A Review on Symbolic 2-Plithogenic Algebraic Structures

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Abstract

The objective of this paper is to give a good review about the 2-plithogenic algebraic structures. Three kinds of algebraic structures will be revisited and discussed, symbolic 2-plithogenic rings, symbolic 2-plithogenic vector spaces, and 2-plithogenic modules.

Key words: 2-plithogenic rings; 2-plithogenic spaces; 2-plithogenic modules; AH-ideals.

1. Introduction

The concept of symbolic n-plithogenic sets was defined by Smarandache. This concept has made a good generalization of classical algebraic structures. Also, these structures have similar structures of neutrosophic and n-refined neutrosophic algebraic structures [10-40].

For n=2, we get symbolic 2-plithogenic algebraic structures, where we find symbolic 2-plithogenic equations, rings, spaces, and modules [1-10].

In this paper, we give the interested reader a good review for three different types of 2-plithogenic algebraic structures, symbolic 2-plithogenic rings, modules, and vector spaces.

2-plithogenic rings [1]

Definition.

Let R be a ring, the symbolic 2-plithogenic ring is defined as follows:

$2-SP_R=\{a_0+a_1P_1+a_2P_2;\ a_i\in R, {P_j}^2=P_j, P_1\times P_2=P_{max(1,2)}=P_2\}$.

Smarandache has defined algebraic operations on $2-SP_R$ as follows:

Addition:

$[a_0+a_1P_1+a_2P_2]+[b_0+b_1P_1+b_2P_2]=(a_0+b_0)+(a_1+b_1)P_1+(a_2+b_2)P_2$.

Multiplication:

$[a_0+a_1P_1+a_2P_2].[b_0+b_1P_1+b_2P_2]=a_0b_0+a_0b_1P_1+a_0b_2P_2+a_1b_0{P_1}^2+a_1b_2P_1P_2+a_2b_0P_2+a_2b_1P_1P_2+a_2b_2{P_2}^2+a_1b_1P_1P_1=a_0b_0+(a_0b_1+a_1b_0+a_1b_1)P_1+(a_0b_2+a_1b_2+a_2b_0+a_2b_1+a_2b_2)P_2$.

It is clear that $(2-SP_R)$ is a ring.

Also, if R is commutative, then $2-SP_R$ is commutative, and if R has a unity (1), than $2-SP_R$ has the same unity (1).

Example.

Consider the ring $R=Z_4=\{0,1,2,3\}$, the corresponding $2-SP_R$ is:

$2-SP_R=\{a+bP_1+cP_2; a,b,c\in Z_4\}$.

If $X=1+2P_1+3P_2, Y=P_1+2P_2$, then:

$X+Y=1+3P_1+P_2, X-Y=1+P_1+P_2, X.Y=P_1+2P_2+2P_1+4P_2+3P_2+6P_2=3P_1+3P_2$.

Definition.

Let Q_0,Q_1,Q_2 be ideals of the ring R, we define the symbolic 2-plithogenic AH-ideal as follows:

$Q=Q_0+Q_1P_1+Q_2P_2=\{x_0+x_1P_1+x_2P_2;\ x_i\in Q_i\}$.

If $Q_0=Q_1=Q_2$, then Q is called an AHS-ideal.

Example.

Let $R=Z$ be the ring of integers, then $Q_0=2Z, Q_1=3Z, Q_2=5Z$ are ideals of R.

$Q=\{2m+3nP_1+5tP_2;\ m.n.t\in Z\}$ is an AHS-ideal of $2-SP_Z$.

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$M = \{2m + 2nP_1 + 2tP_2;\ m.n.t \in Z\}$ is an AHS-ideal of $2 - SP_Z$.

Theorem.

Let Q be an AHS- ideal of $2 - SP_R$, then Q is an ideal by the classical meaning.

Definition.

Let R, T be two rings, $2 - SP_R, 2 - SP_T$ are the corresponding symbolic 2-plithogenic rings, let $f_0, f_1, f_2: R \to T$ be three homomorphisms, we define the AH-homomorphism as follows:

$f: 2 - SP_R \to 2 - SP_T$ such that:

$f(a + bP_1 + cP_2) = f_0(a) + f_1(b)P_1 + f_2(c)P_2$

If $f_0 = f_1 = f_2$, then f is called AHS-homomorphism.

