_1^p, \tilde{s}_2^p, \tilde{s}_3^p, \tilde{s}_4^p : \omega)$ also known as WTpFn.

Definition 3.4. [10] Weighted Triangular Fuzzy Numbers: (WTrFn) : The WTpFn $\tilde{S}_p = (\tilde{s}_1^p, \tilde{s}_2^p, \tilde{s}_3^p, \tilde{s}_4^p : \omega)$ can be transformed into WTrFn $\tilde{S}^{tr} = \left(\tilde{s}_i^{tr}, \tilde{s}_j^{tr}, \tilde{s}_k^{tr} : \omega\right)$ if and only if $\tilde{s}_3^p = \tilde{s}_4^p$

Definition 3.5. [11] LR flat Triangular Fuzzy Number : Let $\tilde{\psi} = \left(\tilde{\psi}_1, \tilde{\psi}_l, \tilde{\psi}_r,\right)_{LR}$ be LR flat TrFn iff the membership function $\mu_{\tilde{\psi}}(t)$ is defined as:

$$\mu_{\tilde{\psi}}(t) = \left\{ \begin{array}{c} L\left(\frac{\tilde{\psi}_1 - t}{\tilde{\psi}_l}\right) \text{ For t} \leq \tilde{\psi}_1 \\ R\left(\frac{t-\tilde{\psi}_1}{\tilde{\psi}_r}\right) \text{ For t} \geq \tilde{\psi}_1 \end{array} \right\}$$

Where $\tilde{\psi}_1$ is the mean value of $\tilde{\psi}$ and $\tilde{\psi}_l$defines left and $\tilde{\psi}_r$ defines right spreads respc.

Therefore, Fuzzy Logic offers a wider scope of factors and embraces uncertainty and vagueness during the optimization process, making it a more favourable option than LP Models in specific situations. In Section 2, it became evident that classical LP fails to effectively handle uncertainty, resulting in reduced accuracy when identifying optimal solutions. As a response to this limitation, Zimmermann[12] introduced fuzzy LPP, which we will explore in detail in Section 3.1

3.1 Introduction of FLPP

Fuzzy linear programming is a powerful mathematical technique that extends traditional linear programming by incorporating uncertainty and vagueness into the optimization process. The FLP model, initially presented by Zimmermann,[12] offers a technique for addressing Linear Programming (LP) problems incorporating fuzzy linear constraints while monitoring the progression of related studies and advancements. It is a valuable tool for decision-making in situations where precise data or crisp constraints may not be available or applicable. FLPP allows for the modeling and analysis of complex systems that involve imprecise or uncertain parameters, enabling more realistic and flexible solutions to real-world problems. In traditional linear programming, all parameters and constraints are assumed to be precise and deterministic. However, in many practical situations, the data or parameters involved may be subject to ambiguity, imprecision, or vagueness. FLPP recognizes and accommodates this uncertainty by representing variables and constraints as fuzzy sets, which capture the degree of membership or possibility of an element belonging to a set. This allows decision-makers to incorporate subjective judgments, expert opinions, or incomplete information into the optimization process. The objective of FLPP is to find the best possible solution that optimizes a given objective function while satisfying a set of fuzzy constraints. By employing fuzzy logic and fuzzy set theory, this approach enables decision-makers to account for various levels of uncertainty and ambiguity, resulting in more robust and flexible solutions. FLP has been successfully applied to a wide range of domains, including engineering, finance, supply chain management, environmental management, and many others. It has proven to be particularly useful in situations where the decision-making process involves qualitative or subjective factors, incomplete or imprecise data, or conflicting objectives. In this paper, we will examine several real-world applications to demonstrate their practical relevance and effectiveness in dealing with uncertainty and imprecision. By embracing the fuzzy nature of real-world problems, FLPP provides decision-makers with a powerful tool for tackling complex optimization challenges. Table 1 and Figure 1 provide a valuable resource for gaining a comprehensive understanding of FLPP and its significance in various domains concerning uncertainty.

https://doi.org/10.54216/IJNS.210410

Received: January 26, 2023 Revised: May 27, 2023 Accepted: July 18, 2023

Table 1: The impact of FLPP on real-life applications has been explored by various researchers across different domains

