Application of Neutrosophic implicative filters and Neutrosophic positive implicative filters in Lattice implication algebra

V. S. Naga Malleswari*[1], G. Luka[2], Bhagyalakshmi Kothuru[3], T. Srinivasa Rao[4], V. Amarendra Babu[5]

[1] Department of Freshman Engineering, PVP Siddhartha Institute of Technology, Vijayawada, India
[2] Department of Mathematics, Government Polytechnic for minorities, Guntur, India
[3]Department of Mathematics, KKR& KSR Institute of Technology& Sciences, Guntur, India
[4]Department of Mathematics, KL university, Vaddeswaram, India
[5]Department of Mathematics, Acharya Nagarjuna University, Guntur, India

Emails: vsnm.maths@gmail.com; gunturuluka@gmail.com; mblakshmi12@gmail.com; tsr_2505@kluniversity.in; amar521@gmail.com

Abstract

We introduced Neutrosophic implicative filters and Neutrosophic positive implicative filters in Lattice implication algebra. We proved some properties and equivalent conditions of both the filters. Finally we proved that "Every Neutrosophic positive implicative filter is a Neutrosophic implicative filter" and "Every Neutrosophic positive implicative filter is a Neutrosophic filter".

Keywords: Neutrosophic set (NS); Neutrosophic filter (NF); Neutrosophic implicative filter (NIF); Neutrosophic positive implicative filter (NPIF); Lattice implication algebra (LIA).

1.Introduction

Non-classical logic has developed into a major formal tool in computer science and artificial intelligence to deal with ambiguous and uncertain information. Non-classical logic has consistently advanced significantly in the direction of many valued logic, a substantial extension and development of classical logic [4]. In an effort to study the many-valued logical system whose propositional value is provided in a lattice, Xu [5] developed the concept of lattice implication algebras and examined some of its aspects in 1990. This logical algebra has since been extensively investigated by many academics. Lattice H implication algebras were first discussed by Xu and Qin in [31], and they represent an important class of lattice implication algebras. In [32], authors Xu and Qin examined filters and implicative filters in addition to presenting lattice inference algebras.

L.A.Zadeh[26] introduced the fuzzy set which studies about only truthiness and K.T.Atanassov[33] introduced intuitionistic fuzzy set which studies about truthiness and falsity. Gau et al. [34] introduced vague set which studies about truthiness and indeterminacy while the neutrosophic studies about truthiness, indeterminacy and falsity. The main distinction between Neutrosophic Set (NS) and all previous set theories are: a) the independence of all three neutrosophic components {truth-membership (T), indeterminacy-membership (I), falsehood-non membership (F)} with respect to each other in NS - while in the previous set theories their components are dependent of each other; and b) the importance of indeterminacy in NS - while in previous set theories indeterminacy is completely or partially ignored.

A filter or order filter is a special subset of a partially ordered set (poset). Many authors [27,28,29] introduced fuzzy filters, intuitionistic filters and vague filters in lattice implication algebra. We introduced Neutrosophic filters in Lattice implication algebra. In this paper we introduce Neutrosophic implicative

DOI: https://doi.org/10.54216/IJNS.210106
Received: January 24, 2023 Revised: April 12, 2023 Accepted: May 09, 2023

filters and Neutrosophic positive implicative filters in Lattice implication algebra. We proved some properties and equivalent conditions of both the filters. Finally we proved that “Every Neutrosophic positive implicative filter is a Neutrosophic implicative filter” and “Every Neutrosophic positive implicative filter is a Neutrosophic filter”.

2 Related Work

2.1. Definition [4]: Let X be the space of points (objects), with a generic element in X denoted by x. A Neutrosophic set A in X is characterized by a truth membership function T_A, indeterminate membership function I_A and falsity membership function F_A where T_A, I_A and F_A are real standard elements of [0,1]. It can be written as A=$\{\langle σ, (T_A(σ), I_A(σ), F_A(σ)/σ \in E, T_A, I_A, F_A \in]0^-, 1^+[\rangle\}$. There is no restriction on the sum of $T_A(σ), I_A(σ)$ and $F_A(σ)$ and so $0^- \leq T_A(σ) + I_A(σ) + F_A(σ) \leq 3^+$.

2.2. Definition [32]: By a lattice implication algebra we mean a bounded lattice (L,∨,∧,0,1) with order-reversing involution “ ’ ” and a binary operation “→” satisfying the following axioms:

(I1) σ → (ρ → δ) = ρ → (σ → δ)

(I2) σ → σ = 1,

(I3) σ → ρ = ρ’ → σ′

(I4) σ → ρ = ρ → σ = 1 ⇒σ = ρ

(I5) (σ → ρ) → ρ = (ρ →σ) → σ

(L1) (σ∨ρ) → δ = (σ → δ) ∧ (ρ → δ)

(L2) (σ∧ρ) → δ = (σ → δ) ∨ (ρ → δ), for allσ,ρ,δ∈ L.

If (L,∨,∧,0,1) satisfies the conditions (I1), (I2), (I3), (I4), and (I5), is called quasi-lattice implication algebra.

2.3. Definition [32]: Lattice implication algebra is called lattice H implication algebra if it satisfies σ∨ρ∨((σ∧ρ) → δ) = 1 for allσ,ρ,δ∈ L.

We can define a partial ordering ≤ on L by condition σ ≤ ρ if and only ifσ → ρ = 1. In a lattice implication algebra L, the following conditions hold[45]:

(a1) 0 → σ = 1, 1 → σ = σ and σ→ 1 = 1.

(a2) σ → ρ≤ (ρ → δ) → (σ→ ρ).

(a3) σ≤ ρ implies ρ → δ ≤ σ→ δ and δ → σ≤ δ → ρ.

(a4) σ′ = σ→ 0.

(a5) σ≤ (σ → ρ) → ρ.

And also in a lattice implication algebra L, the following hold [4]: for allσ,ρ,δ∈ L,

(a6) (σ → ρ) → ((ρ → δ) → (σ → δ)) = 1,

(a7) σ → ((σ → ρ) → ρ) = 1,

(a8) ((σ → ρ) → ρ) → ρ = σ → ρ.

2.4. Definition[32]:

A partial ordering ≤ on a lattice implication algebra L can be define by σ≤ρ if and only if σ→ρ=1, for allσ,∈L.

