Analytical Study of Neutrosophic Fuzzy Unobservable On-Off Fluid Queues in Equipoise Strategies

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Abstract

An unobservable fluid queuing model with alternately occurring on and off states is being examined in this study. The sojourn times differ from one another and are dispersed in a distribution that is exponential. Flow of fluid into the buffer's system accompanied by a few waiting procedures according to the first service is given to those who come first. In a neutrosophic fuzzy environment, the information acquired when the fluid enters the system can be split into fully and partially observable cases. The arrival and outflow rates are both neutrosophic trapezoidal fuzzy numbers. We calculate the average fluid level and sojourn duration per unit of time for the buffer.

Keywords: Buffer's average fluid level; Buffer's average sojourn time; Centroid ranking; Equipoise strategies; Fully unobservable; Partially unobservable; Trapezoidal Neutrosophic fuzzy number; On-off fluid queue model.

1. Introduction

Queuing techniques play a significant role in modern living. It can be challenging to keep hold of current clients while also motivating them to recommend you to others in the fast-paced business world of today. The exponential increase of people has become a significant concern for domains like broadband network transfer and mass production due to the quick development of information technology. A consumer in a queuing system typically receives a reward for using the system to deliver his service, but he also suffers consequences for the amount of time he had to wait. The sophisticated clients analyze the incentive and punishment to determine whether or not to participate in the system. The initial discrete consumers are approaching the continuous fluid as the inter arrival periods of the customers get progressively smaller, and the arrival rate rises steadily.

Introducing a method for solving fluid models was Couto et al. The balancing tactics in the visible only one server queue with malfunctions and repairs were studied by Kanta. A fluid queue that is observable and has alternating service has been examined by Economou and Manou. This research was expanded upon by Edelson and Hildebrand in a variety of fluid queue scenarios with visible and nonvisible cases. Haviv and Hassin investigated the strategic balance actions of several model queue types. Yu et al. explained the queue model's equilibrium techniques in both observable and unobservable conditions. The fluid queue's equilibrium techniques with working vacations in the observable situation were examined by Wang and Xu.

Fuzzy set principle is a famous idea of intellectual phenomena modelling imprecision or uncertainty. Multiple researchers specifically discussed fuzzy queues. A preferred method for queuing structures in fuzzy surroundings is proposed primarily based on extension principle and Markov chains. Atanassov has created a marvelous intuitionistic fuzzy set that meticulously explains membership and non-membership functions. Smarandache coined the idea of having a neutrosophic set containing three different basic elements: (i) truth, (ii) indeterminate and (iii) falsehood. Each attribute of the neutrosophic aggregation is a very important factor for the real-life model. Neutrosophic set was designed to solve all kinds of complex problems in a highly efficient manner. The Neutrosophic hypothesis has been applied in many areas of science to address problems identified in ambiguity.

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Fuzzy number ranking has imparted a key role in fuzzy inference. It has attracted a lot of research attention. There are many approaches that can be used to solve the problem of neutrosophic fuzzy numbers. Broumi et al. described a method for grading trapezoidal neutrosophic fuzzy numbers with respect to their centre. Suresh et al. ordered trapezoidal neutrosophic fuzzy numbers based on the notion of Euclidean measure. Ye projected a trapezoidal neutrosophic set and applied it to multiple attribute determinations. Interval neutrosophic sets were developed by Zhang et al. and are utilized in a decision with multiple factors.

2. Preliminaries

2.1 Fuzzy Set

C is a fuzzy set defined on U and can be written as a collection of ordered pairs if U is a universe of discourse and x is a particular element of U and $\phi_{\tilde{C}}(x)) : U \to [0,1]$

$$\tilde{C} = \{(x, \phi_{\tilde{C}}(x)) / x \in U\}$$

2.2 Trapezoidal Fuzzy Number

A fuzzy number $\tilde{C}$ is said to be trapezoidal fuzzy number if and only if there exists real numbers $c_1 \le c_2 \le c_3 \le c_4$, Such that:

$$\phi_{\tilde{c}}(x) = \begin{cases} \frac{x-c_1}{c_2-c_1}, & c_1 \le x \le c_2 \\ 1, & c_2 \le x \le c_3 \\ \frac{x-c_4}{c_3-c_4}, & c_3 \le x \le c_4 \\ 0, & otherwise \end{cases}$$

It is denoted by $\tilde{c} = (c_1, c_2, c_3, c_4)$ or $\tilde{c} = (c_1 / c_2 / c_3 / c_4)$.