Authors	Year	Environment	Application	Contribution
Sigarpich et al.[13]	2011	Degeneracy problem under TFNs Environment	Solve Fuzzy Degenerate Solution	Discuss about degeneracy in FLPP Understanding and Application.
Iris and Cevikcan[14]	2014	Inventory theory under TFNs Environment	Aggregate Production Planning	This article presents a mathematical programming framework designed to tackle the ag APP problem in the presence of imprecise data.
Wan et al.[15]	2014	Decision making under TIFNs Environment	Possibility method	The aim of this article is to propose a possibility method for addressing the problem of multi-attribute group decision-making under the TIFN environment with incomplete weight information.
Meng and Cui[16]	2014	Transportation problem under TFNs Environment	Train Repathing System	The author introduces a method that utilizes triangular fuzzy coefficients to transform train pathing into a deterministic linear model, providing a solution for the FLPP.
Ebrahimnejad, Ali[17]	2015	Transportation problem under TFNs Environment	Solve fuzzy transportation problems	Effective algorithms have been proposed for precise transportation problem, while a two-step method is suggested for solving fuzzy transportation problems (FTP) with non-negative triangular fuzzy number TFNs representations for all parameters.

Within this article, Figure 1 presents a graphical illustration of the essential components and processes involved in FLPP. This visual aid enhances readers' comprehension of the discussed concepts and provides practical insights into FLPP's functioning. Additionally, Figure 1 incorporates Table 1 for easy reference, further enriching the reader's understanding.

https://doi.org/10.54216/IJNS.210410

Received: January 26, 2023 Revised: May 27, 2023 Accepted: July 18, 2023

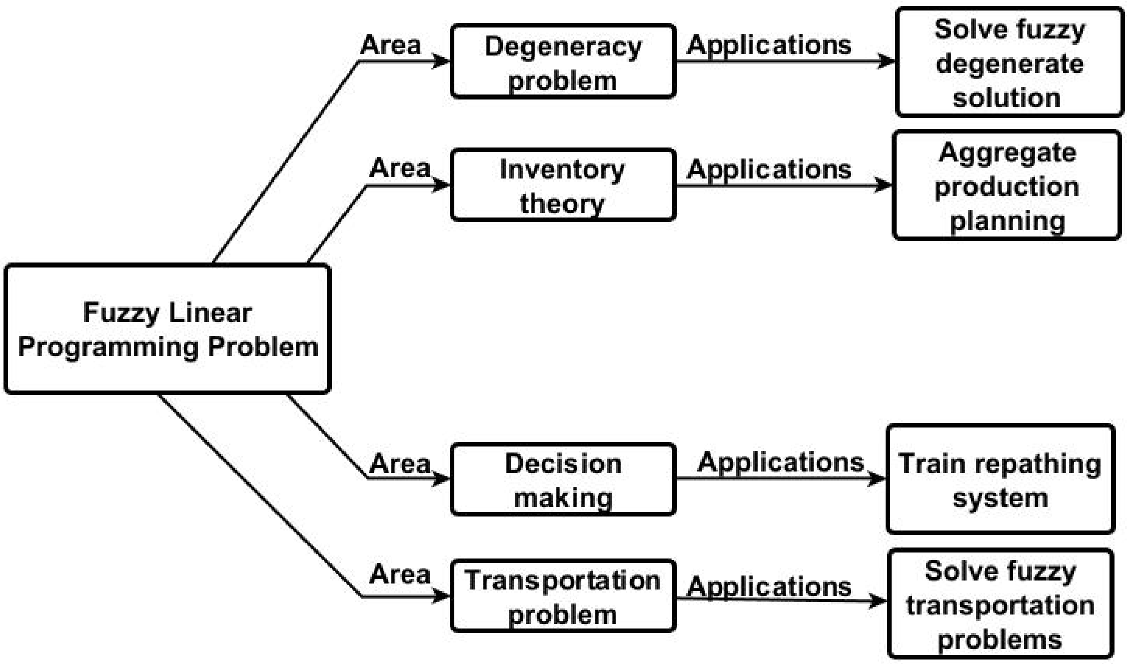

Figure 1: Illustration of the various components and processes in FLPP

In the course of our ongoing investigation, we have extensively examined the unique characteristics of fuzzy logic. Nonetheless, it is crucial to recognize the presence of specific challenges associated with this approach, demanding a thorough discussion in the following section.

3.2 Some Challenges of the Fuzzy Linear Programming (FLP): A Short Discussion

Fuzzy linear programming introduces additional complexity compared to traditional linear programming. The incorporation of fuzzy logic elements, such as fuzzy sets and fuzzy numbers, requires more intricate mathematical formulations and solution techniques. This complexity can make it more challenging to develop and solve fuzzy linear programming problems effectively. Fuzzy linear programming requires the definition of membership functions to represent the degree of membership of elements in fuzzy sets. Selecting appropriate membership functions can be subjective and dependent on expert judgment. This subjectivity introduces a potential source of ambiguity and variability in the model, as different experts may have different interpretations and definitions of membership functions.