2.5. Definition[30]:

Let L be lattice implication algebra. Then the two binary operations ∨ and ∧ on lattice implication algebra L defined as follows, for allσ,ρ∈L,

1) σ∨ρ = (σ→ρ) →ρ = (ρ→σ) →σ
2) σ∧ρ = ((ρ→σ) →ρ′)′=((σ→ρ) →σ′)′

2.6. Definition [30]:

Let L be lattice implication algebra. Then a binary operation ⊗ (read as“times closed”) on L defined σ ⊗ρ = (σ→ρ′)′, σ⊕ρ=σ′→ρfor anyσ,ρ∈L.

2.7. Theorem [30]:

Let L be lattice implication algebra. Then the following holds for anyσ,ρ,δ∈L,

1) (σ⊗ρ′)′ =σ→ρ
2) σ⊗(σ→ρ) =(σ∧ρ) ≤ρ
3) σ⊗ρ=ρ⊗σ

DOI: https://doi.org/10.54216/IJNS.210106
Received: January 24, 2023 Revised: April 12, 2023 Accepted: May 09, 2023

4) (σ→ρ) ⊗σ≤ρ
5) σ→(ρ→(σ⊗ρ)) =1

2.8. Lemma[29]:

1) a ⊗b=(a∧b)≤a
2) a ≤b if and only if a⊗b'=0
3) a ⊗b= b⊗a
4) (a→b) ⊗a≤b
5) (a⊗b) →c= b→(a→c)

2.9. Lemma[29]:

In a Lattice implication algebra L,the following hold
(L8) σ→(ρ→δ)=(σ⊗ρ) →δ
(L9) (σ⊕ρ)'=σ'⊗ρ'
(L10) (σ⊗ρ)'=σ'⊕ρ'
(L11)0⊗σ=0, I⊗σ=σ, σ⊗σ'=0
(L12) 0⊕σ=0, 1⊕σ=σ, σ⊕σ'=0

2.10. Definition[32]:

Let (L,∨,∧, →) be a lattice implication algebra. A subset F of L is called a filter of L if it satisfies for all σ,ρ∈ L
(F1) 1 ∈ F,
(F2) σ∈ F and σ → ρ∈ F implyρ∈ F.
A subset F of L is called an implicative filter of L, if it satisfies (F1) and
(F3) σ → (ρ → δ) ∈ F and σ → ρ∈ F imply σ → δ∈ F for all σ, ρ,δ∈ L.

3. Neutrosophic implicative filters:

3.1. Definition: Let χ be the universe. A NS$A = \{\langle \sigma; T_A(\sigma), I_A(\sigma), F_A(\sigma) / \sigma \in X\}\rangle$ of a LIA L is called a neutrosophic implicative filter of L if it satisfies

(i) $T_A(1) \geq T_A(\sigma)$, $I_A(1) \geq I_A(\sigma)$, $F_A(1) \leq F_A(\sigma)$
(ii) $T_A(\sigma) \geq \min\{T_A(\delta \longrightarrow ((\sigma \longrightarrow \rho) \longrightarrow \sigma)), T_A(\delta)\}$
(iii) $I_A(\sigma) \geq \min\{I_A(\delta \longrightarrow ((\sigma \longrightarrow \rho) \longrightarrow \sigma)), I_A(\delta)\}$
(iv) $F_A(\sigma) \leq \max\{F_A(\delta \longrightarrow ((\sigma \longrightarrow \rho) \longrightarrow \sigma)), F_A(\delta)\}$

3.2. Example:

Let L= {0, a, b, c, 1}. Define partial order on L as 0 <a<b<c< 1 and define$\sigma \wedge \varrho = \min\{\sigma, \varrho\}$ and $\sigma \vee \varrho = \max\{\sigma, \varrho\}$ for all $\sigma, \varrho \in L$ and " ' "and "⟶" as follows.

Table 1: Complement table

Σ	0	a	b	c	1
σ'	1	c	b	a	0

Table 2: Implication table

⟶	**0**	a	b	c	**1**
0	1	1	1	1	1
a	c	1	1	1	1
b	b	c	1	1	1
c	a	b	c	1	1
1	0	a	b	c	1

Then (L,∨,∧,' , ⟶) is LIA.
Define NS Aon L by
Table 3: Truth values of a Neutrosophic set

L	**0**	a	b	c	**1**

DOI: https://doi.org/10.54216/IJNS.210106
Received: January 24, 2023 Revised: April 12, 2023 Accepted: May 09, 2023

$\boldsymbol{T_A}(\sigma)$	0.5	0.5	0.7	0.7	0.7
$I_A(\sigma)$	0.2	0.2	0.3	0.3	0.3
$\boldsymbol{F_A}(\sigma)$	0.7	0.7	0.7	0.5	0.5

Then A is a NIF on L.

3.3. Theorem: In lattice implication algebra, every NIF is a NF.

Proof: Consider a NSA = $\{\langle\sigma; T_A(\sigma), I_A(\sigma), F_A(\sigma)/\sigma \in X\}\rangle$ of a LIA L.

Let A be a NIF of L then

(i) $T_A(1) \geq T_A(\sigma)$, $I_A(1) \geq I_A(\sigma)$, $F_A(1) \leq F_A(\sigma)$

(ii) $T_A(\sigma) \geq \min\{T_A(\delta \longrightarrow ((\sigma \longrightarrow \rho) \longrightarrow \sigma)), T_A(\delta)\}$

(iii) $I_A(\sigma) \geq \min\{I_A(\delta \longrightarrow ((\sigma \longrightarrow \rho) \longrightarrow \sigma)), I_A(\delta)\}$

(iv) $F_A(\sigma) \leq \max\{F_A(\delta \longrightarrow ((\sigma \longrightarrow \rho) \longrightarrow \sigma)), F_A(\delta)\}$

Take $\varrho = \varsigma$ in (ii), (iii) and (iv) then, we have

$T_A(\sigma) \geq \min\{T_A(\delta \longrightarrow ((\sigma \longrightarrow \sigma) \longrightarrow \sigma)), T_A(\delta)\}$

$I_A(\sigma) \geq \min\{I_A(\delta \longrightarrow ((\sigma \longrightarrow \sigma) \longrightarrow \sigma)), I_A(\delta)\}$

$F_A(\sigma) \leq \max\{F_A(\delta \longrightarrow ((\sigma \longrightarrow \sigma) \longrightarrow \sigma)), F_A(\delta)\}$

Since $\sigma \longrightarrow \sigma = 1$ then

$T_A(\sigma) \geq \min\{T_A(\delta \longrightarrow (1 \longrightarrow \sigma)), T_A(\delta)\}$

$I_A(\sigma) \geq r\min\{I_A(\delta \longrightarrow (1 \longrightarrow \sigma)), I_A(\delta)\}$

$F_A(\sigma) \leq \max\{F_A(\delta \longrightarrow (1 \longrightarrow \sigma)), F_A(\delta)\}$

Since $1 \longrightarrow \sigma = \sigma$ then

$T_A(\sigma) \geq \min\{T_A(\delta \longrightarrow \sigma)), T_A(\delta)\}$

$I_A(\sigma) \geq r\min\{I_A(\delta \longrightarrow \sigma)), I_A(\delta)\}$

$F_A(\sigma) \leq \max\{F_A(\delta \longrightarrow \sigma)), F_A(\delta)\}$

Then A is a NF.

