Neutrosophic exponentiated inverse Rayleigh distribution: Properties and Applications

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Abstract

In the field of survival analysis, the exponentiated inverse Rayleigh distribution is used to simulate lifetime data practices of human. In order to describe diverse survival data with indeterminacies, this work aims to create a generalization of the traditional pattern exponentiated inverse Rayleigh distribution, referred to as the neutrosophic exponentiated inverse Rayleigh distribution (NEIRD). In particular, modeling uncertain data that is roughly positively skewed makes use of the established distribution. The key statistical characteristics of the developed NEIRD, such as the neutrosophic survival function, neutrosophic hazard rate and neutrosophic moments, are discussed in this study. Additionally, in a neutrosophic well-known maximum likelihood estimation approach is used to estimate the neutrosophic parameters. A simulation study is conducted to determine whether the estimated neutrosophic parameters were achieved. Last but not least, real data has been used to discuss the potential NEIRD applications in the real world. The effectiveness of the suggested model in comparison to the existing distributions was demonstrated by real data.

Keywords: Neutrosophic statistics; exponentiated inverse Rayleigh distribution; survival analysis; Indeterminacy.

1. Introduction

A subfield of statistics known as neutrosophic statistics deals with the use of neutrosophic logic to handle uncertainty and incompleteness in data. Fuzzy logic was expanded upon by [1] to create neutrosophy, which enables the depiction of uncertainty, ambiguity, and contradiction. Traditional statistics typically assumes that the data is clear, which means that each observation is given a specific value. However, data in real-world situations sometimes includes ambiguous or insufficient information. In order to overcome these drawbacks, neutrosophic statistics offers a paradigm for dealing with ambiguous, insufficient, and inconsistent data [2-4].

Three factors are taken into account by neutrosophic statistics: truth-membership, indeterminacy-membership, and falsity-membership. The degree of truth, ambiguity, or untruth connected with an observation or a hypothesis is represented by each component. Similar to fuzzy sets, membership functions are used to represent these degrees [2, 3].

Numerous industries, such as decision-making, pattern identification, data mining, and image processing, use neutrosophic statistics [4-7]. It offers a versatile mathematical tool for modeling and analyzing complicated systems with a high degree of uncertainty and imprecision.

One of the important applications of neutrosophic data is survival analysis. The survival analysis is a statistical technique that examines the length of time until an event happens [8]. Survival analysis is fully depending on the probability distributions of the time data. The concept of neutrosophic survival probability distribution blends

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survival analysis and neutrosophic logic. The survival probability distribution in the context of neutrosophic represents the possibility of an event occurring at various points in time. In order to take into consideration, the ambiguity and uncertainty in the survival statistics, neutrosophic logic is used. It enables the portrayal of incomplete or partial knowledge of events. The available survival statistics must be taken into account, and neutrosophic elements must be included to reflect the degrees of truth, falsity, and indeterminacy connected with the survival probability at various time points. Neutrosophic logic-specific mathematical models and methods can be used for this. Several papers are related to neutrosophic probability distribution [8-20].

The exponentiated inverse Rayleigh distribution has applications in various fields, such as survival analysis. In this paper, we expanded the uses of the exponentiated inverse Rayleigh distribution when the data is in interval form and has some degree of indeterminacy in the form of neutrosophy. With the aid of simulated and real data application, a number of properties are examined under the newly proposed distribution and their applications are discussed.

2. Neutrosophic exponentiated inverse Rayleigh distribution

The inverse Rayleigh distribution is extended by the exponentiated inverse Rayleigh distribution by [21]. According to [22], the CDF and pdf of the exponentiated inverse Rayleigh distribution (EIRD) is defined as, respectively:

$$F(x)=1-\left(1-e^{-(\sigma/x)^2}\right)^{\alpha};\ x\geq 0,\ \ \sigma>0,\ \alpha>0 \tag{1}$$

$$f(x)=\frac{2\alpha\sigma^2}{x^3}e^{-(\sigma/x)^2}\left(1-e^{-(\sigma/x)^2}\right)^{\alpha-1};x>0,\ \sigma>0,\ \alpha>0 \tag{2}$$

where σ is the scale parameter and α is the shape parameter.

