A Multi Objective Epq Model with Uniform Demand and Productıon Rate Along Wıth Shortages Under Intuitioistic Fuzzy Programmıng Approach

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Abstract

In this paper we have described a multi-objective economic production quantity (EPQ) model with uniform demand rate as well as shortages. In this model we have considered the production rate as finite. Due to uncertainty in the various cost parameters, most of the costs parameters are taken as pentagonal fuzzy number. The model has been solved by Fuzzy Non-Linear Programming Problem (FNLP), Fuzzy Additive Goal Programming Problem (FAGP) and Intuitionistic fuzzy programming approach (IFP).To demonstrate the validity of this model some numerical examples have been given lastly. The sensitivity analysis for some cost parameters has also been given.

Keywords: Deterministic EPQ model; Uniform demand; Finite production rate; Shortages; Pentagonal fuzzy number; Intuitionistic fuzzy programming approach (IFP).

1. Introduction

Now a days we observe that small retailers know roughly the demand of their customers in monthly or weekly basis that is why they can place an order according to their customer's demand. But in big companies, for a departmental manager or a big retailer, it is very much difficult to maintain customer demand and placement of order etc. because stocking in such cases depends upon various factors as example demand, lag between orders and actual receipts, the time of ordering etc. So management of inventory is very crucial things for a big retailer.

Management of inventory is a great aspect in our day to day life. It is a difficult task to maintain inventories for big companies. Sometimes it happens that the demand for a particular item of a company is much higher than the quantities of their production resulting the shortages of their production.Due to this, the company has to bear a huge penalty for shortage. For this reason we need to include the cost due to shortages while developing our model. In reality it is impossible to produce infinite amount of production in a certain time, so in this model we have introduced a finite production rate.

Sometimes it is difficult to give a particular value for a particular cost (shortages cost, carrying cost, etc.), so we have taken costs parameters as fuzzy number.

Upendra dave (1989) introduced a deterministic lot-size inventory model with shortages and a linear trend in demand. Afterwards Hui-Ming Wee (1995) described a deterministic lot-size inventory model for deteriorating items with shortages and a declining market.Arindum Mukhopadhyay and A. Goswami (2017) presented an inventory model with shortages for imperfect items using substitution of two products.

S. sana, S.K Goyal, K.S Choudhuri (2004) have discussed about a production-inventory model for a deterioration item with trended demand and shortages. Hesham k. Alfares (2014) published an research paper where he described a Production-inventory system with finite production rate, stock-dependent demand, and variable holding cost. Bhunia, A and Maiti, M, in 1997 described a deterministic inventory replenishment problem for deteriorating items with time-dependent demand and shortages for the finite time horizon. Goswami, A and Chaudhuri, K in 1991 have introduced an EOQ model for deteriorating items with shortages and a linear trend in demand. .In 2019, Garai. T,Chakraborty. D and Roy. T.K established a multi-objective inventory model in exponential fuzzy environment using chance-operator techniques. Recently in 2021, Soni. H.N and Suthar. S.N

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have considered an EOQ model of deteriorating items for fuzzy demand and learning in fuzziness with finite horizon.

Uncertainty in cost parameters are very much obvious for various companies. To tackle such types of problem (1965), L.A. Zadeh first introduced fuzzy sets theory. There are many types of fuzzy number like triangular fuzzy number, trapezoidal fuzzy number, pentagonal fuzzy number etc. Apurba Panda and Madhumangal Pal (2015) described about pentagonal fuzzy number. Then Pathinathan.T and Mike.Dison E (2018) gave us a defuzzification technique for pentagonal fuzzy numbers. Recently B. Rama and G. Michael Rosario (2021) described pentagonal dense fuzzy set and its defuzzification methods.

It is a difficult job to solve a multi-objective inventory model. R.E.Bellman and L.A.Zadeh (1970) described the Decision making in fuzzy environment. After that many researchers used the fuzzy non-linear programming approach and fuzzy additive goal programming approach to solve multi-objective functions.T.K. Roy and M. Maiti (1998) introduced fuzzy non-linear programming technique (FNLP) and fuzzy goal programming technique (FAGP) in their paper and they have solved a multi-objective inventory models of deterioration items with some constraints in fuzzy environment. In 2020, Pawar.S, Patel.P and Mirajkar.A described a method in intuitionistic fuzzy approach for solving a multi-objective model.Mishra.U, Waliv. R.H and Umap. H.P established how to optimize a multi-objective inventory model by different fuzzy techniques. Recently Anuradha Sahoo and Minakshi Panda (2022) have described novel methods for solving Multi-objective nonlinear inventory model. Banerjee.S., Roy.T.K (2010) have developed the solution of single and multiobjective stochastic inventory models with fuzzy cost components by intuitionistic fuzzy optimization technique.

In this research paper we have developed a deterministic inventory model with shortages. We have taken more practical situation where production rate is finite with uniform rate. In this model we have also considered uniform demand rate which is less than the production rate. In the presence of uncertainty some of the cost parameters have been taken as pentagonal fuzzy number. Finally we have solved the multi-objective EPQ model using FNLP, FAGP and IFP technique.