Note: Converse of the theorem need not be true. It follows from the below example.

3.4. Example: Let the Cayley table of a lattice L = {0, a,b, 1} as follows:

For all $\sigma \in$ L

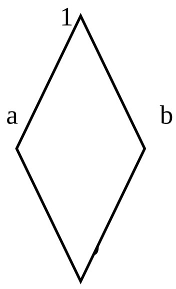

Table 4: Complement table

σ	σ'
0	1
a	b
b	a
1	0

Table 5: Implication table

$\rightarrow$	0	a	b	1
0	1	1	1	1
a	1	1	a	1
b	1	b	1	1
1	0	a	b	1

The operators V and Λ on a Lattice 'L' are defined as given below:

$\sigma \vee \rho = (\sigma \rightarrow \rho) \rightarrow \rho, \sigma \wedge \rho = ((\sigma' \rightarrow \rho') \rightarrow \rho')'$

For all σ, ρ ∈L. Then (L,V,Λ,→,')is a LIA.

Suppose A be an NS in 'L'defined by $T_A(0)=0.4$ and

$T_A(a)= 0.5$, $T_A(b)=T_A(1)=0.7$. And so A is a NF of 'L'.

But it is not an NIF of 'L'since for a, b, 1∈L

$T_A(a) \ngeq \min\{T_A(1 \longrightarrow ((a \longrightarrow b) \longrightarrow a)), T_A(1)\}$

$T_A(a) \ngeq \min\{T_A(1), T_A(1)\}$

DOI: https://doi.org/10.54216/IJNS.210106
Received: January 24, 2023 Revised: April 12, 2023 Accepted: May 09, 2023

$0.5 \ngeq 0.7$
Therefore the converse of the above theorem is not true.
3.5.Theorem: Let A be a NF of a LIA L. If A is a NIF then

(i) $T_A(\sigma) \geq T_A((\sigma \longrightarrow \varrho) \longrightarrow \sigma)$
(ii) $I_A(\sigma) \geq I_A((\sigma \longrightarrow \varrho) \longrightarrow \sigma)$
(iii) $F_A(\sigma) \leq F_A((\sigma \longrightarrow \varrho) \longrightarrow \sigma)$

Proof: Let A be a NIF of L. Then, we have
$T_A(\sigma) \geq \min\{T_A(1 \longrightarrow ((\sigma \longrightarrow \varrho) \longrightarrow \sigma)), T_A(1)\} = M_A((\sigma \longrightarrow \varrho) \longrightarrow \sigma)$
$I_A(\sigma) \geq \mathrm{rmin}\{I_A(1 \longrightarrow ((\sigma \longrightarrow \varrho) \longrightarrow \sigma)), I_A(1)\} = I_A((\sigma \longrightarrow \varrho) \longrightarrow \sigma)$
$F_A(\sigma) \leq \max\{F_A(1 \longrightarrow ((\sigma \longrightarrow \rho) \longrightarrow \sigma)), F_A(1)\} = F_A((\sigma \longrightarrow \rho) \longrightarrow \sigma)$ for all $\sigma, \rho \in L$
Hence proved.
3.6. Theorem: Let A be a NF of a LIA L. If $T_A(\sigma) = T_A((\sigma \longrightarrow \rho) \longrightarrow \sigma), I_A(\sigma) = I_A((\sigma \longrightarrow \rho) \longrightarrow \sigma)$, $F_A(\sigma) = F_A((\sigma \longrightarrow \rho) \longrightarrow \sigma)$ then A is a NIF of L.
Proof: Suppose that $T_A(\sigma) = T_A((\sigma \longrightarrow \rho) \longrightarrow \sigma), I_A(\sigma) = I_A((\sigma \longrightarrow \rho) \longrightarrow \sigma)$,
$F_A(\sigma) = F_A((\sigma \longrightarrow \rho) \longrightarrow \sigma)$
(i)Since A is a NF of a LIA L then we have
$T_A((\sigma \longrightarrow \rho) \longrightarrow \sigma) \geq \min\{T_A(\delta \longrightarrow ((\sigma \longrightarrow \rho) \longrightarrow \sigma)), T_A(\delta)\}$
$I_A\big((\sigma \longrightarrow \rho) \longrightarrow \sigma)\big) \geq \mathrm{rmin}\{I_A(\delta \longrightarrow ((\sigma \longrightarrow \rho) \longrightarrow \sigma)), I_A(\delta)\}$
$F_A\big((\sigma \longrightarrow \rho) \longrightarrow \sigma)\big) \leq \max\{F_A(\delta \longrightarrow ((\sigma \longrightarrow \rho) \longrightarrow \sigma)), F_A(\delta)\}$for all $\sigma, \rho \in L$(ii)
Hence by (i) and (ii)
$T_A(\sigma) \geq \min\{T_A(\delta \longrightarrow ((\sigma \longrightarrow \rho) \longrightarrow \sigma)), T_A(\delta)\}$
$I_A(\sigma) \geq \mathrm{rmin}\{I_A(\delta \longrightarrow ((\sigma \longrightarrow \rho) \longrightarrow \sigma)), I_A(\delta)\}$
$F_A(\sigma) \leq \max\{F(\delta \longrightarrow ((\sigma \longrightarrow \rho) \longrightarrow \sigma)), F_A(\delta)\}$for all $\sigma, \rho \in L$
Therefore A is a NIF of L.
3.7. Theorem: Let L be LIA and A be a NSof L. If A is a NIF then the following statements are satisfied and equivalent.

