On Binary Neutrosophic Crisp Points And Binary Neutrosophic Neighborhoods

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Abstract

As a generalization of crisp topology, neutrosophic crisp topology was introduced. As a progression, binary neutrosophic crisp sets were introduced in this article and their properties were also studied. With the idea that neutrosophic crisp points forms the basis for the neutrosophic neighborhood structures, new points namely binary neutrosophic crisp points were introduced in this article. Owing to the new points, binary neutrosophic neighborhood structure in the new space named as binary neutrosophic crisp topological space is framed. Eventually properties of binary neutrosophic crisp neighborhoods were discussed.
Keywords: BNCS; BNCP; BNC neighborhoods and BNCT.

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1 Introduction

Of the recent mathematical theories developed, neutrosophy plays an inevitable role in applications of its theories in several fields. As a generalization of crisp topology, neutrosophic crisp topology(NCT) was introduced by A.A.Salama and Florentin Samarandache.[8] Relation between neutrosophic crisp sets, their properties and neutrosophic crisp points (NCP) were discussed[7] and neutrosophic crisp neighborhood (NCN) was studied by Gautam Chandra Ray et. al.[6]

Binary topology (BT) was introduced by Nithyanantha Jyothi et.al.[9] and as an extension binary intuitionistic topology (BIT) was introduced along with some properties of binary intuitionistic neighborhoods (BINBD).[10] Binary spaces are required in many areas of research, therefore this article is developed to introduce binary neutrosophic crisp set(BNCS), binary neutrosophic crisp points (BNCP), binary neutrosophic crisp neighborhoods (BNCNBD) and binary neutrosophic crisp topology (BNCT). Eventually neighborhood structures in binary neutrosophic crisp topology is discussed.

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2 Preliminaries

Definition 2.1. [8] Suppose X be a non empty set, a NCS C is an element of form $C = \langle C_1, C_2, C_3\rangle$, where C_1, C_2 and C_3 are subsets of X with $C_1 \cap C_2 = \varphi$, $C_1 \cap C_3 = \varphi$, $C_2 \cap C_3 = \varphi$.

Definition 2.2. [8] The element of form $C = \langle C_1, C_2, C_3\rangle$ is :
(i) A NCS of Type1 if $C_1 \cap C_2 = \varphi$, $C_1 \cap C_3 = \varphi$ and $C_2 \cap C_3 = \varphi$.
(ii) A NCS of Type2 if $C_1 \cap C_2 = \varphi$, $C_1 \cap C_3 = \varphi$ and $C_2 \cap C_3 = \varphi$ and $C_2 \cup C_2 \cup C_3 = X$.
(iii) A NCS of Type3 if $C_1 \cap C_2 \cap C_3 = \varphi$ and $C_2 \cup C_2 \cup C_3 = X$.

Definition 2.3. [8] A NCT on a non empty set X is a family (Λ) of neutrosophic crisp subsets $\in$ X which satisfy below:
(i)$\varphi_N, X_N \in \Lambda$.
(ii)$C_1 \cap C_2 \in \Lambda$ for any $C_1, C_2 \in \Lambda$.
(iii)$\cup C_j \in \Lambda$ for any arbitrary family $\{C_j : j \in J\} \subseteq \Lambda$
(X, Λ) is known as NCTS and the objects in Λ is namely neutrosophic crisp open set (NCOS) and complement of NCOS is neutrosophic crisp closed set (NCCS).

Definition 2.4. [7] Suppose X be a non-empty set and $p \in X$. NCP $p_N = \langle\{p\}, \varphi, \{p\}^c\rangle$ is said to be a NCP in X.

Definition 2.5. [7] Suppose X be a non empty set, $p \in X$ a an object in X. NCS $p_{N_N} = \langle\varphi, \{p\}, \{p\}^c\rangle$ is said to be a vanishing neutrosophic crisp point (VNCP) in X.

Definition 2.6. [7] Suppose $p_N = \langle\{p\}, \varphi, \{p\}^c\rangle$ be a NCP and $C = \langle C_1, C_2, C_3\rangle$ NCS in X.
(a)p_N is $\subseteq$ C iff $p \in C_1$.
(b)Suppose p_{N_N} be a VNCP in X and $C = \langle C_1, C_2, C_3\rangle$ a NCS in X. Then p_{N_N} is $\subseteq$ in C iff $p \notin C_3$.