Remark.

If f_0, f_1, f_2 is isomorphisms, then f is called AH-isomorphism.

Example.

Take $R = Z, T = Z_6, f_0, f_1: R \to T$ such that:

$f_0(x) = x(mod\ 6), f_1(2) = 3x(mod\ 6)$. It is clear that f_0, f_1 are homomorphism.

We define $f: 2 - SP_R \to 2 - SP_T$, where:

$f(x + yP_1 + zP_2) = f_0(x) + f_1(y)P_1 + f_2(z)P_2 = x(mod\ 6) + y(mod\ 6)P_1 + (3z\ mod\ 6)P_2$

Which is an AH-homomorphism.

For example. If $X = 15 + 3P_1 + 4P_2$, we get:

$f(X) = 15(mod\ 6) + (3\ mod\ 6)P_1 + (12\ mod\ 6)P_2 = 3 + 3P_1$

Theorem.

Let $f = f_0 + f_1P_1 + f_2P_2: 2 - SP_R \to 2 - SP_T$ be a mapping, then:

1. If f is an AHS-homomorphism, then f is a ring homomorphism by the classical meaning.
2. If f is an AHS-homomorphism, then it is an isomorphism by the classical meaning.

Definition.

Let $f = f_0 + f_1P_1 + f_2P_2: 2 - SP_R \to 2 - SP_T$ be an AH-homomorphism, we define:

1. AH-$ker(f) = ker(f_0) + ker(f_1)P_1 + ker(f_2)P_2 = \{m_0 + m_1P_1 + m_2P_2;\ m_i \in ker(f_i)\}$.
2. AH-factor $2 - SP_R/\text{AH} - ker(f) = R/ker(f_0) + R/ker(f_1)\,P_1 + R/ker(f_2)\,P_2$

If $f_0 = f_1 = f_2$, then we get an AHS- $ker(f)$ and AHS-factor.

Example.

Take $R = Z_{10}, f_0: R \to T, f_0(x) = (x\ mod\ 10), ker(f_0) = 10Z$.

The corresponding AHS-homomorphism is $f = f_0 + f_1P_1 + f_2P_2: 2 - SP_R \to 2 - SP_T$, such that:

$f(x_0 + x_1P_1 + x_2P_2) = f_0(x_0) + f_0(x_1)P_1 + f_0(x_2)P_2 = (x_0\ mod\ 10) + (x_1\ mod\ 10)P_1 + (x_2 mod\ 10)P_2$

AHS-$ker(f) = 10Z + 10ZP_1 + 10ZP_2 = \{10x + 10yP_1 + 10zP_2;\ x, y, z \in Z\}$

AHS-factor= $Z/10Z + Z/10Z\,P_1 + Z/10Z\,P_2$

Remark.

AH-$ker(f)$ is an AH-ideal of $2 - SP_R$, that is because $ker(f_0), ker(f_1), ker(f_2)$ are ideals of R.

Definition.

Let $(F, +, .)$ be a field, then $(2 - SP_F, +, .)$ Is called a symbolic 2-plithogenic field.

$(2 - SP_F, +, .)$ Is not a field in the algebraic meaning, that is because P_1, P_2 are not invertible, but it is a ring.

Example.

Let $F = Q$ the field of rational numbers, then the corresponding symbolic 2-plithogenic field

$2 - SP_Q = \{a + bP_1 + cP_2; a, b, c \in Q\}$.

Remark.

The $2 - SP_F$ has only the following AH-ideals:

$\{0\}, 2 - SP_F, FP_1 + FP_2, F + FP_1, F + FP_2, FP_1, FP_2, F$.

That is because the field F has only two ideals $\{0\}$ and F.

Example.

Find all AH-ideals in $2 - SP_C$, where C is the complex field.

Solution.

$L_1 = \{0\}, L_2 = C, L_3 = C + CP_1 = \{x + yP_1; x, y \in C\}, L_4 = C + CP_2 = \{x + yP_2; x, y \in C\}, L_5 = 2 - SP_C$

$L_6 = CP_1 + CP_2 = \{xP_1 + yP_2; x, y \in C\}, L_7 = CP_1 = \{xP_1; x \in C\}, L_8 = CP_2 = \{yP_2; y \in C\}$.

