MCGDM based on VIKOR and TOPSIS proposes neutrsophic Fermatean fuzzy soft with aggregation operators

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Abstract

In this study, we presented a new generalization of the Fermatean interval valued fuzzy soft set (FIVFSS) and the neutrosophic interval valued soft set called the neutrsophic Fermatean interval valued soft set (NSFIVSS). The NSFIVSS decision matrix aggregated operations are the topic of our current discussion. Strong points of view for the generalization of the interval valued fuzzy soft set (IVFSS) known as multi-criteria group decision making (MCGDM) are the TOPSIS and VIKOR techniques. We discuss a score function that combines TOPSIS, VIKOR, and NSFIVSS-positive ideal solution (PIS) and NSFIVSS-negative ideal solution (NIS) techniques. The TOPSIS and VIKOR methods also offer decision-making weights. The nearness condition is used to determine the best alternative. An educational trust intends to give some money to those underdeveloped schools since they lack amenities like restrooms, a campus environment that is favorable to learning, sports equipment, and classroom furnishings like desks and lights. In order to lower the factor, they declared a payment to be made in the amounts of 30, 25, 20, 15, and 10. Find the top five under performing schools in the state.

Keywords: NSFIVSS; TOPSIS; VIKOR; aggregation operator

1 Introduction

Numerous ambiguous theories have been proposed, including fuzzy set (FS) by Zadeh,[28] intuitionistic fuzzy set (IFS) by Atanassov,[2] Pythagorean fuzzy set (PFS), soft set with applications by[10, 19–22, 27] and Fermatean fuzzy set (FFS) by.[23] The FS introduced Zadeh, which contends that taking membership degree (MD) into account enables decision-makers to address unclear challenges. Later, Atanassov[2] introduces the idea of an IFS. The notion of PFS, according to Yager,[27] was first established in 2014, and categorized by an MD and non-membership degree (NMD) that meet the requirement that the square sum of its MD and NMD does not exceed unity. However, if the square sum of the MD and NMD for a specific property is more than unity, we can run into an issue while making decisions. The idea of an FFS was put up in 2019 by Senapati et al.[23] It is defined by the requirement that the cubic sum of its MD and NMD does not exceed unity and has been extended by the PFSs. In a decision-making (DM) situation, the best optional alternatives are sought after. The topic of Hwang and Yoon[6] was discussed utilizing the multiple-criteria decision making (MCDM) method.

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These two methods for solving DM problems have been researched by Boran et al.,[3] Eraslan et al.,[4] Gundoodu et al.,[5] Fatma, and Xu et al.,.[26] The idea of a fuzzy soft set was first forth by Maji et al.[11,12] In 2021, Zulqarnain et al. discussed interval valued intuitionistic fuzzy soft sets (IVISS). Additionally, he covered a brand-new correlation coefficient under the IVISS sets,.[29] Trapezoidal neutrosophic aggregation operators and its application by Jana et al. in 2020.[8] Single valued neutrosophic dombi power aggregation operators in 2021 were discussed by Jana et al.[7] Interval-valued intuitionistic fuzzy soft set (IVIFSS), which Zulqarnain et al. discussed in 2021, is included in TOPSIS. Tzeng et al.[25] presentation of the contrast between VIKOR and the TOPSIS approach utilising problems with public transportation. Recently, many authors discussed the application of neutrosophic[1,9,24]

The idea of the PFSS based on TOPSIS and VIKOR under MCGDM ways is extended in this study to the NSFIVSS based on TOPSIS and VIKOR utilising MCGDM approaches, and some of its attributes are derived. The paper is divided into the following seven sections. The first section is referred to as the introduction 1. Basic ideas described in Section 2. Section 3 has a brief explanation of NSFIVSS. The MCGDM based on the NSFIVSS-TOPSIS aggregating operator is discussed in Section 4. A real-world example is used to discuss MCGDM, which is based on the NSFIVSS-VIKOR aggregating operator in Section 5. The benefit is also covered in Section 6. Lastly, In Section 7 refers to conclusion.

2 Preliminaries

Definition 2.1. [5] Let $\mathcal{U}$ be the universe, spherical interval valued fuzzy set (SIVFS) Θ in $\mathcal{U}$ is of the form $\overline{\Theta} = \Big\{\zeta, (\overline{\Delta_\Theta}(\zeta), \overline{\Xi_\Theta}(\zeta), \overline{\Lambda_\Theta}(\zeta)) | \zeta \in \mathcal{U}\Big\}$, where $\overline{\Delta_\Theta}(\zeta) = [\Delta^{\mathcal{L}}_\Theta(\zeta), \Delta^{\mathcal{U}}_\Theta(\zeta)]$ and $\overline{\Xi_\Theta}(\zeta) = [\Xi^{\mathcal{L}}_\Theta(\zeta), \Xi^{\mathcal{U}}_\Theta(\zeta)]$ and $\overline{\Lambda_\Theta}(\zeta) = [\Lambda^{\mathcal{L}}_\Theta(\zeta), \Lambda^{\mathcal{U}}_\Theta(\zeta)]$ is represent the degree of positive membership (PM), neutral membership (NeuM) and negative-membership (NM) of Θ respectively. Consider $\overline{\Delta_\Theta} : \mathcal{U} \to D[0,1]$, $\overline{\Xi_\Theta} : \mathcal{U} \to D[0,1]$, $\overline{\Lambda_\Theta} : \mathcal{U} \to D[0,1]$ and $0 \le (\overline{\Delta_\Theta}(\zeta))^2 + (\overline{\Xi_\Theta}(\zeta))^2 + (\overline{\Lambda_\Theta}(\zeta))^2 \le 1$ that is $0 \le (\Delta^{\mathcal{U}}_\Theta(\zeta))^2 + (\Xi^{\mathcal{U}}_\Theta(\zeta))^2 + (\Lambda^{\mathcal{U}}_\Theta(\zeta))^2 \le 1$, where $\overline{\Delta_\Theta}(\zeta) = [\Delta^{\mathcal{L}}_\Theta(\zeta), \Delta^{\mathcal{U}}_\Theta(\zeta)] = \Big[\sqrt{1 - (\Delta^{\mathcal{U}}_\Theta(\zeta))^2 - (\Xi^{\mathcal{U}}_\Theta(\zeta))^2 - (\Lambda^{\mathcal{U}}_\Theta(\zeta))^2},$ $\sqrt{1 - (\Delta^{\mathcal{L}}_\Theta(\zeta))^2 - (\Xi^{\mathcal{L}}_\Theta(\zeta))^2 - (\Lambda^{\mathcal{L}}_\Theta(\zeta))^2}\,\Big]$.
Since $\overline{\Theta} = \big([\Delta^{\mathcal{L}}_\Theta, \Delta^{\mathcal{U}}_\Theta], [\Xi^{\mathcal{L}}_\Theta, \Xi^{\mathcal{U}}_\Theta], [\Lambda^{\mathcal{L}}_\Theta, \Lambda^{\mathcal{U}}_\Theta]\big)$ is called spherical interval valued fuzzy number(SIVFN).

Definition 2.2. Let $\mathcal{U}$ be the universe, spherical Fermatean fuzzy set (SFFS) Θ in $\mathcal{U}$ is of the form $\Theta = \Big\{\zeta, (\Delta_\Theta(\zeta), \Xi_\Theta(\zeta), \Lambda_\Theta(\zeta)) : \zeta \in \mathcal{U}\Big\}$, where $\Delta_\Theta(\zeta)$ and $\Xi_\Theta(\zeta)$ and $\Lambda_\Theta(\zeta)$ is represent the PM, neuM and NM of Θ, respectively. Consider $\Delta_\Theta : \mathcal{U} \to [0,1]$, $\Xi_\Theta : \mathcal{U} \to [0,1]$, $\Lambda_\Theta : \mathcal{U} \to [0,1]$ and $0 \le (\Delta_\Theta(\zeta))^3 + (\Xi_\Theta(\zeta))^3 + (\Lambda_\Theta(\zeta))^3 \le 1$. Then $\Theta = (\Delta_\Theta, \Xi_\Theta, \Lambda_\Theta)$ is called a spherical Fermatean fuzzy number (SFFN).

