Neutrosophic Crisp Generalized sg-Closed Sets and their Continuity

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Abstract

In this paper, we delivered pioneering notions of closed sets in the neutrosophic crisp sense. In other words, we discussed sg-closed sets, gs-closed sets, and gsg-closed sets in neutrosophic crisp topological space. Moreover, the subsequent innovative ideas are established, for instance, gsg-closure and gsg-interior in neutrosophic crisp topological space, and obtaining numerous of their highlights. Besides, we submitted different kinds of neutrosophic crisp continuous functions and their associations.

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1. Introduction

The notion of neutrosophic crisp for topological space was stated by A. A. Salama et al. [1] and symbolized merely Neu_{CTS}. Next, the various types of crisp nearly open sets were submitted by A. A. Salama et al. [2]. Moreover, the extension of semi-α-closed sets in neutrosophic crisp topological space was presented by R. K. Al-Hamido et al. [3]. Furthermore, the different perceptions of weak forms of open and closed functions in the sense of neutrosophic crisp topological space were displayed by A. H. M. Al-Obaidi et al. [4,5]. Additionally, in neutrosophic topological space, the viewpoint of generalized homeomorphism was represented by Md. Hanif Page et al. [6]. Besides, the weak types of continuity in the sense of neutrosophic crisp topological space were offered by Q. H. Imran et al. [7,8]. Likewise, the intellect of neutrosophic homeomorphism and neutrosophic $\alpha\psi$-homeomorphism were raised by M. Parimala et al. [9]. Subsequent, they set up the thought of neutrosophic $\alpha\psi^*$-homeomorphism, neutrosophic $\alpha\psi$-open and closed mapping and neutrosophic $T\alpha\psi$-space. Consequently, the maps with features αgs-continuity and αgs-irresolute in neutrosophic topological space were inserted by V. Banu Priya et al. [10]. Finally, in neutrosophic topological space, the senses of α-generalized semi-closed and α-generalized semi-open sets were informed by V. Banu Priya et al. [11]. This article seeks to ascertain the neutrosophic crisp topological space perception for sg-closed, gs-closed, and gsg-closed sets and analysis of their essential components. Besides, we detect neutrosophic crisp gsg-closure and neutrosophic crisp gsg-interior sets and accomplish certain of their attributes. Likewise, we give different classes of neutrosophic crisp continuous functions and their interactions.

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2. Preliminaries

All through this work, $(\mathcal{A},\mathcal{T}),(\mathcal{B},\mathcal{L})$ and $(\mathcal{C},\mathcal{J})$ (or simply $\mathcal{A},\mathcal{B}$ and $\mathcal{C}$, respectively) frequently imply Neu_{CTSs}. For any neutrosophic crisp set $\mathcal{U}$ in a Neu_{CTS} $(\mathcal{A},\mathcal{T})$, its closure is signified by $Neu_C cl(\mathcal{U})$, its interior is signified by $Neu_C int(\mathcal{U})$, and its complement is signified by $\underline{\mathcal{U}} = \mathcal{A}_{Neu} - \mathcal{U}$, correspondingly.

Definition 2.1: [1]
Let $\mathcal{A}$ be a non-empty particular fixed space, and let $\mathcal{U}_1,\mathcal{U}_2,\mathcal{U}_3$ be subsets of $\mathcal{A}$ with the mutually exclusive property. An object is a neutrosophic crisp set (or merely Neu_C-set) $\mathcal{U}$ with type $\mathcal{U} = \langle\mathcal{U}_1,\mathcal{U}_2,\mathcal{U}_3\rangle$.

Definition 2.2: [1]
A collection $\mathcal{T}$ of Neu_C-sets in a non-empty particular fixed space $\mathcal{A}$ is called a neutrosophic crisp topology (in short, Neu_{CT}) on $\mathcal{A}$ satisfying the following conditions below:
(i) $\varphi_{Neu},\mathcal{A}_{Neu} \in \mathcal{T}$,
(ii) $\mathcal{U}_1 \cap \mathcal{U}_2 \in \mathcal{T}$ where $\mathcal{U}_1,\mathcal{U}_2 \in \mathcal{T}$,
(iii) $\cup\mathcal{U}_k \in \mathcal{T}$ for all collections $\{\mathcal{U}_k | k \in \Delta\} \subseteq \mathcal{T}$.
In this circumstance, the name of the ordered pair $(\mathcal{A},\mathcal{T})$ is Neu_{CTS} and every Neu_C-set in $\mathcal{T}$ is titled as neutrosophic crisp open set (fleetly, Neu_COS). The complement $\underline{\mathcal{U}}$ of a Neu_COS $\mathcal{U}$ in $\mathcal{A}$ is established as neutrosophic crisp closed set (fleetly, Neu_CCS) in $\mathcal{A}$.

Definition 2.3: [2]
A Neu_C-subset $\mathcal{U}$ of a Neu_{CTS} $(\mathcal{A},\mathcal{T})$ is known to be a neutrosophic crisp semi-open set (shortly, Neu_{Cs}OS) if $\mathcal{U} \subseteq Neu_C cl(Neu_C int(\mathcal{U}))$ and a neutrosophic crisp semi-closed set (shortly, Neu_{Cs}CS) if $Neu_C int(Neu_C cl(\mathcal{U})) \subseteq \mathcal{U}$. The neutrosophic crisp semi-closure of $\mathcal{U}$ of a Neu_{CTS} $(\mathcal{A},\mathcal{T})$ is the intersecting of the all Neu_{Cs} CSs that involve $\mathcal{U}$ and it is signified by $Neu_{Cs}cl(\mathcal{U})$.

Definition 2.4: [12]
Suppose Neu_C-subset $\mathcal{U}$ and Neu_COS $\mathcal{M}$ are in a Neu_{CTS} $(\mathcal{A},\mathcal{T})$ such that $\mathcal{U} \subseteq \mathcal{M}$ then $\mathcal{U}$ is so-called a neutrosophic crisp generalized closed set (in brief, Neu_{Cg}CS) if $Neu_C cl(\mathcal{U}) \subseteq \mathcal{M}$ and the complement of a Neu_{Cg}CS is a Neu_{Cg}-open set (in brief, Neu_{Cg}OS)in $(\mathcal{A},\mathcal{T})$.

Remark 2.5: [2,12]
In a Neu_{CTS} $(\mathcal{A},\mathcal{T})$, then the succeeding declarations grip and the reverse of each declaration is not suitable:
(i) To all Neu_COS (corr. Neu_CCS) are Neu_{Cs}OS (corr. Neu_{Cs}CS).
(ii) To all Neu_COS (corr. Neu_CCS) are Neu_{Cg}OS (corr. Neu_{Cg}CS).

Definition 2.6: [1]
A function $t: (\mathcal{A},\mathcal{T}) \longrightarrow (\mathcal{B},\mathcal{L})$ is understood to be neutrosophic crisp continuous (in short, Neu_C-continuous) if $t^{-1}(\mathcal{U})$ is a Neu_CCS (Neu_COS) in $(\mathcal{A},\mathcal{T})$ for every Neu_CCS (Neu_COS) $\mathcal{U}$ in $(\mathcal{B},\mathcal{L})$.

Definition 2.7: [2]
A function $t: (\mathcal{A},\mathcal{T}) \longrightarrow (\mathcal{B},\mathcal{L})$ is understood to be neutrosophic crisp semi-continuous (in short, Neu_{Cs}-continuous) if $t^{-1}(\mathcal{U})$ is a Neu_{Cs}CS (Neu_{Cs}OS) in $(\mathcal{A},\mathcal{T})$ for every Neu_CCS (Neu_COS) $\mathcal{U}$ in $(\mathcal{B},\mathcal{L})$.

Remark 2.8: [2]
To all Neu_C-continuous are a Neu_{Cs}-continuous; however, the reverse does not reasonable in common.

3. Neutrosophic Crisp gsg-Closed Sets

In this circumstance, we pioneer and investigate the neutrosophic crisp gsg-closed sets with several of their features.

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Definition 3.1:
A Neu_C-subset $\mathcal{U}$ of Neu_{CTS} $(\mathcal{A},\mathcal{T})$ is told to be:
(i) a neutrosophic crisp sg-closed set (shortly, Neu_{Csg}CS) if $Neu_{Cs}cl(\mathcal{U}) \subseteq \mathcal{M}$ whenever $\mathcal{U} \subseteq \mathcal{M}$ and $\mathcal{M}$ is a Neu_{Cs}OS in $(\mathcal{A},\mathcal{T})$. For each Neu_{Csg}CS, its complement is a Neu_{Csg}-open set (in brief, Neu_{Csg}OS) in $(\mathcal{A},\mathcal{T})$.
(ii) a neutrosophic crisp gs-closed set (shortly, Neu_{Cgs}CS) if $Neu_{Cs}cl(\mathcal{U}) \subseteq \mathcal{M}$ whenever $\mathcal{U} \subseteq \mathcal{M}$ and $\mathcal{M}$ is a Neu_COS in $(\mathcal{A},\mathcal{T})$. For each Neu_{Cgs}CS, its complement is a Neu_{Cgs}-open set (in brief, Neu_{Cgs}OS)in $(\mathcal{A},\mathcal{T})$.

Definition 3.2:
A Neu_C-subset $\mathcal{U}$ of a Neu_{CTS} $(\mathcal{A},\mathcal{T})$ is named to be a neutrosophic crisp gsg-closed set (in short, Neu_{Cgsg}CS) if $Neu_Ccl(\mathcal{U}) \subseteq \mathcal{M}$ whenever $\mathcal{U} \subseteq \mathcal{M}$ and $\mathcal{M}$ is a Neu_{Csg}OS in $(\mathcal{A},\mathcal{T})$. The collection of all Neu_{Cgsg}CSs of a Neu_{CTS} $(\mathcal{A},\mathcal{T})$ is symbolized by Neu_{Cgsg}C$(\mathcal{A})$.