Definition 2.7. [5] A BT from T to S is a binary structure $\eta \subseteq Q(T) \times Q(S)$ which satisfy:
i)(φ, φ) and $(T, S) \in \eta$
ii) $(C_1 \cap C_2, D_1 \cap D_2) \in \eta$ whenever $(C_1, C_2) \in \eta$ and $(D_1, D_2) \in \eta$
iii) If $\{(C_\alpha, D_\alpha) : \alpha \in \Delta\}$ is a family of elements of η, then $(\cup C_\alpha, \cup D_\alpha : \alpha \in \Delta) \in \eta$.
If η is a BT from T to S then (T, S, η)is known as the BTS and elements of η are binary open subsets of (T, S, η).
If S = T then η is known as BT on T in which the binary space is written as (T, T, η).

Definition 2.8. [5] Suppose T and S be non empty sets. Suppose $(C, D) \in P(T) \times P(S)$ and $(E, F) \in P(T) \times P(S)$ respectively, then
i)$(C, D) \subseteq (E, F)$ iff $C \subseteq E$ and $D \subseteq F$.
ii)$(C, D) = (E, F)$ iff $C = E$ and $D = F$.
iii)$(C^c, D^c) = (X - C, Y - D)$
iv)$(C, D) - (E, F) = (C, D) \cap (E, F)^c$.

Definition 2.9. [8] Product of 2 NCS T and S is a NCS $T \times S = \langle T_1 \times S_1, T_2 \times S_2, T_3 \times S_3\rangle$.

3 Binary neutrosophic crisp set

Few definitions for several types of BNCS and operators of them are discussed in this section.

Definition 3.1. Suppose (X,Y) be a non empty fixed space. A binary neutrosophic set (BNCS) C is an element of the form $C = \langle C_1, C_2, C_3\rangle$ where C_1, C_2 and C_3 are subsets of (X,Y).

Definition 3.2. The binary element $C = \langle C_1, C_2, C_3\rangle$ is known as:
(i) A BNCS-Type1 if $C_1 \cap C_2 = (\varphi, \varphi)$, $C_1 \cap C_3 = (\varphi, \varphi)$ and $C_2 \cap C_3 = (\varphi, \varphi)$.
(ii) A BNCS-Type2 if $C_1 \cap C_2 = (\varphi, \varphi)$, $C_1 \cap C_3 = (\varphi, \varphi)$ and $C_2 \cap C_3 = (\varphi, \varphi)$ and $C_2 \cup C_2 \cup C_3 = (X, Y)$.
(iii) A binary neutrosophic crisp set of Type3 (BNCS-Type3) if satisfying $C_1 \cap C_2 \cap C_3 = (\varphi, \varphi)$ and $C_2 \cup C_2 \cup C_3 = (X, Y)$.

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Definition 3.3. A BNCS-Type1 $(\varphi_{N_1},\varphi_{N_1}),(X_{N_1},Y_{N_1})$ in (X,Y) is defined to be:
1. $(\varphi_{N_1},\varphi_{N_1})$ is defined to be of 3 types:
a) Type1: $(\varphi_{N_1},\varphi_{N_1})=\langle(\varphi,\varphi),(\varphi,\varphi),(X,Y)\rangle$,
b) Type2: $(\varphi_{N_1},\varphi_{N_1})=\langle(\varphi,\varphi),(X,Y),(\varphi,\varphi)\rangle$,
c) Type3: $(\varphi_{N_1},\varphi_{N_1})=\langle(\varphi,\varphi),(\varphi,\varphi),(\varphi,\varphi)\rangle$.
2. (X_{N_1},Y_{N_1}) is defined to be of 1 type:
a) Type1: $(X_{N_1},Y_{N_1})=\langle(X,Y),(\varphi,\varphi),(\varphi,\varphi)\rangle$.

Definition 3.4. A BNCS-Type2 $(\varphi_{N_2},\varphi_{N_2}),(X_{N_2},Y_{N_2})$ in (X,Y) is defined as:
1. $(\varphi_{N_2},\varphi_{N_2})$ is defined to be of 3 types:
a) Type1: $(\varphi_{N_2},\varphi_{N_2})=\langle(\varphi,\varphi),(\varphi,\varphi),(X,Y)\rangle$,
b) Type2: $(\varphi_{N_2},\varphi_{N_2})=\langle(\varphi,\varphi),(X,Y),(\varphi,\varphi)\rangle$.
2. (X_{N_2},Y_{N_2}) is defined to be of 1 type:
a) Type1: $(X_{N_2},Y_{N_2})=\langle(X,Y),(\varphi,\varphi),(\varphi,\varphi)\rangle$.