Definition 2.3. The cardinal set of the spherical Fermatean fuzzy soft set (SFFSS) Γ_Θ over $\mathcal{U}$ is a SFFSS over E and is defined as $c\Gamma_\Theta = \left\{\frac{\epsilon}{\left(\Delta_{c\tau_\Theta}(\epsilon), \Xi_{cv_\Theta}(\epsilon), \Lambda_{c\varphi_\Theta}(\epsilon)\right)} : \epsilon \in E\right\}$, where $\Delta_{c\tau_\Theta}$, Ξ_{cv_Θ} and $\Lambda_{c\varphi_\Theta} : E \to [0,1]$ are mappings, respectively, where $\Delta_{c\tau_\Theta}(\epsilon) = \frac{|\tau_\Theta(\epsilon)|}{|\mathcal{U}|}$, $\Xi_{cv_\Theta}(\epsilon) = \frac{|v_\Theta(\epsilon)|}{|\mathcal{U}|}$ and $\Lambda_{c\varphi_\Theta}(\epsilon) = \frac{|\varphi_\Theta(\epsilon)|}{|\mathcal{U}|}$, where $|\tau_\Theta(\epsilon)|$, $|v_\Theta(\epsilon)|$ and $|\varphi_\Theta(\epsilon)|$ denotes the scalar cardinalities of the SFFSSs $\tau_\Theta(\epsilon)$, $v_\Theta(\epsilon)$ and $\varphi_\Theta(\epsilon)$, respectively. FFSSs is represents the collection of all cardinal sets of $\mathcal{U}$ as $cSFF(\mathcal{U})$. If $\Theta \subseteq E = \{\epsilon_i : i = 1, 2, ..., n\}$, then $c\Gamma_\Theta \in cSFF(\mathcal{U})$ may be represented in matrix form as $\Big[(p_{1j}, q_{1j}, r_{1j})\Big]_{1\times n} = \Big[(p_{11}, q_{11}, r_{11}), (p_{12}, q_{12}, r_{12}), ..., (p_{1n}, q_{1n}, r_{1n})\Big]$, where $(p_{1j}, q_{1j}, r_{1j}) = \mu_{r\Gamma_\Theta}(\epsilon_j), \forall j = 1, 2, ..., n$.

Definition 2.4. Let $\Gamma_\Theta \in SFFS(\mathcal{U})$ and $c\Gamma_\Theta \in cSFFS(\mathcal{U})$. The SFFSS aggregation operator $SFFSS_{agg} : cSFFS(\mathcal{U}) \times SFFS(\mathcal{U}) \to SFFS(\mathcal{U}, E)$ is defined as $SFFSS_{agg}(c\Gamma_\Theta, \Gamma_\Theta) = \left\{\frac{\zeta}{\mu_{\Gamma^*_\Theta}(\zeta)} : \zeta \in \mathcal{U}\right\} = \left\{\frac{\zeta}{\left(\Delta_{\tau^*_\Theta}(\zeta), \Xi_{v^*_\Theta}(\zeta), \Lambda_{\varphi^*_\Theta}(\zeta)\right)} : \zeta \in \mathcal{U}\right\}$. The aggregate spherical Fermatean fuzzy set of SFFSS Γ_Θ. The PM function $\Delta_{\tau^*_\Theta}(\zeta) : \mathcal{U} \to [0,1]$ by $\Delta_{\tau^*_\Theta}(\zeta) = \frac{1}{|E|}\sum_{\epsilon\in E} \Delta_{c\tau_\Theta}(\epsilon)$, neuM function $\Xi_{v^*_\Theta}(\zeta) : \mathcal{U} \to [0,1]$ by $\Xi_{v^*_\Theta}(\zeta) = \frac{1}{|E|}\sum_{e\in E} \Xi_{cv_\Theta}(\epsilon)$ and NM function $\Lambda_{\varphi^*_\Theta}(\zeta) : \mathcal{U} \to [0,1]$ by $\Lambda_{\varphi^*_\Theta}(\zeta) = \frac{1}{|E|}\sum_{e\in E} \Lambda_{c\varphi_\Theta}(\epsilon)$. The set $SFFSS_{agg}(c\Gamma_\Theta, \Gamma_\Theta)$ is matrix form as $\Big[(p_{i1}, q_{i1}, r_{i1})\Big]_{m\times 1}$, where $\Big[(p_{i1}, q_{i1}, r_{i1})\Big] = \mu_{\Gamma^*_\Theta}(\epsilon_i), \forall\, i = 1, 2, ..., m$.

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3 Main Results

In this section we introduced the new concept of NSFIVSS.

Definition 3.1. Let $\mathcal{U}$ be the universe, neutrsophic Fermatean interval valued set Θ in $\mathcal{U}$ is of the form $\overline{\Theta} = \left\{\zeta, (\overline{\Delta_\Theta}(\zeta), \overline{\Xi_\Theta}(\zeta), \overline{\Lambda_\Theta}(\zeta)) | \zeta \in \mathcal{U}\right\}$, where $\overline{\Delta_\Theta}(\zeta) = [\Delta^{\mathcal{L}}_\Theta(\zeta), \Delta^{\mathcal{U}}_\Theta(\zeta)]$ and $\overline{\Xi_\Theta}(\zeta) = [\Xi^{\mathcal{L}}_\Theta(\zeta), \Xi^{\mathcal{U}}_\Theta(\zeta)]$ and $\overline{\Lambda_\Theta}(\zeta) = [\Lambda^{\mathcal{L}}_\Theta(\zeta), \Lambda^{\mathcal{U}}_\Theta(\zeta)]$ is represents the PM, neuM and NM of Θ, respectively. The function $\overline{\Delta_\Theta} : \mathcal{U} \to D[0,1]$, $\overline{\Xi_\Theta} : \mathcal{U} \to D[0,1]$, $\overline{\Lambda_\Theta} : \mathcal{U} \to D[0,1]$ and $0 \leq (\overline{\Delta_\Theta}(\zeta))^3 + (\overline{\Xi_\Theta}(\zeta))^3 + (\overline{\Lambda_\Theta}(\zeta))^3 \leq 1$ that is $0 \leq (\Delta^{\mathcal{U}}_\Theta(\zeta))^3 + (\Xi^{\mathcal{U}}_\Theta(\zeta))^3 + (\Lambda^{\mathcal{U}}_\Theta(\zeta))^3 \leq 1$, where $\overline{\Delta_\Theta}(\zeta) = [\Delta^{\mathcal{L}}_\Theta(\zeta), \Delta^{\mathcal{U}}_\Theta(\zeta)] = \Big[\sqrt[3]{1 - (\Delta^{\mathcal{U}}_\Theta(\zeta))^3 - (\Xi^{\mathcal{U}}_\Theta(\zeta))^3 - (\Lambda^{\mathcal{U}}_\Theta(\zeta))^3},\ \sqrt[3]{1 -$ Since $\overline{\Theta} = \left([\Delta^{\mathcal{L}}_\Theta, \Delta^{\mathcal{U}}_\Theta], [\Xi^{\mathcal{L}}_\Theta, \Xi^{\mathcal{U}}_\Theta], [\Lambda^{\mathcal{L}}_\Theta, \Lambda^{\mathcal{U}}_\Theta]\right)$ is refered as neutrsophic Fermatean interval valued number(NSFIVN).