Solving fuzzy linear programming problems can be computationally intensive, particularly for complex and large-scale problems. The inclusion of fuzzy logic components often necessitates solving multiple linear programming problems for different fuzzy scenarios or membership function values. This computational burden can increase the time and resources required to obtain optimal or satisfactory solutions. Fuzzy linear programming lacks a standardized approach compared to traditional linear programming. There is no universally accepted method for defining and handling fuzzy sets, membership functions, or fuzzy constraints. As a result, the application and interpretation of fuzzy linear programming can vary across different domains and problem contexts, leading to inconsistencies and difficulties in comparing results or methodologies. Fuzzy linear programming is still a relatively specialized and less commonly implemented technique compared to traditional linear programming. The adoption and acceptance of fuzzy linear programming by decision-makers, practitioners, and software developers may be limited. This limited practical implementation can result from the complexity, lack of standardization, and additional computational requirements associated with fuzzy linear programming. While fuzzy linear programming offers advantages in dealing with uncertainty and imprecision, these demerits highlight the potential challenges and considerations that need to be taken into account when utilizing fuzzy linear programming in decision-making problems. Additionally, upon conducting an extensive literature review, it is clear that relying solely on fuzzy theory is inadequate for effectively addressing

https://doi.org/10.54216/IJNS.210410
Received: January 26, 2023 Revised: May 27, 2023 Accepted: July 18, 2023

uncertainty. As a result, numerous researchers have introduced expanded fuzzy theories, including Intuitionistic, Neutrosophic, Pythagorean and various others. The following paragraph will delve into a more detailed discussion of these extended principles.

4 Some Important Defintions, Introduction Related to Fuzzy Extened Theory

The introductory section of the Neutrosophic Principle offers a comprehensive overview of the core concepts and principles that form the basis of fuzzy logic. Fuzzy logic is a widely employed computational framework designed to tackle the uncertainties and imprecisions inherent in decision-making processes. The extended version of the fuzzy principle surpasses the traditional approach by incorporating additional features and techniques, enhancing its versatility and effectiveness across various domains. This introduction establishes a solid foundation in fuzzy logic, paving the way for comprehending the advancements and contributions of the extended fuzzy principle. Through a blend of theoretical explanations and practical examples, this passage serves as an entry point into the realm of extended fuzzy logic, emphasizing its significance and potential applications in solving intricate real-world problems. Noteworthy developments in the extended theory encompass the following:

a). Intuitionistic

b). Neutrosophic and so on

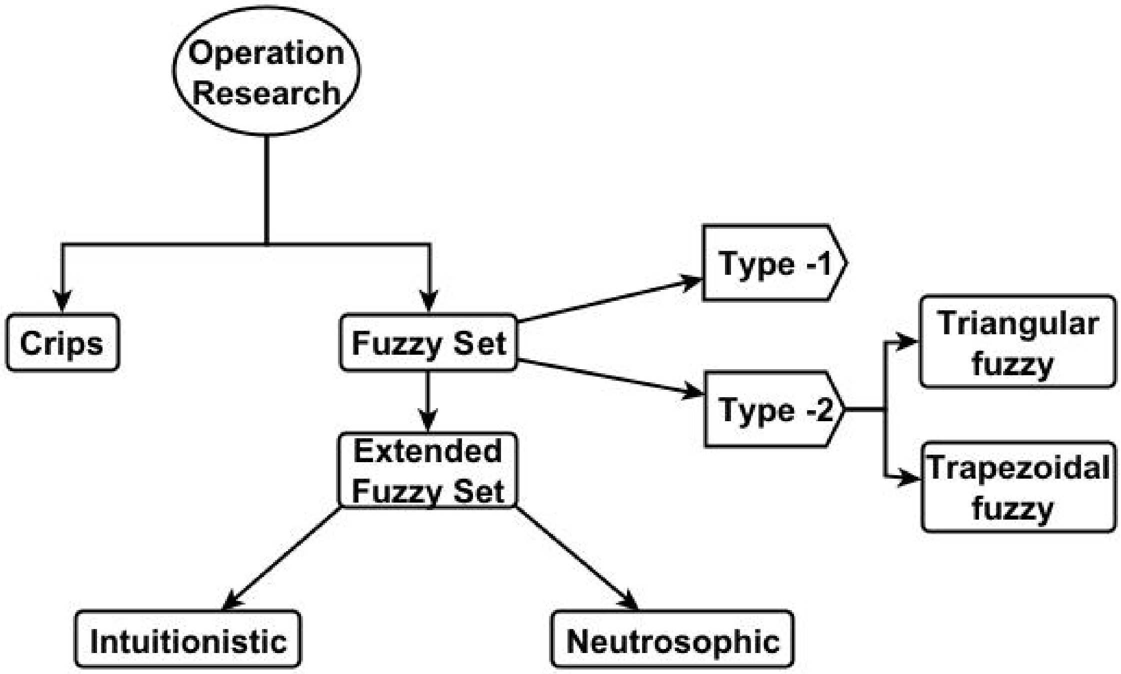

Figure 2: Illustration of Various Types of Fuzzy Extended Principles

Intuitionistic fuzzy: Intuitionistic fuzzy are an extension of classical sets that provide additional information about the degree of uncertainty through non-membership and membership values. Proposed by Atanassov[18] in 1986, intuitionistic fuzzy sets aim to capture the uncertainty and fuzziness inherent in real decision-making processes. In classical fuzzy sets, each element is assigned a membership value ranging from 0 to 1, indicating the degree of belongingness to the set. However, intuitionistic fuzzy sets introduce a new parameter known as the non-membership value, which quantifies the extent to which an element does not belong to the set. This allows for a more comprehensive representation of uncertainty in decision-making.

Definition 4.1. [19] Intuitionistic fuzzy Set : A set$\tilde{\theta}$ on K is defined as $\tilde{\theta} = \{(k, [(\rho(k), \upsilon(k))]) : k \in K\}$ Where $\rho(k) : K \to [0, 1]$ is named as truth function which indicate the degree of assurance and $\upsilon(k) : K \to [0, 1]$ is named as falsity function. And $\rho(k), \upsilon(k)$ satisfies the following relation:

$$0 \le \rho(k) + \upsilon(k) \le 1$$

Definition 4.2. [19] :Intutionistic Fuzzy Number : An intuitionistic fuzzy number $\tilde{A}_d$ with the membership function $\sigma_{\tilde{A}_d}(q)$ and non-membership function $\eta_{\tilde{A}_d}(q)$ is:

I. Concave for the non-membership function, i.e., for all
$q_1, q_2 \in q, \eta_{\tilde{A}_d}(\lambda_d q_1 + (1-\lambda_d)q_2) \le \max\{\eta_{\tilde{A}_d}(q_1), \eta_{\tilde{A}_d}(q_2)\}$, where $\lambda_d \in [0, 1]$.

II. Convex for the membership function, i.e., for all
$q_1, q_2 \in Q, \sigma_{\tilde{A}_d}(\lambda_d q_1 + (1-\lambda_d)q_2) \ge \min\{\sigma_{\tilde{A}_d}(q_1), \sigma_{\tilde{A}_d}(q_2)\}$, where $\lambda_d \in [0, 1]$.

III. Normal, i.e., there is a $q_0 \in Q$ such that $\sigma_{\tilde{A}_d}(q_0) = 1$ and $\eta_{\tilde{A}_d}(q_0) = 0$

IV. An intuitionistic fuzzy subset of the real line.

Definition 4.3. [19] :Triangular Intutionistic Fuzzy Number(TIFN) : If A TIFN $\tilde{A}_d = (\delta_1, \delta_2, \delta_3; \delta_1^{'}, \delta_2, \delta_3^{'})$.is an intuitionistic fuzzy set in R with following non-membership function $(\eta_{\tilde{A}_d}(q))$ and membership function $(\sigma_{\tilde{A}_d}(q))$: $\sigma_{\tilde{A}_d}(q) = \left\{\begin{matrix} \dfrac{q-\delta_1}{\delta_2-\delta_1}, & \text{for}\delta_1 \le q \le \delta_2 \\ \dfrac{\delta_3-q}{\delta_3-\delta_2}, & \text{for}\delta_2 \le q \le \delta_3 \\ 0, & \text{otherwise} \end{matrix}\right\}$, and $\eta_{\tilde{A}_d}(q) = \left\{\begin{matrix} \dfrac{\delta_2-q}{\delta_2-\delta_1^{'}}, & \text{for}\delta_1^{'} \le q \le \delta_2 \\ \dfrac{q-\delta_2}{\delta_3^{'}-\delta_2}, & \text{for}\delta_2 \le q \le \delta_3^{'} \\ 1, & \text{otherwise} \end{matrix}\right\}$
Where, $\delta_1^{'} \le \delta_1 \le \delta_2 \le \delta_3 \le \delta_3^{'}$ and $\eta_{\tilde{A}_d}(q) \le 1, 0 \le \sigma_{\tilde{A}_d}(q)$, for $\sigma_{\tilde{A}_d}(q) = 1 - \eta_{\tilde{A}_d}(q), \forall q \in R$