(i) A is a NF of L and for any $\sigma, \rho, \delta \in L$
$T_A(\sigma \longrightarrow \rho) \geq T_A(\sigma \longrightarrow (\sigma \longrightarrow \rho))$
$I_A(\sigma \longrightarrow \rho) \geq I_A(\sigma \longrightarrow (\sigma \longrightarrow \rho))$
$F_A(\sigma \longrightarrow \rho) \leq F_A(\sigma \longrightarrow (\sigma \longrightarrow \rho))$

(ii) A is a NF of L and for any $\sigma, \rho, \delta \in L$
$T_A\big((\sigma \longrightarrow \rho) \longrightarrow (\sigma \longrightarrow \delta)\big) \geq T_A(\sigma \longrightarrow (\sigma \longrightarrow \rho))$
$I_A\big((\sigma \longrightarrow \rho) \longrightarrow (\sigma \longrightarrow \delta)\big) \geq I_A(\sigma \longrightarrow (\sigma \longrightarrow \rho)$
$F_A\big((\sigma \longrightarrow \rho) \longrightarrow (\sigma \longrightarrow \delta)\big) \leq F_A(\sigma \longrightarrow (\sigma \longrightarrow \rho)$

(iii) $T_A(1) \geq T_A(\sigma)$, $I_A(1) \geq I_A(\sigma)$, $F_A(1) \leq F_A(\sigma)$
$T_A(\sigma \longrightarrow \rho) \geq \min\{T_A(\delta \longrightarrow (\sigma \longrightarrow (\sigma \longrightarrow \rho))), T_A(\delta)\}$
$I_A(\sigma \longrightarrow \rho) \geq \mathrm{rmin}\{I_A(\delta \longrightarrow (\sigma \longrightarrow (\sigma \longrightarrow \rho))), I_A(\delta)\}$
$F_A(\sigma \longrightarrow \rho) \leq \max\{F_A(\delta \longrightarrow (\sigma \longrightarrow (\sigma \longrightarrow \rho))), F_A(\delta)\}$

Proof: A is a NF of L and clearly
$T_A(\sigma \longrightarrow \rho) \geq T_A(\sigma \longrightarrow (\sigma \longrightarrow \rho))$
$I_A(\sigma \longrightarrow \rho) \geq I_A(\sigma \longrightarrow (\sigma \longrightarrow \rho))$
$F_A(\sigma \longrightarrow \rho) \leq F_A(\sigma \longrightarrow (\sigma \longrightarrow \rho))$
Assume that (i) holds
Since for any $\sigma, \rho, \in L$, $T_A(\sigma \longrightarrow \rho) \geq T_A(\sigma \longrightarrow (\sigma \longrightarrow \rho))$
$I_A(\sigma \longrightarrow \rho) \geq I_A(\sigma \longrightarrow (\sigma \longrightarrow \rho))$
$F_A(\sigma \longrightarrow \rho) \leq F_A(\sigma \longrightarrow (\sigma \longrightarrow \rho))$
Then $T_A\big((\sigma \longrightarrow \rho) \longrightarrow (\sigma \longrightarrow \delta)\big) = T_A(\sigma \longrightarrow ((\sigma \longrightarrow \rho) \longrightarrow \delta))$
$\geq T_A(\sigma \longrightarrow (\sigma \longrightarrow ((\sigma \longrightarrow \rho) \longrightarrow \delta))$
$I_A\big((\sigma \longrightarrow \rho) \longrightarrow (\sigma \longrightarrow \delta)\big) = I_A(\sigma \longrightarrow ((\sigma \longrightarrow \rho) \longrightarrow \delta))$
$\geq I_A(\sigma \longrightarrow (\sigma \longrightarrow ((\sigma \longrightarrow \rho) \longrightarrow \delta))$
$F_A\big((\sigma \longrightarrow \rho) \longrightarrow (\sigma \longrightarrow \delta)\big) = F_A(\sigma \longrightarrow ((\sigma \longrightarrow \rho) \longrightarrow \delta))$
$\geq F_A(\sigma \longrightarrow (\sigma \longrightarrow ((\sigma \longrightarrow \rho) \longrightarrow \delta))$
And $(\sigma \longrightarrow (\sigma \longrightarrow ((\sigma \longrightarrow \rho) \longrightarrow \delta)) = \sigma \longrightarrow \big((\sigma \longrightarrow \rho) \longrightarrow (\sigma \longrightarrow \delta)\big)$
$= \sigma \longrightarrow \big((\rho \longrightarrow \sigma) \longrightarrow (\delta \longrightarrow \sigma)\big) \geq \sigma \longrightarrow (\rho \longrightarrow \delta)$

DOI: https://doi.org/10.54216/IJNS.210106
Received: January 24, 2023 Revised: April 12, 2023 Accepted: May 09, 2023