Definition 3.2. The cardinal set of the NSFIVSS $\overline{\Gamma_\Theta}$ over $\mathcal{U}$ is a NSFIVSS of E and $\overline{c\Gamma_\Theta} = \Bigg\{ \frac{\epsilon}{\left(\left[\Delta^{\mathcal{L}}_{c\tau_\Theta}(\epsilon), \Delta^{\mathcal{U}}_{c\tau_\Theta}(\epsilon)\right], \left[\Xi^{\mathcal{L}}_{cv_\Theta}(\epsilon), \Xi^{\mathcal{U}}_{cv_\Theta}(\epsilon)\right.\right.}$ $\epsilon \in E\Bigg\} = \left\{\frac{\epsilon}{\left(\overline{\Delta}_{c\tau_\Theta}(\epsilon), \overline{\Xi}_{cv_\Theta}(\epsilon), \overline{\Lambda}_{c\varphi_\Theta}(\epsilon)\right)} : \epsilon \in E\right\}$, where $\overline{\Delta}_{c\tau_\Theta}$, $\overline{\Xi}_{cv_\Theta}$ and $\overline{\Lambda}_{c\varphi_\Theta} : E \to D[0,1]$, where $\overline{\Delta}_{c\tau_\Theta}(\epsilon) = \frac{|\overline{\tau}_\Theta(\epsilon)|}{|\mathcal{U}|}$, $\overline{\Xi}_{cv_\Theta}(\epsilon) = \frac{|\overline{v}_\Theta(\epsilon)|}{|\mathcal{U}|}$ and $\overline{\Lambda}_{c\varphi_\Theta}(\epsilon) = \frac{|\overline{\varphi}_\Theta(\epsilon)|}{|\mathcal{U}|}$. The collection of all cardinal sets of NSFIVSSs of $\mathcal{U}$ is represented as $cNSFIVF(\mathcal{U})$. If $\Theta \subseteq E = \{\epsilon_i : i = 1, 2, ..., n\}$, then $\overline{c\Gamma_\Theta} \in cNSFIVF(\mathcal{U})$ is matrix form as $\Big[\left([p^{\mathcal{L}}_{1j}, p^{\mathcal{U}}_{1j}], [q^{\mathcal{L}}_{1j}, q^{\mathcal{U}}_{1j}], [r^{\mathcal{L}}_{1j}, r^{\mathcal{U}}_{1j}]\right)\Big]_{1\times n} = \Big[\left([p^{\mathcal{L}}_{11}, p^{\mathcal{U}}_{11}], [q^{\mathcal{L}}_{11}, q^{\mathcal{U}}_{11}], [r^{\mathcal{L}}_{11}, r^{\mathcal{U}}_{11}]\right),$ $\left([p^{\mathcal{L}}_{12}, p^{\mathcal{U}}_{12}], [q^{\mathcal{L}}_{12}, q^{\mathcal{U}}_{12}], [r^{\mathcal{L}}_{12}, r^{\mathcal{U}}_{12}]\right), ..., \left([p^{\mathcal{L}}_{1n}, p^{\mathcal{U}}_{1n}], [q^{\mathcal{L}}_{1n}, q^{\mathcal{U}}_{1n}], [r^{\mathcal{L}}_{1n}, r^{\mathcal{U}}_{1n}]\right)\Big]$,
where $\left([p^{\mathcal{L}}_{1j}, p^{\mathcal{U}}_{1j}], [q^{\mathcal{L}}_{1j}, q^{\mathcal{U}}_{1j}], [r^{\mathcal{L}}_{1j}, r^{\mathcal{U}}_{1j}]\right) = \left[\mu^{\mathcal{L}}_{r\Gamma_\Theta}(e_j), \mu^{\mathcal{U}}_{r\Gamma_\Theta}(e_j)\right], \forall j = 1, 2, ..., n$. The matrix form is $[(\overline{p}_{1j}, \overline{q}_{1j}, \overline{r}_{1j})]_{1\times n} = [(\overline{p}_{11}, \overline{q}_{11}, \overline{r}_{11}), (\overline{p}_{12}, \overline{q}_{12}, \overline{r}_{12}), ..., (\overline{p}_{1n}, \overline{q}_{1n}, \overline{r}_{1n})]$, where $(\overline{p}_{1j}, \overline{q}_{1j}, \overline{r}_{1j}) = \overline{\mu}_{c\Gamma_\Theta}(e_j), j = 1, 2, ..., n$.

Definition 3.3. Let $\overline{\Gamma_\Theta} \in NSFIVF(\mathcal{U})$ and $\overline{c\Gamma_\Theta} \in cNSFIVF(\mathcal{U})$. The NSFIVSS aggregation operator $NSFIVFS_{agg} : cNSFIVF(\mathcal{U}) \times NSFIVF(\mathcal{U}) \to NSFIVFS(\mathcal{U}, E)$ as $NSFIVSS_{agg}(\overline{c\Gamma_\Theta}, \overline{\Gamma_\Theta}) = \left\{\frac{\zeta}{\overline{\mu}_{\Gamma^*_\Theta}(\zeta)} : \zeta \in \mathcal{U}\right\} =$
$\left\{\frac{\zeta}{\left(\overline{\Delta}_{\tau^*_\Theta}(\zeta), \overline{\Xi}_{v^*_\Theta}(\zeta), \overline{\Lambda}_{\varphi^*_\Theta}(\zeta)\right)} : \zeta \in \mathcal{U}\right\}$, where $\overline{\Gamma_\Theta}$ is the collection of aggregate NSFIVSS . The PM $\overline{\Delta}_{\tau^*_\Theta}(\zeta) : \mathcal{U} \to D[0,1]$ by $\overline{\Delta}_{\tau^*_\Theta}(\zeta) = \frac{1}{|E|}\sum_{\epsilon\in E}\left(\overline{\Delta}_{c\tau_\Theta}(\epsilon), \overline{\Delta}_{\tau_\Theta}(\epsilon)\right)(\zeta)$, neuM $\overline{\Xi}_{v^*_\Theta}(\zeta) : \mathcal{U} \to D[0,1]$ by $\overline{\Xi}_{v^*_\Theta}(\zeta) = \frac{1}{|E|}\sum_{\epsilon\in E}\left(\overline{\Xi}_{cv_\Theta}(\epsilon), \overline{\Xi}_{v_\Theta}(\epsilon)\right)(\zeta)$ and NM $\overline{\Lambda}_{\varphi^*_\Theta}(\zeta) : \mathcal{U} \to D[0,1]$ by $\overline{\Lambda}_{\varphi^*_\Theta}(\zeta) = \frac{1}{|E|}\sum_{\epsilon\in E}\left(\overline{\Lambda}_{c\varphi_\Theta}(\epsilon), \overline{\Lambda}_{\varphi_\Theta}(\epsilon)\right)(\zeta)$.
The set $NSFIVSS_{agg}(\overline{c\Gamma_\Theta}, \overline{\Gamma_\Theta})$ is expressed as $\Big[\left([p^{\mathcal{L}}_{i1}, p^{\mathcal{U}}_{i1}], [q^{\mathcal{L}}_{i1}, q^{\mathcal{U}}_{i1}], [r^{\mathcal{L}}_{i1}, r^{\mathcal{U}}_{i1}]\right)\Big]_{m\times 1}$, where $\Big[\left([p^{\mathcal{L}}_{i1}, p^{\mathcal{U}}_{i1}], [q^{\mathcal{L}}_{i1}, q^{\mathcal{U}}_{i1}], [r^{\mathcal{L}}_{i1}, r^{\mathcal{U}}_{i1}]\right)\Big]$ $\left[\mu^{\mathcal{L}}_{\Gamma^*_\Theta}(\Theta_i), \mu^{\mathcal{U}}_{\Gamma^*_\Theta}(\Theta_i)\right], \forall i = 1, 2, ..., m$. This matrix is represent a NSFIVSS aggregate matrix of $NSFIVSS_{agg}(\overline{c\Gamma_\Theta}, \overline{\Gamma_\Theta})$ over $\mathcal{U}$.

4 NSFIVSS-TOPSIS aggregating operator

Algorithm

Step-1: A decision makers $\mathcal{D} = \{\mathcal{D}_i : i \in \mathcal{N}\}$ is a collection of alternatives $\mathcal{C} = \{\varrho_i : i \in \mathcal{N}\}$ and family of parameters $\mathcal{D} = \{e_i : i \in \mathcal{N}\}$.
Step-2: Linguistic variable with a weighted parameter matrix is $\mathcal{O} = \left[o^{\mathcal{L}}_{ij}, o^{\mathcal{U}}_{ij}\right]_{n\times m}$, where o_{ij} denotes $\mathcal{D}_i$ to $\mathcal{O}_j$ by considering linguistic variables.
Step-3: The weighted normalized decision matrix is $\widehat{\mathcal{N}} = \left[\widehat{n}^{\mathcal{L}}_{ij}, \widehat{n}^{\mathcal{U}}_{ij}\right]_{n\times m}$, where $\left[\widehat{n}^{\mathcal{L}}_{ij}, \widehat{n}^{\mathcal{U}}_{ij}\right] = \left[\frac{o^{\mathcal{L}}_{ij}}{\sqrt[3]{\sum_{i=1}^{n} o^{3U}_{ij}}}, \frac{o^{\mathcal{U}}_{ij}}{\sqrt[3]{\sum_{i=1}^{n} o^{3L}_{ij}}}\right]$ is called the normalized parameter and weighted vector $\mathcal{W} = \left([m^{\mathcal{L}}_1, m^{\mathcal{U}}_1], [m^{\mathcal{L}}_2, m^{\mathcal{U}}_2] ..., [m^{\mathcal{L}}_m, m^{\mathcal{U}}_m]\right)$, where $[m^{\mathcal{L}}_i, m^{\mathcal{U}}_i] =$
$\left[\frac{o^{\mathcal{L}}_i}{\sqrt[3]{\sum_{l=1}^{n} o^{\mathcal{U}}_{li}}}, \frac{o^{\mathcal{U}}_i}{\sqrt[3]{\sum_{l=1}^{n} o^{\mathcal{L}}_{li}}}\right]$ is the weight of the j^{th} parameter and $[o^{\mathcal{L}}_j, o^{\mathcal{U}}_j] =$

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$\left[\frac{\sum_{i=1}^{n}\widehat{n}_{ij}^{\mathcal{L}}}{n}, \frac{\sum_{i=1}^{n}\widehat{n}_{ij}^{\mathcal{U}}}{n}\right]$.