Proposition 3.3:
In a Neu_{CTS} $(\mathcal{A},\mathcal{T})$, the following declarations are reasonable:
(i) To all Neu_{Cg}CS are Neu_{Cgs}CS.
(ii) To all Neu_{Cs}CS are Neu_{Csg}CS.
(iii) To all Neu_{Csg}CS are Neu_{Cgs}CS.
Proof:
(i) Suppose Neu_{Cg}CS $\mathcal{U}$ is in a Neu_{CTS} $(\mathcal{A},\mathcal{T})$. Then $Neu_Ccl(\mathcal{U}) \subseteq \mathcal{M}$ whenever $\mathcal{U} \subseteq \mathcal{M}$ and $\mathcal{M}$ is a Neu_COS in $\mathcal{A}$. But $Neu_{Cs}cl(\mathcal{U}) \subseteq Neu_Ccl(\mathcal{U})$ whenever $\mathcal{U} \subseteq \mathcal{M}$, $\mathcal{M}$ is a Neu_COS in $\mathcal{A}$. Now we have $Neu_{Cs}cl(\mathcal{U}) \subseteq \mathcal{M}$, $\mathcal{U} \subseteq \mathcal{M}$, $\mathcal{M}$ is a Neu_COS in $\mathcal{A}$. Therefore $\mathcal{U}$ is a Neu_{Cgs}CS. The proof is evident for others. ▪
The reverse of the exceeding result need not be valid, as seen in the subsequent instances.

Example 3.4:
Suppose $\mathcal{A} = \{v_1, v_2, v_3, v_4\}$. Let $\mathcal{T} = \{\varphi_{Neu}, \langle\{v_1\}, \varphi, \varphi\rangle, \langle\{v_2, v_4\}, \varphi, \varphi\rangle, \langle\{v_1, v_2, v_4\}, \varphi, \varphi\rangle, \mathcal{A}_{Neu}\}$ be a Neu_{CT} on $\mathcal{A}$. Then $\langle\{v_2, v_4\}, \varphi, \varphi\rangle$ is a Neu_{Cgs}CS, just not Neu_{Cg}CS.

Example 3.5:
In example (3.4), then $\langle\{v_3, v_4\}, \varphi, \varphi\rangle$ is a Neu_{Csg}CS, just not Neu_{Cs}CS.

Example 3.6:
Suppose $\mathcal{A} = \{v_1, v_2, v_3\}$. Let $\mathcal{T} = \{\varphi_{Neu}, \langle\{v_1\}, \varphi, \varphi\rangle, \mathcal{A}_{Neu}\}$ be a Neu_{CT} on $\mathcal{A}$. Then $\langle\{v_1, v_3\}, \varphi, \varphi\rangle$ is a Neu_{Cgs}CS, just not Neu_{Csg}CS.

Proposition 3.7:
In a Neu_{CTS} $(\mathcal{A},\mathcal{T})$, the following declarations are reasonable:
(i) Each Neu_CCS is a Neu_{Cgsg}CS.
(ii) Each Neu_{Cgsg}CS is a Neu_{Cg}CS.
Proof:
(i) Suppose that $\mathcal{U}$ is a Neu_CCS in a Neu_{CTS} $(\mathcal{A},\mathcal{T})$ and consider $\mathcal{M}$ is a Neu_{Csg}OS in $\mathcal{A}$ wherever $\mathcal{U} \subseteq \mathcal{M}$. Then $Neu_Ccl(\mathcal{U}) = \mathcal{U} \subseteq \mathcal{M}$. Therefore $\mathcal{U}$ is a Neu_{Cgsg}CS.
(ii) Let $\mathcal{U}$ be a Neu_{Cgsg}CS in a Neu_{CTS} $(\mathcal{A},\mathcal{T})$ and let $\mathcal{M}$ be a Neu_COS in $\mathcal{A}$ such that $\mathcal{U} \subseteq \mathcal{M}$. Since every Neu_COS is a Neu_{Csg}OS, we have $Neu_Ccl(\mathcal{U}) \subseteq \mathcal{M}$. Therefore $\mathcal{U}$ is a Neu_{Cg}CS. ▪
As seen in the subsequent examples, the reverse of the above proposition need not be accurate.

Example 3.8:
Suppose $\mathcal{A} = \{s_1, s_2, s_3, s_4\}$. Let $\mathcal{T} = \{\varphi_{Neu}, \langle\{s_1\}, \varphi, \varphi\rangle, \langle\{s_2, s_3\}, \varphi, \varphi\rangle, \langle\{s_1, s_2, s_3\}, \varphi, \varphi\rangle, \mathcal{A}_{Neu}\}$ be a Neu_{CT} on $\mathcal{A}$. Then $\langle\{s_2, s_3\}, \varphi, \varphi\rangle$ is a Neu_{Cgsg}CS, just not Neu_CCS.

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Example 3.9:
Suppose $\mathcal{A} = \{v_1, v_2, v_3, v_4, v_5\}$.
Let $\mathcal{T} = \{\varphi_{Neu}, \langle\{v_4\}, \varphi, \varphi\rangle, \langle\{v_1, v_2\}, \varphi, \varphi\rangle, \langle\{v_1, v_2, v_4\}, \varphi, \varphi\rangle, \mathcal{A}_{Neu}\}$ be a Neu_{CT} on $\mathcal{A}$. Then $\langle\{v_1, v_3, v_4\}, \varphi, \varphi\rangle$ is a Neu_{Cg}CS, just not Neu_{Cgsg}CS.

Proposition 3.10:
In a Neu_{CTS} $(\mathcal{A}, \mathcal{T})$, the following declarations are reasonable:
(i) Each Neu_{Cgsg}CS is a Neu_{Csg}CS.
(ii) Each Neu_{Cgsg}CS is a Neu_{Cgs}CS.
Proof:
(i) Consider that $\mathcal{U}$ is a Neu_{Cgsg}CS in a Neu_{CTS} $(\mathcal{A}, \mathcal{T})$ and postulate that $\mathcal{M}$ is a Neu_{Cs}OS in $\mathcal{A}$ wherever $\mathcal{U} \subseteq \mathcal{M}$. Since every Neu_{Cs}OS is a Neu_{Csg}OS, we have $Neu_{Cs}cl(\mathcal{U}) \subseteq Neu_{C}cl(\mathcal{U}) \subseteq \mathcal{M}$ implies $Neu_{Cs}cl(\mathcal{U}) \subseteq \mathcal{M}$. Therefore $\mathcal{U}$ is a Neu_{Csg}CS.
(ii) Let $\mathcal{U}$ be a Neu_{Cgsg}CS in a Neu_{CTS} $(\mathcal{A}, \mathcal{T})$ and let $\mathcal{M}$ be a Neu_{C}OS in $\mathcal{A}$ such that $\mathcal{U} \subseteq \mathcal{M}$.Since every Neu_{C}OS is a Neu_{Csg}OS, we have $Neu_{Cs}cl(\mathcal{U}) \subseteq Neu_{C}cl(\mathcal{U}) \subseteq \mathcal{M}$ implies $Neu_{Cs}cl(\mathcal{U}) \subseteq \mathcal{M}$. Therefore $\mathcal{U}$ is a Neu_{Cgs}CS. ▪
The reverse of the above proposition need not be accurate, as shown in the subsequent instance.

Example 3.11:
Suppose $\mathcal{A} = \{s_1, s_2, s_3, s_4\}$. Let $\mathcal{T} = \{\varphi_{Neu}, \langle\{s_1\}, \varphi, \varphi\rangle, \langle\{s_2, s_4\}, \varphi, \varphi\rangle, \langle\{s_1, s_2, s_4\}, \varphi, \varphi\rangle, \mathcal{A}_{Neu}\}$ be a Neu_{CT} on $\mathcal{A}$. Then $\langle\{s_1\}, \varphi, \varphi\rangle$ is a Neu_{Csg}CS and hence Neu_{Cgs}CS, just not Neu_{Cgsg}CS.

Remark 3.12:
The Neu_{Cgsg}CS and Neu_{Cs}CS are independent.

Definition 3.13:
A Neu_{C}-subset $\mathcal{U}$ of a Neu_{CTS} $(\mathcal{A}, \mathcal{T})$ is stated as a neutrosophic crisp gsg-open set (shorty Neu_{Cgsg}OS) iff $\mathcal{A}_{Neu} - \mathcal{U}$ is a Neu_{Cgsg}CS. The collection of all Neu_{Cgsg}OSs of a Neu_{CTS} $(\mathcal{A}, \mathcal{T})$ is symbolized by Neu_{Cgsg}O$(\mathcal{A})$.

Proposition 3.14:
Suppose that Neu_{C}OS $\mathcal{U}$ is in $Neu_{CTS}(\mathcal{A}, \mathcal{T})$, then this set stands as a Neu_{Cgsg}OS in that topological space.
Proof:
Assume that a Neu_{C}OS $\mathcal{U}$ is in a Neu_{CTS} $(\mathcal{A}, \mathcal{T})$, so therefore $\mathcal{A}_{Neu} - \mathcal{U}$ stands as a Neu_{C}CS in $(\mathcal{A}, \mathcal{T})$. By employing proposition (3.7) portion (i), $\mathcal{A}_{Neu} - \mathcal{U}$ is a Neu_{Cgsg}CS. Thus, $\mathcal{U}$ is a Neu_{Cgsg}OS in $(\mathcal{A}, \mathcal{T})$.▪

Proposition 3.15:
Suppose that Neu_{Cgsg}OS $\mathcal{U}$ is in Neu_{CTS} $(\mathcal{A}, \mathcal{T})$, then this set is a Neu_{Cg}OS in that topological space.
Proof:
Let $\mathcal{U}$ be a Neu_{Cgsg}OS in a Neu_{CTS} $(\mathcal{A}, \mathcal{T})$, then $\mathcal{A}_{Neu} - \mathcal{U}$ is a Neu_{Cgsg}CS in $(\mathcal{A}, \mathcal{T})$. By employing proposition (3.7) portion (ii), $\mathcal{A}_{Neu} - \mathcal{U}$ is a Neu_{Cg}CS. Thus, $\mathcal{U}$ is a Neu_{Cg}OS in $(\mathcal{A}, \mathcal{T})$. ▪

Proposition 3.16:
In a Neu_{CTS} $(\mathcal{A}, \mathcal{T})$, the subsequent declarations are reasonable:
(i) To all Neu_{Cgsg}OS are Neu_{Csg}OS.
(ii) To all Neu_{Cgsg}OS are Neu_{Cgs}OS.
Proof:
Similar to the exceeding result. ▪

Theorem 3.17:
Suppose that $\mathcal{U}$ and $\mathcal{V}$ are Neu_{Cgsg}CSs in a Neu_{CTS} $(\mathcal{A}, \mathcal{T})$, then $\mathcal{U} \cup \mathcal{V}$ is a Neu_{Cgsg}CS.