Definition 3.5. A BNCS-Type3 $(\varphi_{N_3},\varphi_{N_3}),(X_{N_3},Y_{N_3})$ in (X,Y) is defined as: 1. $(\varphi_{N_3},\varphi_{N_3})$ is defined to be three types:
a) Type1: $(\varphi_{N_3},\varphi_{N_3})=\langle(\varphi,\varphi),(\varphi,\varphi),(X,Y)\rangle$,
b) Type2: $(\varphi_{N_3},\varphi_{N_3})=\langle(\varphi,\varphi),(X,Y),(\varphi,\varphi)\rangle$,
c) Type3: $(\varphi_{N_3},\varphi_{N_3})=\langle(\varphi,\varphi),(X,Y),(X,Y)\rangle$.
2. (X_{N_3},Y_{N_3}) is defined to be one type:
a) Type1: $(X_{N_3},Y_{N_3})=\langle(X,Y),(\varphi,\varphi),(\varphi,\varphi)\rangle$.
b) Type2: $(X_{N_3},Y_{N_3})=\langle(X,Y),(X,Y),(\varphi,\varphi)\rangle$.
a) Type1: $(X_{N_3},Y_{N_3})=\langle(X,Y),(\varphi,\varphi),(X,Y)\rangle$.

Remark 3.6. A binary neutrosophic crisp set $C=\langle C_1,C_2,C_3\rangle$ can be identified as an ordered triple $\langle C_1,C_2,C_3\rangle$, subsets in (X,Y). BNCS (φ_N,φ_N) and (X_N,Y_N) in (X,Y) may be defined as:
1)(φ_N,φ_N) is defined to be following 4 types:
a)Type1: (φ_N,φ_N)= $\langle(\varphi,\varphi),(\varphi,\varphi),(X,Y)\rangle$,
b)Type2: (φ_N,φ_N)= $\langle(\varphi,\varphi),(X,Y),(X,Y)\rangle$,
c)Type3: (φ_N,φ_N)= $\langle(\varphi,\varphi),(X,Y),(\varphi,\varphi)\rangle$,
d)Type4: (φ_N,φ_N)= $\langle(\varphi,\varphi),(\varphi,\varphi),(\varphi,\varphi)\rangle$.
2)(X_N,Y_N) is defined to be following 4 types:
a)Type1: (X_N,Y_N)= $\langle(X,Y),(\varphi,\varphi),(\varphi,\varphi)\rangle$,
b)Type2: (X_N,Y_N)= $\langle(X,Y),(X,Y),(\varphi,\varphi)\rangle$,
c)Type3: (X_N,Y_N)= $\langle(X,Y),(\varphi,\varphi),(X,Y)\rangle$,
d)Type4: (φ_N,φ_N)= $\langle(X,Y),(X,Y),(X,Y)\rangle$.

Definition 3.7. Assume (X,Y) to be a binary non empty set, $C=\langle C_1,C_2,C_3\rangle$.
1)If C is BNCS-Type1 in (X,Y), then the complement of C (C^c) is defined to be 1 kind of complement Type1:$C^c=\langle C_3,C_2,C_1\rangle$.
2)If C is BNCS-Type2 in (X,Y), then the complement of C (C^c) is defined to be 1 kind of complement Type2:$C^c=\langle C_3,C_2,C_1\rangle$.
3)If C is BNCS-Type3 in (X,Y), then the complement of C (C^c) is defined to be 1 kind of complement is defined to be of 3 kinds of complements:
$(D_1)Type1: C^c=\langle C_1^c,C_2^c,C_3^c\rangle$.
$(D_2)Type2: C^c=\langle C_3,C_2,C_1\rangle$.
$(D_3)Type3: C^c=\langle C_3,C_2^c,C_1\rangle$.

Example 3.8. Suppose $X=\{a\},Y=\{1,2\}$ and binary neutrosophic crisp subset $A=\langle\{(a,2)\},\{(\varphi,\varphi)\},\{(\varphi,\varphi)\}\rangle$ be a BNCSType1,
$B=\langle\{(a,2)\},\{(\varphi,\varphi)\},\{(a,1)\}\rangle$ be a BNCSType2,
$C=\langle\{(a,1),(a,2)\},\{(\varphi,\varphi)\},\{(a,1)\}\rangle$ be a BNCSType3.
Then $A^c=\langle\{(\varphi,\varphi)\},\{(\varphi,\varphi)\},\{(a,2)\}\rangle$;
$B^c=\langle\{(a,1)\},\{(\varphi,\varphi)\},\{(a,2)\}\rangle$;
$C^c=\langle\{(a,1)\},\{(\varphi,\varphi)\},\{(a,1),(a,2)\}\rangle$.