Step-4: NSFIVSS decision matrix is $\mathcal{D}_i = \left[l_{jk}^{\mathcal{L}i}, l_{jk}^{\mathcal{U}i}\right]_{l\times m}$, where $\left[l_{jk}^{\mathcal{L}i}, l_{jk}^{\mathcal{U}i}\right]$ is a NSFIVSS element for i^{th} decision maker $[\mathcal{D}_i^{\mathcal{L}}, \mathcal{D}_i^{\mathcal{U}}]$ for each i. Determine the aggregating matrix is $\left[\mathcal{Y}^{\mathcal{L}}, \mathcal{Y}^{\mathcal{U}}\right] = \frac{\left[\mathcal{D}_1^{\mathcal{L}}, \mathcal{D}_1^{\mathcal{U}}\right]+\left[\mathcal{D}_2^{\mathcal{L}}, \mathcal{D}_2^{\mathcal{U}}\right]+...+\left[\mathcal{D}_n^{\mathcal{L}}, \mathcal{D}_n^{\mathcal{U}}\right]}{n} = \left[\psi_{jk}^{\mathcal{L}}, \psi_{jk}^{\mathcal{U}}\right]_{l\times m}$.

Step 5: The decision weighted NSFIVSS matrix is $[\mathcal{L}^{\mathcal{L}}, \mathcal{L}^{\mathcal{U}}] = \left[\varrho_{jk}^{\mathcal{L}}, \varrho_{jk}^{\mathcal{U}}\right]_{l\times m}$, where $\left[\varrho_{jk}^{\mathcal{L}}, \varrho_{jk}^{\mathcal{U}}\right] = \left[m_k^{\mathcal{L}} \times \psi_{jk}^{\mathcal{L}}, m_k^{\mathcal{U}} \times \psi_{jk}^{\mathcal{U}}\right]$.

Step-6: The NSFIVSS-PIS = $\left(\left[\varrho_1^{\mathcal{L}+}, \varrho_1^{\mathcal{U}+}\right], \left[\varrho_2^{\mathcal{L}+}, \varrho_2^{\mathcal{U}+}\right]..., \left[\varrho_l^{\mathcal{L}+}, \varrho_l^{\mathcal{U}+}\right]\right)$

$= \left\{\left(\max_k\left[\varrho_{jk}^{\mathcal{L}}, \varrho_{jk}^{\mathcal{U}}\right], \min_k\left[\varrho_{jk}^{\mathcal{L}}, \varrho_{jk}^{\mathcal{U}}\right], \min_k\left[\varrho_{jk}^{\mathcal{L}}, \varrho_{jk}^{\mathcal{U}}\right]\right) : k = 1, 2, ..., m\right\}$ and

NSFIVSS-NIS = $\left(\left[\varrho_1^{\mathcal{L}-}, \varrho_1^{\mathcal{U}-}\right], \left[\varrho_2^{\mathcal{L}-}, \varrho_2^{\mathcal{U}-}\right]..., \left[\varrho_l^{\mathcal{L}-}, \varrho_l^{\mathcal{U}-}\right]\right)$

$= \left\{\left(\min_k\left[\varrho_{jk}^{\mathcal{L}}, \varrho_{jk}^{\mathcal{U}}\right], \max_k\left[\varrho_{jk}^{\mathcal{L}}, \varrho_{jk}^{\mathcal{U}}\right], \max_k\left[\varrho_{jk}^{\mathcal{L}}, \varrho_{jk}^{\mathcal{U}}\right]\right) : k = 1, 2, ..., m\right\}$.

Step-7: Hamming distances between the NSFIVSS-PIS and the NSFIVSS-NIS. Since

$$\left[D_j^{\mathcal{L}+}, D_j^{\mathcal{U}+}\right] = \left[\left|\sum_{k=1}^{m}\left\{\left(\Delta_{jk}^{\mathcal{L}} - \Delta_j^{\mathcal{L}+}\right)^3 + \left(\Xi_{jk}^{\mathcal{L}} - \Xi_j^{\mathcal{L}+}\right)^3 + \left(\Lambda_{jk}^{\mathcal{L}} - \Lambda_j^{\mathcal{L}+}\right)^3\right\}\right|,\right.$$
$$\left.\left|\sum_{k=1}^{m}\left\{\left(\Delta_{jk}^{\mathcal{U}} - \Delta_j^{\mathcal{U}+}\right)^3 + \left(\Xi_{jk}^{\mathcal{U}} - \Xi_j^{\mathcal{U}+}\right)^3 + \left(\Lambda_{jk}^{\mathcal{U}} - \Lambda_j^{\mathcal{U}+}\right)^3\right\}\right|\right]$$

and
$$\left[D_j^{\mathcal{L}-}, D_j^{\mathcal{U}-}\right] = \left[\left|\sum_{k=1}^{m}\left\{\left(\Delta_{jk}^{\mathcal{L}} - \Delta_j^{\mathcal{L}-}\right)^3 + \left(\Xi_{jk}^{\mathcal{L}} - \Xi_j^{\mathcal{L}-}\right)^3 + \left(\Lambda_{jk}^{\mathcal{L}} - \Lambda_j^{\mathcal{L}-}\right)^3\right\}\right|,\right.$$
$$\left.\left|\sum_{k=1}^{m}\left\{\left(\Delta_{jk}^{\mathcal{U}} - \Delta_j^{\mathcal{U}-}\right)^3 + \left(\Xi_{jk}^{\mathcal{U}} - \Xi_j^{\mathcal{U}-}\right)^3 + \left(\Lambda_{jk}^{\mathcal{U}} - \Lambda_j^{\mathcal{U}-}\right)^3\right\}\right|\right],$$

where $j = 1, 2, ..., n$.

Step-8: Compute the values for nearness are $\left[C^{\mathcal{L}*}(\varrho_j), C^{\mathcal{U}*}(\varrho_j)\right] = \left[\frac{D_j^{\mathcal{L}-}}{d_j^{\mathcal{U}+}+D_j^{\mathcal{U}-}}, \frac{D_j^{\mathcal{U}-}}{D_j^{\mathcal{L}+}+D_j^{\mathcal{L}-}}\right]$ and $C^{*}(\varrho_j) = \frac{C^{\mathcal{L}*}(\varrho_j)+C^{\mathcal{U}*}(\varrho_j)}{2} \in [0, 1]$.

Step-9: To depict the rank of alternatives using nearness coefficients in decreasing (or increasing) order.

Step-10: The final stage is to output the best choice.

Example 4.1. Due to a states underdeveloped schools lack of amenities like restrooms, a campus climate conducive to learning, sports equipment, and classroom furnishings like desks and lights, an educational trust plans to send some money to those schools. They announced a payment to give the percentages of 30, 25, 20, 15, and 10 in order to reduce the factor. Find the states top five underdeveloped schools.

Step-1: The decision makers as $\mathcal{D} = \{\mathcal{D}_i : i = 1, 2, 3, 4, 5\}$, the collection of schools/alternatives is $\mathcal{C} = \{\varrho_i : i = 1, 2, ..., 10\}$ and finite family of parameters is $\mathcal{D} = \{e_i : i = 1, 2, ..., 5\}$, put e_1 = campus environment, e_2 = sports equipment, e_3 = classroom equipment, e_4 = career development, e_5 = academic quality.

Step-2: Determine weighted parameter matrix based on the linguistic variables. Very Good Deliberate,(VGD)=[0.9,0.95], Good Deliberate (GD)=[0.8,0.9], Average Deliberate (AD)=[0.65,0.8], Poor Deliberate (PD)=[0.5,0.65], Very Poor Deliberate (VPD)=[0.35,0.5].