2-plithogenic vector spaces [2]

Definition.

Let V be a vector space over the field F, let $2 - SP_F$ be the corresponding symbolic 2-plithogenic field.

$2 - SP_F = \{x + yP_1 + zP_2;\ x, y, z \in F, {P_i}^2 = P_i, P_1P_2 = P_2P_1 = P_2\}$.

We define the symbolic 2-plithogenic vector space as follows:

$2 - SP_V = V + VP_1 + VP_2 = \{a + bP_1 + cP_2;\ a, b, c \in V\}$.

Operations on $2 - SP_V$ can be defined as follows:

Addition: $(+): 2 - SP_V \to 2 - SP_V$, such that:

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$[x_0 + x_1P_1 + x_2P_2] + [y_0 + y_1P_1 + ys_2P_2] = (x_0 + y_0) + (x_1 + y_1)P_1 + (x_2 + y_2)P_2$
Multiplication: $(.): 2 - SP_F \times 2 - SP_V \to 2 - SP_V$, such that:
$$[a + bP_1 + cP_2].[x_0 + x_1P_1 + x_2P_2] = ax_0 + (ax_1 + bx_0 + bx_1)P_1 + (ax_2 + bx_2 + cx_0 + cx_1 + cx_2)P_2$$
where $x_i, y_i \in V, a, b, c \in F$

Theorem.

Let $(2 - SP_V, +, .)$ Is a module over the ring $2 - SP_F$.

Example.

Let $V = R^3$ be the Euclidean space over the field $F = R$.
The corresponding symbolic 2-plithogenic vector space over $2 - SP_F$ is:
$2 - SP_{R^3} = \{(x_0, y_0, z_0) + (x_1, y_1, z_1)P_1 + (x_2, y_2, z_2)P_2; x_i, y_i, z_i \in R\}$
Consider $X = (1,1,0) + (2, -1,1)P_1 + (0,1, -1)P_2 \in 2 - SP_{R^3}, A = 2 + P_1 + P_2 \in 2 - SP_R$. We have:
$$A.X = (2,2,0) + [(4, -2,2) + (1,1,0) + (2, -1,1)]P_1 + [(0,2,2) + (0,1,1) + (1,1,0) + (2, -1,1) + (0,1,1)]P_2$$
$$= (2,2,0) + (7, -2,3)P_1 + (3,4,5)P_2$$

Definition.

Let $2 - SP_V$ be a symbolic 2-plithogenic vector space over $2 - SP_F$, let V_0, V_1, V_2 be the three subspaces of V, we define the AH-subspace as follows:
$$W = V_0 + V_1P_1 + V_2P_2 = \{x + yP_1 + zP_2;\ x \in V_0, y \in V_1, z \in V_2\}$$
If $V_0 = V_1 = V_2$, then W is called an AHS-subspace.

Example.

Consider $2 - SP_{R^3}$, we have $V_0 = \{(a, 0,0);\ a \in R\}, V_1 = \{(0, b, 0);\ b \in R\}, V_2 = \{(0,0, c);\ c \in R\}$ are three subspaces of $V = R^3$.
$W = V_0 + V_1P_1 + V_2P_2 = \{(a, 0,0) + (0, b, 0)P_1 + (0,0, c)P_2;\ a, b, c \in R\}$ is an AH-subspace of $2 - SP_{R^3}$.
$T = V_1 + V_1P_1 + V_1P_2 = \{(0, a, 0) + (0, b, 0)P_1 + (0, c, 0)P_2;\ a, b, c \in R\}$ is an AHS-subspace.

Theorem.

Let $2 - SP_V$ be a symbolic 2-plithogenic vector space over $2 - SP_F$, let W be an AHS-subspace of $2 - SP_V$, then W is a submodule of $2 - SP_V$.

Definition.

Let V, W be two vector spaces over the field F. Let $2 - SP_V$, $2 - SP_W$ be the corresponding symbolic 2-plithogenic vector spaces over $2 - SP_F$.
Let $L_0, L_1, L_2: V \to W$ be three linear transformations, we define the AH-linear transformation as follows:
$L: 2 - SP_V \to 2 - SP_W, L = L_0 + L_1P_1 + L_2P_2\ ; L(x + yP_1 + zP_2) = L_0(x) + L_1(y)P_1 + L_2(z)P_2$.
If $L_0 = L_1 = L_2$, then L is called AHS-linear transformation.