The survival and the hazard functions of EIRD are as follows:

$$S(x)=\left(1-e^{-(\sigma/x)^2}\right)^{\alpha} \tag{3}$$

$$h(x)=2\alpha\sigma^2x^{-3}e^{-(\sigma/x)^2}\left(1-e^{-(\sigma/x)^2}\right)^{-1} \tag{4}$$

The concept of neutrosophic probability as a function $NP:\rightarrow\ [0,\ 1]^3$ was originally presented by [2], where U is a neutrosophic sample space and defined the probability mapping to take the form $NP(S)\ =\ \left(ch(S),\ ch(neut\ S),\ ch(anti\ S)\right)\ =(\eta,\ \beta,\ \tau\ )$ $where\ 0\ \leq\ \eta,\ \beta,\ \tau\ \leq\ 1\ and\ 0\ \leq\ \eta\ +\ \beta\ +\ \tau\ \leq\ 3$. The term Ψ represents the set of sample space, R represents the set of real numbers, and Υ denotes a sample space event, X_N and Y_N denote neutrosophic r.v. Furthermore, we demonstrate certain renowned definitions and characteristics of neutrosophic probability and logic that will be important in creating this neutrosophic probability model.

Definition 1:

Consider the real-valued crisp r.v. X , which has the following definition: $X\ :\Psi\rightarrow\ R$

where Ψ is the event space and X_N neutrosophic r.v. as follows:

$$X_N\ :\Psi\rightarrow\ R(I)$$

and

$$X_N\ =\ X\ +I$$

The term I represents indeterminacy.

Theorem 1:

Let the neutrosophic r.v. $X_N\ =\ X\ +I$ and the CDF of X is $F_X\ (x)\ =\ P(X\ \leq\ x)$ [13]. The following assertions are correct:

$$FX_N\ (x)\ =\ F_X\ (x\ -I),$$

$$f_{X_N}\ (x)\ =\ f_X\ (x\ -I),$$

where F_{X_N} and f_{X_N} are the CDF and PDF of a neutrosophic r.v. X_N , respectively.

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Theorem 2 :

Let $X_N = X + I$, is the neutrosophic r.v., then the expected value and variance can be derived as follows: $E(X_N) = E(X) + I$ and $V(X_N) = V(X)$ [13].

Suppose the neutrosophic variable could be expressed as: $x_N = x_L + x_U I_N$ where $I_N \in \{I_L, I_U\}$ and x_L and $x_U I_N$ denote the determined and indeterminate parts, respectively. Assume that the neutrosophic random variable $x_N \in \{x_L, x_U\}$ follows the EIRD having neutrosophic scale parameter $\sigma_N \in \{\sigma_L, \sigma_U\}$ and neutrosophic shape parameter $\alpha_N \in \{\alpha_L, \alpha_U\}$ where the letters L and U are the lower values and the upper values, respectively. Then, the neutrosophic probability density function (NPDF) of neutrosophic exponentiated inverse Rayleigh distribution (NEIRD) is given by

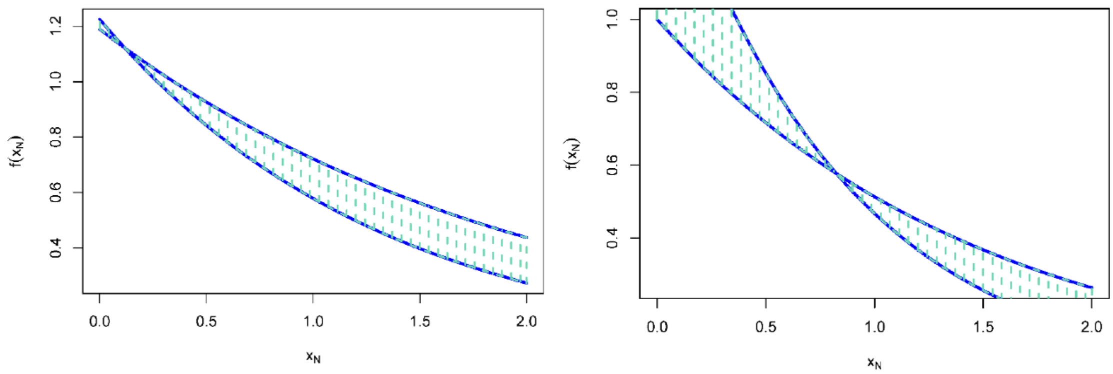
$$f(x_N) = \frac{2\alpha_N \sigma_N^{\ 2}}{x_N^3} e^{-(\sigma_N/x_N)^2} \left(1 - e^{-(\sigma_N/x_N)^2}\right)^{\alpha_N - 1}; x_N > 0,\ \sigma_N, \alpha_N > 0 \qquad (5)$$

Figures 1 shows the NPDF for different values of σ and α. The neutrosophic cumulative density function (NCDF), the neutrosophic survival, and neutrosophic hazard functions are given below, respectively:

$$F(x_N) = 1 - \left(1 - e^{-(\sigma_N/x_N)^2}\right)^{\alpha_N} \qquad (6)$$

$$S(x_N) = \left(1 - e^{-(\sigma_N/x_N)^2}\right)^{\alpha_N} \qquad (7)$$

$$h(x_N) = 2\alpha_N \sigma_N^{\ 2} x_N^{-3} e^{-(\sigma_N/x_N)^2} \left(1 - e^{-(\sigma_N/x_N)^2}\right)^{-1} \qquad (8)$$



(a) (b)

Figure 1: (a) The pdf of NEIRD when $\sigma_N \in [1.5, 2]$ and $\alpha_N \in [2, 4]$, (b) The pdf of NEIRD when $\sigma_N \in [1, 3]$ and $\alpha_N \in [1.5, 2.5]$.