2. Mathematical Preliminaries:

2.1 Fuzzy set(L.A. Zadeh, 1965)

Let X be a universe of discourse. A fuzzy set which is denoted by Ã∈X and is defined with the ordered pairs Ã= {(x,$T_{\tilde{A}}$(x)): x∈ X}.

Here $T_{\tilde{A}}$: X→[0,1] is a function known as truth membership function of the fuzzy set Ã.

2.2 Pentagonal Fuzzy Number (TFN) (Apurba Panda and Madhumangal Pal , 2015)

The pentagonal fuzzy number $\widetilde{A^p} = (a, b, c, d, e; p, w)$ is a subset of real number, where a, b, c, d, e all are real number and k be the α-cut value for the pentagonal fuzzy number with $0 < p < 1$; w be the height of the pentagonal fuzzy number with$0 \leq w \leq 1$.

Then the pentagonal fuzzy membership function is defined as follows:

$$\mu_{\widetilde{A^p}}(x) = \begin{cases} p\left(\frac{x-a}{b-a}\right) & for\ a \leq x \leq b \\ p + (w-p)\left(\frac{x-b}{c-b}\right) & for\ b \leq x \leq c \\ w & for\ x = c \\ p + (w-p)\left(\frac{d-x}{d-c}\right) & for\ c \leq x \leq d \\ p\left(\frac{e-x}{e-d}\right) & for\ d \leq x \leq e \\ 0 & otherwise \end{cases} \quad (2.1.1)$$

2.3. Defuzzification Method for Pentagonal Fuzzy Number(Pathinathan T and Mike Dison E , 2018; B. Rama and G. Michael Rosario, 2021)

In this section we will discuss about the defuzzification technique known as Weighted Average Method.

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Weighted approximation method is mainly used for symmetrical fuzzy numbers. The weighted method is formulated by taking the average of core or maximum membership function. The formula for this technique is given as follows

$$P^* = \frac{\sum \overline{p_0}\, \mu_{\tilde{G}}(\overline{p_0})}{\mu_{\tilde{G}}(\overline{p_0})}$$

$\overline{p_0}$ be the centroid of the pentagonal symmetric fuzzy number, which is defined as follows

$$\overline{p_0} = \frac{a+b+5c+d+e}{9} \qquad (2.3.1)$$

Illustration example:

Let $\widetilde{X_p} = (6,7,8,9,10,;0.8)$, $\widetilde{Y_p} = (7,8,9,10,11;0.5)$ be two symmetric pentagonal fuzzy number with core value 0.8 and 0.5 respectively. Then the defuzzyfied value for these two pentagonal fuzzy number is given by

$$P^* = \frac{(8 \times 0.8) + (9 \times 0.5)}{(0.8 + 0.5)} = 8.384$$

3. Mathematical Model:

The following assumptions and notations are used in developing the EPQ model (for i'th item per unit).

3.1 Assumptions:

1. The lead time is negligible.

2. Shortages are allowed and fully back-logged.

3. The EPQ models deals with multi-objective function.

3.2 Notations:

D_i : The uniform demand per unit time.

P_i: The production rate is finite per unit time.

C_{1i}: The inventory carrying cost or holding cost per unit time.

C_{2i}: The inventory shortage cost per unit time.

C_{3i}: The inventory set up cost per unit time.

Q_{1i}: The height positive inventory in the time interval $[0, t_{2i}]$

Q_{2i}: The height negative inventory in the time interval $[0, t_{4i}]$

$\widetilde{C_{1\iota}}$: The fuzzy inventory carrying cost or holding cost per unit time.

$\widetilde{C_{2\iota}}$: The fuzzy inventory shortage cost or holding cost per unit time.

$\widetilde{C_{3\iota}}$: The fuzzy inventoryset − up cost or holding cost per unit time.