Neutrosophic: Neutrosophic logic is a unique form of logic that combines elements from classical and fuzzy logic to effectively handle uncertain or incomplete data. Since its introduction in the 1999s by Florentin Smarandache,[20] it has garnered attention from mathematicians, computer scientists, AI researchers, decision-makers, and fusion researchers. In neutrosophic logic, each proposition can be categorized as true (T), false (F), or indeterminate (I). The inclusion of indeterminacy in neutrosophic logic allows for greater flexibility and expression when reasoning with limited or ambiguous information. Figure 2 presents a visual representation illustrating the crucial components and processes that form the foundation of the Neutrosophic Principle. This graphical depiction serves to enhance readers' understanding of the concepts discussed and provides a practical glimpse into the operational aspects of the Neutrosophic Principle. By offering a visual aid, readers can grasp the key elements more effectively and gain practical insights into the application of the Neutrosophic Principle.

Definition 4.4. [20] Neutrosophic Set : A set $\widetilde{neu\text{A}}$ in the universe of discourse X, is called Neutrosophic Set if $\widetilde{neu\text{A}} = \left\{\left(x, \left[t_{\widetilde{neu\text{A}}}(x), i_{\widetilde{neu\text{A}}}(x), f_{\widetilde{neu\text{A}}}(x)\right]\right) : x \in X\right\}$ Where truth, indeterminacy, Falsity, membership function which has the degree of belongingness $t_{\widetilde{neu\text{A}}}(x) : X \to [0, 1]$, $i_{\widetilde{neu\text{A}}}(x) : X \to [0, 1]$, and $f_{\widetilde{neu\text{A}}}(x) : X \to [0, 1]$ of the decision maker. $t_{\widetilde{neu\text{A}}}(x), i_{\widetilde{neu\text{A}}}(x), f_{\widetilde{neu\text{A}}}(x)$ satisfy the following relation. $0 \le sup\left\{t_{\widetilde{neu\text{A}}}(x)\right\} + sup\left\{i_{\widetilde{neu\text{A}}}(x)\right\} + sup\left\{f_{\widetilde{neu\text{A}}}(x)\right\} \le 3$.

Definition 4.5. [21] Triangular Neutrosophic number (TNNs) : If A TNNs is denoted by $\xi^X = <(\gamma^u, \gamma^v, \gamma^w), (\omega, \lambda, \chi)>$ who's membership function represents for the truth, indeterminacy, and falsity of x can be defined as follows:

$$\tau_{\xi^X}(x) = \left\{\begin{matrix} \frac{(x-\gamma^u)}{(\gamma^v-\gamma^u)}\mu,\ \gamma^u \le x \le \gamma^v, \\ \mu,\ x = \gamma^v \\ \frac{(\gamma^w-x)}{(\gamma^w-\gamma^v)}\mu,\ \gamma^v \le x \le \gamma^w \\ 0\ , otherwise \end{matrix}\right\}$$

,

$$l_{\xi^X}(x) = \left\{\begin{matrix} \frac{(\gamma^v-x)}{(\gamma^v-\gamma^u)}i,\ \gamma^u \le x \le \gamma^v, \\ \text{i},\ x = \gamma^v \\ \frac{(x-\gamma^w)}{(\gamma^w-\gamma^v)}i,\ \gamma^v \le x \le \gamma^w \\ 1, otherwise \end{matrix}\right\}$$

Received: January 26, 2023 Revised: May 27, 2023 Accepted: July 18, 2023

$$\nu_{\xi^X}(x) = \left\{ \begin{array}{c} \frac{(\gamma^v - x)}{(\gamma^v - \gamma^u)}\chi, \ \gamma^u \le x \le \gamma^v, \\ \chi, x = \gamma^v \\ \frac{(x - \gamma^w)}{(\gamma^w - \gamma^v)}\chi, \ \gamma^v \le x \le \gamma^w \\ 1, otherwise \end{array} \right\}$$

Where,

$$0 \le \tau_{\xi^X}(x) + l_{\xi^X}(x) + \nu_{\xi^X}(x) \le 3, x \in \xi^X.$$

4.1 Different Methodologies of the Neutrosophic Linear Programming Problem in different real-life problems

Neutrosophic principles exhibit versatility across numerous sectors and domains, presenting a myriad of practical applications. Highlighted below are some prevalent use cases: Table 2 presents an in-depth exploration of the remarkable progressions in Neutrosophic principles, exemplifying their diverse applications across various disciplines. The table accentuates the fundamental attributes and benefits of each approach, serving as a user-friendly point of reference.