3. Statistical Properties of NEIRD

In this section, statistical properties of the NEIRD are covered.

Moments: The r^{th} moment about origin is given by

$$\mu_r' = E(x_N^r) = \int_0^\infty x_N^r f(x_N) dx_N \qquad (9)$$

Then, the mean is:

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Mean= $E(x_N) = \alpha_N \sum_{j=0}^{\infty} \binom{\alpha_N - 1}{j} (-1)^j \frac{(\sigma_N^2 (j+1))^{1/2}}{j+1} \Gamma\left(\frac{1}{2}\right)$ (10)

Moment Generating Function:

$$M_{x_N}(t) = E(e^{tx_N}) = \int_0^{\infty} e^{tx_N} f(x_N)\, dx_N$$
$$= \alpha_N \sum_{r=0}^{\infty} \binom{\alpha_N - 1}{r} \frac{(-1)^r}{r!} \frac{(\sigma_N^2 (r+1))^{r/2}}{r+1} \Gamma\left(1 - \frac{r}{2}\right), \quad r < 2 \qquad (11)$$

4. Parameter Estimation of NEIRD

Maximum likelihood estimation (MLE) method is mostly used in estimating NEIRD parameters. Let $x_{N1}, x_{N2}, \ldots, x_{Nn}$ be random sample of size n drawn from NEIRD distribution, then the MLE can be obtained as follows:

$$L = 2^n \alpha_N^2 \sigma_N^{2n} \prod_{i=1}^{n} x_{Ni}^{-3} e^{-\sum_{i=1}^{n} (\sigma_N / x_{Ni})^2} \prod_{i=1}^{n} \left(1 - e^{-(\sigma_N / x_{Ni})^2}\right)^{\alpha_N - 1} \qquad (12)$$

$$\log L = n \ln 2 + n \ln \alpha_N + 2n \ln \sigma_N - \sum_{i=1}^{n} \ln x_{Ni}^{-3} - \sum_{i=1}^{n} \left(\frac{\sigma_N}{x_{Ni}}\right)^2 + (\alpha_N - 1) \sum_{i=1}^{n} \ln\left(1 - e^{-(\sigma_N / x_{Ni})^2}\right) \qquad (13)$$

$$\frac{\partial \log L}{\partial \alpha_N} = \frac{n}{\alpha_N} + \sum_{i=1}^{n} \ln\left(1 - e^{-(\sigma_N / x_{Ni})^2}\right) \qquad (14)$$

$$\frac{\partial \log L}{\partial \alpha_N} = 0 \Rightarrow \hat{\alpha}_N = \frac{-n}{\sum_{i=1}^{n} \ln\left(1 - e^{-(\sigma_N / x_{Ni})^2}\right)} \qquad (15)$$

$$\frac{\partial \log L}{\partial \sigma_N} = \frac{2n}{\sigma_N} - 2\sigma_N \sum_{i=1}^{n} \frac{1}{x_{Ni}^2} + 2\sigma_N (\alpha_N - 1) \sum_{i=1}^{n} \frac{e^{-(\sigma_N / x_{Ni})^2}}{x_{Ni}^2 \left(1 - e^{-(\sigma_N / x_{Ni})^2}\right)} \qquad (16)$$

Equation (16) can be solved by Newton Raphson method to obtain the MLE for σ_N.

5. Simulation results

A Monte Carlo simulation is run in R software with several sample sizes, $n = 30, 50, 150, 250$ and neutrosophic parameters in two cases: (1) $\sigma_N \in [1,3]$ and $\alpha_N \in [1.5, 2.5]$ and (2) $\sigma_N \in [1.5, 2]$ and $\alpha_N \in [2,4]$. The simulation is replicated for 1000 times. Performance measures, such as the neutrosophic average of the estimators, the neutrosophic average bias (NAB) and neutrosophic Mean Square Error (NMSE) are attained for all values of n. The results are given in Tables 1 and 2. From Tables 1 and 2, It is seen that, as expected, the NAB and NMSE fall for both neutrosophic parameters as sample sizes rise. Furthermore, according to the study's findings, the neutrosophic MLE for the NEIRD offers accurate estimation with a higher sample size.