3.3 Formulation of EPQ model: (Upendra dave, 1989; Arindum Mukhopadhyay and A. Goswami, 2017)

INVENTORY

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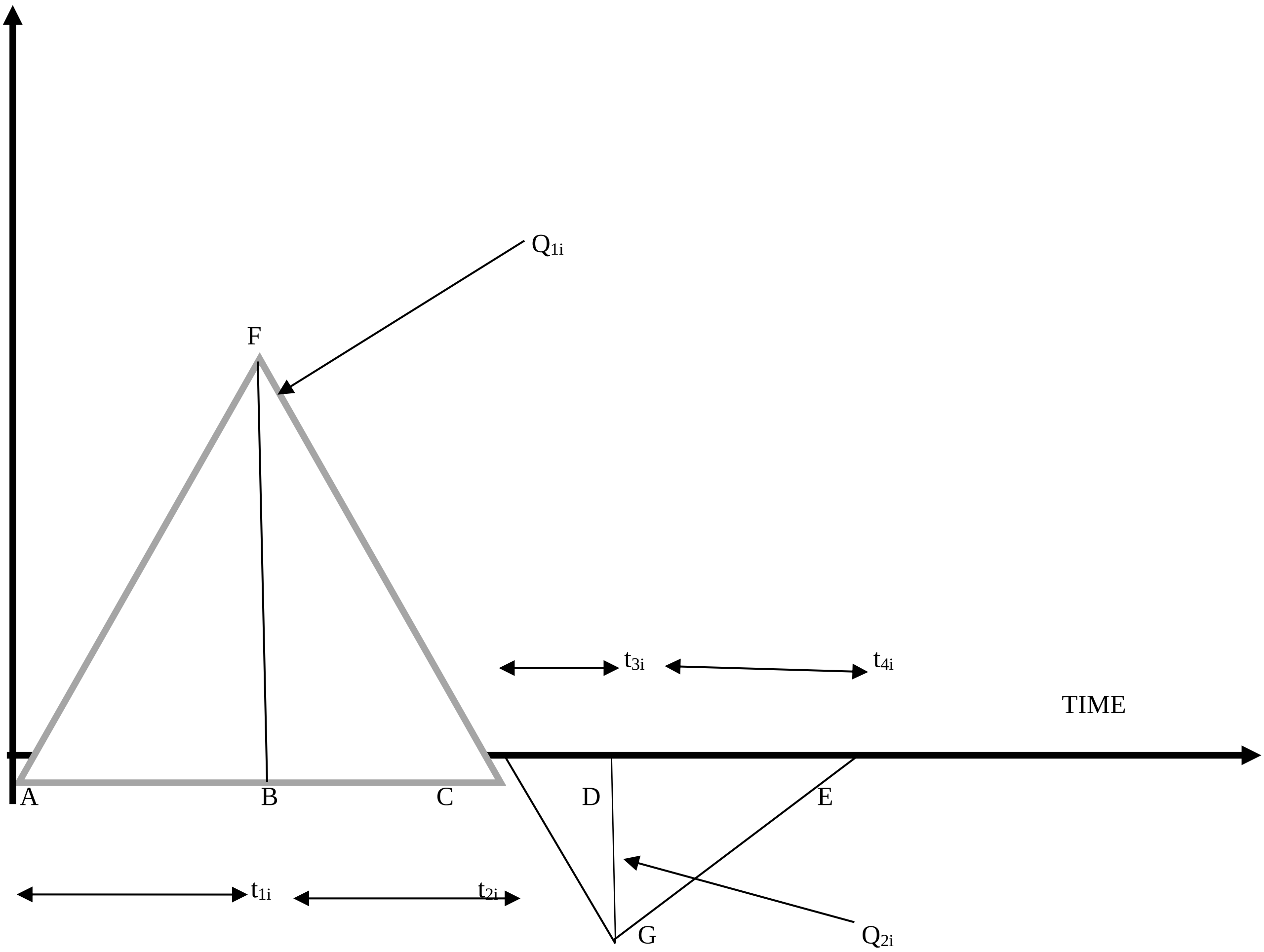

From this above figure we can see that inventory starts from zero level and increases for the time interval$[\,0, t_{1i}]$. Then it started to decreases for demand in the time interval $[\,0, t_{2i}]$.At the time $t_{2i}$the inventory level comes to zero and backlogging starts at $t = t_{3i}$. Backlogging continued for the time $t = t_{4i}$.

Now we have the following costs involved in this EPQ model.

Holding Cost $= C_{1i} \times \Delta AFC = C_{1i}.\frac{1}{2}.Q_{1i}(t_{1i} + t_{2i})$

Shortages Costs $= C_{2i} \times \Delta CEG = C_{2i}.\frac{1}{2}.Q_{2i}(t_{3i} + t_{4i})$

Set up Cost $= C_{3i}$

Therefore the total average cost per unit time is given by the following equation.

$$TAC_i = \frac{C_{3i}+C_{1i}.\frac{1}{2}.Q_{1i}(t_{1i}+t_{2i})+C_{2i}.\frac{1}{2}.Q_{1i}(t_{3i}+t_{4i})}{t_{1i}+t_{2i}+t_{3i}+t_{4i}}. \qquad (3.3.1)$$

Here total average cost are function involving six variable.$(TAC_i = f(Q_{1i}, Q_{2i}, t_{1i}, t_{2i}, t_{3i}t_{4i}))$

In the time interval $[\,0, t_{1i}]$ the net amount of inventory is given by

$$Q_{1i} = P_i t_{1i} - D_i t_{1i}\ (P_i > D_i)\ , \qquad (3.3.2)$$

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Where P_i represents the production rate and D_i represents the demand rate for a particular item.

After time t_{1i} the production for an item has stopped and the inventory stock Q_{1i} are used for the rest of the time.