Table 2: Exploring the Extensive area of Neutrosophic Principles: A Literature Survey

Authors	Year	Environment	Application	Contribution
Pramanik et al.[25]	2016	Decision making SVNNs	TOPSIS method	The paper presents a TOPSIS approach for multi-attribute decision making in a refined neutrosophic environment with SVNNs.
Prabha and Vimala[22]	2018	Assignment problem under TNNs	Branch and Bound (BnB) algorithm	This paper introduces the application of the branch and bound technique to solve the triangular fuzzy Neutrosophic assignment problem (TFNAP).
Bera and Mahapatra[26]	2020	Assignment problem TNNs	Centroid method	The objective of this article is to address an assignment problem to tackle a scenario where the cost matrix comprises single-valued TNNs in an uncertain environment.
Bera and Mahapatra[23]	2021	Linear Programming Problem under TNNs	Duality algorithm	Solving LPP using the Duality approach in a Neutrosophic triangular environment.
Yao and Ran[24]	2023	Decision making under TNNs	TFNN-GRA method	Evaluating the operational efficiency of the basic pension insurance system for urban and rural residents using the TFNN-GRA Method.

This article incorporates Figure 3, offering a clear and insightful visual representation of the fundamental components and processes underlying Neutrosophic Principle. This graphical illustration significantly aids readers in grasping the discussed concepts and provides a practical glimpse into the application of Neutrosophic Principle. Furthermore, Figure 3 seamlessly integrates Table 2, enabling readers to conveniently refer to relevant information.

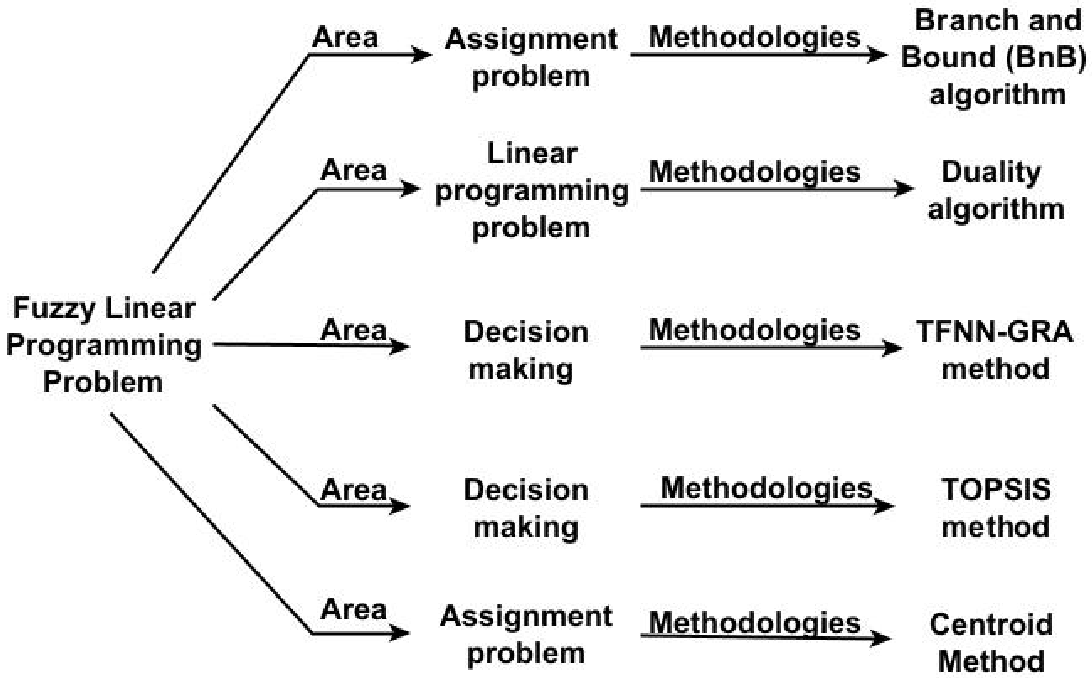

Figure 3: Visualizing the Diversity of Environments and Applications in Neutrosophic Fuzzy Principlee

These examples merely scratch the surface of the diverse applications of Neutrosophic principles. Their exceptional flexibility and capacity to manage uncertainties render them suitable for various domains where the analysis and decision-making processes necessitate the consideration of imprecise and vague information.