By the above theorem $T_A((\sigma \to \rho) \to (\sigma \to \delta)) \geq T_A(\sigma \to (\rho \to \rho))$
$I_A((\sigma \to \rho) \to (\sigma \to \delta)) \geq I_A(\sigma \to (\rho \to \rho)$
$F_A((\sigma \to \rho) \to (\sigma \to \delta)) \leq F_A(\sigma \to (\rho \to \rho)$
Therefore (ii) holds
Now consider (ii) then $T_A(1) \geq T_A(\sigma)$, $I_A(1) \geq I_A(\sigma)$, $F_A(1) \leq F_A(\sigma)$ is obvious
For any$\sigma, \rho, \delta \in L$, Since$T_A((\sigma \to \rho) \to (\sigma \to \delta)) \geq T_A(\sigma \to (\rho \to \rho))$
$I_A((\sigma \to \rho) \to (\sigma \to \delta)) \geq I_A(\sigma \to (\rho \to \rho)$
$F_A((\sigma \to \rho) \to (\sigma \to \delta)) \leq F_A(\sigma \to (\rho \to \rho)$
We get $T_A(\sigma \to \delta) \geq T_A(\sigma \to (\sigma \to \delta))$
$I_A(\sigma \to \delta) \geq I_A(\sigma \to (\sigma \to \delta))$
$F_A(\sigma \to \delta) \leq F_A(\sigma \to (\sigma \to \delta))$
Where $\rho = \sigma$
Hence for any$\sigma, \rho, \delta \in L$,
$T_A(\sigma \to \rho) \geq T_A(\sigma \to (\sigma \to \rho))$
$I_A(\sigma \to \rho) \geq I_A(\sigma \to (\sigma \to \rho))$
$F_A(\sigma \to \rho) \leq F_A(\sigma \to (\sigma \to \rho))$
Since A is a NF of L then $T_A(\sigma \to (\sigma \to \rho)) \geq \min\{T_A\left(\delta \to ((\sigma \to (\sigma \to \rho))\right), T_A(\delta)\}$
$I_A(\sigma \to (\sigma \to \rho)) \geq \min\{I_A\left(\delta \to ((\sigma \to (\sigma \to \rho))\right), I_A(\delta)\}$
$F_A(\sigma \to (\sigma \to \rho)) \leq \max\{F_A\left(\delta \to ((\sigma \to (\sigma \to \rho))\right), F_A(\delta)\}$
It follows that for any $\sigma, \rho, \delta \in L$
$T_A(\sigma \to \rho) \geq \min\{T_A(\delta \to (\sigma \to (\sigma \to \rho))), T_A(\delta)\}$
$I_A(\sigma \to \rho) \geq \min\{I_A(\delta \to (\sigma \to (\sigma \to \rho))), I_A(\delta)\}$
$F_A(\sigma \to \rho) \leq \max\{F_A(\delta \to (\sigma \to (\sigma \to \rho))), F_A(\delta)\}$
Hence (iii) holds.
Assume that (iii) holds
For any $\sigma, \rho, \delta \in L$,
$T_A(\sigma \to \rho) \geq \min\{T_A(\delta \to (\sigma \to (\sigma \to \rho))), T_A(\delta)\}$
$I_A(\sigma \to \rho) \geq \min\{I_A(\delta \to (\sigma \to (\sigma \to \rho))), I_A(\delta)\}$
$F_A(\sigma \to \rho) \leq \max\{F_A(\delta \to (\sigma \to (\sigma \to \rho))), F_A(\delta)\}$
If $\sigma = 1$ it follows that
$T_A(1 \to \rho) \geq \min\{T_A(\delta \to (1 \to (1 \to \rho))), T_A(\delta)\}$
$I_A(1 \to \rho) \geq \min\{I_A(\delta \to (1 \to (1 \to \rho))), I_A(\delta)\}$
$F_A(1 \to \rho) \leq \max\{F_A(\delta \to (1 \to (1 \to \rho))), F_A(\delta)\}$
Then $T_A(\rho) \geq \min\{T_A(\delta \to \rho), T_A(\delta)\}$
$I_A(\rho) \geq \min\{I_A(\delta \to \rho), I_A(\delta)\}$
$F_A(\rho) \leq \max\{F_A(\delta \to \rho), F_A(\delta)\}$
And $T_A(1) \geq T_A(\sigma)$, $I_A(1) \geq I_A(\sigma)$, $F_A(1) \leq F_A(\sigma)$
Hence A is a NF of L.
Since for $\sigma, \rho, \delta \in L$,
$T_A(\sigma \to \rho) \geq \min\{T_A(\delta \to (\sigma \to (\sigma \to \rho))), T_A(\delta)\}$
$I_A(\sigma \to \rho) \geq \min\{I_A(\delta \to (\sigma \to (\sigma \to \rho))), I_A(\delta)\}$
$F_A(\sigma \to \rho) \leq \max\{F_A(\delta \to (\sigma \to (\sigma \to \rho))), F_A(\delta)\}$
Take $\delta = 1$ then
$T_A(\sigma \to \rho) \geq \min\{T_A(1 \to (\sigma \to (\sigma \to \rho))), T_A(1)\}$
$I_A(\sigma \to \rho) \geq r\min\{I_A(1 \to (\sigma \to (\sigma \to \rho))), I_A(1)\}$
$F_A(\sigma \to \rho) \leq \max\{F_A(1 \to (\sigma \to (\sigma \to \rho))), F_A(1)\}$
Then $T_A(\sigma \to \rho) \geq T_A((\sigma \to (\sigma \to \rho))$
$I_A(\sigma \to \rho) \geq I_A((\sigma \to (\sigma \to \rho))$
$J_A(\sigma \to \rho) \leq J_A(\sigma \to (\sigma \to \rho)))$
Hence (i) holds.

3.8.Theorem: Let L be LIA. A NSA in L is a NIF if and only if it satisfies $\delta \otimes u \leq (\sigma \to \rho) \to \sigma$ implies

(i) $T_A(\sigma) \geq \min\{T_A(\delta), T_A(u)\}$

DOI: https://doi.org/10.54216/IJNS.210106
Received: January 24, 2023 Revised: April 12, 2023 Accepted: May 09, 2023

(ii) $I_A(\sigma) \geq r \min\{I_A(\delta), I_A(u)\}$
(iii) $F_A(\sigma) \leq \max\{F_A(\delta), F_A(u)\}$ for all $\sigma, \rho, \delta, u \in L$

Proof: Suppose that A is a NIF of L and let $\sigma, \rho, \delta, u \in L$ be such that $\delta \otimes u \leq (\sigma \longrightarrow \rho) \longrightarrow \sigma$.
Since A is a NF of L by above theorem it follows that $T_A(\sigma) \geq T_A((\sigma \longrightarrow \rho) \longrightarrow \sigma) \geq T_A(\delta \otimes u) \geq \min\{T_A(\delta), T_A(u)\}$
$I_A(\sigma) \geq I_A((\sigma \longrightarrow \rho) \longrightarrow \sigma) \geq I_A(\delta \otimes u) \geq r\min\{I_A(\delta), I_A(u)\}$
$F_A(\sigma) \leq F_A((\sigma \longrightarrow \rho) \longrightarrow \sigma) \leq F_A(\delta \otimes u) \leq \max\{F_A(\delta), F_A(u)\}$
Conversely suppose that if $\delta \otimes u \leq (\sigma \longrightarrow \rho) \longrightarrow \sigma$ implies
(i) $T_A(\sigma) \geq \min\{T_A(\delta), T_A(u)\}$
(ii) $I_A(\sigma) \geq \min\{I_A(\delta), I_A(u)\}$
(iii) $F_A(\sigma) \leq \max\{F_A(\delta), F_A(u)\}$ for all $\sigma, \rho, \delta, u \in L$
Obviously A satisfies $T_A(1) \geq T_A(\sigma)$, $I_A(1) \geq I_A(\sigma)$, $F_A(1) \leq F_A(\sigma)$
Since $(\delta \longrightarrow ((\sigma \longrightarrow \rho) \longrightarrow \sigma) \otimes \delta \leq (\sigma \longrightarrow \rho) \longrightarrow \sigma$ implies that
$T_A(\sigma) \geq \min\{T_A(\delta \longrightarrow ((\sigma \longrightarrow \rho) \longrightarrow \sigma)), T_A(\delta)\}$
$I_A(\sigma) \geq \min\{I_A(\delta \longrightarrow ((\sigma \longrightarrow \rho) \longrightarrow \sigma)), I_A(\delta)\}$
$F_A(\sigma) \leq \max\{F_A(\delta \longrightarrow ((\sigma \longrightarrow \rho) \longrightarrow \sigma)), F_A(\delta)\}$
Therefore A is NIF of L.