Definition.

Let $L = L_0 + L_1P_1 + L_2P_2: 2 - SP_V \to 2 - SP_W$ be an AH-linear transformation, we define:
1. $AH - ker(L) = ker(L_0) + ker(L_1)P_1 + ker(L_2)P_2 = \{x + yP_1 + zP_2\}; x \in ker(L_0), y \in ker(L_1), z \in ker(L_2)$.
2. $AH - Im(L) = Im(L_0) + Im(L_1)P_1 + Im(L_2)P_2 = \{a + bP_1 + cP_2\}; a \in Im(L_0), b \in Im(L_1), c \in Im(L_2)$

If L is AHS-linear transformation, then we get $AHS - kernel$, $AHS - Image$.

Theorem.

Let $L = L_0 + L_1P_1 + L_2P_2: 2 - SP_V \to 2 - SP_W$ be an AH-linear transformation, then:
1. $AH - ker(L)$ is AH-subspace of $2 - SP_V$.
2. $AH - Im(L)$ is AH-subspace of $2 - SP_W$.

Remark.

If L_0, L_1, L_2 are isomorphism, then $ker(L_0) = ker(L_1) = ker(L_2) = \{0\}, Im(L_0) = Im(L_1) = Im(L_2) = W$, thus $AH - ker(L) = \{0\}, AH - Im(L) = 2 - SP_W$.

Example.

Take $V = R^3, W = R^3, L_0, L_1, L_2: V \to W$ such that:
$L_0(x, y, z) = (x, y), L_1(x, y, z) = (2x, z), L_2(x, y, z) = (x - y, y - z)$
The corresponding AH-linear transformation is:
$L = L_0 + L_1P_1 + L_2P_2: 2 - SP_{R^3} \to 2 - SP_{R^2}$:
$$L[(x_0, y_0, z_0) + (x_1, y_1, z_1)P_1 + (x_2, y_2, z_2)P_2] = L_0(x_0, y_0, z_0) + L_1(x_1, y_1, z_1)P_1 + L_2(x_2, y_2, z_2)P_2$$
$$= (x_0, y_0) + (2x_1, z_1)P_1 + (x_2 - y_2, y_2 - z_2)P_2$$
For example, take $X = (1,2,1) + (4,3, -5)P_1 + (1,1,1)P_2$, then:
$L(X) = (1,2) + (8, -5)P_1 + (0,0)P_2 = (1,2) + (8, -5)P_1$.
$$\begin{cases} ker(L_0) = \{(0,0, z_0);\ z_0 \in R\} \\ ker(L_1) = \{(0, y_1, 0);\ y_1 \in R\} \\ ker(L_2) = \{(x_2, x_2, x_2);\ x_2 \in R\} \\ AH - ker(L) = \{(0,0, z_0) + (0, y_1, 0)P_1 + (x_2, x_2, x_2)P_2; z_0, y_1, x_2 \in R\} \end{cases}$$

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Also,

$$\begin{cases} Im(L_0) = R^2 \\ Im(L_1) = R^2 \\ Im(L_2) = R^2 \\ AH - Im(L) = R^2 + R^2P_1 + R^2P_2 = 2 - SP_W \end{cases}$$

Example.

Take $W = V = R^2$, $L_0, L_1, L_2: V \to W$ such that:

$L_0(x, y) = (3x, -2x), L_1(x, y) = (x - y, 2x), L_2(x, y, z) = (x + 2y, y)$

The corresponding AH-linear transformation is $L = L_0 + L_1P_1 + L_2P_2: 2 - SP_V \to 2 - SP_W$;

$$L[(x_0, y_0) + (x_1, y_1)P_1 + (x_2, y_2)P_2] = L_0(x_0, y_0) + L_1(x_1, y_1)P_1 + L_2(x_2, y_2)P_2$$
$$= (3x_0, -2x_0) + (x_1 - y_1, 2x_1)P_1 + (x_2 + 2y_2, y_2)P_2$$