The primary aim of addressing Table 2 was to emphasize that fuzzy is not the exclusive principle for managing uncertainty. Apart from fuzzy, numerous other Neutrosophic principles provide superior and more accurate solutions. Furthermore, these principles have experienced a rapid increase in influence across diverse areas and applications in recent times. Although it is not feasible to delve into an exhaustive discussion of all these applications within this paper's constraints, our focus will be on exploring the impact of the Neutrosophic on LPP.

5 The LPP under the Neutrosophic Principle

Jun Ye (2017)[27] presented a comprehensive study on neural networks (NNs) and introduced the concept of NN functions. The author then proposed a novel approach called NNLP, specifically designed to address NN optimization problems in uncertain environments. The primary advantages of this method are twofold. Firstly, the NNLP method offers a range of possible optimal solutions for decision variables and neutrosophic objective functions when confronted with indeterminacy "I," thus surpassing the limitations of existing uncertain LP methods that can only provide a single crisp optimal solution. Secondly, the NNLP method extends the capabilities of classical LP methods, making it more versatile and practical in dealing with indeterminate environments. By enriching the field of uncertain LP methods and offering an effective solution for indeterminate optimization problems, this research significantly contributes to the domain. To exemplify the proposed NNLP method, a numerical example was provided, demonstrating its application in solving optimization problems. Furthermore, the developed NNLP method was successfully employed in a production planning problem. Looking ahead, future efforts will focus on extending the NNLP method to address NN nonlinear programming problems and applying it in various fields, including design, management, and engineering.

Received: January 26, 2023 Revised: May 27, 2023 Accepted: July 18, 2023

S. A. Edalatpanah[28] introduced a novel algorithm for solving Neutrosophic LPP. In this algorithm, TNNs are used to represent the variables and the right-hand side. The primary objectives of the paper are twofold. Firstly, it proposes a new direct model that incorporates neutrosophic variables and the right-hand side. Secondly, it presents a solution method specifically designed for neutrosophic LP problems. The author aims to maximize the degrees of acceptance while minimizing indeterminacy and rejection of objectives. To demonstrate the effectiveness of the proposed procedure, numerical experiments are conducted. The results obtained from these experiments illustrate that the new algorithm is both straightforward and applicable for guiding the modeling and design of various neutrosophic optimization scenarios.

Bera and Mahapatra[29] proposed the use of single-valued Neutrosophic numbers as an extension to the concept of crisp linear programming problems (LP-problems) in a neutrosophic environment. In this approach, SVNNs are utilized to represent the coefficients in the objective function, technical coefficients, right-hand side coefficients, and decision variables of an LP-problem. This novel formulation is referred to as a NLPP. The authors developed an algorithm based on the Big-M simplex method to solve such NLPPs. They also applied the proposed method to a real-life problem, showcasing its practical application.

Conclusion

This review article delves into the amalgamation of Neutrosophic Set Theory and Neutrosophic principles theory within the realm of Linear Programming (LP) problems. Neutrosophic set theory, dealing adeptly with uncertain, imprecise, and indeterminate information, pairs harmoniously with Neutrosophic principles theory, which expands the classical fuzzy set theory to handle intricate membership degrees. The symbiosis of these two frameworks equips decision-makers with a robust toolset for modeling and solving LP problems in environments rife with ambiguity and uncertainty. By thoroughly analyzing and summarizing existing literature on Neutrosophic Linear Programming in Neutrosophic environments, this review aims to explore the theoretical foundations and practical applications. Ultimately, the article seeks to foster a deeper understanding of these integrated approaches and their potential to address complex decision-making problems under uncertain conditions.

References

[1] Taha HA. *Operations Research: An Introduction*. Pearson Education India, 2013.

[2] VK Kapoor. *Operations research techniques for management*. Sultan Chand & Sons., New Delhi, 2003.

[3] Yaghini M; Momeni M; Sarmadi M. Solving train formation problem using simulated annealing algorithm in a simplex framework. *Journal of Advanced Transportation*, 48(5):402–416, 2014.

[4] Li Y; Tarlow D; Brockschmidt M; Zemel R. Gated graph sequence neural networks. 2015.

[5] Liu M; Wang S;Zheng F;Chu C. Algorithms for the joint multitasking scheduling and common due date assignment problem. *International Journal of Production Research*, 55(20):6052–6066, 2017.

[6] S; Oberoi T; Sharma T; Thakkar A Jhawar, S; Agarwaal. Application of game theory in water resource management. *International Journal of Advance Research and Development*, 3(10):63–8, 2018.