4. Neutrosophic positive implicative filters:

4.1. Definition: Let χ be the universe. A NSA $= \{\langle \sigma; T_A(\sigma), I_A(\sigma), F_A(\sigma) / \sigma \in X \rangle\}$ of a LIA L is called a Neutrosophic positive implicative filter of L if it satisfies

i. $T_A(1) \geq T_A(\sigma)$, $I_A(1) \geq I_A(\sigma)$, $F_A(1) \leq F_A(\sigma)$
ii. $T_A(\rho) \geq \min\{T_A(\sigma \longrightarrow ((\rho \longrightarrow \delta) \longrightarrow \rho)), T_A(\sigma)\}$
iii. $I_A(\rho) \geq \min\{I_A(\sigma \longrightarrow ((\rho \longrightarrow \delta) \longrightarrow \rho)), I_A(\sigma)\}$
iv. $F_A(\rho) \leq \max\{F_A(\sigma \longrightarrow ((\rho \longrightarrow \delta) \longrightarrow \rho)), F_A(\sigma)\}$

4.2. Example: Let L= $\{0, a, b, c, d, 1\}$. Define partial order on L as $0 < a < b < c < d < 1$,
$\sigma \wedge \varrho = \min\{\sigma, \varrho\}$, $\sigma \vee \varrho = \max\{\sigma, \varrho\}$ for all $\sigma, \varrho \in L$, "'" and "⟶" as follows

Table 6: Complement table

Σ	0	a	b	c	d	1
σ'	1	c	d	a	b	0

Table 7: Implication table
Then (L,∨,∧,′ ,⟶) is LIA.

⟶	**0**	***a***	***b***	***c***	***d***	**1**
0	1	1	1	1	1	1
a	c	1	b	c	b	1
b	d	a	1	b	a	1
c	a	b	c	1	1	1
d	b	1	1	b	1	1
1	0	a	b	c	d	1

Define NS Aon L by is a NPIF on L.
Table 8: Truth values of a Neutrosophic set

L	**0**	***a***	***b***	***c***	d	**1**
$T_A(\sigma)$	0.4	0.8	0.4	0.4	0.4	0.8
$I_A(\sigma)$	0.6	0.2	0.2	0.2	0.2	0.6
$F_A(\sigma)$	0.3	0.1	0.3	0.3	0.3	0.1

DOI: https://doi.org/10.54216/IJNS.210106
Received: January 24, 2023 Revised: April 12, 2023 Accepted: May 09, 2023

4.3. Theorem: Every NPIF of LIA is a NF.

Proof: Let A be a NPIFof a LIA L. Then $T_A(1) \geq T_A(\sigma)$, $I_A(1) \geq I_A(\sigma)$, $F_A(1) \leq F_A(\sigma)$ and Let δ =ρ then $T_A(\rho) \geq \min\{T_A(\sigma \longrightarrow ((\rho \longrightarrow \rho) \longrightarrow \rho)), T_A(\sigma)\}$ $=\min\{T_A(\sigma \longrightarrow (1 \longrightarrow \rho)), T_A(\sigma)\}$
$=\min\{T_A(\sigma \longrightarrow \rho), T_A(\sigma)\}$

$I_A(\rho) \geq \min\{I_A(\sigma \longrightarrow ((\rho \longrightarrow \rho) \longrightarrow \rho)), I_A(\sigma)\} = \min\{I_A(\sigma \longrightarrow (1 \longrightarrow \rho)), I_A(\sigma)\}$
$=\min\{I_A(\sigma \longrightarrow \rho), I_A(\sigma)\}$

$F_A(\rho) \leq \max\{F_A(\sigma \longrightarrow ((\rho \longrightarrow \rho) \longrightarrow \rho)), F_A(\sigma)\} = \max\{F_A(\sigma \longrightarrow (1 \longrightarrow \rho)), F_A(\sigma)\}$
$=\max\{F_A(\sigma \longrightarrow \rho), F_A(\sigma)\}$

Therefore A is a Neutrosophic filter.

Note: Converse of the above theorem is need not be true. It will be followed by the below example.

4.4.Example:

Let the Cayley table of a lattice L = {0,a, b, 1} as follows:

For all $\varsigma \in$ L

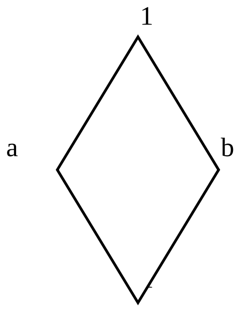

Table 9: Complement set

ς	ς'
0	1
a	b
b	a
1	0

Table 10: Implication table

→	**0**	a	b	**1**
0	1	1	1	1
a	1	1	a	1
b	1	b	1	1
1	0	a	b	1

The operators V and Λon a Lattice 'L' are defined as given below:
$\sigma \vee \rho = (\sigma \rightarrow \rho) \rightarrow \rho, \sigma \wedge \rho = ((\sigma' \rightarrow \rho') \rightarrow \rho')'$
For all σ, ρ ∈ L. Then (L,V,Λ,→,′) is a LIA.
Suppose A be anN set in 'L'defined by $T_A(0)$=0.4 and
$T_A(a)$= 0.7, $T_A(b)$= 0.5, $T_A(1)$=0.7. And so A is a NFof 'L'.
But it is not a NPIF of 'L' since
For a, b, 1∈ L
$T_A(b) \ngeq \min\{T_A(a \longrightarrow ((b \longrightarrow 1) \longrightarrow b)), T_A(a)\}$
$T_A(b) \ngeq \min\{T_A(a)), T_A(a)\}$
$T_A(b) \ngeq \min\{T_A(a)\}$
$0.5 \ngeq 0.7$

DOI: https://doi.org/10.54216/IJNS.210106
Received: January 24, 2023 Revised: April 12, 2023 Accepted: May 09, 2023

Then A is not a NPIF on L though it is a NF.