Determine the weighted parameter matrix is called as

$$\begin{aligned}\mathcal{O} &= [o_{ij}^{\mathcal{L}}, o_{ij}^{\mathcal{U}}]_{5\times 5}\\ &= \begin{bmatrix} GD & AD & VGD & PD & VPD\\ PD & AD & GD & VPD & VGD\\ PD & VGD & VPD & GD & AD\\ VGD & GD & AD & VPD & PD\\ VPD & AD & PD & VGD & GD \end{bmatrix}\end{aligned}$$

Here $[o_{ij}^{\mathcal{L}}, o_{ij}^{\mathcal{U}}]$ is weight of the $\mathcal{D}_i$ to $\mathcal{O}_j$.

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Step-3: The weighted normalized decision matrix is

$$
\begin{aligned}
\widehat{\mathcal{N}} &= \left[\widehat{n}_{ij}^{\mathcal{L}}, \widehat{n}_{ij}^{\mathcal{U}}\right]_{5\times 5} \\
&= \begin{bmatrix}
[0.6096, 0.6858] & [0.4447, 0.5473] & [0.6633, 0.7002] & [0.3898, 0.5067] & [0.258, 0.3685] \\
[0.381, 0.4953] & [0.4447, 0.5473] & [0.5896, 0.6633] & [0.2728, 0.3898] & [0.6633, 0.7002] \\
[0.381, 0.4953] & [0.6158, 0.65] & [0.258, 0.3685] & [0.6236, 0.7016] & [0.4791, 0.5896] \\
[0.6858, 0.7238] & [0.5473, 0.6158] & [0.4791, 0.5896] & [0.2728, 0.3898] & [0.3685, 0.4791] \\
[0.1548, 0.381] & [0.4447, 0.5473] & [0.3685, 0.4791] & [0.7016, 0.7406] & [0.5896, 0.6633]
\end{bmatrix}
\end{aligned}
$$

and $\mathcal{W} = ([0.1223, 0.1738], [0.1218, 0.1662], [0.1150, 0.1556], [0.1350, 0.2059], [0.1241, 0.1750])$.

Step-4: The aggregated decision matrix is $\left[\times^{\mathcal{L}}, \times^{\mathcal{U}}\right] = \frac{\left[\mathcal{D}_1^{\mathcal{L}}, \mathcal{D}_1^{\mathcal{U}}\right] + \left[\mathcal{D}_2^{\mathcal{L}}, \mathcal{D}_2^{\mathcal{U}}\right] + \ldots + \left[\mathcal{D}_5^{\mathcal{L}}, \mathcal{D}_5^{\mathcal{U}}\right]}{5}$

$$
= \begin{bmatrix}
([0.56, 0.58], [0.7, 0.72], [0.54, 0.65]) & ([0.59, 0.6], [0.48, 0.65], [0.37, 0.45]) & ([0.68, 0.7], [0.34, 0.62], [0.58, 0.6]) \\
([0.62, 0.63], [0.28, 0.3], [0.5, 0.6]) & ([0.53, 0.59], [0.47, 0.48], [0.55, 0.6]) & ([0.46, 0.49], [0.53, 0.55], [0.68, 0.69]) \\
([0.5, 0.52], [0.51, 0.54], [0.55, 0.64]) & ([0.51, 0.6], [0.47, 0.65], [0.35, 0.6]) & ([0.45, 0.65], [0.31, 0.6], [0.57, 0.8]) \\
([0.52, 0.62], [0.42, 0.65], [0.55, 0.6]) & ([0.6, 0.63], [0.45, 0.6], [0.5, 0.59]) & ([0.37, 0.54], [0.39, 0.6], [0.4, 0.45]) \\
([0.28, 0.45], [0.1, 0.55], [0.3, 0.35]) & ([0.23, 0.33], [0.55, 0.58], [0.55, 0.59]) & ([0.41, 0.7], [0.45, 0.65], [0.6, 0.75]) \\
([0.7, 0.73], [0.44, 0.47], [0.55, 0.65]) & ([0.59, 0.61], [0.47, 0.49], [0.5, 0.54]) & ([0.38, 0.65], [0.43, 0.6], [0.5, 0.68]) \\
([0.45, 0.49], [0.27, 0.31], [0.4, 0.41]) & ([0.25, 0.34], [0.52, 0.53], [0.55, 0.6]) & ([0.51, 0.54], [0.54, 0.6], [0.45, 0.5]) \\
([0.5, 0.8], [0.65, 0.7], [0.52, 0.65]) & ([0.4, 0.5], [0.47, 0.65], [0.35, 0.6]) & ([0.45, 0.65], [0.3, 0.6], [0.5, 0.55]) \\
([0.5, 0.55], [0.25, 0.31], [0.36, 0.39]) & ([0.4, 0.45], [0.48, 0.49], [0.3, 0.55]) & ([0.47, 0.48], [0.57, 0.65], [0.7, 0.75]) \\
([0.4, 0.45], [0.45, 0.6], [0.34, 0.35]) & ([0.45, 0.55], [0.45, 0.58], [0.35, 0.37]) & ([0.43, 0.7], [0.44, 0.65], [0.5, 0.55]) \\
 & & \\
([0.43, 0.5], [0.45, 0.5], [0.5, 0.55]) & ([0.27, 0.3], [0.31, 0.33], [0.3, 0.31]) & \\
([0.63, 0.64], [0.49, 0.5], [0.61, 0.62]) & ([0.64, 0.66], [0.57, 0.59], [0.55, 0.56]) & \\
([0.41, 0.65], [0.25, 0.75], [0.42, 0.6]) & ([0.23, 0.3], [0.4, 0.6], [0.49, 0.5]) & \\
([0.64, 0.66], [0.49, 0.5], [0.4, 0.5]) & ([0.55, 0.57], [0.48, 0.53], [0.5, 0.52]) & \\
([0.65, 0.68], [0.46, 0.47], [0.65, 0.68]) & ([0.6, 0.62], [0.4, 0.45], [0.42, 0.46]) & \\
([0.68, 0.73], [0.49, 0.63], [0.46, 0.59]) & ([0.58, 0.59], [0.49, 0.52], [0.5, 0.65]) & \\
([0.7, 0.75], [0.5, 0.55], [0.6, 0.67]) & ([0.65, 0.7], [0.58, 0.6], [0.5, 0.51]) & \\
([0.4, 0.66], [0.25, 0.77], [0.42, 0.62]) & ([0.23, 0.3], [0.4, 0.45], [0.48, 0.5]) & \\
([0.67, 0.68], [0.45, 0.5], [0.59, 0.62]) & ([0.63, 0.64], [0.58, 0.59], [0.3, 0.35]) & \\
([0.61, 0.68], [0.47, 0.49], [0.35, 0.4]) & ([0.66, 0.7], [0.38, 0.4], [0.58, 0.65]) &
\end{bmatrix}
$$

$$= \left[\psi_{jk}^{\mathcal{L}}, \psi_{jk}^{\mathcal{U}}\right]_{10\times 5}$$

Step-5: The weighted decision NSFIVSS matrix is $\left[\mathcal{Y}^{\mathcal{L}}, \mathcal{Y}^{\mathcal{U}}\right] = \left[m_k^{\mathcal{L}} \times \psi_{jk}^{\mathcal{L}}, m_k^{\mathcal{U}} \times \psi_{jk}^{\mathcal{U}}\right]$