For example $X = (1,4) + (2,8)P_1 + (3, -1)P_2$

$L(X) = (1,4) + (2,8)P_1 + (3, -1)P_2$.

$$\begin{cases} ker(L_0) = \{(0, y_0);\ y_0 \in R\} \\ ker(L_1) = \{0\} \\ ker(L_2) = \{0\} \\ AH - ker(L) = \{(0, y_0) + 0P_1 + 0P_2; y_0 \in R\} \end{cases}$$

Also,

$$\begin{cases} Im(L_0) = \{(a, 0);\ a \in R\} \\ Im(L_1) = R^2 \\ Im(L_2) = R^2 \\ AH - Im(L) = \{(a, 0) + (a_1, b_1)P_1 + (a_2, b_2)P_2; a, a_1, a_2, b_2, b_1 \in R\} \end{cases}$$

Theorem.

Let $L = f + fP_1 + fP_2: 2 - SP_V \to 2 - SP_W$ be an AHS-linear transformation, then L is a module homomorphism.

2-plithogenic modules [3]

Definition.

Let M be a module over the ring R, let $2 - SP_R$ be the corresponding symbolic 2-plithogenic ring.

$2 - SP_R = \{x + yP_1 + zP_2;\ x, y, z \in R, P_i^2 = P_i, P_1P_2 = P_2P_1 = P_2\}$.

We define the symbolic 2-plithogenic module as follows:

$2 - SP_M = M + MP_1 + MP_2 = \{a + bP_1 + cP_2;\ a, b, c \in M\}$.

Operations on $2 - SP_M$ can be defined as follows:

Addition: $(+): 2 - SP_M \to 2 - SP_M$, such that:

$[x_0 + x_1P_1 + x_2P_2] + [y_0 + y_1P_1 + ys_2P_2] = (x_0 + y_0) + (x_1 + y_1)P_1 + (x_2 + y_2)P_2$

Multiplication: $(.): 2 - SP_R \times 2 - SP_M \to 2 - SP_M$, such that:

$[a + bP_1 + cP_2].[x_0 + x_1P_1 + x_2P_2] = ax_0 + (ax_1 + bx_0 + bx_1)P_1 + (ax_2 + bx_2 + cx_0 + cx_1 + cx_2)P_2$.

where $x_i, y_i \in M, a, b, c \in R$

Theorem.

Let $(2 - SP_M, +, .)$ Is a module over the ring $2 - SP_R$.

Example.

Let $M = Z^3$ be the module over the ring $R =$.

The corresponding symbolic 2-plithogenic vector space over $2 - SP_Z$ is:

$2 - SP_{Z^3} = \{(x_0, y_0, z_0) + (x_1, y_1, z_1)P_1 + (x_2, y_2, z_2)P_2; x_i, y_i, z_i \in Z\}$

Consider $X = (1,1,0) + (2, -1,1)P_1 + (0,1, -1)P_2 \in 2 - SP_{Z^3}, A = 2 + P_1 + P_2 \in 2 - SP_Z$. We have:

$A.X = (2,2,0) + [(4, -2,2) + (1,1,0) + (2, -1,1)]P_1 + [(0,2,2) + (0,1,1) + (1,1,0) + (2, -1,1) + (0,1,1)]P_2 = (2,2,0) + (7, -2,3)P_1 + (3,4,5)P_2$.

Definition.

Let $2 - SP_M$ be a symbolic 2-plithogenic module over $2 - SP_R$, let M_0, M_1, M_2 be the three sub-modules of V, we define the AH-submodule as follows:

$W = M_0 + M_1P_1 + M_2P_2 = \{x + yP_1 + zP_2;\ x \in M_0, y \in M_1, z \in M_2\}$.

If $M_0 = M_1 = M_2$, then W is called an AHS-sub-module.

Example.

Consider $2 - SP_{Z^3}$, we have $M_0 = \{(a, 0,0);\ a \in R\}, M_1 = \{(0, b, 0);\ b \in R\}, M_2 = \{(0,0, c);\ c \in Z\}$ are three sub-modules of $M = Z^3$.

$W = M + M_1P_1 + M_2P_2 = \{(a, 0,0) + (0, b, 0)P_1 + (0,0, c)P_2;\ a, b, c \in Z\}$ is an AH-submodule of $2 - SP_{Z^3}$.