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Proof:
Assume that $\mathcal{U}$ and $\mathcal{V}$ be two Neu_{Cgsg}CSs in a Neu_{CTS} $(\mathcal{A},\mathcal{T})$ and assume that $\mathcal{M}$ be any Neu_{Csg}OS in $\mathcal{A}$ where $\mathcal{U} \subseteq \mathcal{M}$ and $\mathcal{V} \subseteq \mathcal{M}$. Therefore, we get $\mathcal{U} \cup \mathcal{V} \subseteq \mathcal{M}$. Later $\mathcal{U}$ and $\mathcal{V}$ are Neu_{Cgsg}CSs in $\mathcal{A}$, $Neu_C cl(\mathcal{U}) \subseteq \mathcal{M}$ and $Neu_C cl(\mathcal{V}) \subseteq \mathcal{M}$. Now, $Neu_C cl(\mathcal{U} \cup \mathcal{V}) = Neu_C cl(\mathcal{U}) \cup Neu_C cl(\mathcal{V}) \subseteq \mathcal{M}$ and so $Neu_C cl(\mathcal{U} \cup \mathcal{V}) \subseteq \mathcal{M}$. Hence $\mathcal{U} \cup \mathcal{V}$ stands in $\mathcal{A}$ as a Neu_{Cgsg}CS. ▪

Proposition 3.18:
Assume Neu_{Cgsg}CS $\mathcal{U}$ is in a Neu_{CTS} $(\mathcal{A},\mathcal{T})$, then $Neu_C cl(\mathcal{U}) - \mathcal{U}$ contains no non-empty Neu_CCS in $(\mathcal{A},\mathcal{T})$.
Proof:
Postulate that $\mathcal{U}$ is a Neu_{Cgsg}CS in a Neu_{CTS} $(\mathcal{A},\mathcal{T})$ and let $\mathcal{F}$ be any Neu_CCS in $(\mathcal{A},\mathcal{T})$ where $\mathcal{F} \subseteq Neu_C cl(\mathcal{U}) - \mathcal{U}$. Since $\mathcal{U}$ is a Neu_{Cgsg}CS, we have $Neu_C cl(\mathcal{U}) \subseteq \mathcal{A}_{Neu} - \mathcal{F}$. This implies $\mathcal{F} \subseteq \mathcal{A}_{Neu} - Neu_C cl(\mathcal{U})$. Then $\mathcal{F} \subseteq Neu_C cl(\mathcal{U}) \cap (\mathcal{A}_{Neu} - Neu_C cl(\mathcal{U})) = \varphi_{Neu}$. Thus, $\mathcal{F} = \varphi_{Neu}$. Hence $Neu_C cl(\mathcal{U}) - \mathcal{U}$ involves no non-null Neu_CCS in $(\mathcal{A},\mathcal{T})$. ▪

Proposition 3.19:
A Neu_C-set $\mathcal{U}$ is Neu_{Cgsg}CS in a Neu_{CTS} $(\mathcal{A},\mathcal{T})$ iff $Neu_C cl(\mathcal{U}) - \mathcal{U}$ contains no non-empty Neu_{Csg}CS in $(\mathcal{A},\mathcal{T})$.
Proof:
Postulate that $\mathcal{U}$ is a Neu_{Cgsg}CS in a Neu_{CTS} $(\mathcal{A},\mathcal{T})$ and let $\mathcal{K}$ be any Neu_{Csg}CS in $(\mathcal{A},\mathcal{T})$ where $\mathcal{K} \subseteq Neu_C cl(\mathcal{U}) - \mathcal{U}$. Meanwhile, $\mathcal{U}$ is a Neu_{Cgsg}CS, we have $Neu_C cl(\mathcal{U}) \subseteq \mathcal{A}_{Neu} - \mathcal{K}$. This implies $\mathcal{K} \subseteq \mathcal{A}_{Neu} - Neu_C cl(\mathcal{U})$. Then $\mathcal{K} \subseteq Neu_C cl(\mathcal{U}) \cap (\mathcal{A}_{Neu} - Neu_C cl(\mathcal{U})) = \varphi_{Neu}$. Thus, $\mathcal{K}$ is null.
In contrast, suppose that $Neu_C cl(\mathcal{U}) - \mathcal{U}$ involves no non-null Neu_{Csg}CS in $(\mathcal{A},\mathcal{T})$. Let $\mathcal{U} \subseteq \mathcal{M}$ and $\mathcal{M}$ is Neu_{Csg}OS. If $Neu_C cl(\mathcal{U}) \subseteq \mathcal{M}$ then $Neu_C cl(\mathcal{U}) \cap (\mathcal{A}_{Neu} - \mathcal{M})$ is non-empty. Because $Neu_C cl(\mathcal{U})$ is Neu_CCS and $\mathcal{A}_{Neu} - \mathcal{M}$ is Neu_{Csg}CS, we have $Neu_C cl(\mathcal{U}) \cap (\mathcal{A}_{Neu} - \mathcal{M})$ is non-empty Neu_{Csg}CS of $Neu_C cl(\mathcal{U}) - \mathcal{U}$, which is a contradiction. Therefore $Neu_C cl(\mathcal{U}) \nsubseteq \mathcal{M}$. Hence $\mathcal{U}$ is a Neu_{Cgsg}CS. ▪

Theorem 3.20:
If $\mathcal{U}$ is a Neu_{Cgsg}CS in a Neu_{CTS} $(\mathcal{A},\mathcal{T})$ where $\mathcal{U} \subseteq \mathcal{V} \subseteq Neu_C cl(\mathcal{U})$, then $\mathcal{V}$ is a Neu_{Cgsg}CS in $(\mathcal{A},\mathcal{T})$.
Proof:
Postulate that $\mathcal{U}$ is a Neu_{Cgsg}CS in a Neu_{CTS} $(\mathcal{A},\mathcal{T})$. Postulate $\mathcal{M}$ be a Neu_{Csg}OS in $(\mathcal{A},\mathcal{T})$ such that $\mathcal{V} \subseteq \mathcal{M}$. Then $\mathcal{U} \subseteq \mathcal{M}$ and because $\mathcal{U}$ stands as a Neu_{Cgsg}CS, it follows that $Neu_C cl(\mathcal{U}) \subseteq \mathcal{M}$. Now, $\mathcal{V} \subseteq Neu_C cl(\mathcal{U})$ implies $Neu_C cl(\mathcal{V}) \subseteq Neu_C cl(Neu_C cl(\mathcal{U})) = Neu_C cl(\mathcal{U})$. Thus, $Neu_C cl(\mathcal{V}) \subseteq \mathcal{M}$. Hence $\mathcal{V}$ is a Neu_{Cgsg}CS. ▪

Proposition 3.21:
Let $\mathcal{U} \subseteq \mathcal{B} \subseteq \mathcal{A}$ and if $\mathcal{U}$ is a Neu_{Cgsg}CS in $\mathcal{A}$, then $\mathcal{U}$ is a Neu_{Cgsg}CS relative to $\mathcal{B}$.
Proof:
$\mathcal{U} \subseteq \mathcal{B} \cap \mathcal{M}$ everywhere $\mathcal{M}$ is a Neu_{Csg}OS in $\mathcal{A}$. So therefore $\mathcal{U} \subseteq \mathcal{M}$ and consequently $Neu_C cl(\mathcal{U}) \subseteq \mathcal{M}$. It indicates that $\mathcal{B} \cap Neu_C cl(\mathcal{U}) \subseteq \mathcal{B} \cap \mathcal{M}$. Thus $\mathcal{U}$ is a Neu_{Cgsg}CS analogous to $\mathcal{B}$. ▪

Proposition 3.22:
Assume that $\mathcal{U}$ is a Neu_{Csg}OS and a Neu_{Cgsg}CS in Neu_{CTS} $(\mathcal{A},\mathcal{T})$, then $\mathcal{U}$ is a Neu_CCS in $(\mathcal{A},\mathcal{T})$.
Proof:
Consider $\mathcal{U}$ is a Neu_{Csg}OS and a Neu_{Cgsg}CS in Neu_{CTS} $(\mathcal{A},\mathcal{T})$, so therefore $Neu_C cl(\mathcal{U}) \subseteq \mathcal{U}$ and since $\mathcal{U} \subseteq Neu_C cl(\mathcal{U})$. Thus, $Neu_C cl(\mathcal{U}) = \mathcal{U}$. For this reason, $\mathcal{U}$ is a Neu_CCS. ▪

Theorem 3.23:
If $\mathcal{U}$ and $\mathcal{V}$ are Neu_{Cgsg}OSs in a Neu_{CTS} $(\mathcal{A},\mathcal{T})$, then $\mathcal{U} \cap \mathcal{V}$ is a Neu_{Cgsg}OS.
Proof:
Let $\mathcal{U}$ and $\mathcal{V}$ be Neu_{Cgsg}OSs in a Neu_{CTS} $(\mathcal{A},\mathcal{T})$. Then $\mathcal{A}_{Neu} - \mathcal{U}$ and $\mathcal{A}_{Neu} - \mathcal{V}$ are Neu_{Cgsg}CSs. By theorem (3.17), $(\mathcal{A}_{Neu} - \mathcal{U}) \cup (\mathcal{A}_{Neu} - \mathcal{V})$ is a Neu_{Cgsg}CS. Since $(\mathcal{A}_{Neu} - \mathcal{U}) \cup (\mathcal{A}_{Neu} - \mathcal{V}) = \mathcal{A}_{Neu} - (\mathcal{U} \cap \mathcal{V})$. For this reason, $\mathcal{U} \cap \mathcal{V}$ is a Neu_{Cgsg}OS. ▪