$$
= \begin{bmatrix}
([0.0652, 0.1008], [0.0815, 0.1251], [0.0629, 0.113]) & ([0.0719, 0.0997], [0.0585, 0.108], [0.0451, 0.0748]) \\
([0.0722, 0.1095], [0.0326, 0.0521], [0.0582, 0.1043]) & ([0.0646, 0.098], [0.0573, 0.0798], [0.067, 0.0997]) \\
([0.0582, 0.0904], [0.0594, 0.0939], [0.064, 0.1112]) & ([0.0621, 0.0997], [0.0573, 0.108], [0.0426, 0.0997]) \\
([0.0605, 0.1078], [0.0489, 0.113], [0.064, 0.1043]) & ([0.0731, 0.1047], [0.0548, 0.0997], [0.0609, 0.098]) \\
([0.0342, 0.0782], [0.0122, 0.0956], [0.0367, 0.0608]) & ([0.028, 0.0548], [0.067, 0.0964], [0.067, 0.098]) \\
([0.0815, 0.1269], [0.0512, 0.0817], [0.064, 0.113]) & ([0.0719, 0.1014], [0.0573, 0.0814], [0.0609, 0.0897]) \\
([0.0524, 0.0852], [0.0314, 0.0539], [0.0466, 0.0713]) & ([0.0305, 0.0565], [0.0633, 0.0881], [0.067, 0.0997]) \\
([0.0612, 0.1391], [0.0757, 0.1217], [0.0605, 0.113]) & ([0.0487, 0.0831], [0.0573, 0.108], [0.0426, 0.0997]) \\
([0.0582, 0.0956], [0.0291, 0.0539], [0.0419, 0.0678]) & ([0.0487, 0.0748], [0.0585, 0.0814], [0.0365, 0.0914]) \\
([0.0466, 0.0782], [0.0524, 0.1043], [0.0396, 0.0608]) & ([0.0548, 0.0914], [0.0548, 0.0964], [0.0426, 0.0615]) \\
 & \\
([0.0782, 0.1089], [0.0391, 0.0965], [0.0667, 0.0934]) & ([0.058, 0.103], [0.0607, 0.103], [0.0675, 0.1133]) \\
([0.0529, 0.0762], [0.061, 0.0856], [0.0782, 0.1074]) & ([0.085, 0.1318], [0.0661, 0.103], [0.0823, 0.1277]) \\
([0.0518, 0.1011], [0.0357, 0.0934], [0.0656, 0.1245]) & ([0.0553, 0.1338], [0.0337, 0.1544], [0.0567, 0.1235]) \\
([0.0426, 0.084], [0.0449, 0.0934], [0.046, 0.07]) & ([0.0864, 0.1359], [0.0661, 0.103], [0.054, 0.103]) \\
([0.0472, 0.1089], [0.0518, 0.1011], [0.069, 0.1167]) & ([0.0877, 0.14], [0.0621, 0.0968], [0.0877, 0.14]) \\
([0.0437, 0.1011], [0.0495, 0.0934], [0.0575, 0.1058]) & ([0.0918, 0.1503], [0.0661, 0.1297], [0.0621, 0.1215]) \\
([0.0587, 0.084], [0.0621, 0.0934], [0.0518, 0.0778]) & ([0.0945, 0.1544], [0.0675, 0.1133], [0.081, 0.138]) \\
([0.0518, 0.1011], [0.0345, 0.0934], [0.0575, 0.0856]) & ([0.054, 0.1359], [0.0337, 0.1586], [0.0567, 0.1277]) \\
([0.0541, 0.0747], [0.0656, 0.1011], [0.0805, 0.1167]) & ([0.0904, 0.14], [0.0607, 0.103], [0.0796, 0.1277]) \\
([0.0495, 0.1089], [0.0506, 0.1011], [0.0575, 0.0856]) & ([0.0823, 0.14], [0.0634, 0.1009], [0.0472, 0.0824]) \\
 & \\
([0.0335, 0.0525], [0.0385, 0.0578], [0.0372, 0.0543]) & \\
([0.0794, 0.1155], [0.0708, 0.1033], [0.0683, 0.098]) & \\
([0.0285, 0.0525], [0.0497, 0.105], [0.0608, 0.0875]) & \\
([0.0683, 0.0998], [0.0596, 0.0928], [0.0621, 0.091]) & \\
([0.0745, 0.1085], [0.0497, 0.0788], [0.0521, 0.0805]) & \\
([0.072, 0.1033], [0.0608, 0.091], [0.0621, 0.1138]) & \\
([0.0807, 0.1225], [0.072, 0.105], [0.0621, 0.0893]) & \\
([0.0285, 0.0525], [0.0497, 0.0788], [0.0596, 0.0875]) & \\
([0.0782, 0.112], [0.072, 0.1033], [0.0372, 0.0613]) & \\
([0.0819, 0.1225], [0.0472, 0.07], [0.072, 0.1138]) &
\end{bmatrix}
$$

$$= \left[\varrho_{jk}^{\mathcal{L}}, \varrho_{jk}^{\mathcal{U}}\right]_{10\times 5}$$

Step-6: The NSFIVSS-PISs are

$\left[\varrho_1^{\mathcal{L}+}, \varrho_1^{\mathcal{U}+}\right] = ([0.0782, 0.1089], [0.0385, 0.0578], [0.0372, 0.0543])$,

$\left[\varrho_2^{\mathcal{L}+}, \varrho_2^{\mathcal{U}+}\right] = ([0.085, 0.1318], [0.0342, 0.0521], [0.0612, 0.098])$,

$\left[\varrho_3^{\mathcal{L}+}, \varrho_3^{\mathcal{U}+}\right] = ([0.0621, 0.1338], [0.0337, 0.0934], [0.0426, 0.0875])$,

$\left[\varrho_4^{\mathcal{L}+}, \varrho_4^{\mathcal{U}+}\right] = ([0.0864, 0.1359], [0.0449, 0.0928], [0.046, 0.07])$,

$\left[\varrho_5^{\mathcal{L}+}, \varrho_5^{\mathcal{U}+}\right] = ([0.0877, 0.14], [0.0122, 0.0788], [0.0367, 0.0608])$,

$\left[\varrho_6^{\mathcal{L}+}, \varrho_6^{\mathcal{U}+}\right] = ([0.0918, 0.1503], [0.0495, 0.0814], [0.0575, 0.0897])$,

$\left[\varrho_7^{\mathcal{L}+}, \varrho_7^{\mathcal{U}+}\right] = ([0.0945, 0.1544], [0.033, 0.0539], [0.0489, 0.0713])$,

$\left[\varrho_8^{\mathcal{L}+}, \varrho_8^{\mathcal{U}+}\right] = ([0.0612, 0.1391], [0.0337, 0.0788], [0.0426, 0.0856])$,