Now for the time period $[t_{1i}, t_{2i}]$ we have,

$$Q_{1i} = D_i t_{2i} \quad (3.3.3)$$

Therefore, $t_{1i} = \frac{Q_{1i}}{P_i - D_i} = \frac{D_i t_{2i}}{P_i - D_i}$ (3.3.4)

For the time period $[t_{2i}, t_{3i}]$, the shortages accumulate at the rate of D_i.

$$Q_{2i} = D_i t_{3i} \quad (3.3.5)$$

For the time period $[t_{3i}, t_{4i}]$, the demand rate and production rate are D_i, P_i rspectively. So the net rate of reduction of the inventory at the time of shortages is $P_i - D_i$ .Therefore we have

$$Q_{2i} = (P_i - D_i) t_{4i} \quad (3.6.3)$$

Therefore $t_{4i} = \frac{Q_{2i}}{P_i - D_i} = \frac{D_i t_{3i}}{P_i - D_i}$ (3.3.7)

Now eliminating $t_{1i}, t_{2i}, t_{3i} t_{4i}$ from the above equation (3.3.1) and using the relation (3.3.4) and (3.3.7) we get the total average costs per item as given by the following equation.

$$TAC_i = \frac{\frac{1}{2}\,(C_{1i}t_{2i}^2 + C_{2i}t_{3i}^2).\frac{D_i\,P_i}{P_i - D_i} + C_{3i}}{(t_{2i} + t_{3i})\left(\frac{P_i}{P_i - D_i}\right)} \quad (3.3.8)$$

$$= \frac{1}{2}\frac{(C_{1i}Q_{1i}^2 + C_{2i}Q_{2i}^2)}{Q_{1i} + Q_{2i}} + \frac{C_{3i}D_i}{Q_{1i} + Q_{2i}} - \frac{C_{3i}D_i^2}{K_i(Q_{1i} + Q_{2i})}$$

Now our objective is to find the minimum total average cost for the above EPQ model.

So, our multi-item EPQ model transform into

Minimize $\{TAC_1(Q_{11}, Q_{21}), TAC_2(Q_{12}, Q_{22}), \dots\dots\dots\dots\dots\dots\dots., TACn(Q_{1n}, Q_{2n})\}\ for\ i = 1,2,3,4, \dots\dots, n$

4. Fuzzy Model:

Since the presence of uncertainty we take some costs parameters as pentagonal fuzzy number.

$$\widetilde{C_{1i}} = (c_{11}, c_{12}, c_{13}, d_{14}, e_{15}; w_1)$$

$$\widetilde{C_{2i}} = (c_{21}, c_{22}, c_{23}, d_{24}, e_{25}; w_2)$$

$$\widetilde{C_{3i}} = (c_{31}, c_{32}, c_{33}, d_{34}, e_{35}; w_2)$$

Now our multi-objective EPQ model transform into a fuzzy model as follows.

$Minimize\ \{(TAC_1(Q_{11}, Q_{21}), TAC_2(Q_{12}, Q_{22}), \dots\dots\dots\dots\dots\dots\dots., TACn(Q_{1n}, Q_{2n})\}\ for\ i = 1,2,3,4, \dots\dots, n$

Where,

$$\widetilde{TAC_i} = \frac{1}{2}\frac{(\widetilde{C_{1i}}Q_{1i}^2 + \widetilde{C_{2i}}Q_{2i}^2)}{Q_{1i} + Q_{2i}} + \frac{\widetilde{C_{3i}}D_i}{Q_{1i} + Q_{2i}} - \frac{\widetilde{C_{3i}}D_i^2}{K_i(Q_{1i} + Q_{2i})} \quad (4.1)$$

Using defuzzification technique discussed at 2.3, our PENTAGOPAL fuzzy costs parameters $(\widetilde{C_{1i}}, \widetilde{C_{2i}}, \widetilde{C_{3i}})$ transform into the crisp value $(\widehat{C_{1i}}, \widehat{C_{2i}}, \widehat{C_{3i}})$ and the corresponding fuzzy EPQ model transform in the following crisp model as.

$Minimize\ \{(\widehat{TAC_1}(Q_{11}, Q_{21}), \widehat{TAC_2}(Q_{12}, Q_{22}), \dots\dots\dots\dots\dots\dots\dots., \widehat{TACn}(Q_{1n}, Q_{2n})\}\ for\ i = 1,2,3,4, \dots\dots, n$

With, $\widehat{TAC_i} = \frac{1}{2}\frac{(\widehat{C_{1i}}Q_{1i}^2 + \widehat{C_{2i}}Q_{2i}^2)}{Q_{1i} + Q_{2i}} + \frac{\widehat{C_{3i}}D_i}{Q_{1i} + Q_{2i}} - \frac{\widehat{C_{3i}}D_i^2}{K_i(Q_{1i} + Q_{2i})},\ \ for\ i = 1,2,3,4, \dots\dots, n$ (4.2)

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5. Different techniques for solving Multi-Objective EPQ Model.

To solving the above multi objective inventory (4.2) problem we consider single objective at a time and the others objectives are ignored.