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Theorem 3.24:
A Neu_C-set $\mathcal{U}$ is Neu_{Cgsg}OS iff $\mathcal{W} \subseteq Neu_C int(\mathcal{U})$ where $\mathcal{W} \subseteq \mathcal{U}$ besides $\mathcal{W}$ stands as a Neu_{Cgsg}CS.
Proof:
Suppose that $\mathcal{W} \subseteq Neu_C int(\mathcal{U})$ where $\mathcal{W}$ is a Neu_{Cgsg}CS and $\mathcal{W} \subseteq \mathcal{U}$. Then $\mathcal{A}_{Neu} - \mathcal{U} \subseteq \mathcal{A}_{Neu} - \mathcal{W}$ and $\mathcal{A}_{Neu} - \mathcal{W}$ is a Neu_{Csg}OS by proposition (3.16). Now, $Neu_C cl(\mathcal{A}_{Neu} - \mathcal{U}) = \mathcal{A}_{Neu} - Neu_C int(\mathcal{U}) \subseteq \mathcal{A}_{Neu} - \mathcal{W}$. Then $\mathcal{A}_{Neu} - \mathcal{U}$ is a Neu_{Cgsg}CS. Hence $\mathcal{U}$ is a Neu_{Cgsg}OS.
Conversely, let $\mathcal{U}$ be a Neu_{Cgsg}OS and $\mathcal{W}$ be a Neu_{Cgsg}CS and $\mathcal{W} \subseteq \mathcal{U}$. Then $\mathcal{A}_{Neu} - \mathcal{U} \subseteq \mathcal{A}_{Neu} - \mathcal{W}$. Since $\mathcal{A}_{Neu} - \mathcal{U}$ is a Neu_{Cgsg}CS and $\mathcal{A}_{Neu} - \mathcal{W}$ is a Neu_{Csg}OS, we have $Neu_C cl(\mathcal{A}_{Neu} - \mathcal{U}) \subseteq \mathcal{A}_{Neu} - \mathcal{W}$. Then $\mathcal{W} \subseteq Neu_C int(\mathcal{U})$. ▪

Theorem 3.25:
If $\mathcal{U}$ is a Neu_{Cgsg}OS in a Neu_{CTS} $(\mathcal{A}, \mathcal{T})$ and $Neu_C int(\mathcal{U}) \subseteq \mathcal{V} \subseteq \mathcal{U}$, then $\mathcal{V}$ is a Neu_{Cgsg}OS in $(\mathcal{A}, \mathcal{T})$.
Proof:
Consider $\mathcal{U}$ is a Neu_{Cgsg}OS in a Neu_{CTS} $(\mathcal{A}, \mathcal{T})$ and $Neu_C int(\mathcal{U}) \subseteq \mathcal{V} \subseteq \mathcal{U}$. Then $\mathcal{A}_{Neu} - \mathcal{U}$ remains a Neu_{Cgsg}CS such that $\mathcal{A}_{Neu} - \mathcal{U} \subseteq \mathcal{A}_{Neu} - \mathcal{V} \subseteq Neu_C cl(\mathcal{A}_{Neu} - \mathcal{U})$. Then $\mathcal{A}_{Neu} - \mathcal{V}$ is a Neu_{Cgsg}CS by theorem (3.20). Hence, $\mathcal{V}$ is a Neu_{Cgsg}OS. ▪

Theorem 3.26:
For a Neu_C-set $\mathcal{U}$ of a Neu_{CTS} $(\mathcal{A}, \mathcal{T})$, afterwards, the subsequent assertions are the duplicate:
(i) $\mathcal{U}$ is a Neu_{Cgsg}CS.
(ii) $Neu_C cl(\mathcal{U}) - \mathcal{U}$ includes no non-null Neu_{Csg}OS.
(iii) $Neu_C cl(\mathcal{U}) - \mathcal{U}$ is a Neu_{Cgsg}OS.

Proof:
Observe by employing proposition (3.19) and proposition (3.21). ▪

Remark 3.27:
The succeeding chart covers up the comparison involving the numerous kinds of Neu_CCSs:

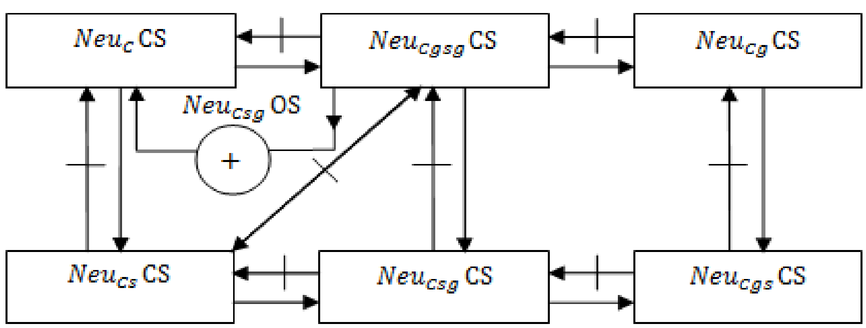

Fig.3.1

4. Neutrosophic Crisp gsg-Closure and Neutrosophic Crisp gsg-Interior

We represent Neu_{Cgsg}-closure and Neu_{Cgsg}-interior and attain various of their advantages in the current part.

Definition 4.1:
The crossing of all Neu_{Cgsg}CSs in a Neu_{CTS} $(\mathcal{A}, \mathcal{T})$ involving $\mathcal{U}$ is titled Neu_{Cgsg}-closure of $\mathcal{U}$ and is denoted by $Neu_{Cgsg}cl(\mathcal{U})$, $Neu_{Cgsg}cl(\mathcal{U}) = \cap\{\mathcal{V}: \mathcal{U} \subseteq \mathcal{V}, \mathcal{V}$ stands as a Neu_{Cgsg}CS$\}$.

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Definition 4.2:
The coalition of all Neu_{Cgsg}OSs in a Neu_{CTS} $(\mathcal{A},\mathcal{T})$ contained in $\mathcal{U}$ is titled Neu_{Cgsg}-interior of $\mathcal{U}$ and is denoted by $Neu_{Cgsg}int(\mathcal{U})$, $Neu_{Cgsg}int(\mathcal{U}) = \cup\{\mathcal{V}: \mathcal{U} \supseteq \mathcal{V}, \mathcal{V}$ is a Neu_{Cgsg}OS$\}$.

Proposition 4.3:
Assume that $\mathcal{U}$ is any Neu_C-set in a Neu_{CTS} $(\mathcal{A},\mathcal{T})$. Next, the subsequent features stand:
(i) $Neu_{Cgsg}int(\mathcal{U}) = \mathcal{U}$ iff $\mathcal{U}$ is a Neu_{Cgsg}OS.
(ii) $Neu_{Cgsg}cl(\mathcal{U}) = \mathcal{U}$ iff $\mathcal{U}$ is a Neu_{Cgsg}CS.
(iii) $Neu_{Cgsg}int(\mathcal{U})$ is the massive Neu_{Cgsg}OS included in $\mathcal{U}$.
(iv) $Neu_{Cgsg}cl(\mathcal{U})$ is the minimum Neu_{Cgsg}CS, including $\mathcal{U}$.
Proof:
The evidence of the points above is apparent. ■

Proposition 4.4:
Suppose that $\mathcal{U}$ be any Neu_C-set in a Neu_{CTS} $(\mathcal{A},\mathcal{T})$. So therefore, the subsequent features determined:
(i) $Neu_{Cgsg}int(\mathcal{A}_{Neu} - \mathcal{U}) = \mathcal{A}_{Neu} - (Neu_{Cgsg}cl(\mathcal{U}))$,
(ii) $Neu_{Cgsg}cl(\mathcal{A}_{Neu} - \mathcal{U}) = \mathcal{A}_{Neu} - (Neu_{Cgsg}int(\mathcal{U}))$.
Proof:
(i) By definition, $Neu_{Cgsg}cl(\mathcal{U}) = \cap\{\mathcal{V}: \mathcal{U} \subseteq \mathcal{V}, \mathcal{V}$ stands as a Neu_{Cgsg}CS$\}$
$\mathcal{A}_{Neu} - (Neu_{Cgsg}cl(\mathcal{U})) = \mathcal{A}_{Neu} - \cap\{\mathcal{V}: \mathcal{U} \subseteq \mathcal{V}, \mathcal{V}$ is a Neu_{Cgsg}CS$\}$
$= \cup\{\mathcal{A}_{Neu} - \mathcal{V}: \mathcal{U} \subseteq \mathcal{V}, \mathcal{V}$ is a Neu_{Cgsg}CS$\}$
$= \cup\{\mathcal{M}: \mathcal{A}_{Neu} - \mathcal{U} \supseteq \mathcal{M}, \mathcal{M}$ is a Neu_{Cgsg}OS$\}$
$= Neu_{Cgsg}int(\mathcal{A}_{Neu} - \mathcal{U})$.
(ii) The facts is comparable to (i). ■