4.5. Theorem: Let A be a NF of a LIA L. Then A is a NPIF of L if and only if it satisfies

$$T_A(\sigma) \geq T_A((\sigma \longrightarrow \rho) \longrightarrow \sigma)$$
$$I_A(\sigma) \geq I_A((\sigma \longrightarrow \rho) \longrightarrow \sigma)$$
$$F_A(\sigma) \leq F_A((\sigma \longrightarrow \rho) \longrightarrow \sigma)$$

Proof: Assume that A is a NPIF of L and let $\sigma, \rho \in$ L.

Then $T_A(\sigma) \geq \min\{T_A(1 \longrightarrow ((\sigma \longrightarrow \rho) \longrightarrow \sigma)), T_A(1)\}$=min $\{T_A((\sigma \longrightarrow \rho) \longrightarrow \sigma)), T_A(1)\}$
$=T_A((\sigma \longrightarrow \rho) \longrightarrow \sigma)$

$I_A(\sigma) \geq \min\{I_A(1 \longrightarrow ((\sigma \longrightarrow \rho) \longrightarrow \sigma)), I_A(1)\}$= min $\{I_A((\sigma \longrightarrow \rho) \longrightarrow \sigma)), I_A(1)\}$
$=I_A((\sigma \longrightarrow \rho) \longrightarrow \sigma)$

$F_A(\sigma) \leq \max\{F_A(1 \longrightarrow ((\sigma \longrightarrow \rho) \longrightarrow \sigma)), F_A(1)\}$=max $\{F_A((\sigma \longrightarrow \rho) \longrightarrow \sigma)), F_A(1)\}$
$=F_A((\sigma \longrightarrow \rho) \longrightarrow \sigma)$

Conversely suppose that $T_A(\sigma) \geq T_A((\sigma \longrightarrow \rho) \longrightarrow \sigma)$
$I_A(\sigma) \geq I_A((\sigma \longrightarrow \rho) \longrightarrow \sigma)$
$F_A(\sigma) \leq F_A((\sigma \longrightarrow \rho) \longrightarrow \sigma)$

Since $T_A(\sigma) \geq T_A((\sigma \longrightarrow \rho) \longrightarrow \sigma) \geq$min $\{T_A(\sigma \longrightarrow ((\rho \longrightarrow \delta) \longrightarrow \rho)), T_A(\sigma)\}$
Since $I_A(\sigma) \geq I_A((\sigma \longrightarrow \rho) \longrightarrow \sigma) \geq$min $\{I_A(\sigma \longrightarrow ((\rho \longrightarrow \delta) \longrightarrow \rho)), I_A(\sigma)\}$
Since $F_A(\sigma) \leq F_A((\sigma \longrightarrow \rho) \longrightarrow \sigma) \leq$max $\{F_A(\sigma \longrightarrow ((\rho \longrightarrow \delta) \longrightarrow \rho)), F_A(\sigma)\}$
HenceA is a NPIF of L.

4.6. Theorem: Every NPIF of LIA is a NIF.

Proof: Let A be a NPIFof a LIA L.
Then A is a NFof a LIA L.
We note that $(\sigma \longrightarrow(\varrho \longrightarrow \varphi)) \rightarrow ((\sigma \rightarrow \varrho) \rightarrow (\sigma \rightarrow (\sigma \rightarrow \varphi))) = 1$
And $T_A(\delta) \geq \min\{T_A(\sigma), T_A(\rho)\}$
$I_A(\delta) \geq \min\{I_A(\sigma), I_A(\rho)\}$,
$F_A(\delta) \leq \max\{F_A(\sigma), F_A(\rho)\}$
On the other hand $((\sigma \rightarrow \delta) \rightarrow \delta) \rightarrow (\sigma \rightarrow \delta) = \sigma \rightarrow (\sigma \rightarrow \delta)$
we obtain $T_A(\sigma \longrightarrow \delta) \geq T_A(((\sigma \longrightarrow \delta) \rightarrow \delta) \rightarrow (\sigma \longrightarrow \delta))$
$= T_A(\sigma \rightarrow (\sigma \rightarrow \delta)) \geq \min\{T_A(\sigma \rightarrow (\rho \rightarrow \delta), T_A(\sigma \rightarrow \rho)\}$
$I_A(\sigma \longrightarrow \delta) \geq I_A(((\sigma \longrightarrow \delta) \rightarrow \delta) \rightarrow (\sigma \longrightarrow \delta))$
$=I_A(\sigma \rightarrow (\sigma \rightarrow \delta)) \geq \min\{I_A(\sigma \rightarrow (\rho \rightarrow \delta), I_A(\sigma \rightarrow \rho)\}$
$F_A(\sigma \longrightarrow \delta) \leq F_A(((\sigma \longrightarrow \delta) \rightarrow \delta) \rightarrow (\sigma \longrightarrow \delta))$
$=F_A(\sigma \rightarrow (\sigma \rightarrow \delta)) \leq \max\{F_A(\sigma \rightarrow (\rho \rightarrow \delta), F_A(\sigma \rightarrow \rho)\}$
Hence A is a NIF.

5. Conclusion

In this paper we introduced Neutrosophic implicative filters and Neutrosophic positive implicative filters in lattice implication algebra. We derive relation between both the filters. We have given some characteristics and some equivalent properties for both the filters. Probing more profound, the results in this paper also provide a strong foundation for future work in logical algebraic structure and in Neutrosophic set. One area of future work is in combining some other kind of subalgebra fantastic filter and associative filter etc with Neutrosophic sets. Another area is in applying the results studied here to the other algebraic structures like BCI/BCK algebras. Future work will be in these two areas.

Funding: "This research received no external funding"
Conflicts of Interest: "The authors declare no conflict of interest."

References

[1] C. A. C. Sweety, I. Arockiarani, Rough sets in neutrosophic approximation space, Annals of Fuzzy Mathematics and Informatics, 13 (4) (2017), 449–463.
[2] E. H. Roh, S. Y. Kim, Y. Xu and Y. B. Jun, Some operations on lattice implication algebras, International J. Of Mathematics and Mathematical Sciences 27(2001), no. 1,45-52.
[3] F. Smarandache, A unifying field in logics. Neutrosophy: Neutrosophic probability, set and logic, Rehoboth: American Research Press (1999).
[4] F. Smarandache, Neutrosophic set, a generalization of intuitionistic fuzzy sets, International Journal of Pure and Applied Mathematics, 24(5) (2005), 287–297.