[7] N; Schmidt M Bouman, P; Agatz. Dynamic programming approaches for the traveling salesman problem with drone. *Networks*, 72(4):528–542, 2018.

[8] Lotfi A Zadeh. Fuzzy sets. *Information and Control*, 8(3):338–353, 1965.

[9] Nidhi Singh, Avishek Chakraborty, Soma Bose Biswas, and Malini Majumdar. Impact of social media in banking sector under triangular neutrosophic arena using mcgdm technique. *Neutrosophic Sets And Systems*, 35:153–176, 2020.

[10] RR Yager. A procedure for ordering fuzzy subsets of the unit interval. *Information sciences*, 24(2):143–161, 1981.

https://doi.org/10.54216/IJNS.210410

Received: January 26, 2023 Revised: May 27, 2023 Accepted: July 18, 2023

[11] DJ Dubois. *Fuzzy sets and systems: theory and applications*, volume 144. Academic press, 1980.

[12] HJ Zimmermann. Fuzzy programming and linear programming with several objective functions. *Fuzzy Sets and Systems*, 1(1):45–55, 1978.

[13] FH;Kiani NA Sigarpich LA; Allahviranloo, T;Lotfi. Degeneracy in fuzzy linear programming and its application. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, 19(06):999–1012, 2011.

[14] Cagatay Iris and Emre Cevikcan. A fuzzy linear programming approach for aggregate production planning. *Supply Chain Management Under Fuzziness: Recent Developments and Techniques*, pages 355–374, 2014.

[15] Shu-Ping Wan and Jiu-Ying Dong. Possibility method for triangular intuitionistic fuzzy multi-attribute group decision making with incomplete weight information. *International Journal of Computational Intelligence Systems*, 7(1):65–79, 2014.

[16] Xuelei Meng, Bingmou Cui, et al. Train repathing in emergencies based on fuzzy linear programming. *The Scientific World Journal*, 2014, 2014.

[17] JL Ebrahimnejad, A; Verdegay. A survey on models and methods for solving fuzzy linear programming problems. *Fuzzy Logic in Its 50th Year New Developments, Directions and Challenges*, pages 327–368, 2016.

[18] Krassimir T Atanassov and S Stoeva. Intuitionistic fuzzy sets. *Fuzzy sets and Systems*, 20(1):87–96, 1986.

[19] N;Mishra Vishnu N Sharma, MK;Dhiman. Mediative fuzzy logic mathematical model: A contradictory management prediction in covid-19 pandemic. *Applied Soft Computing*, 105:107285, 2021.

[20] Florentin Smarandache. A unifying field in logics. neutrosophy: Neutrosophic probability, set and logic, 1999.

[21] M;Mohamed M;Smarandache F Abdel-Basset, M;Gunasekaran. A novel method for solving the fully neutrosophic linear programming problems. *Neural Computing And Applications*, 31:1595–1605, 2019.

[22] S Prabha, SK; Vimala. Neutrosophic assignment problem via bnb algorithm. In *Advances in Algebra and Analysis: International Conference on Advances in Mathematical Sciences, Vellore, India, December 2017-Volume I*, pages 323–330. Springer, 2018.

[23] NK Bera T;Mahapatra. On solving linear programming problem by duality approach in neutrosophic environment. *International Journal of Mathematics in Operational Research*, 18(3):310–335, 2021.

[24] H Yao, Z; Ran. Operational efficiency evaluation of urban and rural residents' basic pension insurance system based on the triangular fuzzy neutrosophic gra method. *Journal of Intelligent and Fuzzy Systems*, (Preprint):1–12, 2023.

[25] BC Pramanik S; banerjee D; giri. *TOPSIS approach for multi attribute group decision making in refined neutrosophic environment*. Infinite Study, 2016.

[26] Tuhin Bera and Nirmal Kumar Mahapatra. To solve assignment problem by centroid method in neutrosophic environment. *TIF*, page 84, 2020.

[27] Jun Ye. Neutrosophic number linear programming method and its application under neutrosophic number environments. *Soft Computing*, 22:4639–4646, 2018.

[28] SA Edalatpanah et al. A direct model for triangular neutrosophic linear programming. *International Journal Of Neutrosophic Science*, 1(1):19–28, 2020.

[29] Tuhin Bera and Nirmal Kumar Mahapatra. Neutrosophic linear programming problem and its application to real life. *Afrika Matematika*, 31(3-4):709–726, 2020.

https://doi.org/10.54216/IJNS.210410

Received: January 26, 2023 Revised: May 27, 2023 Accepted: July 18, 2023