Theorem 4.5:
Assume that $\mathcal{U}$ and $\mathcal{V}$ are two Neu_C-sets in a Neu_{CTS} $(\mathcal{A},\mathcal{T})$. Next, the subsequent features stand:
(i) $Neu_{Cgsg}cl(\varphi_{Neu}) = \varphi_{Neu}$, $Neu_{Cgsg}cl(\mathcal{A}_{Neu}) = \mathcal{A}_{Neu}$.
(ii) $\mathcal{U} \subseteq Neu_{Cgsg}cl(\mathcal{U})$.
(iii) $\mathcal{U} \subseteq \mathcal{V} \Longrightarrow Neu_{Cgsg}cl(\mathcal{U}) \subseteq Neu_{Cgsg}cl(\mathcal{V})$.
(iv) $Neu_{Cgsg}cl(\mathcal{U}\cap\mathcal{V}) \subseteq Neu_{Cgsg}cl(\mathcal{U})\cap Neu_{Cgsg}cl(\mathcal{V})$.
(v) $Neu_{Cgsg}cl(\mathcal{U}\cup\mathcal{V}) = Neu_{Cgsg}cl(\mathcal{U})\cup Neu_{Cgsg}cl(\mathcal{V})$.
(vi) $Neu_{Cgsg}cl(Neu_{Cgsg}cl(\mathcal{U})) = Neu_{Cgsg}cl(\mathcal{U})$.
Proof:
The first two points are recognizable.
(iii) By applying portion (ii), $\mathcal{V} \subseteq Neu_{Cgsg}cl(\mathcal{V})$. While $\mathcal{U} \subseteq \mathcal{V}$, we get $\mathcal{U} \subseteq Neu_{Cgsg}cl(\mathcal{V})$. However, $Neu_{Cgsg}cl(\mathcal{V})$ is a Neu_{Cgsg}CS. In consequence, $Neu_{Cgsg}cl(\mathcal{V})$ is a Neu_{Cgsg}CS including $\mathcal{U}$. While $Neu_{Cgsg}cl(\mathcal{U})$ is the minimum Neu_{Cgsg}CS including $\mathcal{U}$, we have $Neu_{Cgsg}cl(\mathcal{U}) \subseteq Neu_{Cgsg}cl(\mathcal{V})$.
(iv) We know that $\mathcal{U}\cap\mathcal{V} \subseteq \mathcal{U}$ and $\mathcal{U}\cap\mathcal{V} \subseteq \mathcal{V}$. Therefore, by part (iii), $Neu_{Cgsg}cl(\mathcal{U}\cap\mathcal{V}) \subseteq Neu_{Cgsg}cl(\mathcal{U})$ and also we have $Neu_{Cgsg}cl(\mathcal{U}\cap\mathcal{V}) \subseteq Neu_{Cgsg}cl(\mathcal{V})$. Hence $Neu_{Cgsg}cl(\mathcal{U}\cap\mathcal{V}) \subseteq Neu_{Cgsg}cl(\mathcal{U})\cap Neu_{Cgsg}cl(\mathcal{V})$.
(v) Since $\mathcal{U} \subseteq \mathcal{U}\cup\mathcal{V}$ and $\mathcal{V} \subseteq \mathcal{U}\cup\mathcal{V}$, it results from part (iii) that $Neu_{Cgsg}cl(\mathcal{U}) \subseteq Neu_{Cgsg}cl(\mathcal{U}\cup\mathcal{V})$ and also we have $Neu_{Cgsg}cl(\mathcal{V}) \subseteq Neu_{Cgsg}cl(\mathcal{U}\cup\mathcal{V})$. Hence $Neu_{Cgsg}cl(\mathcal{U})\cup Neu_{Cgsg}cl(\mathcal{V}) \subseteq Neu_{Cgsg}cl(\mathcal{U}\cup\mathcal{V})$……… (1)
Since $Neu_{Cgsg}cl(\mathcal{U})$ and $Neu_{Cgsg}cl(\mathcal{V})$ are Neu_{Cgsg}CSs, $Neu_{Cgsg}cl(\mathcal{U})\cup Neu_{Cgsg}cl(\mathcal{V})$ is also Neu_{Cgsg}CS by theorem (3.17). Also $\mathcal{U} \subseteq Neu_{Cgsg}cl(\mathcal{U})$ and $\mathcal{V} \subseteq Neu_{Cgsg}cl(\mathcal{V})$ implies that $\mathcal{U}\cup\mathcal{V} \subseteq Neu_{Cgsg}cl(\mathcal{U})\cup Neu_{Cgsg}cl(\mathcal{V})$. Thus $Neu_{Cgsg}cl(\mathcal{U})\cup Neu_{Cgsg}cl(\mathcal{V})$ is a Neu_{Cgsg}CS containing $\mathcal{U}\cup\mathcal{V}$. Since $Neu_{Cgsg}cl(\mathcal{U}\cup\mathcal{V})$ is the smallest Neu_{Cgsg}CS containing $\mathcal{U}\cup\mathcal{V}$, we get $Neu_{Cgsg}cl(\mathcal{U}\cup\mathcal{V}) \subseteq Neu_{Cgsg}cl(\mathcal{U})\cup Neu_{Cgsg}cl(\mathcal{V})$……….. (2)
From (1) and (2), we get $Neu_{Cgsg}cl(\mathcal{U}\cup\mathcal{V}) = Neu_{Cgsg}cl(\mathcal{U})\cup Neu_{Cgsg}cl(\mathcal{V})$.

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(vi) Since $Neu_{Cgsg}cl(\mathcal{U})$ is a Neu_{Cgsg}CS, we have by proposition (4.3) part (ii), $Neu_{Cgsg}cl(Neu_{Cgsg}cl(\mathcal{U})) = Neu_{Cgsg}cl(\mathcal{U})$. ▪

Theorem 4.6:
Assume that $\mathcal{U}$ and $\mathcal{V}$ are two Neu_C-sets in a Neu_{CTS} $(\mathcal{A},\mathcal{T})$. So therefore, the subsequent features stand:
(i) $Neu_{Cgsg}int(\varphi_{Neu}) = \varphi_{Neu}$, $Neu_{Cgsg}int(\mathcal{A}_{Neu}) = \mathcal{A}_{Neu}$.
(ii) $Neu_{Cgsg}int(\mathcal{U}) \subseteq \mathcal{U}$.
(iii) $\mathcal{U} \subseteq \mathcal{V} \Longrightarrow Neu_{Cgsg}int(\mathcal{U}) \subseteq Neu_{Cgsg}int(\mathcal{V})$.
(iv) $Neu_{Cgsg}int(\mathcal{U} \cap \mathcal{V}) = Neu_{Cgsg}int(\mathcal{U}) \cap Neu_{Cgsg}int(\mathcal{V})$.
(v) $Neu_{Cgsg}int(\mathcal{U} \cup \mathcal{V}) \supseteq Neu_{Cgsg}int(\mathcal{U}) \cup Neu_{Cgsg}int(\mathcal{V})$.
(vi) $Neu_{Cgsg}int(Neu_{Cgsg}int(\mathcal{U})) = Neu_{Cgsg}int(\mathcal{U})$.
Proof:
The above points are noticeable. ▪

Definition 4.7: [12]
A Neu_{CTS} $(\mathcal{A},\mathcal{T})$ is stated to be a neutrosophic crisp $T_{\frac{1}{2}}$-space (fleetingly, $Neu_C T_{\frac{1}{2}}$-space) if for all Neu_{Cg}CS in it are Neu_CCS.

Definition 4.8:
A Neu_{CTS} $(\mathcal{A},\mathcal{T})$ is stated to be a neutrosophic crisp T_{gsg}-space (fleetingly, $Neu_C T_{gsg}$-space) if for all Neu_{Cgsg}CS in it are Neu_CCS.

Proposition 4.9:
Each $Neu_C T_{\frac{1}{2}}$-space is a $Neu_C T_{gsg}$-space.
Proof:
Consider $(\mathcal{A},\mathcal{T})$ is a $Neu_C T_{\frac{1}{2}}$-space and Assume $\mathcal{U}$ is a Neu_{Cgsg}CS in $\mathcal{A}$. Therefore, $\mathcal{U}$ is a Neu_{Cg}CS, by employing proposition (3.7) part (ii). While $(\mathcal{A},\mathcal{T})$ is a $Neu_C T_{\frac{1}{2}}$-space, then $\mathcal{U}$ is a Neu_CCS in $\mathcal{A}$. Thus, $(\mathcal{A},\mathcal{T})$ is a $Neu_C T_{gsg}$-space. ▪
The subsequent occurrence discloses that the beyond proposition's reverse is not valid.

Example 4.10:
Suppose $\mathcal{A} = \{v_1, v_2, v_3\}$. Let $\mathcal{T} = \{\varphi_{Neu}, \langle\{v_1\},\varphi,\varphi\rangle, \langle\{v_2,v_3\},\varphi,\varphi\rangle, \mathcal{A}_{Neu}\}$ be a Neu_{CT} on $\mathcal{A}$. Then $(\mathcal{A},\mathcal{T})$ is a $Neu_C T_{gsg}$-space but not $Neu_C T_{\frac{1}{2}}$-space.

5. Neutrosophic Crisp gsg-Continuous Functions

In this circumstance, we pioneer and investigate the neutrosophic crisp gsg-continuous functions with several of their features.

Definition 5.1:
A function $\mathcal{t}: (\mathcal{A},\mathcal{T}) \longrightarrow (\mathcal{B},\mathcal{L})$ is named neutrosophic crisp g-continuous and symbolized by Neu_{Cg}-continuous if $\mathcal{t}^{-1}(\mathcal{U})$ is a Neu_{Cg}CS (Neu_{Cg}OS) in $(\mathcal{A},\mathcal{T})$ for each Neu_CCS (Neu_COS) $\mathcal{U}$ in $(\mathcal{B},\mathcal{L})$.

Definition 5.2:
A function $\mathcal{t}: (\mathcal{A},\mathcal{T}) \longrightarrow (\mathcal{B},\mathcal{L})$ is named neutrosophic crisp sg-continuous and symbolized by Neu_{Csg}-continuous if $\mathcal{t}^{-1}(\mathcal{U})$ is a Neu_{Csg}CS (Neu_{Csg}OS) in $(\mathcal{A},\mathcal{T})$ for each Neu_CCS (Neu_COS) $\mathcal{U}$ in $(\mathcal{B},\mathcal{L})$.

Definition 5.3:
A function $\mathcal{t}: (\mathcal{A},\mathcal{T}) \longrightarrow (\mathcal{B},\mathcal{L})$ is named neutrosophic crisp gs-continuous and symbolized by Neu_{Cgs}-continuous if $\mathcal{t}^{-1}(\mathcal{U})$ is a Neu_{Cgs}CS (Neu_{Cgs}OS) in $(\mathcal{A},\mathcal{T})$ for each Neu_CCS (Neu_COS) $\mathcal{U}$ in $(\mathcal{B},\mathcal{L})$.

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Definition 5.4:
A function $t:(\mathcal{A},\mathcal{T}) \longrightarrow (\mathcal{B},\mathcal{L})$ is named neutrosophic crisp gsg-continuous and symbolized by Neu_{Cgsg}-continuous if $t^{-1}(\mathcal{U})$ is a Neu_{Cgsg}CS (Neu_{Cgsg}OS) in $(\mathcal{A},\mathcal{T})$ for each Neu_CCS (Neu_COS) $\mathcal{U}$ in $(\mathcal{B},\mathcal{L})$.