DOI: https://doi.org/10.54216/IJNS.210106
Received: January 24, 2023 Revised: April 12, 2023 Accepted: May 09, 2023

[5] J. Liu and Y. Xu, Filters and structure of lattice implication algebras, Chinese Science Bulletin 42(18), 1517–1520, 1997.
[6] Y.B. Jun, Neutrosophic subalgebras of several types in BCK/BCI-algebras, Annals of Fuzzy Mathematics and Informatics, 14(1) (2017), 75–86.
[7] Y.B. Jun, S.J. Kim and F. Smarandache, Interval neutrosophic sets with applications in BCK/BCI-algebra, Axioms 2018, 7, 23; doi:10.3390/axioms7020023.
[8] Y.B. Jun, F. Smarandache and H. Bordbar, Neutrosophic N -structures applied to BCK/BCIalgebras, Information 2017, 8, 128; doi:10.3390/info8040128.
[9] Y.B. Jun, F. Smarandache, S.Z. Song and M. Khan, Neutrosophic positive implicative N - ideals in BCK/BCI-algebras, Axioms 2018, 7, 3; doi:10.3390/axioms7010003.
[10] R.A. Borzooei, X.H. Zhang, F. Smarandache and Y.B. Jun, Commutative generalized neutrosophic ideals in BCK-algebras, Symmetry 2018, 10, 350; doi:10.3390/sym10080350.
[11] R. A. Borzooei, H. Farahani and M. Moniri, Neutrosophic Deductive Filters on BLAlgebras, Journal of Intel- ligent and Fuzzy Systems, 26(6), (2014), 2993-3004.
[12] R.A. Borzooei, M. Mohseni Takallo, F. Smarandache nad Y.B. Jun, Positive implicative BMBJ-neutrosophic ideals in BCK-algebras, Neutrosophic Sets and Systems, 23 (2018), 148-163.
[13] Y.B. Jun, On LI-ideals and prime LI-ideals of lattice implication algebras, Journal of the Korean Mathematical Society, 36(2) (1999), 369–380.
[14] V. Chang, M. Abdel-Basset, M. Ramachandran, Towards a reuse strategic decision pattern framework–from theories to practices, Information Systems Frontiers, 21(1) (2019), 27-44. International journal of neutrosophic sceince(IJNS)Vol. x, No. y, PP. a-b, year Doi :https://doi.org/10.54216/IJNS.xxxxxx Received: July 20, 2021Accepted: Jan 04, 2022 47
[15] N. A. Nabeeh, F. Smarandache, M. Abdel-Basset, H. A. El-Ghareeb, A. Aboelfetouh, An integrated neutrosophic-topsis approach and its application to personnel selection: A new trend in brain processing and analysis, IEEE Access, 7 (2019), 29734-29744.
[16] N. A. Nabeeh, M. Abdel-Basset, H. A. El-Ghareeb, A. Aboelfetouh, Neutrosophic multicriteria decision making approach for iot-based enterprises, IEEE Access, 7 (2019), 59559- 59574.
[17] Y.B. Jun, E.H. Roh and Y. xu, LI-ideals in lattice implication algebras, Bulletin of the Korean Mathematical Society, 35(1) (1998), 13–24.
[18] M.A. Ozt¨urk ¨and Y.B. Jun,Neutrosophic ideals in BCK/BCI-algebras based on neutrosophic points, Journal of the International Mathematical Virtual Institute, 8 (2018), 1– 17.
[19] S.Z. Song, M. Khan, F. Samarandache and Y.B. Jun, A novel extension of neutrosophic sets and its application in BCK/BI-algebras, (2),(2018)308-318
[20] S.Z. Song, F. Smarandache and Y.B. Jun, Neutrosophic commutative N -ideals in BCKalgebras, Information, 8 (2017), 130; doi:10.3390/info8040130.
[21] Y. Xu, Homomorphisms in lattice implication algebras, Proc. of 5th Many-Valued Logical Congress of China, (1992), 206–211.
[22] Y. Xu, Lattice implication algebras, Journal of Southwest Jiaotong University, 1 (1993), 20– 27.
[23] Y. Xu, D. Ruan, K.Y. Qin and J. Liu, Lattice-valued logic, Studies in Fuzzyness and Soft Computing, Vol. 132, Springer-Verlag, Berlin Heidelberg, New York, 2003.
[24] L. Z. Liu, Generalized intuitionistic fuzzy filters on residuated lattices, Journal of Intelligent & Fuzzy Systems, 28 (2015), 1545–1552
[25] J. M. Zhan, Q. Liu and Hee Sik Kim, Rough fuzzy (fuzzy rough) strong h-ideals of hemirings, Italian Journal of Pure and Applied Mathematics, 34(2015), 483–496.
[26] Zadeh, L.A. : Fuzzy sets. Infor and Control 8:94-102,1965.
[27] V.Amarendra babu, V.Siva naga malleswari, K.Abida begum, MBJ-filters on lattice implication algebras,Journal of Physics: Conference Series, Volume 2332, Issue 1, id.012007, 8 pp.
[28] Y.B.Jun, Implicative filters of lattice implication algebra, Bull. Korean Math. Soc.34(2):193-198, 1997. International journal of neutrosophic sceince(IJNS)Vol. x, No. y, PP. a-b, year Doi :https://doi.org/10.54216/IJNS.xxxxxx Received: July 20, 2021Accepted: Jan 04, 2022 48
[29] Y.B.Jun, Fuzzy positive implicative and fuzzy associative filters of lattice implication algebras, Fuzzy sets an systems. 121:353- 357, 2001.
[30] K.Y.Qin, Y.Xu, On some properties of fuzzy filters of lattice implication algebra, Liu. Y.M.(Ed.) Fuzzy set Theory and its Application. Press of Hebi University, Baoding, China,179- 182,1998
[31] Y.Xu, K.Y.Qin, Fuzzy lattice implication algebras, J. Southwest Jiaotong Univ. 30:121-127,1995.
[32] Y.Xu, K.Y.Qin, On filters of lattice implication algebras, J. Fuzzy Math. 1:251-260, 1993.
[33] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy setsa and systems,20(1986), pp.87-96.

DOI: https://doi.org/10.54216/IJNS.210106
Received: January 24, 2023 Revised: April 12, 2023 Accepted: May 09, 2023

[34]Gau.W.L,Bueher.D.J, Vague sets, IEEE Transactions on Systems, Man and Cybernetics 23(2),April 1993, Pages: 610-614.

DOI: https://doi.org/10.54216/IJNS.210106
Received: January 24, 2023 Revised: April 12, 2023 Accepted: May 09, 2023