$T = M_1 + MP_1 + M_1P_2 = \{(0, a, 0) + (0, b, 0)P_1 + (0, c, 0)P_2;\ a, b, c \in Z\}$ is an AHS-submodule.

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Theorem.
Let $2-SP_M$ be a symbolic 2-plithogenic module over $2-SP_R$, let W be an AHS-submodule of $2-SP_M$, then W is a submodule of $2-SP_M$.

Definition.
Let V, W be two modules over the ring R. Let $2-SP_V$, $2-SP_W$ be the corresponding symbolic 2-plithogenic modules over $2-SP_R$.
Let $L_0, L_1, L_2: V \to W$ be three homomorphisms, we define the AH-homomorphism as follows:
$L: 2-SP_V \to 2-SP_W, L = L_0 + L_1P_1 + L_2P_2 \,; L(x + yP_1 + zP_2) = L_0(x) + L_1(y)P_1 + L_2(z)P_2$.
If $L_0 = L_1 = L_2$, then L is called AHS-homomorphism.

Definition.
Let $L = L_0 + L_1P_1 + L_2P_2: 2-SP_V \to 2-SP_W$ be an AH-homomorphism, we define:

3. $AH-ker(L) = ker(L_0) + ker(L_1)P_1 + ker(L_2)P_2 = \{x + yP_1 + zP_2\}; x \in ker(L_0), y \in ker(L_1), z \in ker(L_2)$.
4. $AH-Im(L) = Im(L_0) + Im(L_1)P_1 + Im(L_2)P_2 = \{a + bP_1 + cP_2\}; a \in Im(L_0), b \in Im(L_1), c \in Im(L_2)$

If L is AHS-linear homomorphism, then we get $AHS-kernel$, $AHS-Image$.

Theorem.
Let $L = L_0 + L_1P_1 + L_2P_2: 2-SP_V \to 2-SP_W$ be an AH-homomorphism, then:

3. $AH-ker(L)$ is AH-submodule of $2-SP_V$.
4. $AH-Im(L)$ is AH-submodule of $2-SP_W$.

Remark.
If L_0, L_1, L_2 are isomorphisms, then $ker(L_0) = ker(L_1) = ker(L_2) = \{0\}, Im(L_0) = Im(L_1) = Im(L_2) = W$, thus $AH-ker(L) = \{0\}, AH-Im(L) = 2-SP_W$.

Example.
Take $V = Z^3, W = Z$, $L_0, L_1, L_2: V \to W$ such that:
$L_0(x, y, z) = (x), L_1(x, y, z) = (y), L_2(x, y, z) = (z)$
The corresponding AH-homomorphism is:
$L = L_0 + L_1P_1 + L_2P_2: 2-SP_{Z^3} \to 2-SP_Z$:
$L[(x_0, y_0, z_0) + (x_1, y_1, z_1)P_1 + (x_2, y_2, z_2)P_2] = L_0(x_0, y_0, z_0) + L_1(x_1, y_1, z_1)P_1 + L_2(x_2, y_2, z_2)P_2 = (x_0) + (y_1)P_1 + (z_2)P_2$.
For example, take $X = (1,9,8) + (9,10,-9)P_1 + (3,2,1)P_2$, then:
$L(X) = 1 + (10)P_1 + P_2$.

$$\begin{cases} ker(L_0) = \{ (0, y_0, z_0);\ y_0, z_0 \in Z\} \\ ker(L_1) = \{(x_1, 0, z_1);\ x_1, z_1 \in Z\} \\ ker(L_2) = \{(x_2, y_2, 0);\ x_2, y_2 \in Z\} \\ AH-ker(L) = \{(0, y_0, z_0) + (x_1, 0, z_1)P_1 + (x_2, y_2, 0)P_2; y_0, z_0, x_1, z_1, x_2, y_2 \in Z\} \end{cases}$$

Also,

$$\begin{cases} Im(L_0) = Z \\ Im(L_1) = Z \\ Im(L_2) = Z \\ AH-Im(L) = Z + ZP_1 + ZP_2 = 2-SP_W \end{cases}$$

Theorem.
Let $L = f + fP_1 + fP_2: 2-SP_V \to 2-SP_W$ be an AHS-homomorphism, then L is a module homomorphism.

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