2.3 Neutrosophic fuzzy set

A neutrosophic fuzzy set $\tilde{N}$ be the subset of universe of discourse U is denoted by $\tilde{R}$ and defined by $\tilde{R} = \{x, M_{\tilde{R}}(x), I_{\tilde{R}}(x), N_{\tilde{R}}(x); x \in U\}$. Each element of R has a truth, indeterminacy, falsity functions and $M_{\tilde{R}}(x), I_{\tilde{R}}(x), N_{\tilde{R}}(x) : U \to [0,1]$, Such that $0 \le M_{\tilde{R}}(x), I_{\tilde{R}}(x), N_{\tilde{R}}(x) \le 3$.

2.4 Trapezoidal Neutrosophic Fuzzy Number

A trapezoidal neutrosophic fuzzy number $\tilde{r}$ with parameters $m_1 \le m_2 \le m_3 \le m_4$, $i_1 \le i_2 \le i_3 \le i_4$ & $n_1 \le n_2 \le n_3 \le n_4$ is denoted as $\tilde{r} = \langle (m_1, m_2, m_3, m_4), (i_1, i_2, i_3, i_4), (n_1, n_2, n_3, n_4) \rangle$. Its truth-Membership function $M_{\tilde{R}}(x)$, Indeterminacy-membership function $I_{\tilde{R}}(x)$ and falsity membership function $N_{\tilde{R}}(x)$ are defined as follows

$$M_{\tilde{R}}(x) = \begin{cases} \frac{x-m_1}{m_2-m_1}, & m_1 \le x \le m_2 \\ 1, & m_2 \le x \le m_3 \\ \frac{x-m_4}{m_3-m_4}, & m_3 \le x \le m_4 \\ 0, & otherwise \end{cases}$$

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$$I_{\tilde{R}}(x)=\begin{cases}\frac{i_2-x}{i_2-i_1} & , i_1\le x\le i_2\\ 0 & , i_2\le x\le i_3\\ \frac{x-i_3}{i_4-i_3} & , i_3\le x\le i_4\\ 1 & , otherwise\end{cases}$$

and

$$N_{\tilde{R}}(x)=\begin{cases}\frac{n_2-x}{n_2-n_1} & , n_1\le x\le n_2\\ 0 & , n_2\le x\le n_3\\ \frac{x-n_3}{n_4-n_3} & , n_3\le x\le n_4\\ 1 & , otherwise\end{cases}$$

In the above definition, when $m_2=m_3$, $i_2=i_3$ and $n_2=n_3$ then we will get a neutrosophic triangular fuzzy number.

3. Model Description: Unobservable On-off Fluid queuing models

3.1 Assumptions

(i) At time t, the fluid enters the buffer at a rate of η, and this input procedure can be seen as the customer's arrival.

(ii) The buffer has two states, given by $J(t)=0,1$ indicating that the states of on-off.

(iii) An exponential distribution with appropriate parameters $\omega_i, i=0,1$ and two alternating states are each possible. If the buffer is disabled, the system is terminated and there is no longer any drain.

(iv) When the buffer is turned on, the system's drain rate is taken into account as the service rate ζ. $(\zeta>0)$.

(v) The buffer's probability in state i is $\pi_i, i=0,1$ with $\pi_0=\frac{\omega_1}{\omega_0+\omega_1}$ and $\pi_1=\frac{\omega_0}{\omega_0+\omega_1}$.