Definition 3.9. Suppose (X,Y) be a non-empty set and BNCSs C and D be in the form $C=\langle C_1,C_2,C_3\rangle$, $D=\langle D_1,D_2,D_3\rangle$ .Two possible definitions for subsets $(C\subseteq D)$ are defined as:

Type1:$C \subseteq D \Leftrightarrow C_1 \subseteq D_1, C_2 \subseteq D_2$ and $C_3 \supseteq D_3$,
Type2:$C \subseteq D \Leftrightarrow C_1 \subseteq D_1, C_2 \supseteq D_2$ and $C_3 \supseteq D_3$.

Example 3.10. Suppose $X = \{a\}, Y = \{1, 2\}$ with binary neutrosophic crisp subsets $E = \langle\{(a, 2)\}, \{(\varphi, \varphi)\}, \{(a, 1)\}\rangle$ and $F = \langle\{(a, 1), (a, 2)\}, \{(\varphi, \varphi)\}, \{(\varphi, \varphi)\}\rangle$,
Here Type2 definition of 'contained in' is true (i.e).,
$E \subseteq F \Leftrightarrow \{(a, 2)\} \subseteq \{(a, 1), (a, 2)\}, \{(\varphi, \varphi)\} \subseteq \{(\varphi, \varphi)\}, \{(\varphi, \varphi)\} \subseteq \{(a, 1)\}$.

Proposition 3.11. *For any BNCS E :*
a)$(\varphi_N, \varphi_N) \subseteq E, (\varphi_N, \varphi_N) \subseteq (\varphi_N, \varphi_N)$
b)$E \subseteq (X_N, Y_N), (X_N, Y_N) \subseteq (X_N, Y_N)$.

Example 3.12. Suppose $X = \{a\}, Y = \{1, 2\}$ with binary neutrosophic crisp subset $A = \langle\{(a, 2)\}, \{(\varphi, \varphi)\}, \{(a, 1)\}\rangle$. Here we find that in the above proposition both (a) and (b) hold.

Definition 3.13. Assume (X,Y) as a non empty set and BNCSs C and D be $C = \langle C_1, C_2, C_3\rangle$, $D = \langle D_1, D_2, D_3\rangle$ then:
1)$C \cap D$ may be defined as two types:
Type1: $C \cap D = \langle C_1 \cap D_1, C_2 \cap D_2, C_3 \cup D_3\rangle$,
Type2: $C \cap D = \langle C_1 \cap D_1, C_2 \cup D_2, C_3 \cup D_3\rangle$.

2)$C \cup D$ may be defined as two types:
Type1: $C \cup D = \langle C_1 \cup D_1, C_2 \cap D_2, C_3 \cup D_3\rangle$,
Type2: $C \cup D = \langle C_1 \cup D_1, C_2 \cap D_2, C_3 \cap D_3\rangle$.

3)$[]C = \langle C_1, C_2, C_1^c\rangle$.
4)$\langle\rangle\, C = \langle C_3^c, C_2, C_3\rangle$.

Proposition 3.14. *For two BNCS C and D in (X,Y), the following holds:*
$(a)(C \cap D)^c = C^c \cup D^c$;
$(b)(C \cup D)^c = C^c \cap D^c$;

Example 3.15. Consider BNCS of Type3, $C = \langle\{(\varphi, \varphi)\}, \{(\varphi, \varphi)\}, \{(a, 1), (a, 2)\}\rangle$
and $D = \langle\{(a, 2)\}, \{(\varphi, \varphi)\}, \{(a, 1)\}\rangle$.
Here $C \cap D = \langle\{(\varphi, \varphi)\}, \{(\varphi, \varphi)\}, \{(a, 1), (a, 2)\}\rangle$ (using type1 definition of 'intersection') thus
$(C \cap D)^c = \langle\{(a, 1), (a, 2)\}, \{(\varphi, \varphi)\}, \{(\varphi, \varphi)\}\rangle$.
Now $C^c = \langle\{(a, 1), (a, 2)\}, \{(\varphi, \varphi)\}, \{(\varphi, \varphi)\}\rangle$ and
$D^c = \langle\{(a, 1)\}, \{(\varphi, \varphi)\}, \{(a, 2)\}\rangle$, $C^c \cup D^c = \langle\{(a, 1), (a, 2)\}, \{(\varphi, \varphi)\}, \{(\varphi, \varphi)\}\rangle$ (using type2 definition of 'union'). Therefore (a) is true in above proposition.
Consider BNCS of Type3, $C = \langle\{(a, 1)\}, \{(\varphi, \varphi)\}, \{(a, 1), (a, 2)\}\rangle$ and $D = \langle\{(a, 1)\}, \{(\varphi, \varphi)\}, \{(a, 2)\}\rangle$.
Here $C \cup D = \langle\{(a, 1)\}, \{(\varphi, \varphi)\}, \{(a, 2)\}\rangle$ (using type2 definition of 'union') thus
$(C \cap D)^c = \langle\{(a, 2)\}, \{(\varphi, \varphi)\}, \{(a, 1)\}\rangle$.
Now $C^c = \langle\{(a, 1), (a, 2)\}, \{(\varphi, \varphi)\}, \{(a, 1)\}\rangle$ and
$D^c \langle\{(a, 2)\}, \{(\varphi, \varphi)\}, \{(a, 1)\}\rangle$,
$C^c \cap D^c = \langle\{(a, 2)\}, \{(\varphi, \varphi)\}, \{(a, 1)\}\rangle$ (using type1 definition of 'intersection').
Therefore (b) is true in above proposition.