Proposition 5.5:
(i) Each Neu_C-continuous is a Neu_{Cg}-continuous.
(ii) Each Neu_{Cg}-continuous is a Neu_{Cgs}-continuous.
(iii) Each Neu_{Cs}-continuous is a Neu_{Csg}-continuous.
(iv) Each Neu_{Csg}-continuous is a Neu_{Cgs}-continuous.
Proof:
(i) Let $t:(\mathcal{A},\mathcal{T}) \longrightarrow (\mathcal{B},\mathcal{L})$ be a Neu_C-continuous function and let $\mathcal{U}$ be a Neu_CCS in $(\mathcal{B},\mathcal{L})$, since t is a Neu_C-continuous then $t^{-1}(\mathcal{U})$ is a Neu_CCS in $(\mathcal{A},\mathcal{T})$, which implies $t^{-1}(\mathcal{U})$ is a Neu_{Cg}CS in $(\mathcal{A},\mathcal{T})$. Hence t is a Neu_{Cg}-continuous.
(ii) Let $t:(\mathcal{A},\mathcal{T}) \longrightarrow (\mathcal{B},\mathcal{L})$ be a Neu_{Cg}-continuous function and let $\mathcal{U}$ be a Neu_CCS in $(\mathcal{B},\mathcal{L})$, since t is a Neu_{Cg}-continuous then $t^{-1}(\mathcal{U})$ is a Neu_{Cg}CS in $(\mathcal{A},\mathcal{T})$, which implies $t^{-1}(\mathcal{U})$ is a Neu_{Cgs}CS in $(\mathcal{A},\mathcal{T})$. Hence t is a Neu_{Cgs}-continuous. The proof is evident to others. ▪
The contrast of the upper proposition need not be accurate, as indicated in the subsequent instances.

Example 5.6:
Suppose $\mathcal{A} = \{s_1, s_2, s_3, s_4\}$ and $\mathcal{B} = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4\}$. Then $\mathcal{T} = \{\varphi_{Neu}, \langle\{s_1\},\varphi,\varphi\rangle, \langle\{s_2,s_3\},\varphi,\varphi\rangle, \langle\{s_1,s_2,s_3\},\varphi,\varphi\rangle, \mathcal{A}_{Neu}\}$ and $\mathcal{L} = \{\varphi_{Neu}, \langle\{\sigma_1\},\varphi,\varphi\rangle, \langle\{\sigma_2,\sigma_3\},\varphi,\varphi\rangle, \langle\{\sigma_1,\sigma_2,\sigma_3\},\varphi,\varphi\rangle, \mathcal{B}_{Neu}\}$ are Neu_{CTSs} on $\mathcal{A}$ and $\mathcal{B}$, respectively. Define the function $t:(\mathcal{A},\mathcal{T}) \longrightarrow (\mathcal{B},\mathcal{L})$ via $t(\langle\{s_1\},\varphi,\varphi\rangle) = \langle\{\sigma_2\},\varphi,\varphi\rangle, t(\langle\{s_2\},\varphi,\varphi\rangle) = \langle\{\sigma_1\},\varphi,\varphi\rangle, t(\langle\{s_3\},\varphi,\varphi\rangle) = \langle\{\sigma_4\},\varphi,\varphi\rangle, t(\langle\{s_4\},\varphi,\varphi\rangle) = \langle\{\sigma_3\},\varphi,\varphi\rangle$. Then t is a Neu_{Cg}-continuous, just not Neu_C-continuous.

Example 5.7:
Suppose $\mathcal{A} = \{s_1, s_2, s_3, s_4\}$ and $\mathcal{B} = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4\}$. Then $\mathcal{T} = \{\varphi_{Neu}, \langle\{s_1\},\varphi,\varphi\rangle, \langle\{s_2,s_4\},\varphi,\phi\rangle, \langle\{s_1,s_2,s_4\},\varphi,\varphi\rangle, \mathcal{A}_{Neu}\}$ and $\mathcal{L} = \{\phi_{Neu}, \langle\{\sigma_1\},\varphi,\varphi\rangle, \langle\{\sigma_2,\sigma_3\},\varphi,\varphi\rangle, \langle\{\sigma_1,\sigma_2,\sigma_3\},\varphi,\varphi\rangle, \mathcal{B}_{Neu}\}$ are Neu_{CTSs} on $\mathcal{A}$ and $\mathcal{B}$, correspondingly. Define the function $t:(\mathcal{A},\mathcal{T}) \longrightarrow (\mathcal{B},\mathcal{L})$ via $t(\langle\{s_1\},\varphi,\varphi\rangle) = \langle\{\sigma_2\},\varphi,\varphi\rangle,\ t(\langle\{s_2\},\varphi,\varphi\rangle) = \langle\{\sigma_1\},\varphi,\varphi\rangle, t(\langle\{s_3\},\varphi,\varphi\rangle) = \langle\{\sigma_3\},\varphi,\varphi\rangle, t(\langle\{s_4\},\varphi,\varphi\rangle) = \langle\{\sigma_4\},\varphi,\varphi\rangle$. Then t is a Neu_{Cgs}-continuous, just not Neu_{Cg}-continuous.

Example 5.8:
Suppose $\mathcal{A} = \{s_1, s_2, s_3, s_4\}$ and $\mathcal{B} = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4\}$. Then $\mathcal{T} = \{\varphi_{Neu}, \langle\{s_4\},\varphi,\varphi\rangle, \langle\{s_1,s_3\},\varphi,\varphi\rangle, \langle\{s_1,s_3,s_4\},\varphi,\varphi\rangle, \mathcal{A}_{Neu}\}$ and $\mathcal{L} = \{\varphi_{Neu}, \langle\{\sigma_1\},\varphi,\varphi\rangle, \langle\{\sigma_2,\sigma_3\},\varphi,\varphi\rangle, \langle\{\sigma_1,\sigma_2,\sigma_3\},\varphi,\varphi\rangle, \mathcal{B}_{Neu}\}$ are Neu_{CTSs} on $\mathcal{A}$ and $\mathcal{B}$, correspondingly. Define the function $t:(\mathcal{A},\mathcal{T}) \longrightarrow (\mathcal{B},\mathcal{L})$ via $t(\langle\{s_1\},\varphi,\varphi\rangle) = \langle\{\sigma_1\},\varphi,\varphi\rangle,\ t(\langle\{s_2\},\varphi,\varphi\rangle) = \langle\{\sigma_4\},\varphi,\varphi\rangle, t(\langle\{s_3\},\varphi,\varphi\rangle) = \langle\{\sigma_2\},\varphi,\varphi\rangle, t(\langle\{s_4\},\varphi,\varphi\rangle) = \langle\{\sigma_3\},\varphi,\varphi\rangle$. Then t is a Neu_{Csg}-continuous, just not Neu_{Cs}-continuous.

Example 5.9:
Suppose $\mathcal{A} = \{s_1, s_2, s_3\}$ and $\mathcal{B} = \{\sigma_1, \sigma_2, \sigma_3\}$. Then $\mathcal{T} = \{\varphi_{Neu}, \langle\{s_1\},\varphi,\varphi\rangle, \mathcal{A}_{Neu}\}$ and $\mathcal{L} = \{\varphi_{Neu}, \langle\{\sigma_2\},\varphi,\varphi\rangle, \mathcal{B}_{Neu}\}$ are Neu_{CTSs} on $\mathcal{A}$ and $\mathcal{B}$, correspondingly. Define the function $t:(\mathcal{A},\mathcal{T}) \longrightarrow (\mathcal{B},\mathcal{L})$ via $t(\langle\{s_1\},\varphi,\varphi\rangle) = \langle\{\sigma_1\},\varphi,\varphi\rangle,\ t(\langle\{s_2\},\varphi,\varphi\rangle) = \langle\{\sigma_3\},\varphi,\varphi\rangle, t(\langle\{s_3\},\varphi,\varphi\rangle) = \langle\{\sigma_2\},\varphi,\varphi\rangle$. Then t is a Neu_{Cgs}-continuous, just not Neu_{Csg}-continuous.

Theorem 5.10:
Suppose that the following $t:(\mathcal{A},\mathcal{T}) \longrightarrow (\mathcal{B},\mathcal{L})$ is given such that $(\mathcal{A},\mathcal{T})$ is
(i) a $Neu_C T_{\frac{1}{2}}$-space, therefore t is a Neu_{Cg}-continuous iff t is a Neu_{Cgsg}-continuous.
(ii) a $Neu_C T_{gsg}$-space, therefore t is a Neu_C-continuous iff t is a Neu_{Cgsg}-continuous.

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Proof:

(i) Assume $\mathcal{U}$ be a Neu_CCS in $(\mathcal{B},\mathcal{L})$. Because t is a Neu_{Cg}-continuous, then $t^{-1}(\mathcal{U})$ in $(\mathcal{A},\mathcal{T})$ remains a Neu_{Cg}CS. By $(\mathcal{A},\mathcal{T})$ is a $Neu_C T_{\frac{1}{2}}$-space, which implies $t^{-1}(\mathcal{U})$ is a Neu_CCS. By proposition (3.7) part (i), $t^{-1}(\mathcal{U})$ is a Neu_{Cgsg}CS in $(\mathcal{A},\mathcal{T})$. Hence, t is a Neu_{Cgsg}-continuous.

Conversely, suppose that t is a Neu_{Cgsg}-continuous. Let $\mathcal{U}$ be a Neu_CCS in $(\mathcal{B},\mathcal{L})$. Then $t^{-1}(\mathcal{U})$ is a Neu_{Cgsg}CS in $(\mathcal{A},\mathcal{T})$. By proposition (3.7) part (ii), $t^{-1}(\mathcal{U})$ is a Neu_{Cg}CS in $(\mathcal{A},\mathcal{T})$. Hence t is a Neu_{Cg}-continuous.

(ii) Let $t:(\mathcal{A},\mathcal{T}) \longrightarrow (\mathcal{B},\mathcal{L})$ be a Neu_C-continuous function and let $\mathcal{U}$ be a Neu_CCS in $(\mathcal{B},\mathcal{L})$, since t is a Neu_C-continuous then $t^{-1}(\mathcal{U})$ is a Neu_CCS in $(\mathcal{A},\mathcal{T})$, which implies $t^{-1}(\mathcal{U})$ is a Neu_{Cgsg}CS in $(\mathcal{A},\mathcal{T})$. Hence t is a Neu_{Cgsg}-continuous.