(vi) Let $X(t)$ be the fluid level in the buffer at time t and the net entry rate of the on-off fluid queue model can be expressed as

$$\frac{dX(t)}{dt}=\begin{cases}\eta-\zeta, & X(t)>0,\quad J(t)=1,\\ \eta, & X(t)\ge 0,\quad J(t)=0,\\ \max(\eta-\zeta,0), & X(t)=0,\quad J(t)=1.\end{cases}$$

The aforementioned assumptions are the common trait of both scenarios of complete and partial unobservability in the model.

3.2 Fully Unobservable Case

This input process can be thought of as the arrival of the fluid, since at time t, the fluid enters the buffer at a fuzzy rate $\tilde{\lambda}$. Both the buffer's fluid level X (t) and the buffer's state J(t) are unidentified. We presum that incoming consumers cannot view both the count of system customers and the server states for a queue that is entirely unobservable. Therefore, arriving customers can be queued using a mixed strategy q. This indicates that when liquid enters the system, it will flow into the buffer with irreversible decision and the probability will be q.

3.2.1 Performance Indices of fully unobservable case

When liquid enters system's buffer, liquids follow the q- strategy and the state J (t) and level of liquid X (t) are unknown.

(i) The buffer's average fluid level

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$$\tilde{E}_q(X) = \frac{q\tilde{\lambda}((\tilde{\omega}_0+\tilde{\omega}_1)^2+\tilde{\mu}\tilde{\omega}_1)}{(\tilde{\omega}_0+\tilde{\omega}_1)(\tilde{\mu}\tilde{\omega}_0-q\tilde{\lambda}\tilde{\omega}_0-q\tilde{\lambda}\tilde{\omega}_1)}$$

(ii) The buffer's average sojourn time

$$\tilde{E}_q(S) = \frac{(\tilde{\omega}_0+\tilde{\omega}_1)^2+\tilde{\mu}\tilde{\omega}_1}{(\tilde{\omega}_0+\tilde{\omega}_1)(\tilde{\mu}\tilde{\omega}_0-q\tilde{\lambda}\tilde{\omega}_0-q\tilde{\lambda}\tilde{\omega}_1)}$$

3.3 Partially Unobservable Case

Assuming the fluid reaches the buffer at time t with fuzzy rate of $\tilde{\lambda}$, we know the state $J(t)$ of the buffer at that moment, but we do not know the fluid level $X(t)$ of the buffer at the current state. Assume the fluid follows the strategy $\tilde{q}=(q_0,\ q_1)$. This indicates that when fluid enters the system buffer and the probability will be q_i when the buffer is in state i. $S_i(q_0\ ,q_1)$ is the mean sojourn time after the liquid enters the buffer having the probability q_i when the buffer state i.

3.3.1 Performance Indices of partially unobservable case

If fluids adopt a strategy of mixing (i.e.) $\tilde{q}=(q_0\ ,q_i)$ then

(i) The buffer's average fluid level at state $i=0$ is

$$\tilde{E}_q(X/0) = \frac{\tilde{\lambda}q_0\tilde{\omega}_1+\tilde{\lambda}q_1\tilde{\omega}_0+\tilde{\lambda}\tilde{\mu}q_0-\tilde{\lambda}^2q_0q_1}{\tilde{\mu}\tilde{\omega}_0-q_0\tilde{\lambda}\tilde{\omega}_1-q_1\tilde{\lambda}\tilde{\omega}_0}$$

(ii) The buffer's average fluid level at state $i=1$ is

$$\tilde{E}_q(X/1) = \frac{\tilde{\lambda}^2(q_0)^2\tilde{\omega}_1+\tilde{\lambda}q_1(\tilde{\omega}_0)^2+\tilde{\lambda}q_0\tilde{\omega}_0\tilde{\omega}_1}{\tilde{\omega}_0(\tilde{\mu}\tilde{\omega}_0-q_0\tilde{\lambda}\tilde{\omega}_1-q_1\tilde{\lambda}\tilde{\omega}_0)}$$