Union and intersection operations are discussed in the following definitions.

Definition 3.16. Let $\{E_i : i \in I\}$ be family of binary neutrosophic crisp subsets in (X,Y) then:
1) $\cap E_i$ is defined to be of 2 types:
a)Type1:$\cap E_i = \langle \cap E_{i1}, \cap E_{i2}, \cup E_{i3}\rangle$,
b)Type2:$\cap E_i = \langle \cap E_{i1}, \cup E_{i2}, \cup E_{i3}\rangle$.
2)$\cup E_i$ is defined to be of 2 types:
a)Type1:$\cup E_i = \langle \cup E_{i1}, \cap E_{i2}, \cap E_{i3}\rangle$,
b)Type2:$\cup E_i = \langle \cup E_{i1}, \cup E_{i2}, \cap E_{i3}\rangle$.

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Definition 3.17. If $F = \langle F_1, F_2, F_3 \rangle$ is a BNCS in (X_2, Y_2) then pre-image of F under h, denoted as $h^{-1}(F)$ is a BNCS in (X,Y) defined as $h^{-1}(F)$ is a BNCS in (X,Y) defined as $h^{-1}(F) = \langle h^{-1}(F_1), h^{-1}(F_2), h^{-1}(F_3) \rangle$ If $E = \langle E_1, E_2, E_3 \rangle$ is a BNCS in (X_1, Y_1) then image of E under h, denoted as $h^{-1}(E)$ is a BNCS in (X,Y) defined as $h(E)$ is a BNCS in (X,Y) defined as $h(E) = \langle h(E_1), h(E_2), h(E_3^c) \rangle$.

Corollary 3.18. *Suppose $\{E_i : i \in J\}$ be family of BNCSs in (X_1, Y_1), $\{F_j : j \in K\}$ a BNCS in (X_2, Y_2) and $h : (X_1, Y_1) \to (X_2, Y_2)$ a map, then:*
i) $E_1 \subseteq E_2 \Leftrightarrow h(E_1) \subseteq h(E_2), F_1 \subseteq F_2 \Leftrightarrow h^{-1}(F_1) \subseteq h^{-1}(E_2)$,
ii) $E \subseteq h^{-1}(h(E))$ and if h is injective, then $E = h^{-1}(h(E))$,
iii) $h^{-1}(h(F)) \subseteq F$ and if h is surjective then $h^{-1}(h(F)) = F$,
iv) $h^{-1}(\cup F_j) = h^{-1}(F_j), h^{-1}(\cap F_j) = \cap h^{-1}(F_j)$,
v) $h(\cup E_i) = \cup h(E_i) \subseteq \cap h((E_i)$ and if h is injective then $h(\cap E_i) = \cap h(E_i)$;
vi) $h^{-1}(X_N, Y_N) = (X_N, Y_N), h^{-1}(\varphi_N, \varphi_N)$,
vii) $h(\varphi_N, \varphi_N) = (\varphi_N, \varphi_N), h(X_N, Y_N)$, if h is surjective.

4 Binary neutrosophic crisp points and its properties

Binary neutrosophic crisp points(BNCP) and binary vanishing neutrosophic crisp points(BVNCP) corresponding to (X,Y) is defined as:

Definition 4.1. Suppose (X,Y) be a non empty set and $p \in (X, Y)$ a fixed element in (X,Y). The BNCS, $\langle \{p\}, \varphi, \{p\}^c \rangle$ is said to be binary neutrosophic crisp point (BNCP) in (X,Y) denoted as $Bp_N = \langle \{p\}, \varphi, \{p\}^c \rangle$.

In terms of BNCP's in some cases BNCS in (X,Y) is expressed which may be inappropriate hence the following binary neutrosophic crisp vanishing points (BVNCP)is defined.