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$\left[\varrho_9^{\mathcal{L}+}, \varrho_9^{\mathcal{U}+}\right] = ([0.0904, 0.14], [0.0306, 0.0539], [0.0365, 0.0613]),$
$\left[\varrho_{10}^{\mathcal{L}+}, \varrho_{10}^{\mathcal{U}+}\right] = ([0.0823, 0.14], [0.0472, 0.07], [0.0416, 0.0608]).$
Similarly, the values of NSFIVSS-NISs are
$\left[\varrho_1^{\mathcal{L}-}, \varrho_1^{\mathcal{U}-}\right] = [0.0335, 0.0525], [0.0856, 0.1251], [0.0675, 0.1133],$
$\left[\varrho_2^{\mathcal{L}-}, \varrho_2^{\mathcal{U}-}\right] = [0.0529, 0.0762], [0.0708, 0.1033], [0.0823, 0.1277],$
$\left[\varrho_3^{\mathcal{L}-}, \varrho_3^{\mathcal{U}-}\right] = [0.0285, 0.0525], [0.0624, 0.1544], [0.0673, 0.1245],$
$\left[\varrho_4^{\mathcal{L}-}, \varrho_4^{\mathcal{U}-}\right] = [0.0426, 0.084], [0.0661, 0.113], [0.0673, 0.1043],$
$\left[\varrho_5^{\mathcal{L}-}, \varrho_5^{\mathcal{U}-}\right] = [0.028, 0.0548], [0.067, 0.1011], [0.0877, 0.14],$
$\left[\varrho_6^{\mathcal{L}-}, \varrho_6^{\mathcal{U}-}\right] = [0.0437, 0.1011], [0.0661, 0.1297], [0.0673, 0.1215],$
$\left[\varrho_7^{\mathcal{L}-}, \varrho_7^{\mathcal{U}-}\right] = [0.0305, 0.0565], [0.072, 0.1133], [0.081, 0.138],$
$\left[\varrho_8^{\mathcal{L}-}, \varrho_8^{\mathcal{U}-}\right] = [0.0285, 0.0525], [0.0795, 0.1586], [0.0636, 0.1277],$
$\left[\varrho_9^{\mathcal{L}-}, \varrho_9^{\mathcal{U}-}\right] = [0.0487, 0.0747], [0.072, 0.1033], [0.0805, 0.1277],$
$\left[\varrho_{10}^{\mathcal{L}-}, \varrho_{10}^{\mathcal{U}-}\right] = [0.0489, 0.0782], [0.0634, 0.1043], [0.072, 0.1138].$
Step-7: The hamming distances from NSFIVSS-PIS and NSFIVSS-NIS. Now,
$\left[D_1^{\mathcal{L}+}, D_1^{\mathcal{U}+}\right] = [0.0001, 0.0009]$, $\left[D_1^{\mathcal{L}-}, D_1^{\mathcal{U}-}\right] = [0.00008, 0.00009]$,
$\left[D_2^{\mathcal{L}+}, D_2^{\mathcal{U}+}\right] = [0.00008, 0.0001]$, $\left[D_2^{\mathcal{L}-}, D_2^{\mathcal{U}-}\right] = [0.000003, 0.00006]$,
$\left[D_3^{\mathcal{L}+}, D_3^{\mathcal{U}+}\right] = [0.00004, 0.0003]$, $\left[D_3^{\mathcal{L}-}, D_3^{\mathcal{U}-}\right] = [0.00004, 0.00007]$,
$\left[D_4^{\mathcal{L}+}, D_4^{\mathcal{U}+}\right] = [0.00007, 0.0001]$, $\left[D_4^{\mathcal{L}-}, D_4^{\mathcal{U}-}\right] = [0.0001, 0.0001]$,
$\left[D_5^{\mathcal{L}+}, D_5^{\mathcal{U}+}\right] = [0.00002, 0.0002]$, $\left[D_5^{\mathcal{L}-}, D_5^{\mathcal{U}-}\right] = [0.00005, 0.0001]$,
$\left[D_6^{\mathcal{L}+}, D_6^{\mathcal{U}+}\right] = [0.0001, 0.0002]$, $\left[D_6^{\mathcal{L}-}, D_6^{\mathcal{U}-}\right] = [0.0002, 0.0002]$,
$\left[D_7^{\mathcal{L}+}, D_7^{\mathcal{U}+}\right] = [0.0002, 0.0009]$, $\left[D_7^{\mathcal{L}-}, D_7^{\mathcal{U}-}\right] = [0.0003, 0.0004]$,
$\left[D_8^{\mathcal{L}+}, D_8^{\mathcal{U}+}\right] = [0.0001, 0.0002]$, $\left[D_8^{\mathcal{L}-}, D_8^{\mathcal{U}-}\right] = [0.0002, 0.0002]$,
$\left[D_9^{\mathcal{L}+}, D_9^{\mathcal{U}+}\right] = [0.0002, 0.0002]$, $\left[D_9^{\mathcal{L}-}, D_9^{\mathcal{U}-}\right] = [0.0002, 0.0003]$,
$\left[D_{10}^{\mathcal{L}+}, D_{10}^{\mathcal{U}+}\right] = [0.00006, 0.00009]$, $\left[D_{10}^{\mathcal{L}-}, D_{10}^{\mathcal{U}-}\right] = [0.000006, 0.00003]$.
Step-8: The nearness coefficients from NSFIVSS-PIS and NSFIVSS-NIS, $C_1^* = 0.2915$, $C_2^* = 0.3385$, $C_3^* = 0.5090$, $C_4^* = 0.5284$, $C_5^* = 0.4045$, $C_6^* = 0.6112$, $C_7^* = 0.4901$, $C_8^* = 0.6702$, $C_9^* = 0.6451$, $C_{10}^* = 0.2769$.
Step-9: Order of the alternatives is C_i^* is $\varrho_8 \geq \varrho_9 \geq \varrho_6 \geq \varrho_4 \geq \varrho_3 \geq \varrho_7 \geq \varrho_5 \geq \varrho_2 \geq \varrho_1 \geq \varrho_{10}$.
Step-10: We conclude that the educational trust donates some payment. The ϱ_8 school receives 30 %, the ϱ_9 school receives 25 %, the ϱ_6 school receives 20 %, the ϱ_4 school receives 15 %, and the ϱ_3 school receives 10 %. As a result, the schools are able to provide students with a proper education.

5 NSFIVSS-VIKOR aggregating operator

Algorithm

Step-1 to **Step-5** as the same in TOPSIS approach. Now, we start the other steps.
Step-6: NSFIVSS-PIS = $\left(\left[\varrho_1^{\mathcal{L}+}, \varrho_1^{\mathcal{U}+}\right], \left[\varrho_2^{\mathcal{L}+}, \varrho_2^{\mathcal{U}+}\right] ..., \left[\varrho_l^{\mathcal{L}+}, \varrho_l^{\mathcal{U}+}\right]\right)$
$= \left\{ \left(\max_k \left[\varrho_{jk}^{\mathcal{L}}, \varrho_{jk}^{\mathcal{U}}\right], \min_k \left[\varrho_{jk}^{\mathcal{L}}, \varrho_{jk}^{\mathcal{U}}\right], \min_k \left[\varrho_{jk}^{\mathcal{L}}, \varrho_{jk}^{\mathcal{U}}\right]\right) : j = 1, 2, ..., l \right\}$ and
NSFIVSS-NIS = $\left(\left[\varrho_1^{\mathcal{L}-}, \varrho_1^{\mathcal{U}-}\right], \left[\varrho_2^{\mathcal{L}-}, \varrho_2^{\mathcal{U}-}\right] ..., \left[\varrho_l^{\mathcal{L}-}, \varrho_l^{\mathcal{U}-}\right]\right)$

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$$= \left\{ \left(\min_k \left[\varrho_{jk}^{\mathcal{L}}, \varrho_{jk}^{\mathcal{U}} \right], \max_k \left[\varrho_{jk}^{\mathcal{L}}, \varrho_{jk}^{\mathcal{U}} \right], \max_k \left[\varrho_{jk}^{\mathcal{L}}, \varrho_{jk}^{\mathcal{U}} \right] \right) : j = 1, 2, ..., l \right\}.$$

Step-7: The utilities $[\mathcal{S}_i^{\mathcal{L}}, \mathcal{S}_i^{\mathcal{U}}]$, individual regret $[\mathcal{R}_i^{\mathcal{L}}, \mathcal{R}_i^{\mathcal{U}}]$ and compromise $\mathcal{Q}_i$, where $[\mathcal{S}_i^{\mathcal{L}}, \mathcal{S}_i^{\mathcal{U}}] = \left[\sum_{j=1}^m m_j^{\mathcal{L}} \cdot \left| \frac{\varrho_{ij}^{3L} - \varrho_j^{3U+}}{\varrho_j^{3U+} - \varrho_j^{3L-}} \right|, \sum_{j=1}^m m_j^{\mathcal{U}} \cdot \left| \frac{\varrho_{ij}^{3U} - \varrho_j^{3L+}}{\varrho_j^{3L+} - \varrho_j^{3U-}} \right| \right]$ and

$$[\mathcal{R}_i^{\mathcal{L}}, \mathcal{R}_i^{\mathcal{U}}] = \left[\max_{j=1}^m m_j^{\mathcal{L}} \cdot \left| \frac{\varrho_{ij}^{3L} - \varrho_j^{3U+}}{\varrho_j^{3U+} - \varrho_j^{3L-}} \right|, \max_{j=1}^m m_j^{\mathcal{U}} \cdot \left| \frac{\varrho_{ij}^{3U} - \varrho_j^{3L+}}{\varrho_j^{3L+} - \varrho_j^{3U-}} \right| \right] \text{ and}$$

$$Q_i = \frac{1}{2} \left[\kappa \left(\frac{\mathcal{S}_i^{\mathcal{L}} - \mathcal{S}^{\mathcal{U}-}}{\mathcal{S}^{\mathcal{U}+} - \mathcal{S}^{\mathcal{L}-}} \right) + (1 - \kappa) \left(\frac{\mathcal{R}_i^{\mathcal{L}} - \mathcal{R}^{\mathcal{U}-}}{\mathcal{R}^{\mathcal{U}+} - \mathcal{R}^{\mathcal{L}-}} \right) + \kappa \left(\frac{\mathcal{S}_i^{\mathcal{U}} - \mathcal{S}^{\mathcal{L}-}}{\mathcal{S}^{\mathcal{L}+} - \mathcal{S}^{\mathcal{U}-}} \right) + (1 - \kappa) \left(\frac{\mathcal{R}_i^{\mathcal{U}} - \mathcal{R}^{\mathcal{L}-}}{\mathcal{R}^{\mathcal{L}+} - \mathcal{R}^{\mathcal{U}-}} \right) \right],$$