Applying this technique we find out the value of each objective function separately and by tracking this technique we will formulate the following pay-of-matrix.

$$\begin{array}{c} \\ (Q_{11}^1, Q_{21}^1) \\ (Q_{12}^2, Q_{22}^2) \\ \dots\dots\dots \\ (Q_{1n}^n, Q_{2n}^n) \end{array} \begin{array}{c} \begin{array}{cccc} TAC_1(Q_{11}, Q_{21}) & TAC_2(Q_{12}, Q_{22}) & \dots\dots\dots & TAC_n(Q_{1n}, Q_{2n}) \end{array} \\ \begin{bmatrix} TAC_1^*(Q_{11}^1, Q_{21}^1) & TAC_2(Q_{11}^1, Q_{21}^1) & \dots\dots & TAC_n(Q_{11}^1, Q_{21}^1) \\ TAC_1(Q_{12}^2, Q_{22}^2) & TAC_2^*(Q_{12}^2, Q_{22}^2) & \dots\dots\dots & TAC_n(Q_{12}^2, Q_{22}^2) \\ \dots\dots\dots & \dots\dots\dots & \dots\dots\dots & \dots\dots\dots \\ TAC_1(Q_{1n}^n, Q_{2n}^n) & TAC_2(Q_{1n}^n, Q_{2n}^n) & \dots\dots\dots & TAC_n^*(Q_{1n}^n, Q_{2n}^n) \end{bmatrix} \end{array}$$

Now we set $U_r^T = \max\{TAC_r(Q_{1i}^i, Q_{2i}^i), i = 1,2,3, \dots\dots, n\}$, for r= 1,2,3,…..,n

And $L_r^T = \{TAC_r^*(Q_{1r}^r, Q_{2r}^r), r = 1,2,3, \dots\dots, n\}$

Where $L_r^T \leq TAC_r(Q_{1i}^i, Q_{2i}^i) \leq U_r^T$; for $i = 1,2,3, \dots\dots, n$; and r= 1,2,3,…..,n; (5.1)

5.1. Fuzzy Non-Linear Programming Problems (FNLP) and Fuzzy Additive Goal Programming Problems (FAGP) (R.E.Bellman and L.A.Zadeh , 1970)

In FNLP first we solve one objective at a time ignoring the others.

Now we take linear fuzzy membership function $\mu_{TAC_r}(TAC_r(t_{1r}, T_r))$ for the r'th objective function $TAC_r(t_{1r}, T_r)$ as follows.

$$\mu_{TAC_r}(TAC_r(Q_{1r}, Q_{2r})) = \begin{cases} 1 & for \quad TAC_r(Q_{1r}, Q_{2r}) \leq L_r^T \\ \frac{U_r^T - TAC_r(Q_{1r}, Q_{2r})}{U_r^T - L_r^T} & for \; L_r^T \leq TAC_r(Q_{1r}, Q_{2r}) \leq U_r^T \\ 0 & for \quad TAC_r(Q_{1r}, Q_{2r}) \geq U_r^T \end{cases} \qquad (5.1.1)$$

For r=1,2,3,…..,n;

Using (5.1) we established the fuzzy non-linear programming problems (FNLP) based on minimum operator.

Max= p

Subject to,

$p(U_r^T - L_r^T) + TAC_r(Q_{1r}, Q_{2r}) \leq U_r^T$ For r=1,2,3,……,n

$0 \leq p \leq 1 \quad Q_{1r} \geq 0, Q_{2r} \geq 0;$ (5.1.2)

And the same restriction and constraints as in the problem (4.2)

Now we formulated Fuzzy additive goal programming (FAGP) based on max-additive operator as given below:

Max $\sum_{r=1}^{n} \frac{U_r^T - TAC_r(Q_{1r}, Q_{2r})}{U_r^T - L_r^T}$

Subject to,$0 \leq \mu_{TAC_r}(TAC_r(Q_{1r}, Q_{2r})) \leq 1$, for r=1,2,3,…….,n (5.1.3)

And the same restriction and constraints as in the problem (4.2)

Now we are finding the optimal solution for the above reduced problem (5.1.2) and (5.1.3) with the help of above FNLP and FAGP method.

5.2. Weighted Fuzzy Non-Linear Programming technique and Weighted Fuzzy Goal Programming Technique (WFNLP AND WFAGP): (R.E.Bellman and L.A.Zadeh , 1970)

We are taking here a positive weight ω_r for every objective $(TAC_r(Q_{1r}, Q_{2r}))$

(Where r=1,2,3,…..,n) and $\sum_{r=1}^{n} \omega_r = 1$.

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Having these normalized weights and the membership function (5.1.1), the FNLP technique becomes

Max p

Subject to,

$\omega_r.\mu_{TAC_r}\left(TAC_r(Q_{1r},Q_{2r})\right) \geq$ p For r=1, 2, 3,...., n

$0 \leq p \leq 1, \quad Q_{1r} \geq 0, Q_{2r} \geq 0$ and $\sum_{r=1}^{n} \omega_r = 1.$ (5.2.1)

And the same restriction and constraints as in the problem (4.2)

Having these normalized weights and the membership function (5.1.1), the FAGP technique becomes

Max $\sum_{R=1}^{n} \omega_k.\mu_{TAC_r}\left(TAC_r(Q_{1r},Q_{2r})\right)$

Subject to,$0 \leq \mu_{TAC_r}\left(TAC_r(Q_{1r},Q_{2r})\right) \leq 1$, for r=1,2,3,.......,n and

$Q_{1r} \geq 0, Q_{2r} \geq 0;\ \sum_{r=1}^{n} \omega_r = 1$ (5.2.2)

And the same restriction and constraints as in the problem (4.2)

Now we are finding the optimal solution with the help of above WFNLP and WFAGP method.