Definition 4.2. Suppose (X,Y) be a non empty set and $p \in (X, Y)$ a fixed element in (X,Y). Then the BNCS $Bp_{N_N} = \langle \varphi, \{p\}, \{p\}^c \rangle$ is called a vanishing binary neutrosophic crisp point ($BVNCP$) in (X,Y).

Definition 4.3. (a) Let $p \in (X, Y)$ and $C = \langle C_1, C_2, C_3 \rangle$ an $BNCS$ in (X,Y). Bp_N is said to be contained in C($Bp_N \subseteq C$) iff $p \in C_1$.
(b) Let Bp_{N_N} be a BVNCP in (X,Y) and $C = \langle C_1, C_2, C_3 \rangle$ an BNCS in (X,Y). Bp_{N_N} is said to be contained in C ($Bp_{N_N} \subseteq C$) iff $p \notin C_3$.

Note : If C is a BNCS then it is an ordered pair which can be represented as $(C^1, C^2) = C = \langle C_1, C_2, C_3 \rangle$. Also if p is a BNCP in (X,Y) then it is an ordered pair which can be represented as (p^1, p^2).

Proposition 4.4. *Suppose $\{C_j : j \in J\}$ be a family of BNCS's in (X,Y) where J, an indexed set. Then*
$(i) Bp_N \in \bigcap_{j \in J} C_j$ iff $Bp_N \in C_i$ for every $j \in J$.
$(ii) Bp_{N_N} \in \bigcap_{j \in J} C_j$ iff $Bp_{N_N} \in C_j$ for every $j \in J$.
$(iii) Bp_N \in \bigcup_{j \in J} C_j$ iff there exists $j \in J$ s.t $Bp_N \in C_j$
$(iv) Bp_{N_N} \in \bigcap_{j \in J} C_j$ iff there exists $j \in J$ s.t $Bp_{N_N} \in C_j$.

Proof. Assume that $Bp_N \in C_{j \in J}$ that is $\langle \{p\}, \varphi, \{p\}^c \rangle \in C_j \in J$by the definition of intersection it's clear that for every $j \in J$, $Bp_N \in C_i$. Similarly using the definition of intersection, (ii) is proved. Again by assuming that $Bp_N \in \bigcup_{j \in J} C_j$ and $j \in J$, $Bp_N \in C_j$ (using definition of BNCP). Similarly (iv) is proved. □

Proposition 4.5. *Suppose C and D be two BNCS in (X,Y). Then*
(a) $C \subseteq D$ if for every Bp_N, we have $Bp_N \in C \Leftrightarrow Bp_N \in D$ and for every Bp_{N_N}, $Bp_N \in C \Rightarrow Bp_{N_N} \in D$.
(b) $A = B$ if for each Bp_N we have $Bp_N \in C \Rightarrow Bp_N \in D$ and for every Bp_{N_N}, $Bp_N \in C \Leftrightarrow Bp_{N_N} \in D$.

Proof. Assume that C and D to be two BNCS in (X,Y). By the definition of Bp_N, Bp_{N_N} and by the properties of BNCS (a) and (b) is proved. □

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Proposition 4.6. *Suppose C be a BNCS in (X,Y).Then*
$C = (\cup\{Bp_N : Bp_N \in C\}) \cup (\cup\{Bp_{N_N} : Bp_{N_N} \in C\})$.

Proof. It's enough to S.T: $C_1 = (\cup\{p\} : Bp_N \in C\}) \cup (\cup\{\varphi : Bp_{N_N} \in C\}), C_2 = \varphi$ and $C_3 = (\cap\{\{p^c : Bp_{N_N} \in C\}) \cap (\cap\{\{p\}^c : p_{N_N} \in C\})$ which are clear from the definitions. □

Definition 4.7. Let $g : (X_1 \times Y_1) \to (X_2 \times Y_2)$ be a map
(a) Suppose Bp_N be a BNCP in (X,Y)(say (x_1, y_1)).Then image of Bp_N under g, denoted as $g(p)$, is defined as $g(p_N) = \langle\{q\}, \varphi, \{q\}^c\rangle$, where $q = g(p)$.
(b) Suppose Bp_{N_N} be a BNCP in (X,Y)(say (x_1, y_1)). Then image of Bp_{N_N} under g, denoted as $g(Bp_{N_N})$, is defined as $g(Bp_{N_N}) = \langle\varphi, \{q\}, \{q\}^c\rangle$, where $q = g(p)$.