Table 1: Average NAB and NMSE for case 1

n	NAB		NMSE	
	σ_N	α_N	σ_N	α_N
30	[0.0173, 0.0181]	[0.0204, 0.0212]	[0.0383, 0.0391]	[0.0414, 0.0422]
50	[0.0123, 0.0132]	[0.0154, 0.0162]	[0.0331, 0.0342]	[0.0364, 0.0372]
150	[0.0111, 0.0119]	[0.0149, 0.0151]	[0.0321, 0.0329]	[0.0359, 0.0361]

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250	[0.0054, 0.0067]	[0.0075, 0.0088]	[0.0264, 0.0278]	[0.0283, 0.0298]

Table 2: Average NAB and NMSE for case 2

n	NAB		NMSE	
	σ_N	α_N	σ_N	α_N
30	[0.0254, 0.0267]	[0.0285, 0.0293]	[0.0464, 0.0473]	[0.0495, 0.0503]
50	[0.0204, 0.0213]	[0.0235, 0.0243]	[0.0415, 0.0423]	[0.0445, 0.0453]
150	[0.0192, 0.0201]	[0.0232, 0.0235]	[0.0402, 0.0415]	[0.0338, 0.0312]
250	[0.0136, 0.0148]	[0.0157, 0.0169]	[0.0345, 0.0359]	[0.0264, 0.0271]

6. Applications

The carefully crafted data set relates to information on alloy melting points that was obtained from [23] and used for the first time by [24]. An alloy is a mixture of material components, containing at least one metal. These alloys may possess properties that let them stand out from pure metals, which helps them increase strength or hardness while also bringing down the price of the material. Red gold, made of a copper and gold alloy, white gold, made of a silver and gold alloy, etc. are a few examples of alloys. Manufacturing engineers involved in the production of bimetals frequently take the information on alloy melting points from a distribution with a set of aggregate melting values. Because it might be difficult to determine melting points in general, observations are indeterministic and can be reported in intervals. For quick reference, the following is a list of the 18 questionable data observations of alloy melting points: [563.3, 545.5], [529.4, 511.6], [523.1, 503.5], [470.1, 449.2], [506.7, 489.0], [495.6, 479.1], [495.3, 467.9], [520.9, 495.6], [496.9, 472.8], [542.9, 519.1], [505.4, 484.0], [550.7, 525.9], [517.7, 500.9], [499.2, 483.0], [500.6, 480.0], [516.8, 499.6], [535.0, 515.1], [489.3, 464.4].

The model adequacy of the proposed NEIRD is compared with the neutrosophic exponential distribution (NED) applications for complicated data analysis investigated by [12] and neutrosophic Log-Logistic distribution (NLLD) by [17]. The log-likelihood value (LogL), Akaike Information Criteria (AIC), Bayesian Information Criteria (BIC), and Kolmogorov-Smirnov (KS) test are the criteria selection methods used to determine which model fits the data the best. The criteria for the best fitting model are the highest LogL values and the lowest AIC, BIC, and KS statistic values. Additionally, a higher p-value suggests that the model that best fits the neutrosophic data. Table 3 lists the neutrosophic maximum likelihood estimators and model sufficiency metrics. The findings show that the NEIRD is more effective than the NED and NLLD for data on alloy melting points. The table's bold values demonstrate the effectiveness of the suggested model.

Table 3: The criteria selection neutrosophic distributions for alloy melting points data

	NED	NLLD	NEIRD
Parameter	$\sigma_N=[461.958, 491.642]$	$\sigma_N=[492.696, 512.952]$	$\sigma_N=[492.696, 512.952]$
		$\beta_N=[36.418,\ 38.934]$	$\alpha_N=[36.418,\ 38.934]$
Log	[129.6745, 130.392]	[82.1435, 82.581]	**[80.3025, 81.1497]**
AIC	[261.349, 262.784]	[168.287, 169.162]	**[156.605, 158.2994]**
BIC	[262.239, 263.674]	[170.067, 170.943]	**[154.8234, 156.5187]**
KS-value	[0.6156, 0.62181]	[0.101, 0.117]	**[0.124, 0.132]**
KS- p-value	[3.032×10^{-7}, 4.289×10^{-7}]	[0.942, 0.984]	**[0.955, 0.987]**

7. Conclusions

In this paper, a neutrosophic exponentiated inverse Rayleigh distribution (NEIRD) has been suggested. This established distribution is useful for a variety of application data for indeterminacies in survival and dependability. The neutrosophic survival function, neutrosophic hazard rate, and neutrosophic moments have all been explored as the main statistical characteristics of the evolved NEIRD. For different sample sizes, the neutrosophic MLEs have been developed and have shown neutrosophic average bias and MSEs. The simulation study was carried out to examine whether the computed neutrosophic parameters were achieved. Simulation results show that the sample

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size and neutrosophic parametric value are important factors in accurately estimating an unknown parameter. The melting point of alloy materials used further supports the use of the NEIRD in neutrosophic instances.

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