5.3. Intuitionistic Fuzzy Programming Approach (IFP) for solving multi-objective EPQ model: (Anuradha Sahoo and Minakshi Panda, 2022)

According to intuitionistic fuzzy optimization technique we need to minimize the degree of rejection of our fuzzy objective function and maximize the degree of acceptance of fuzzy objective function.

Let us denote the degree of rejection by γ_1 *and* degree of acceptance δ_1.

By (5.1) we are defining two membership function known as degree of truth and degree of falsity membership functions as follows

$$\alpha_{TAC_r}(TAC_r(t_{1r},T_r)) = \begin{cases} 1 & for \quad TAC_r(t_{1r},T_r) \leq L_r^{ACT} \\ \frac{U_r^{ACT} - TAC_r(t_{1r},T_r)}{U_r^T - L_r^T} & for \ L_r^{ACT} \leq TAC_r(t_{1r},T_r) \leq U_r^{ACT} \\ 0 & for \quad TAC_r(t_{1r},T_r) \geq U_r^{ACT} \end{cases}$$

$$\beta_{TAC_r}(TAC_r(t_{1r},T_r)) = \begin{cases} 1 & for \quad TAC_r(t_{1r},T_r) \geq U_r^{REJT} \\ \frac{TAC_r(t_{1r},T_r) - L_r^{REJT}}{U_r^T - L_r^{REJT}} & for \ L_r^{REJT} \leq TAC_r(t_{1r},T_r) \leq U_r^{REJT} \\ 0 & for \quad TAC_r(t_{1r},T_r) \leq L_r^{REJT} \end{cases} \quad (5.7)$$

For r=1,2,3,.....,n;

From (Roy and Banerjee, 2010) the corresponding lower and upper bound for falsity membership functions are as follows.

$$L_r^{REJT} = L_r^{ACT} + f * (U_r^{ACT} - L_r^{ACT})$$
$$U_r^{REJT} = U_r^{ACT} + e * (U_r^{ACT} - L_r^{ACT})$$

Now considering linear and non-linear membership function our intuitionistic fuzzy optimization EPQ model transform into.[Bellman-Zadeh, 1970]

Max γ_1, Min δ_1

Subject to $\mu_{TAC_r}\left(TAC_r(t_{1r},T_r)\right) \geq \gamma_1$

$$\partial_{TAC_r}\left(TAC_r(t_{1r},T_r)\right) \leq \delta_1$$
$$\gamma_1 + \delta_1 \leq 1;\ \gamma_1 \geq \delta_1$$
$$\gamma_1, \delta_1 \geq 0;$$

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$$t_{1r} \geq 0, T_r \geq 0; \quad (5.8)$$

With the same constraints and restriction as in (4.2)

For r=1,2,3,…..,n

So, our multi-objective Economic production model (MOEPQ) reduces to the following form

$$\text{Max } (\gamma_1 - \delta_1)$$

$$\text{Subject to, } \mu_{TAC_r}\left(TAC_r(t_{1r}, T_r)\right) \geq \gamma_1$$

$$\partial_{TAC_r}\left(TAC_r(t_{1r}, T_r)\right) \leq \delta_1$$

$$\gamma_1 + \delta_1 \leq 1; \ \gamma_1 \geq \delta_1$$

$$\gamma_1, \delta_1 \in [0,1];$$

$$t_{1r} \geq 0, T_r \geq 0; \quad (5.9)$$

With the same constraints and restriction as in (4.2)

For r=1,2,3,…..,n

6. Numerical Example:

To illustrate the validity of the EPQ model we describe a numerical example by taking two item as two-objective.

$Minimize$ { $(\widehat{TAC_1}(Q_{11}, Q_{21}), \widehat{TAC_2}(Q_{12}, Q_{22})\}$ we take i=1,2

Where, $C_i = \frac{1}{2}\frac{(\widetilde{C_{1i}}Q_{1i}^2 + \widetilde{C_{2i}}Q_{2i}^2)}{Q_{1i}+Q_{2i}} + \frac{\widetilde{C_{3i}}D_i}{Q_{1i}+Q_{2i}} - \frac{\widetilde{C_{3i}}D_i^2}{K_i(Q_{1i}+Q_{2i})}$, $for\ i = 1,2,3,4, \dots \dots, n$

Now we take the costs parameters which are not fuzzy that is crisp values

$D_1 = 1500, P_1 = 3000$, $D_2 = 2000, P_2 = 3500$

The cost parameters which are pentagonal fuzzy number.