It's clear, $g(Bp_N)$ is a BNCP in Y, s.t $g(Bp_N) = Bq_N$, where $q = g(p)$, and it's exactly the image of a BNCP under the function g. Bp_{N_N} is a BVNCP in Y, known as g $(Bp_{N_N}) = Bq_{N_N}$, where $q = g(p)$.

Proposition 4.8. *Any BNCS, C in (X,Y) can be written as* $C_N \cup C_{N_N} \cup C_{N_{N_N}}$*, where* $C_N = \cup\{Bp_N : Bp_N \in C\}$*,* $C_N = \varphi_N$ *and* $C_{N_{N_N}} = \cup\{Bp_{N_N} : Bp_{N_N} \in C\}$.

Proof. It is obvious that, if $C = \langle C_1, C_2, C_3\rangle$, then $C_N = \langle C_1, (\varphi, \varphi), C_1^c\rangle$ and $C_{N_N} = \langle(\varphi, \varphi), C_2, C_3\rangle$. □

Proposition 4.9. *Suppose* $g : (X_1 \times Y_1) \to (X_2 \times Y_2)$ *be a map and C is a BNCS in (X,Y). Then* $g(C) = g(C_N) \cup g(C_{N_N}) \cup g(C_{N_{N_N}})$.

Proof. It is clear from the fact that $C = C_N \cup C_{N_N} \cup C_{N_{N_N}}$, then $g(C) = g(C_N) \cup g(C_{N_N}) \cup g(C_{N_{N_N}})$. □

5 Binary neutrosophic crisp topology and neighborhood

Binary and neutrosophic crisp topological spaces are combined to form a new topological structure known as binary neutrosophic crisp topological structure. Binary neutrosophic crisp neighborhood is defined and the properties of this is studied in the section below.

Definition 5.1. A binary neutrosophic crisp topology (BNCT) from X to Y is a binary structure $B_{NC} \subseteq P(X) \times P(Y)$ that follows below:
i) If $(X, Y) \in B_{NC}$ and $(\varphi, \varphi) \in B_{NC}$.
ii) If $\{(C_\alpha, D_\alpha) : \alpha \in \Delta\}$ is a family of objects of B_{NC} then $(\cup C_\alpha, \cup D_\alpha) \in B_{NC}$.
iii) If $\{(C_1 \cap D_1), (C_2 \cap D_2) \in B_{NC}\}$ whenever $(C_1, D_1) \in B_{NC}$ and $(C_2, D_2) \in B_{NC}$
If B_{NC} is a BNCT from X to Y then the triple (X, Y, B_{NC}) is said as binary neutrosophic crisp open sets(BNCOS). The complement of BNCOS are known as binary neutrosophic crisp closed sets(BNCCS).

Definition 5.2. Suppose (X, Y, B_{NC}) be a BNCT and let Bp_N be a BNCP. A BNCNBD of Bp_N if $\exists$ a binary neutrosophic crisp open set $(U, V) \in X$ s.t $Bp_N \in (U, V) \subseteq (C, D)$.

Example 5.3. Suppose $X = \{1\}$ and $Y = \{2, 3\}$ with binary neutrosophic crisp topology
$B_{NC} = \{\langle\{(x, y)\}, \{(\varphi, \varphi)\}, \{(\varphi, \varphi)\}\rangle$,
$\langle\{(\varphi, \varphi)\}, \{(\varphi, \varphi)\}, \{(x, y)\}\rangle$, $\langle\{(1, 2)\}, \{(\varphi, \varphi)\}, \{(\varphi, \varphi)\}\rangle$,
$\langle\{(\varphi, \varphi)\}, \{(1, 2)\}, \{(1, 3)\}\rangle$,
$\langle\{(1, 2\}, \{(\varphi, \varphi)\}, \{(1, 3)\}\rangle$, $\langle\{(\varphi, \varphi)\}, \{(\varphi, \varphi)\}, \{(1, 3)\}\rangle\}$.
Hence $\{(X, Y), \{(\varphi, \varphi)\}, \{(\varphi, \varphi)\}), \langle\{(1, 2)\}, \{(\varphi, \varphi)\}, \{(\varphi, \varphi)\}\rangle$,
$\langle\{(1, 2)\}, \{(\varphi, \varphi), \{(1, 3)\}\}\rangle\}$ are binary neutrosophic crisp neighborhoods of a BNCP $\langle\{(1, 2)\}, \{(\varphi, \varphi)\}, \{(1, 3)\}\rangle$.