Conversely, suppose that t is a Neu_{Cgsg}-continuous. Let $\mathcal{U}$ be a Neu_CCS in $(\mathcal{B},\mathcal{L})$. Then $t^{-1}(\mathcal{U})$ is a Neu_{Cgsg}CS in $(\mathcal{A},\mathcal{T})$. By $(\mathcal{A},\mathcal{T})$ is a $Neu_C T_{gsg}$-space, which implies $t^{-1}(\mathcal{U})$ is a Neu_CCS in $(\mathcal{A},\mathcal{T})$. Hence, t is a Neu_C-continuous. ▪

Proposition 5.11:

(i) Each Neu_{Cgsg}-continuous is a Neu_{Csg}-continuous.

(ii) Each Neu_{Cgsg}-continuous is a Neu_{Cgs}-continuous.

Proof:

(i) Let $t:(\mathcal{A},\mathcal{T}) \longrightarrow (\mathcal{B},\mathcal{L})$ be a Neu_{Cgsg}-continuous function and let $\mathcal{U}$ be a Neu_CCS in $(\mathcal{B},\mathcal{L})$, since t is a Neu_{Cgsg}-continuous then $t^{-1}(\mathcal{U})$ is a Neu_{Cgsg}CS in $(\mathcal{A},\mathcal{T})$, which implies $t^{-1}(\mathcal{U})$ is a Neu_{Csg}CS in $(\mathcal{A},\mathcal{T})$. Hence t is a Neu_{Csg}-continuous.

(ii) Let $t:(\mathcal{A},\mathcal{T}) \longrightarrow (\mathcal{B},\mathcal{L})$ be a Neu_{Cgsg}-continuous function and let $\mathcal{U}$ be a Neu_CCS in $(\mathcal{B},\mathcal{L})$, since t is a Neu_{Cgsg}-continuous then $t^{-1}(\mathcal{U})$ is a Neu_{Cgsg}CS in $(\mathcal{A},\mathcal{T})$, which implies $t^{-1}(\mathcal{U})$ is a Neu_{Cgs}CS in $(\mathcal{A},\mathcal{T})$. Hence t is a Neu_{Cgs}-continuous. ▪

As the subsequent example indicates, the beyond proposition's reverse need not be accurate.

Example 5.12:

Suppose $\mathcal{A} = \{s_1, s_2, s_3, s_4\}$ and $\mathcal{B} = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4\}$. Then $\mathcal{T} = \{\varphi_{Neu}, \langle\{s_1\}, \varphi, \varphi\rangle, \langle\{s_2, s_4\}, \varphi, \varphi\rangle, \langle\{s_1, s_2, s_4\}, \varphi, \varphi\rangle, \mathcal{A}_{Neu}\}$ and $\mathcal{L} = \{\varphi_{Neu}, \langle\{\sigma_2\}, \varphi, \varphi\rangle, \langle\{\sigma_1, \sigma_3\}, \varphi, \varphi\rangle, \langle\{\sigma_1, \sigma_2, \sigma_3\}, \varphi, \varphi\rangle, \mathcal{B}_{Neu}\}$ are Neu_{CTSs} on $\mathcal{A}$ and $\mathcal{B}$, correspondingly. Define the function $t:(\mathcal{A},\mathcal{T}) \longrightarrow (\mathcal{B},\mathcal{L})$ where $t(\langle\{s_1\}, \varphi, \varphi\rangle) = \langle\{\sigma_3\}, \varphi, \varphi\rangle$, $t(\langle\{s_2\}, \varphi, \varphi\rangle) = \langle\{\sigma_1\}, \varphi, \varphi\rangle, t(\langle\{s_3\}, \varphi, \varphi\rangle) = \langle\{\sigma_4\}, \varphi, \varphi\rangle, t(\langle\{s_4\}, \varphi, \varphi\rangle) = \langle\{\sigma_2\}, \varphi, \varphi\rangle$. Then t is a Neu_{Csg}-continuous and Neu_{Cgs}-continuous but not Neu_{Cgsg}-continuous.

Theorem 5.13:

A function $t:(\mathcal{A},\mathcal{T}) \longrightarrow (\mathcal{B},\mathcal{L})$ is Neu_{Cgsg}-continuous iff $t(Neu_{Cgsg}cl(\mathcal{U})) \subseteq Neu_{Cgsg}cl(t(\mathcal{U}))$, for every $\mathcal{U} \subseteq \mathcal{A}$.

Proof:

Let t be Neu_{Cgsg}-continuous and $\mathcal{U} \subseteq \mathcal{A}$. Then $t(\mathcal{U}) \subseteq \mathcal{B}$. Since t is Neu_{Cgsg}-continuous and $Neu_{Cgsg}cl(t(\mathcal{U}))$ is Neu_CCS in $(\mathcal{B},\mathcal{L})$, $t^{-1}(Neu_{Cgsg}cl(t(\mathcal{U})))$ is a Neu_{Cgsg}CS in $(\mathcal{A},\mathcal{T})$. Since $t(\mathcal{U}) \subseteq Neu_{Cgsg}cl(t(\mathcal{U}))$, $t^{-1}(t(\mathcal{U})) \subseteq t^{-1}(Neu_{Cgsg}cl(t(\mathcal{U})))$, then $Neu_{Cgsg}cl(\mathcal{U}) \subseteq Neu_{Cgsg}cl(t^{-1}(Neu_{Cgsg}cl(t(\mathcal{U})))$ $=$ $t^{-1}(Neu_{Cgsg}cl(t(\mathcal{U}))$. Thus $Neu_{Cgsg}cl(\mathcal{U}) \subseteq t^{-1}(Neu_{Cgsg}cl(t(\mathcal{U}))$. Therefore $t(Neu_{Cgsg}cl(\mathcal{U})) \subseteq Neu_{Cgsg}cl(t(\mathcal{U}))$, for every $\mathcal{U} \subseteq \mathcal{A}$.

Conversely, let $t(Neu_{Cgsg}cl(\mathcal{U})) \subseteq Neu_{Cgsg}cl(t(\mathcal{U}))$, for every $\mathcal{U} \subseteq \mathcal{A}$. If $\mathcal{V}$ is Neu_CCS in $(\mathcal{B},\mathcal{L})$, since $t^{-1}(\mathcal{V}) \subseteq \mathcal{A}$, $t(Neu_{Cgsg}cl(t^{-1}(\mathcal{V}))) \subseteq Neu_{Cgsg}cl(t(t^{-1}(\mathcal{V}))) = Neu_{Cgsg}cl(\mathcal{V}) = \mathcal{V}$. That is $t(Neu_{Cgsg}cl(t^{-1}(\mathcal{V}))) \subseteq \mathcal{V}$, hence $Neu_{Cgsg}cl(t^{-1}(\mathcal{V})) \subseteq t^{-1}(\mathcal{V})$ but $t^{-1}(\mathcal{V}) \subseteq Neu_{Cgsg}cl(t^{-1}(\mathcal{V}))$. This mean $Neu_{Cgsg}cl(t^{-1}(\mathcal{V})) = t^{-1}(\mathcal{V})$. Therefore $t^{-1}(\mathcal{V})$ is Neu_{Cgsg}CS in $(\mathcal{A},\mathcal{T})$. Hence t is Neu_{Cgsg}-continuous. ▪

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Definition 5.14:
A function $\mathcal{t}:(\mathcal{A},\mathcal{T})\longrightarrow(\mathcal{B},\mathcal{L})$ is named neutrosophic crisp gsg^*-continuous and symbolized by Neu_{Cgsg^*}-continuous if $\mathcal{t}^{-1}(\mathcal{U})$ is a Neu_CCS (Neu_COS) in $(\mathcal{A},\mathcal{T})$ for each Neu_{Cgsg}CS (Neu_{Cgsg}OS) $\mathcal{U}$ in $(\mathcal{B},\mathcal{L})$.

Definition 5.15:
A function $\mathcal{t}:(\mathcal{A},\mathcal{T})\longrightarrow(\mathcal{B},\mathcal{L})$ is named neutrosophic crisp gsg^{**}-continuous and symbolized by $Neu_{Cgsg^{**}}$-continuous if $\mathcal{t}^{-1}(\mathcal{U})$ is a Neu_{Cgsg}CS (Neu_{Cgsg}OS) in $(\mathcal{A},\mathcal{T})$ for each Neu_{Cgsg}CS (Neu_{Cgsg}OS) $\mathcal{U}$ in $(\mathcal{B},\mathcal{L})$.

Proposition 5.16:
(i) Each Neu_{Cgsg^*}-continuous is a $Neu_{Cgsg^{**}}$-continuous.
(ii) Each Neu_{Cgsg}-continuous is a $Neu_{Cgsg^{**}}$-continuous.
Proof:
(i) Let $\mathcal{t}:(\mathcal{A},\mathcal{T})\longrightarrow(\mathcal{B},\mathcal{L})$ be a Neu_{Cgsg^*}-continuous function and suppose that $\mathcal{U}$ is a Neu_{Cgsg}CS in $(\mathcal{B},\mathcal{L})$. Since $\mathcal{t}$ is a Neu_{Cgsg^*}-continuous, then $\mathcal{t}^{-1}(\mathcal{U})$ is a Neu_CCS in $(\mathcal{A},\mathcal{T})$, which implies $\mathcal{t}^{-1}(\mathcal{U})$ is a Neu_{Cgsg}CS in $(\mathcal{A},\mathcal{T})$. Hence $\mathcal{t}$ is a $Neu_{Cgsg^{**}}$-continuous.
(ii) Let $\mathcal{t}:(\mathcal{A},\mathcal{T})\longrightarrow(\mathcal{B},\mathcal{L})$ be a Neu_{Cgsg}-continuous function and let $\mathcal{U}$ be a Neu_CCS in $(\mathcal{B},\mathcal{L})$, which implies $\mathcal{U}$ is a Neu_{Cgsg}CS in $(\mathcal{B},\mathcal{L})$. Since $\mathcal{t}$ is a Neu_{Cgsg}-continuous, then $\mathcal{t}^{-1}(\mathcal{U})$ is a Neu_{Cgsg}CS in $(\mathcal{A},\mathcal{T})$. Hence $\mathcal{t}$ is a $Neu_{Cgsg^{**}}$-continuous. ▪
The contrast of the upper proposition need not be true, as seen in the subsequent instances.