Table 1

Cost parameters	Items	
	1st item's cost (PFN)	2nd item's cost (PFN)
$\widetilde{C_{1i}}$	(0.7,,0.10,0.13,0.17,0.20;1) (0.13,0.15,0.17,0.19,0.21;1)	(0.12,0.15,0.18,0.21,0.24;1) (0.14,0.18,0.22,0.26,0.30;1)
$\widetilde{C_{2i}}$	(19,20,21,22,23;1) (11,15,19,24,28;1)	(19,22,25,28,31;1) (21,23,25,27,29;1)
$\widetilde{C_{3i}}$	(450,470,490,510,530;1) (490,500,510,520,530;1)	(480,520,560,580,600;1) (480,510,540,570,600;1)

So the optimal solution in different techniques (FNLP, FAGP, WFNLP, WFAGP)

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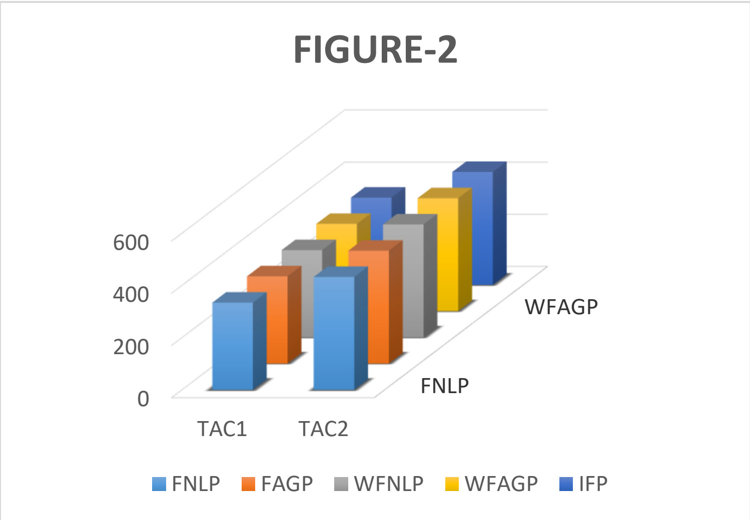
Table 2

Techniques	$TAC_1^*(Q_{11}^*, Q_{21}^*)$	Q_{11}^*	Q_{21}^*	$TAC_2^*(Q_{12}^*, Q_{22}^*)$	Q_{12}^*	Q_{22}^*
FNLP	334.1594	2227.745	16.71510	432.5215	2162.606	17.31340
FAGP	334.1595	2227.914	16.63957	432.5215	2163.857	17.34284
WFNLP	334.2437	2230.200	21.05901	432.5215	2162.607	17.30086
WFAGP	334.1594	2227.730	16.70797	432.5215	2162.658	17.24074
IFP	334.3070	2229.571	21.19676	432.5979	2162.607	17.30086



From this figure-2 we can say that FNLP and WFAFP gives the minimum value of total average cost for the 1st item and for 2nd item all the methods as FNLP, FAGP, WFNLP, WFAGP, IFP gives the same values for total average costs.

Graph for the T.A.C in different technique for two items.

7. Sensitivity analysis:

7.1. For different rate of demand rate how the total average costs changes have been shown in the following table.

Here we are taking all the cost's parameters value from table-1 and taking $P_1 = 3000$, $P_2 = 3500$

Table 3

Different methods	Demand for 1st item	Demand for 2nd item	$TAC_1(Q_{11}, Q_{12})$	$TAC_2(Q_{12}, Q_{22})$
FNLP				
	1400	1900	333.4160	435.3954
	1500	2000	334.1594	432.5215
FAGP	1400	1900	333.4160	435.3954
	1500	2000	334.1595	432.5215
WFNLP	1400	1900	333.4372	435.3954
	1500	2000	334.2437	432.5215
WFAGP	1400	1900	333.4160	435.3954
	1500	2000	334.1594	432.5215

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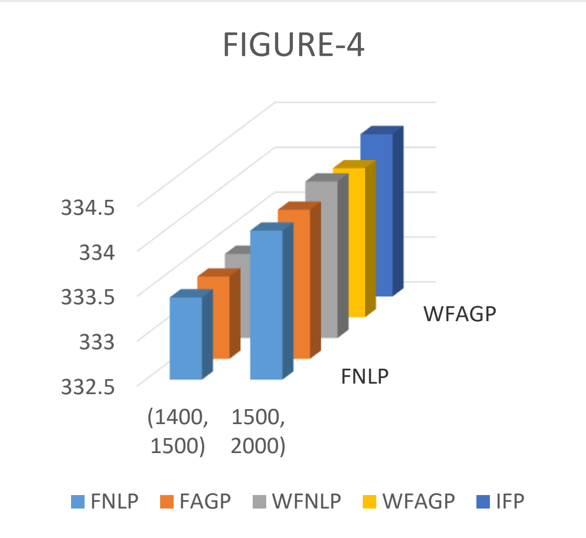
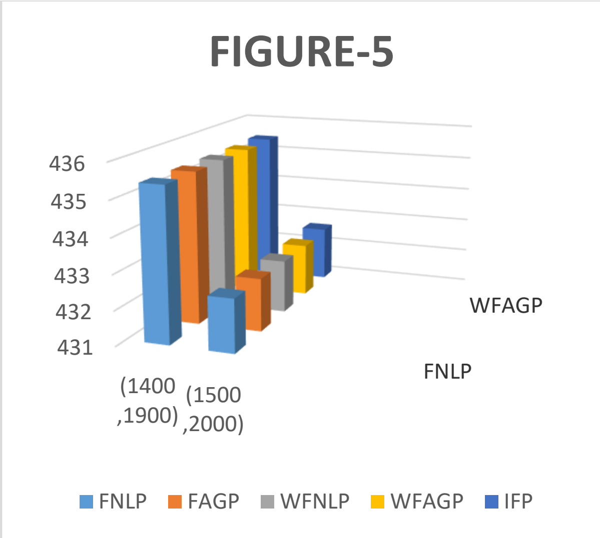
IFP	1400	1900	333.4371	435.4341
	1500	2000	334.3070	432.5979

Here we take the weighs (0.7, 0.3) for both of the methods WFNLP, WFAGP.