Definition 5.4. Suppose (X, Y, B_{NC}) be a binary neutrosophic crisp topological space and suppose $BN(p_{N_N})$ be a binary neutrosophic crisp vanishing point known as BNCNBD of $BN(p_{N_N})$ in (X,Y) is an BVNCP if $\exists$ a binary neutrosophic crisp open set $(U, V) \in X$ s.t $BN(p_{N_N}) \in (U, V) \subseteq (C, D)$.

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Example 5.5. Suppose $X = \{1\}$ and $Y = \{2, 3\}$ with binary neutrosophic crisp topology
$B_{NC} = \{\langle\{(x, y)\}, \{(\varphi, \varphi)\}, \{(\varphi, \varphi)\}\rangle$,
$\langle\{(\varphi, \varphi)\}, \{(\varphi, \varphi)\}, \{(x, y)\}\rangle$, $\langle\{(1, 2)\}, \{(\varphi, \varphi)\}, \{(\varphi, \varphi)\}\rangle$,
$\langle\{(\varphi, \varphi)\}, \{(1, 2)\}, \{(1, 3)\}\rangle$,
$\langle\{(1, 2)\}, \{(\varphi, \varphi)\}, \{(1, 3)\}\rangle$, $\langle\{(\varphi, \varphi)\}, \{(\varphi, \varphi)\}, \{(1, 3)\}\rangle\}$.
Hence $\{\langle\{(\varphi, \varphi)\}, \{(1, 2)\}, \{(1, 3)\}\rangle\}$ is binary neutrosophic crisp neighborhood of a BVNCP $\langle\{(\varphi, \varphi)\}, \{(1, 2)\}, \{(1, 3)\}\rangle$.

Theorem 5.6. *Suppose (X, Y, B_{NC}) be a BNCTS of X. The binary neutrosophic crisp set C of X is binary neutrosophic crisp open set (BNCOS) iff C is a BNCNB of p $\forall$ BNCS $p \in C$.*

Proof. Suppose C be BNCOS of X. Thus C is a BNCNBD of any $p \in C$. Conversely suppose $p \in C$. $\because$ C is a BNCND of p, $\exists$ a BNCOS D in X s.t $p \in D \subseteq C$. So we have $C = \cup\{p; p \in C\} \subseteq \cup\{D : p \in C\} \subseteq C$ and therefore $C = \cup\{D : p \in C\}$. $\because$ each D is BNCOS. □

Proposition 5.7. *The binary neutrosophic crisp neighborhood system $BN(p_N)$ in the BNCTS (X, Y, B_{NC}) holds the following:*
(N1) If $N \in BN(p_N)$, then $Bp_N \in N$.
(N2) If $N \in BN(p_N)$ and $N \subseteq M$, then $M \in BN(p_N)$.
(N3) If $N_1, N_2 \in BN(p_N)$, then $N_1 \cap N_2 \in BN(p_N)$.
(N4) If $N \in BN(p_N)$, then there exists $M \in BN(p_N)$ $N \in BN(q_N)$such that for each $(q_N) \in M$.

Proof. By definition (N1),(N2) and (N4) by the definition are true. For (N3) let $N_1, N_2 \in (Bp_N)$. Then there exits the BNCOS's G_1 and G_2 such that $(Bp_N) \in G_i \subseteq N_i (i = 1, 2)$.For the BNCOS $G := G_1 \cap G_2$, also $(Bp_N) \in G \subseteq N_1 \cap N_2$,and so $N_1 \cap N_2 \in BN(p_N)$. □

Proposition 5.8. *The binary neutrosophic crisp neighborhood system $B{-}N(p_{N_N})$ in the BNCTS (X, Y, B_{NC}) holds the following:*
(N1) If $N \in BN(p_{N_N})$, then $(Bp_{N_N}) \in N$.
(N2) If $N \in BN(p_{N_N})$ and $N \subseteq M$, then $M \in BN(p_{N_N})$.
(N3) If $N_1, N_2 \in BN(p_{N_N})$, then $N_1 \cap N_2 \in BN(p_{N_N})$.
(N4) If $N \in BN(p_{N_N})$, then $\exists$ $M \in BN(p_{N_N})$ $N \in BN(q_{N_N})$ s.t for each $(q_{N_N}) \in M$.

Proof. By definition (N1),(N2) and (N4) by the definition are true. For (N3) let $N_1, N_2 \in (Bp_{N_N})$. Then there exits the BNCOS's G_1 and G_2 such that $(Bp_{N_N}) \in G_i \subseteq N_i (i = 1, 2)$. For the binary NCOS $G := G_1 \cap G_2$, also $(Bp_{N_N}) \in G \subseteq N_1 \cap N_2$, and so $N_1 \cap N_2 \in BN(p_{N_N})$. □

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