(iii) The buffer's average sojourn time at state $i=0$ is

$$\tilde{E}_q(S/0) = \left(\frac{\tilde{\omega}_0+\tilde{\omega}_1}{\tilde{\mu}\tilde{\omega}_0}\right)\frac{\tilde{\lambda}q_0\tilde{\omega}_1+\tilde{\lambda}q_1\tilde{\omega}_0+\tilde{\lambda}\tilde{\mu}q_0-\tilde{\lambda}^2q_0q_1}{\tilde{\mu}\tilde{\omega}_0-q_0\tilde{\lambda}\tilde{\omega}_1-q_1\tilde{\lambda}\tilde{\omega}_0}+\frac{1}{\tilde{\omega}_0}$$

(iv) The buffer's average sojourn time at state $i=1$ is

$$\tilde{E}_q(S/1) = \left(\frac{\tilde{\omega}_0+\tilde{\omega}_1}{\tilde{\mu}\tilde{\omega}_0}\right)\frac{\tilde{\lambda}^2(q_0)^2\tilde{\omega}_1+\tilde{\lambda}q_1(\tilde{\omega}_0)^2+\tilde{\lambda}q_0\tilde{\omega}_0\tilde{\omega}_1}{\tilde{\omega}_0(\tilde{\mu}\tilde{\omega}_0-q_0\tilde{\lambda}\tilde{\omega}_1-q_1\tilde{\lambda}\tilde{\omega}_0)}$$

4. Centroid Ranking Technique for Trapezoidal Neutrosophic Fuzzy number

Let $\tilde{r}=\langle(m_1,m_2,m_3,m_4),(i_1,i_2,i_3,i_4),(n_1,n_2,n_3,n_4)\rangle$ be a trapezoidal neutrosophic fuzzy number. The centroids of real, hypothetical, and false trapezoids are treated as equilibrium points of the trapezoid. The Euclidean measure can be used to create a centroid-based measure of distance.

The centroid point of the truth membership function of a given trapezoidal neutrosophic fuzzy number $\tilde{r}$ be $C^T=\left(p_0{}^T(\tilde{r}),q_0{}^T(\tilde{r})\right)$

$$p_0{}^T(\tilde{r}) = \frac{1}{3}\left[(m_1+m_2+m_3+m_4)-\frac{m_3m_4-m_1m_2}{(m_3+m_4)-(m_1+m_2)}\right]$$

$$q_0{}^T(\tilde{r}) = \frac{1}{3}\left[1+\frac{m_3-m_2}{(m_3+m_4)-(m_1+m_2)}\right]$$

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The centroid point of the indeterminacy membership function of the trapezoidal neutrosophic fuzzy number $\tilde{r}$ be $C^I = \left(p_0{}^I(\tilde{r}), q_0{}^I(\tilde{r})\right)$

$$p_0{}^I(\tilde{r}) = \frac{1}{3}\left[(i_1+i_2+i_3+i_4) - \frac{i_3 i_4 - i_1 i_2}{(i_3+i_4)-(i_1+i_2)}\right]$$

$$q_0{}^I(\tilde{r}) = \frac{1}{3}\left[1 + \frac{i_3 - i_2}{(i_3+i_4)-(i_1+i_2)}\right]$$

The centroid point of the falsity membership function of neutrosophic trapezoidal fuzzy number $\tilde{r}$ be $C^F = \left(p_0{}^F(\tilde{r}), q_0{}^F(\tilde{r})\right)$

$$p_0{}^F(\tilde{r}) = \frac{1}{3}\left[(n_1+n_2+n_3+n_4) - \frac{n_3 n_4 - n_1 n_2}{(n_3+n_4)-(n_1+n_2)}\right]$$

$$q_0{}^F(\tilde{r}) = \frac{1}{3}\left[1 + \frac{n_3 - n_2}{(n_3+n_4)-(n_1+n_2)}\right]$$