Example 5.17:
Suppose $\mathcal{A}=\{s_1,s_2,s_3,s_4\}$ and $\mathcal{B}=\{\sigma_1,\sigma_2,\sigma_3,\sigma_4\}$.
Then $\mathcal{T}=\{\varphi_{Neu},\langle\{s_4\},\varphi,\varphi\rangle,\langle\{s_1,s_3\},\varphi,\varphi\rangle,\langle\{s_1,s_3,s_4\},\varphi,\varphi\rangle,\mathcal{A}_{Neu}\}$ and $\mathcal{L}=\{\varphi_{Neu},\langle\{\sigma_1\},\varphi,\varphi\rangle,\langle\{\sigma_2,\sigma_3\},\varphi,\varphi\rangle,\langle\{\sigma_1,\sigma_2,\sigma_3\},\varphi,\varphi\rangle,\mathcal{B}_{Neu}\}$ are Neu_{CTSs} on $\mathcal{A}$ and $\mathcal{B}$, respectively. Identify the function $\mathcal{t}:(\mathcal{A},\mathcal{T})\longrightarrow(\mathcal{B},\mathcal{L})$ such that $\mathcal{t}(\langle\{s_1\},\varphi,\varphi\rangle)=\langle\{\sigma_1\},\varphi,\varphi\rangle$, $\mathcal{t}(\langle\{s_2\},\varphi,\varphi\rangle)=\langle\{\sigma_4\},\varphi,\varphi\rangle$, $\mathcal{t}(\langle\{s_3\},\varphi,\varphi\rangle)=\langle\{\sigma_2\},\varphi,\varphi\rangle$, $\mathcal{t}(\langle\{s_4\},\varphi,\varphi\rangle)=\langle\{\sigma_3\},\varphi,\varphi\rangle$. Then $\mathcal{t}$ is a $Neu_{Cgsg^{**}}$-continuous, just not Neu_{Cgsg^*}-continuous.

Example 5.18:
Suppose $\mathcal{A}=\{s_1,s_2,s_3,s_4\}$ and $\mathcal{B}=\{\sigma_1,\sigma_2,\sigma_3,\sigma_4\}$. Then $\mathcal{T}=\{\varphi_{Neu},\langle\{s_3\},\varphi,\varphi\rangle,\langle\{s_1,s_4\},\varphi,\varphi\rangle,\langle\{s_1,s_3,s_4\},\varphi,\varphi\rangle,\mathcal{A}_{Neu}\}$ and $\mathcal{L}=\{\varphi_{Neu},\langle\{\sigma_4\},\varphi,\varphi\rangle,\langle\{\sigma_1,\sigma_3\},\varphi,\varphi\rangle,\langle\{\sigma_1,\sigma_3,\sigma_4\},\varphi,\varphi\rangle,\mathcal{B}_{Neu}\}$ are Neu_{CTSs} on $\mathcal{A}$ and $\mathcal{B}$, respectively. Identify the function $\mathcal{t}:(\mathcal{A},\mathcal{T})\longrightarrow(\mathcal{B},\mathcal{L})$ such that $\mathcal{t}(\langle\{s_1\},\varphi,\varphi\rangle)=\langle\{\sigma_1\},\varphi,\varphi\rangle$, $\mathcal{t}(\langle\{s_2\},\varphi,\varphi\rangle)=\langle\{\sigma_2\},\varphi,\varphi\rangle$, $\mathcal{t}(\langle\{s_3\},\varphi,\varphi\rangle)=\langle\{\sigma_3\},\varphi,\varphi\rangle$, $\mathcal{t}(\langle\{s_4\},\varphi,\varphi\rangle)=\langle\{\sigma_4\},\varphi,\varphi\rangle$. Then $\mathcal{t}$ is a $Neu_{Cgsg^{**}}$-continuous, just not Neu_{Cgsg}-continuous.

Theorem 5.19:
Let $\mathcal{t}_1:(\mathcal{A},\mathcal{T})\longrightarrow(\mathcal{B},\mathcal{L})$ and $\mathcal{t}_2:(\mathcal{B},\mathcal{L})\longrightarrow(\mathcal{C},\mathcal{I})$ be two functions, then:
(i) If $\mathcal{t}_1$ and $\mathcal{t}_2$ are Neu_{Cgsg^*}-continuous, then $\mathcal{t}_2\circ\mathcal{t}_1:(\mathcal{A},\mathcal{T})\longrightarrow(\mathcal{C},\mathcal{I})$ is a Neu_{Cgsg^*}-continuous function.
(ii) If $\mathcal{t}_1$ and $\mathcal{t}_2$ are $Neu_{Cgsg^{**}}$-continuous, then $\mathcal{t}_2\circ\mathcal{t}_1:(\mathcal{A},\mathcal{T})\longrightarrow(\mathcal{C},\mathcal{I})$ is a $Neu_{Cgsg^{**}}$-continuous function.
(iii) If $\mathcal{t}_1$ is a $Neu_{Cgsg^{**}}$-continuous and $\mathcal{t}_2$ is a Neu_{Cgsg^*}-continuous, then $\mathcal{t}_2\circ\mathcal{t}_1:(\mathcal{A},\mathcal{T})\longrightarrow(\mathcal{C},\mathcal{I})$ is a $Neu_{Cgsg^{**}}$-continuous function.
(iv) If $\mathcal{t}_1$ is a Neu_C-continuous and $\mathcal{t}_2$ is a Neu_{Cgsg}-continuous (Neu_{Cgsg^*}-continuous, $Neu_{Cgsg^{**}}$-continuous), then $\mathcal{t}_2\circ\mathcal{t}_1:(\mathcal{A},\mathcal{T})\longrightarrow(\mathcal{C},\mathcal{I})$ is a Neu_{Cgsg}-continuous (Neu_{Cgsg^*}-continuous, $Neu_{Cgsg^{**}}$-continuous) function.
Proof:
(i) Let $\mathcal{K}\subseteq\mathcal{C}$ be a Neu_{Cgsg}CS, since $\mathcal{t}_2$ is a Neu_{Cgsg^*}-continuous then ${\mathcal{t}_2}^{-1}(\mathcal{K})$ stands a Neu_CCS in $\mathcal{B}$. Since every Neu_CCS is a Neu_{Cgsg}CS, therefore ${\mathcal{t}_2}^{-1}(\mathcal{K})$ stands a Neu_{Cgsg}CS in $\mathcal{B}$. Since $\mathcal{t}_1$ is Neu_{Cgsg^*}-continuous, ${\mathcal{t}_1}^{-1}({\mathcal{t}_2}^{-1}(\mathcal{K}))$ is a Neu_CCS in $\mathcal{A}$. Thus $(\mathcal{t}_2\circ\mathcal{t}_1)^{-1}(\mathcal{K})$ is a Neu_CCS in $\mathcal{A}$. Hence $\mathcal{t}_2\circ\mathcal{t}_1$ is a Neu_{Cgsg^*}-continuous.

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(ii) Let $\mathcal{K} \subseteq \mathcal{C}$ be a Neu_{Cgsg}CS, given that t_2 remains a $Neu_{Cgsg^{**}}$-continuous then ${t_2}^{-1}(\mathcal{K})$ stays a Neu_{Cgsg}CS in $\mathcal{B}$. Since t_1 is $Neu_{Cgsg^{**}}$-continuous, ${t_1}^{-1}({t_2}^{-1}(\mathcal{K}))$ is a Neu_{Cgsg}CS in $\mathcal{A}$. Thus $(t_2 \circ t_1)^{-1}(\mathcal{K})$ is a Neu_{Cgsg}CS in $\mathcal{A}$. Hence $t_2 \circ t_1$ is a $Neu_{Cgsg^{**}}$-continuous. The proof is evident for others. ▪

Remark 5.20:
The succeeding illustration reveals the relation involving the numerous types of Neu_C-continuous functions:

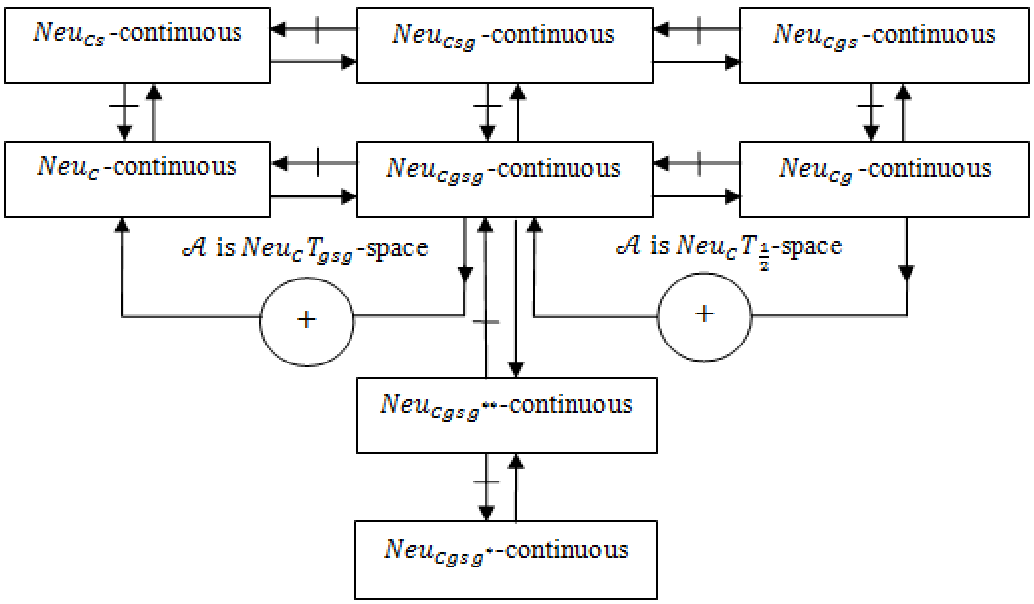

Fig. 5.1

6. Conclusion

The concept of Neu_{Cgsg}CS is described by employing Neu_{Csg}CS with structures a Neu_{CT} and deceptions between the concepts of Neu_CCS and Neu_{Cg}CS. We are exhibited well illustration of Neu_{Cgsg}-continuous functions by applying Neu_{Cgsg}CS. In the future, we anticipate that many additional studies will be able to be conducted in the using these concepts from Neu_{CTS}.

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