Graph for the average of total cost of 1st item by different Technique with different Demand.



Graph for the average of total cost of 2nd item by different Technique with different Demand.

From the above figures (Figure-4, Figure-5), we can see the changes of total average cost depending on various demand per unit time for two items.

7.2. For different rate of Production how the total average costs changes have been shown in the following table.

Here we are taking all the cost's parameters value from table-1 and taking $D_1 = 3000$, $D_2 = 3500$ and the weights for WFNLP and WFAGP are 0.7 *and* 0.3 respectively.

Table-3

Different methods	Production for 1st item	Production for 2nd item	$TAC_1(Q_{11}, Q_{12})$	$TAC_2(Q_{12}, Q_{22})$
FNLP	2900	3400	328.3475	423.9558
	3000	3500	334.1594	432.5215
	3100	3600	339.5063	440.4583
FAGP	2900	3400	328.3475	423.9558
	3000	3500	334.1595	432.5215
	3100	3600	339.5063	440.4583
WFNLP	2900	3400	328.4444	423.9558
	3000	3500	334.2437	432.5215
	3100	3600	339.5794	440.4583
WFAGP	2900	3400	328.3475	423.9558

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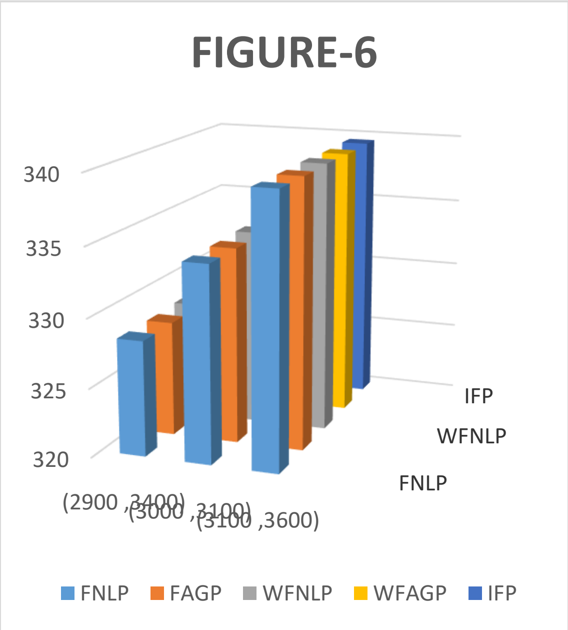
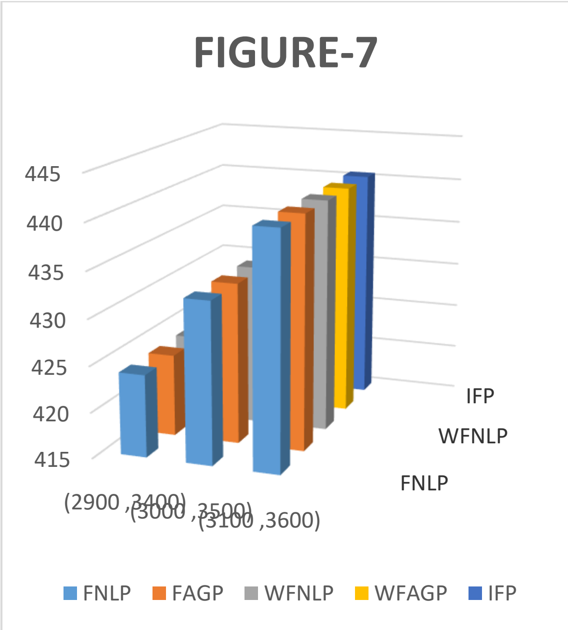
	3000	3500	334.1594	432.5215
	3100	3600	339.5063	440.4583
IFP	2900	3400	328.4323	424.0653
	3000	3500	334.2491	432.5979
	3100	3600	339.5703	440.5412

From this table-3 we can see that as the production rate increases the corresponding total average cost for two items are also increases.



T.A.C for the 1st item in various methods with various production rate.



T.A.C for the 2nd item in various methods with various production rate.

5. Conclusion

In this research paper we have discussed a multi-objective deterministic EPQ model with uniform demand rate as well as production rate. The shortages have been taken as fully-backlogged. To deal with the uncertainty all the cost parameters have been taken as pentagonal fuzzy number. Finally the model has been solved by fIVE different techniques as FNLP, FAGP, WFNLP, WFAGP and IFP. The result shows us the validity and existence of our EPQ model.

In future-research someone can take the demand function and production rate depending on time. Deterioration may also be considered hereto. To handle the uncertainty the cost parameters may also be considered as triangular fuzzy number, trapezoidal fuzzy number, stochastic fuzzy, nuetrosophic, intuitionistic fuzzy number etc.

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