Centroids of C^T, C^I, C^F represented as $C = \left(p(\tilde{r}), q(\tilde{r})\right)$ which is defined as

$$p(\tilde{r}) = \frac{p_0{}^T(\tilde{r}) + p_0{}^I(\tilde{r}) + p_0{}^F(\tilde{r})}{3}$$

$$= \frac{1}{9}\left(\sum_{s=1}^{4} m_s + \sum_{s=1}^{4} i_s + \sum_{s=1}^{4} n_s - \frac{m_3 m_4 - m_1 m_2}{(m_3+m_4)-(m_1+m_2)} - \frac{i_3 i_4 - i_1 i_2}{(i_3+i_4)-(i_1+i_2)} - \frac{n_3 n_4 - n_1 n_2}{(n_3+n_4)-(n_1+n_2)}\right)$$

$$q(\tilde{r}) = \frac{q_0{}^T(\tilde{r}) + q_0{}^I(\tilde{r}) + q_0{}^F(\tilde{r})}{3}$$

$$= \frac{1}{9}\left(3 + \frac{m_3 - m_2}{(m_3+m_4)-(m_1+m_2)} + \frac{i_3 - i_2}{(i_3+i_4)-(i_1+i_2)} + \frac{n_3 - n_2}{(n_3+n_4)-(n_1+n_2)}\right)$$

The score function is given by $R(\tilde{r}) = \sqrt{p(r)^2 + q(r)^2}$

5. Numerical Illustration

The arrival rate $\tilde{\lambda}$, service rate $\tilde{\mu}$ and two alternate state parameters $\tilde{\omega}_i$, $i = 0,1$ are trapezoidal neutrosophic fuzzy numbers. The average fluid level and sojourn time can be obtained for fully unobservable and partially unobservable cases with $\tilde{\lambda} q < \tilde{\mu}$ & $\tilde{\lambda} q > \tilde{\mu}$

Case I:

$\tilde{\lambda} q < \tilde{\mu}$ and for any $q \in [0,1]$

Consider,

$$\tilde{\lambda} = \left\langle (0.3, 0.4, 0.5, 0.7), (0.1, 0.3, 0.6, 0.9), (0.2, 0.4, 0.6, 1) \right\rangle$$

$$\tilde{\mu} = \left\langle (1, 1.1, 1.2, 1.4), (0.9, 1.1, 1.4, 1.6), (0.8, 1.1, 1.5, 1.7) \right\rangle$$

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$\tilde{\omega}_0 = \langle (0.6, 0.7, 0.8, 0.9), (0.5, 0.6, 0.8, 1), (0.3, 0.7, 0.9, 1.2) \rangle$

$\tilde{\omega}_1 = \langle (0.15, 0.25, 0.3, 0.35), (0, 0.1, 0.2, 0.4), (0.1, 0.15, 0.35, 0.5) \rangle$

From the centroid method, the score values are

$R(\tilde{\lambda}) = 0.6503$; $R(\tilde{\mu}) = 1.3045$; $R(\tilde{\omega}_0) = 0.8558$; $R(\tilde{\omega}_1) = 0.4788$.

The average fluid level and sojourn time can be obtained for fully unobservable and partially unobservable cases.

Table 1 Case I: Performance Measures with respect to q

Fully Unobservable Case

Q	$\tilde{E}_q(X)$	$\tilde{E}_q(S)$
0	0	1.6147
0.1	0.1139	1.7509
0.2	0.2487	1.9120
0.3	0.4108	2.1059
0.4	0.6096	2.3435
0.5	0.8589	2.6416
0.6	1.1809	3.0265
0.7	1.6127	3.5427
0.8	2.2221	4.2712
0.9	3.1470	5.3769
1	4.7180	7.2551