where $[\mathcal{S}^{\mathcal{L}+}, \mathcal{S}^{\mathcal{U}+}] = \max_i[\mathcal{S}_i^{\mathcal{L}}, \mathcal{S}_i^{\mathcal{U}}]$, $[\mathcal{S}^{\mathcal{L}-}, \mathcal{S}^{\mathcal{U}-}] = \min_i[\mathcal{S}_i^{\mathcal{L}}, \mathcal{S}_i^{\mathcal{U}}]$, $[\mathcal{R}^{\mathcal{L}+}, \mathcal{R}^{\mathcal{U}+}] = \max_i[\mathcal{R}_i^{\mathcal{L}}, \mathcal{R}_i^{\mathcal{U}}]$ and $[\mathcal{R}^{\mathcal{L}-}, \mathcal{R}^{\mathcal{U}-}] = \min_i[\mathcal{R}_i^{\mathcal{L}}, \mathcal{R}_i^{\mathcal{U}}]$. A decision mechanism's coefficient is the real number κ. When $\kappa > 0.5$, a majority compromise solution is reached; when $\kappa = 0.5$, a consensus compromise solution is reached; and when $\kappa < 0.5$, a veto compromise solution is reached. Let $[m_j^{\mathcal{L}}, m_j^{\mathcal{U}}]$ denotes the weight of the j^{th} parameter.

Step-8: Establish the importance of the options, then arrive at a workable compromise. Create a ranking list by placing $\mathcal{Q}_i$ in ascending order. It is determined that the alternative $\varrho_\triangle$ is a compromise solution if it ranks first (has the least value) in $\mathcal{Q}_i$ and both of the following conditions are satisfied at the same time:

$C1$: If $\varrho_\triangle$ and ϱ_Ξ represent best alternatives in $\mathcal{Q}$, then $\mathcal{Q}(\varrho_\Xi) - \mathcal{Q}(\varrho_\triangle) \geq \frac{1}{n-1}$, where n is the number of parameters.

$C2$: The alternative $\varrho_\triangle$ should be best ranked by $[\mathcal{S}_i^{\mathcal{L}}, \mathcal{S}_i^{\mathcal{U}}]$ and /or $[\mathcal{R}_i^{\mathcal{L}}, \mathcal{R}_i^{\mathcal{U}}]$.

If $C1$ and $C2$ do not satisfy each other, then there are multiple compromise solutions:

(i) If $C1$ is correct, then the alternatives $\varrho_\triangle$ and ϱ_Ξ are called compromise solutions:

(ii) If $C1$ is false, then the alternatives $\varrho_\triangle$, ϱ_Ξ,..., ϱ_υ are refered to as the multiple compromise solutions, where ϱ_υ is determined by $\mathcal{Q}(\varrho_\upsilon) - \mathcal{Q}(\varrho_\triangle) \geq \frac{1}{n-1}$.

Example 5.1. Let us start using the VIKOR approach with step 6.

Step-6: The values for NSFIVSS-PIS and NSFIVSS-NIS are listed below. Now,

$\left[\varrho_1^{\mathcal{L}+}, \varrho_1^{\mathcal{U}+}\right] = ([0.0856, 0.1391], [0.0122, 0.0521], [0.0367, 0.0608])$,

$\left[\varrho_2^{\mathcal{L}+}, \varrho_2^{\mathcal{U}+}\right] = ([0.0731, 0.1047], [0.0548, 0.0798], [0.0365, 0.0615])$,

$\left[\varrho_3^{\mathcal{L}+}, \varrho_3^{\mathcal{U}+}\right] = ([0.0782, 0.1089], [0.0345, 0.0856], [0.046, 0.07])$,

$\left[\varrho_4^{\mathcal{L}+}, \varrho_4^{\mathcal{U}+}\right] = ([0.0945, 0.1544], [0.0337, 0.0968], [0.0472, 0.0824])$,

$\left[\varrho_5^{\mathcal{L}+}, \varrho_5^{\mathcal{U}+}\right] = ([0.0819, 0.1225], [0.0385, 0.0578], [0.0372, 0.0543])$.

Similarly,

$\left[\varrho_1^{\mathcal{L}-}, \varrho_1^{\mathcal{U}-}\right] = [0.0342, 0.0782], [0.0856, 0.1251], [0.0673, 0.1130]$,

$\left[\varrho_2^{\mathcal{L}-}, \varrho_2^{\mathcal{U}-}\right] = [0.0280, 0.0548], [0.0670, 0.1080], [0.0670, 0.0997]$,

$\left[\varrho_3^{\mathcal{L}-}, \varrho_3^{\mathcal{U}-}\right] = [0.0426, 0.0747], [0.0656, 0.1011], [0.0805, 0.1245]$,

$\left[\varrho_4^{\mathcal{L}-}, \varrho_4^{\mathcal{U}-}\right] = [0.0540, 0.1030], [0.0675, 0.1586], [0.0877, 0.1400]$,

$\left[\varrho_5^{\mathcal{L}-}, \varrho_5^{\mathcal{U}-}\right] = [0.0285, 0.0525], [0.0720, 0.1050], [0.0720, 0.1138]$.

Step-7: Using $\kappa = 0.5$,

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ϱ	$\left[\mathcal{S}_i^{\mathcal{L}}, \mathcal{S}_i^{\mathcal{U}}\right]$	$\left[\mathcal{R}_i^{\mathcal{L}}, \mathcal{R}_i^{\mathcal{U}}\right]$	$\mathcal{Q}_i$
ϱ_1	$[0.3288, 0.3929]$	$[0.1229, 0.1742]$	0.4713
ϱ_2	$[0.1804, 0.4252]$	$[0.0731, 0.1304]$	0.1011
ϱ_3	$[0.4838, 0.6712]$	$[0.1612, 0.1766]$	0.9114
ϱ_4	$[0.2753, 0.4376]$	$[0.1437, 0.1486]$	0.4244
ϱ_5	$[0.4231, 0.4413]$	$[0.1350, 0.1354]$	0.4666
ϱ_6	$[0.2111, 0.5019]$	$[0.1255, 0.1471]$	0.3771
ϱ_7	$[0.2753, 0.4093]$	$[0.1112, 0.1226]$	0.2250
ϱ_8	$[0.5175, 0.6317]$	$[0.1665, 0.1903]$	0.9686
ϱ_9	$[0.2272, 0.3637]$	$[0.0661, 0.1296]$	0.0668
ϱ_{10}	$[0.2872, 0.3561]$	$[0.0980, 0.1333]$	0.1985

Step-8: The ranking of alternatives for $\mathcal{Q}_i$ is $\varrho_9 \leq \varrho_2 \leq \varrho_{10} \leq \varrho_7 \leq \varrho_6 \leq \varrho_4 \leq \varrho_5 \leq \varrho_1 \leq \varrho_3 \leq \varrho_8$.
Now, $\mathcal{Q}(\varrho_2) - \mathcal{Q}(\varrho_9) = 0.0343 \not\geq \frac{1}{4}$. Thus C1 is false, further more $\mathcal{Q}(\varrho_6) - \mathcal{Q}(\varrho_9) = 0.3103 \geq \frac{1}{4}$. Therefore, we establish that $\varrho_9, \varrho_2, \varrho_{10}, \varrho_7, \varrho_6$ are multiple compromise solutions. We conclude that the educational trust donates some payment. Hence, the school gets 30% on ϱ_9, 25% on ϱ_2, 20% on ϱ_{10}, 15% on ϱ_7 and 10% on ϱ_6. As a result, the schools are able to provide students with a proper education.

6 Advantage:

A novel generalization of the FIVSS is the NSFIVSS. The VIKOR approach's top-ranked option is the one that comes closest to being the greatest choice. The TOPSIS approach's top-ranked option, while it is the best according to the ranking index, falls short of the perfect answer. Consequently, the VIKOR approach's benefit is that it provides a compromise option.

7 Conclusion:

Under the aggregation operator, the NSFIVSS linguistic TOPSIS and VIKOR methods respectively follow these two algorithms. We communicate with the NSFIVSS aggregation operator and compute function scores using a particular method. These two methods differ from the normalization method in that they both assume a scalar component for each criterion. Use a linear normalizing strategy for VIKOR and a vector normalization approach for TOPSIS. The aggregation function is where the two approaches diverge most. Using an aggregating function, we may determine the order of values.

Conflicts of Interest: The authors declare no conflict of interest.

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