Table 2 Case I: Performance Measures with respect to $\tilde{q} = (q_0, q_i)$

Partially Unobservable Case

q_0	q_1	$\tilde{E}_q(X/0)$	$\tilde{E}_q(X/1)$	$\tilde{E}_q(S/0)$	$\tilde{E}_q(S/1)$
0	1	0.9939	0.9942	2.3567	1.1886
0.1	0.9	0.9903	0.9145	2.3524	1.0933
0.2	0.8	1.0009	0.8491	2.3651	1.0151
0.3	0.7	1.0240	0.7962	2.3927	0.9519
0.4	0.6	1.0583	0.7545	2.4337	0.9020
0.5	0.5	1.1025	0.7226	2.4865	0.8639
0.6	0.4	1.1555	0.6997	2.5499	0.8365
0.7	0.3	1.2166	0.6848	2.6229	0.8187
0.8	0.2	1.2849	0.6771	2.7046	0.8095
0.9	0.1	1.3598	0.6760	2.7941	0.8082
1	0	1.4406	0.6808	2.8907	0.8139

Case II:

$\tilde{\lambda} q > \tilde{\mu}$ and for any $q \in [0,1]$

consider,

$\tilde{\lambda} = \langle (0.95, 1, 1.15, 1.25), (0.75, 0.95, 1.25, 1.45), (0.65, 0.95, 1.35, 1.55) \rangle$

$\tilde{\mu} = \langle (0.65, 0.7, 0.85, 0.90), (0.55, 0.65, 0.95, 1), (0.35, 0.55, 1.25, 1.5) \rangle$

$\tilde{\omega}_0 = \langle (0.6, 0.7, 0.8, 0.9), (0.5, 0.6, 0.8, 1), (0.3, 0.7, 0.9, 1.2) \rangle$

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$\tilde{\omega}_1 = \langle (0.15, 0.25, 0.3, 0.35), (0, 0.1, 0.2, 0.4), (0.1, 0.15, 0.35, 0.5) \rangle$

The score values are $R(\tilde{\lambda}) = 1.1873$; $R(\tilde{\mu}) = 0.9453$; $R(\tilde{\omega}_0) = 0.8558$; $R(\tilde{\omega}_1) = 0.4788$.

Table 3: Case II: Performance Measures with respect to q

Fully Unobservable Case

Q	$\tilde{E}_q(X)$	$\tilde{E}_q(S)$
0	0	2.0689
0.1	0.3055	2.5729
0.2	0.8077	3.4013
0.3	1.7870	5.0168
0.4	4.5379	9.5551
0.5	59.4664	100.1704

Table 4: Case II: Performance Measures with respect to q

Partially Unobservable Case

q_1	q_0	$\tilde{E}_q(X/0)$	$\tilde{E}_q(X/1)$	$\tilde{E}_q(S/0)$	$\tilde{E}_q(S/1)$
0	1	7.0308	5.6438	12.7672	9.3106
0.1	0.9	7.6456	6.3978	13.7814	10.5545
0.2	0.8	8.8117	7.7034	15.7052	12.7083
0.3	0.7	11.2260	10.2584	19.6880	16.9233
0.4	0.6	17.6156	16.7949	30.2290	27.7065
0.5	0.5	59.9446	59.4246	100.0591	98.0328

Looking at the data in the table above, it is evident that the strategy q increases, and likewise the buffer's average fluid level and the buffer's average sojourn time increases considerably in fully unobservable cases. In partially unobservable case, if $\tilde{\lambda}q < \tilde{\mu}$ then the buffer's average fluid level and the buffer's average sojourn time increases in state $i = 0$ and decreases in state $i = 1$. If $\tilde{\lambda}q > \tilde{\mu}$ then the buffer's average fluid level and the buffer's average sojourn time increases in both the states $i = 0$ and $i = 1$.

6. Conclusion

This article proposes a centroid grading approach for trapezoidal neutrosophic fuzzy numbers. This classifies neutrosophic fuzzy numbers as tangible and possible. The continuous fluid split up into the discrete parts using the batch fluid method. Performance measures of the neutrosophic fuzzy fluid turn-on-off queue model with equilibrium strategies that can be obtained for fully unobservable and partially unobservable cases. An example is also supplied to highlight the effectiveness of the given on-off fluid queue paradigm.

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